MICROMETER ADJUSTMENT OF MIRRORS AND PRISMS

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As far as the dimensioning of the reflecting surfaces is concerned, distinction must be made between prisms and mirrors, on account of the differences between refraction and reflection, irrespectively of the fact that, from the point of view of physics, refraction is but a special case of reflection. For an incident pencil of rays, parallel to the optical axis, as represented in Fig. 1, the length x of the reflecting faces of mirrors and prisms are equal. The situation is, however, different for parallel pencils constructing an angle with the optical axis (Fig. 2). The incident and emerging pencils are refracted

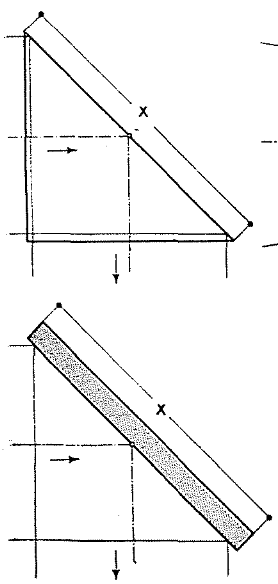

Fig. 1. The effective (free) reflecting face of prisms and mirrors has equal dimensions for pencils of rays entering parallel with the optical axis

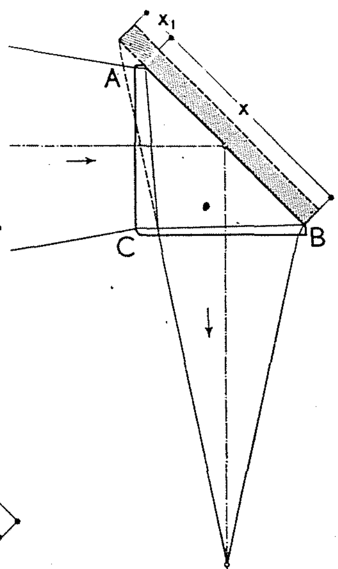

Fig. 2. The path of a pencil making an angle with the optical axis, in a prism, and if prism is replaced by a mirror

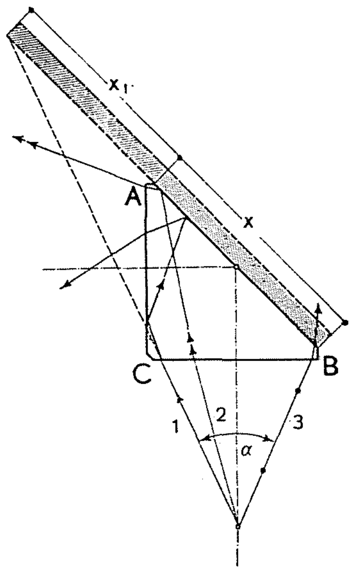

Fig. 3. Partial and total reflection as well as refraction of a wide-angle pencil in a prism. The mirror substituted for the prism has to be increased so as to be covered by marginal rays

at the faces AC and BC, respectively, cutting a useful surface x as reflecting portion from the face AB. Let us now substitute a mirror for the face AB. The pencil will reach the mirror unrefracted, but in this case, supposing an

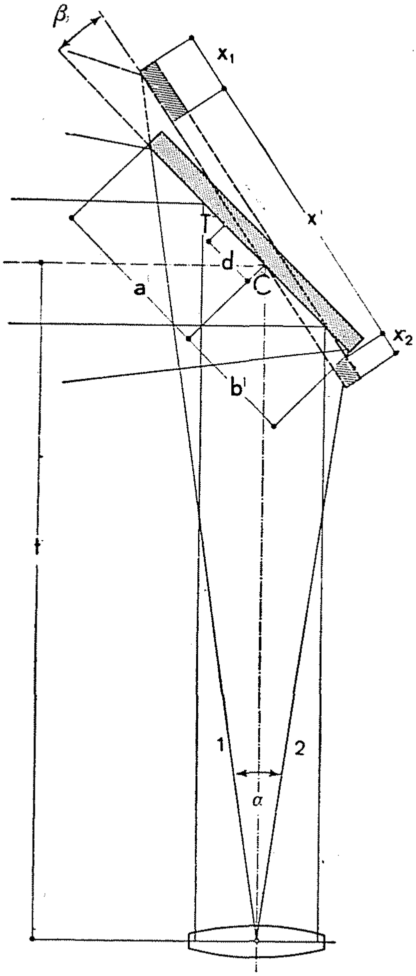

Fig. 4. Effect of mirror rotation. Mirror faces have to be increased in either direction by tilting, because of the marginal rays. In order to reduce the screening due to the mirror, the rotation centre was shifted from C to T

inverted path of rays, a reflecting face x_1 larger than the prism is required for receiving a pencil of full opening. Yet, the advantages inherent in refraction also involve certain disadvantages for wide-angle pencils (Fig. 3). The rays of a pencil emerging at angle α are refracted in the prism upon incidence. Before ray *1* should reach face AB, it is totally reflected on face AC so that it does not take part in image formation, and may entail inconvenient reflections which one may find impossible to eliminate. The ray *3* reaches face AB at an angle smaller than the boundary angle of the reflection, so that it is not only reflected but also refracted. In order to remove the resulting inconvenient reflections, the prism has to be replaced by a mirror x_1 which is larger than the length x of the prism.

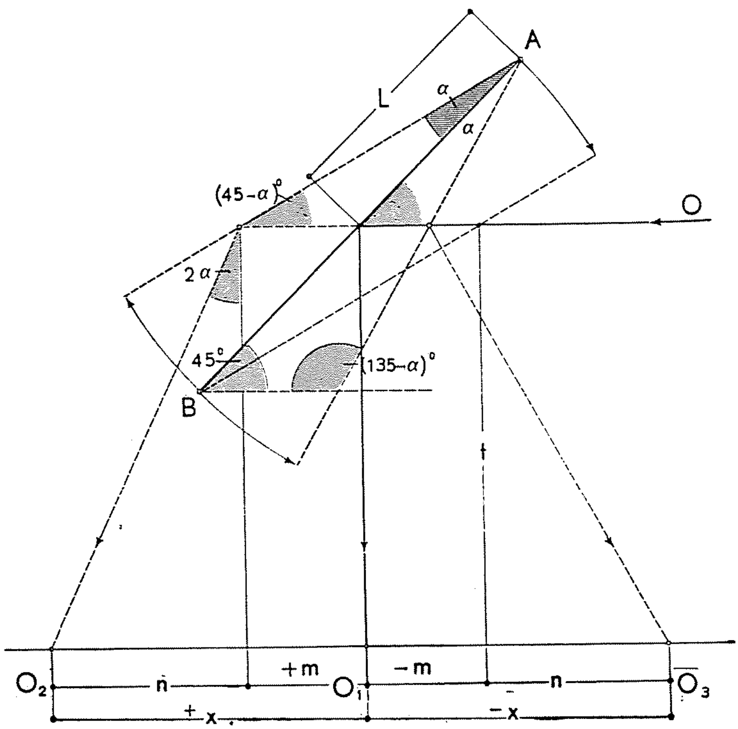

Fig. 5. Phenomena arising upon tilting the mirror, and displacement of the image point

On rotating the mirror (Fig. 4) the pencil travelling with inverted path of rays will cut out unequal portions from the face of the mirror $[a \neq b]$. The larger the angle of rotation β is, the longer the mirror has to be, so that the ray l may reach the edge of the mirror. A similar phenomenon is encountered at the other end of the mirror. Thus, one has to enlarge the original length x by adding x_1 at the upper and x_2 at the lower edge.

In the case of precision instruments the tilting of the mirror has to be computed taking the direction of the ray into consideration, that is, whether the ray travels from the image space to the object space, or vice versa (Fig. 5). The only difference between the two cases lies in the mirror being swung about either A or B, as rotation centres. For the sake of simplicity, only the axial ray has been represented. Let us suppose that the ray travelling from the object point O is reflected as O_1, and the mirror is tilted through a 45° angle. If the mirror is swung vertically from the basis position through an angle α, the reflected ray will arrive to O_2, and, on an opposite rotation, to O_3. Let us denote displacement of the incident ray by $(+m)$ and $(-m)$, respectively, then the displacements observed at distance t will consist of two parts: a constant (n) and a variable part composed of the two (m)-s.

According to Fig. 5:

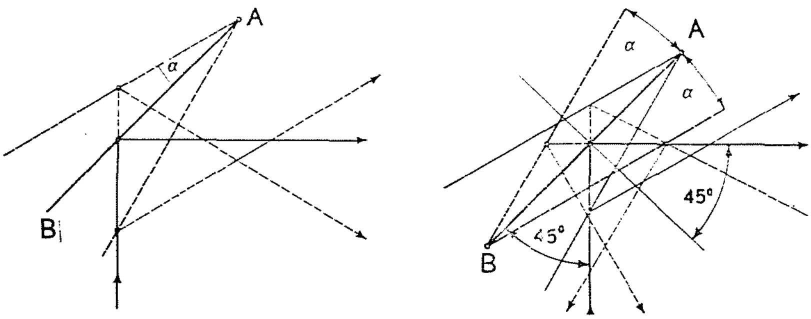

Fig. 6. Shifting of the ray for a mirror tilted about the upper and lower axis

From the scalene I:

$$m : \mathrm{L} = \sin \alpha : \sin \ (45°\text{-}\alpha]$$

$$m = L \frac{\sin \alpha}{\sin [45° - \alpha]}$$

From the scalene II:

$$[-m] : \mathrm{L} = \sin \alpha : \ \sin \ [135°\text{-}\alpha]$$

$$n = t \cdot \tan 2\alpha$$

$$[-x] = L \frac{\sin \alpha}{[\sin 135° - \alpha]} + t \cdot \tan 2\alpha$$

The displacement is $m + n = x$

$$x = L \frac{\sin \alpha}{\sin [45° - \alpha]} + t \cdot \tan 2\alpha$$

In other words, for a ray travelling in the optical axis, the two cases above referred to differ only in that the same axis of rotation is employed for rays of either direction, when tilting the mirror upwards or downwards from the basic position (Fig. 6). The derivation gives the following conclusions:

1. The image displacements are not equal: $x \neq (-x)$
2. The direction of the displacements is not similar: x being positive and $(-x)$ negative.

3. The reason for the difference in the image displacement is: $(-m) \neq$ $\neq (+m)$.

Thus, we can see from the foregoing that the two axes of rotation represent opposite hand views.

Example:
$$\begin{aligned} \alpha_{max} &= 22°30' \\ L &= 60 \text{ mm} \\ t &= 100 \text{ mm} \\ x &= 124.6 \text{ mm and} \\ (-x) &= 160 \text{ mm.} \end{aligned}$$

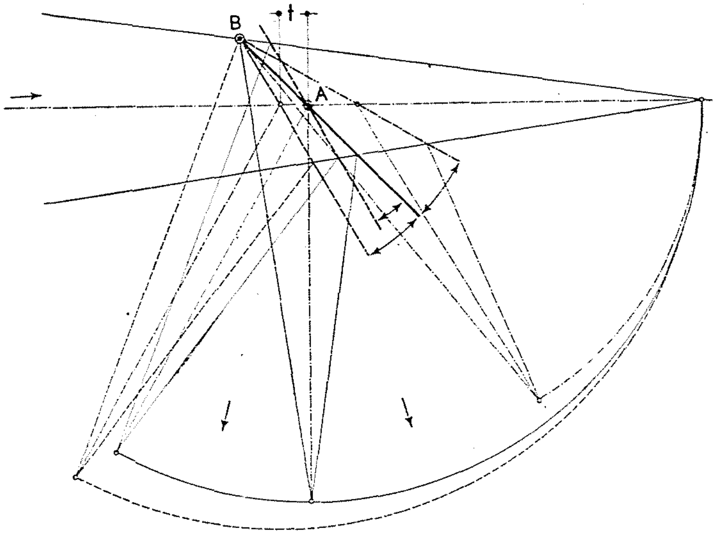

Fig. 7. Displacement of a pencil coming from infinity and made convergent by means of a lens, upon swinging the mirror about the rotation centre B

In Fig. 7 the axial pencil is directed by lens *l* convergently on to mirror *2*. The mirror is swung about point *B*, and not about the axial point *A*. The following conclusions may be established:

1. Swinging the mirror about axis *A* is more advantageous than about axis *B*, for in the case of equally small angles α it causes no stopping out of rays, but

2. this advantage is offset by the large amount of image displacement encountered.

Fig. 8 shows an arrangement in which the mirror is swung about axis *B*, placed rather low, hence, the motion is kinematically opposed to the previous one. We may thus make the statements that

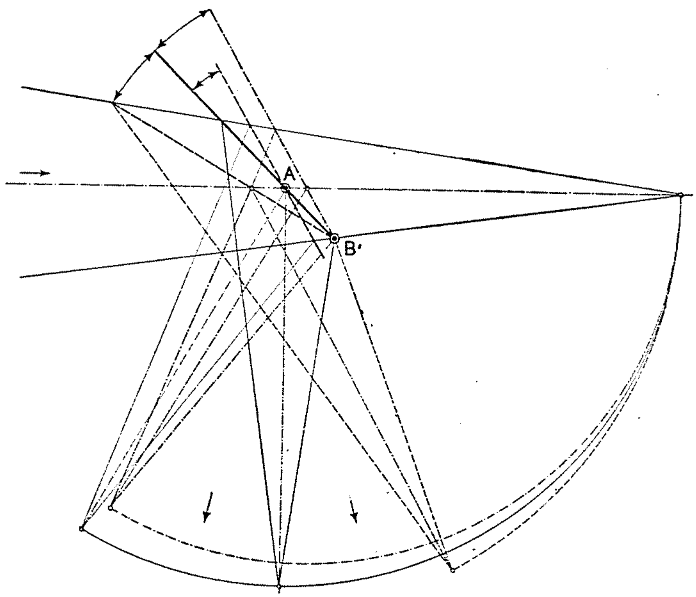

Fig. 8. Same as Fig. 7, with the mirror swung about axis A

1. swinging the mirror about point B is detrimental, as it leads to heavier screening or stopping out, than does the motion about point A.

2. In certain cases image displacement may be very slight.

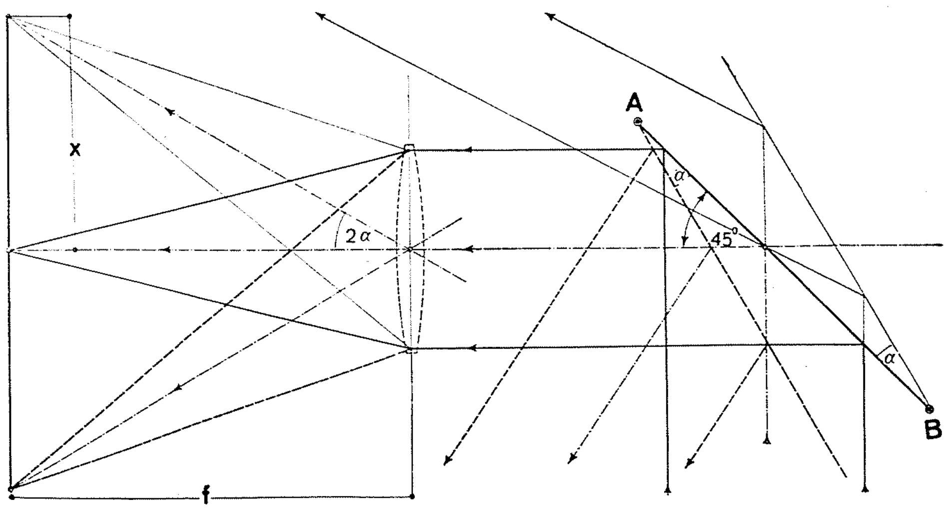

Fig. 9. For a pencil from infinity, the position of the image plane is independent of the position of the centre

If a parallel pencil reaches the mirror, as represented in Fig. 9, the position of the image plane is independing from the position of the rotation centre, since $\tan 2\alpha = x : f$. Image displacement is therefore: $f \cdot \tan 2\alpha$.

The foregoing derivations are true on the assumption that a front-coated mirror is used, but it is equally possible to employ a back-coated mirror which, naturally, has to consist of a plane-parallel glass plate. However, such mirrors can only be used when the path of rays is parallel, as pencils arriving from object points nearer than infinity are not collected by the lens in one image point. In addition, one has to consider the astigmatism due to oblique rays.

Reverting to Fig. 4, the inevitable screening of the rays, due to mirror rotation, may be lessened if the mirror is swung about axis T which is shifted towards the longer mirror portion cut out by the pencil, instead of being swung about the axial point. The distance d of the axis from the optical centre C is $a : 3$, a ratio which was found satisfactory for all practical purposes.

Micrometer control of mirrors

Let us now establish the correlation between the displacement of the micrometer screw serving as adjustor to the mirror, and the angular displacement of the lever carrying the mirror (Fig. 10). Let us suppose that lever *2* having a length L of plane mirror *3* rotating about point C is shifted, each revolution of the screw *1*, by uniform distances m. Since the angular displacement α of the mirror is not proportional to the shifting of the leading screw,

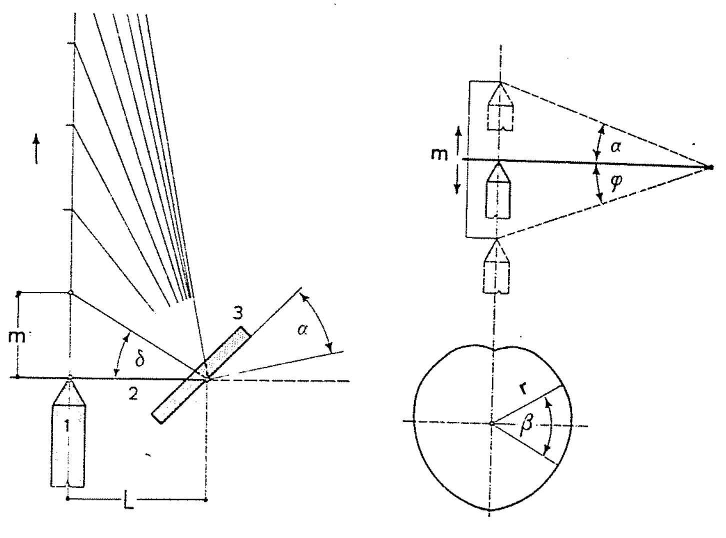

Fig. 10. Phenomena arising in connection with a mirror rotated by a micrometer leading screw, with the aid of a push rod

the drum cannot have linear scale divisions. It is therefore necessary to insert a means apt for ensuring linear readings, such as a cam disc, for displacing the leading screw. For the purpose of making the derivation of the curve of the disc's mantle, let δ represent the angular displacement of the disc, a the uniform angular displacement of the mirror, and L the length of the lever

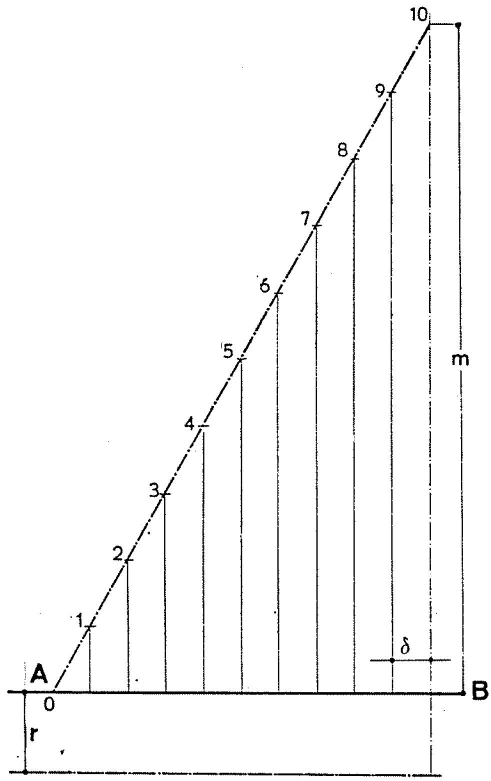

Fig. 11. Auxiliary diagram for the graphical construction of the curve of a control disc inserted to secure linear drum graduations

rotating the mirror. Then, in accordance with the indications of the figure:

$$m \cdot \tan \alpha + m \cdot \tan \varphi = r$$

$$r = r[\beta]$$

$$\frac{d\alpha}{d\beta} = c \qquad c \leqq \beta \leqq \pi$$

$$\alpha = c\,\beta + c'$$

$$r = m\,[\tan(c\,\beta + c') + \tan \varphi]$$

Computation of the constant member:

$$\text{at } \beta = 0 \quad r = 0$$

$$0 = m\,[\tan c' + \tan \varphi]$$

$$\tan c' = -\tan \varphi$$

$$r = m\,[\tan (c\,\beta - \varphi) + \tan \varphi]$$

$$\text{at } \beta = \pi \quad r = 2\,m \cdot \tan \varphi$$

$$2\,m \cdot \tan \varphi = [\tan (cr - \varphi) + \tan \varphi]$$

$$\tan \varphi = \tan\,[c\,\pi - \varphi]$$

$$\varphi + y\,\pi = c\,\pi - \varphi$$

$$c = \frac{2\,\varphi}{\pi} + y$$

at $y = 0$:

$$r = m\left[\tan\left(\frac{2\,\varphi}{\pi}\beta - \varphi\right) + \tan \varphi\right]$$

where β and φ are expressed in radians.

The formula reveals that the curve is not cardioid, because with B as the origin of the system of coordinates, and BC representing the direction of the axis x,

$$[y^2 + x^2 - 2\,rx]^2 = 4\,r^2\,[x^2 + y^2]$$

or, with γ and ε expressed in polar coordinates:

$$\gamma = 2\,r\,[1 + \cos \varepsilon]$$

For the graphical construction of the curve of the controlling disc, one can proceed as follows, as is illustrated in Fig. 11: Computing the values of r (multiplied by any arbitrary factor), parallel lines are traced by means of the normal at point B of the line AB across uniformly spaced points, corresponding in number to the angular displacements β of the disc. The values listed in the table below are transferred to these parallels.

A mirror rotation of $+10^\circ$ =	17.63
	15.83
	14.05
	12.27
	10.51
	8.74
	6.99
	5.24

3.49
1.74
0.00
1.74
3.49
.
.
mirror rotation of -10° = 17,63

The zero point corresponds to a 45° angle included between the mirror and the incident ray. The positive and negative values of the table being equal, the graphical construction will present a symmetrical formation.

The curve is obtained by first tracing a circle of an arbitrary radius r (Fig. 12). Then, optionally chosen but equal angles β are traced, starting from the centre 0 of the circle. In the example illustrated in the figure, β is $= 18^\circ$. Then the distances m obtained according to Fig. 11 are transferred to the shanks of the angles. A leading screw engaging the periphery of a cardioid disc, so shaped, will secure uniform angular displacement of the mirror, hence permits to provide the disc with equidistant angular graduations. However, solutions of this type permit the utilization of only part of the disc's periphery. To avoid this, the curve has been extended to cover 328°, as represented in Fig. 13. In the basic position of the leading screw its pointed end engages the curve at A. A_1 is the terminal of the curve.

The above-described controlling cam disc secures uniform vertical graduations only in the case of a pointed leading screw engaging the lever *2* of length L at A. If the screw is vertically displaced by m, the length of the lever changes to L_2. As in practice the leading screw's end cannot be given a pointed shape, it is generally either rounded off, or a roller is pivoted in its forked end, preferably by means of ball bearings. Such an arrangement, however, modifies the manner of control. Let us suppose that in the basic position the pointed end of the leading screw engages the lever of the mirror at A, and the mirror constructs a 45° angle with the horizontally travelling radiation. Let us now replace the pointed end by a roller of radius r. If the roller is vertically displaced by m, the lever will engage point A_3 of the roller mantle, instead of A_1. The mirror will therefore be rotated to such an extent as if the pointed end had been displaced by a length y from A_1 to A_2. Thus, the roller control gives rise to an angular error δ, the size of which depends on radius r of the roller, and length L of the lever. This error can only be removed by an appropriate modification of the shape of the cam. From the triangle CA_2A_3:

$$r = \tan \alpha = x$$

$$y + r = \sqrt{r^2 + x^2} = \sqrt{r^2 + r^2 \cdot \tan^2 \alpha - r}$$

If r is 3 mm and $\alpha = 10°$, y will be 0.046, or 0.05 mm.

The value y associated with α has to be deduced from the scale length corresponding to each angular value.

The angular error δ is

$$\delta = \operatorname{arc\,tan} \frac{m - r}{L} + \operatorname{arc\,sin} \frac{r}{\sqrt{(m - r)^2 + L^2}} - \operatorname{arc\,tan} \frac{m}{L}$$

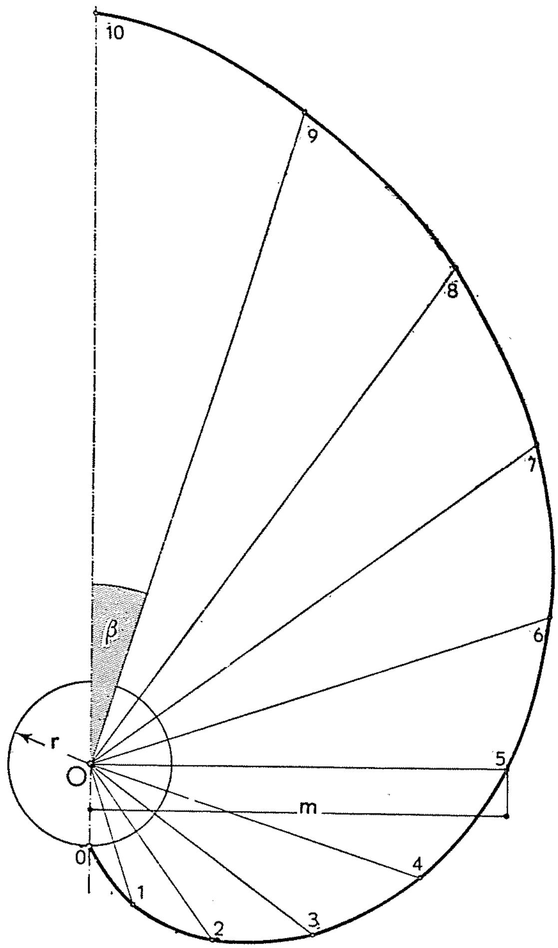

Fig 12. Graphical construction of the curve of the control disc

Plotting the values on a system of polar co-ordinates, the periphery of the cam disc can be graphically constructed.

Whatever the shape of the cam disc is, it will only operate precisely, that is, supply exact measurements, if its periphery is machined with high precision. Accordingly, the manufacture of such discs, particularly if they are small and have to be produced in series, is rather expensive and cumbersome. It has, therefore, been suggested to eliminate the use of such discs by employing a parallel gear drive as represented in Fig. 15. Vertical adjustment is effected by means of a gear-driven rod system, and connected with a gear segment.

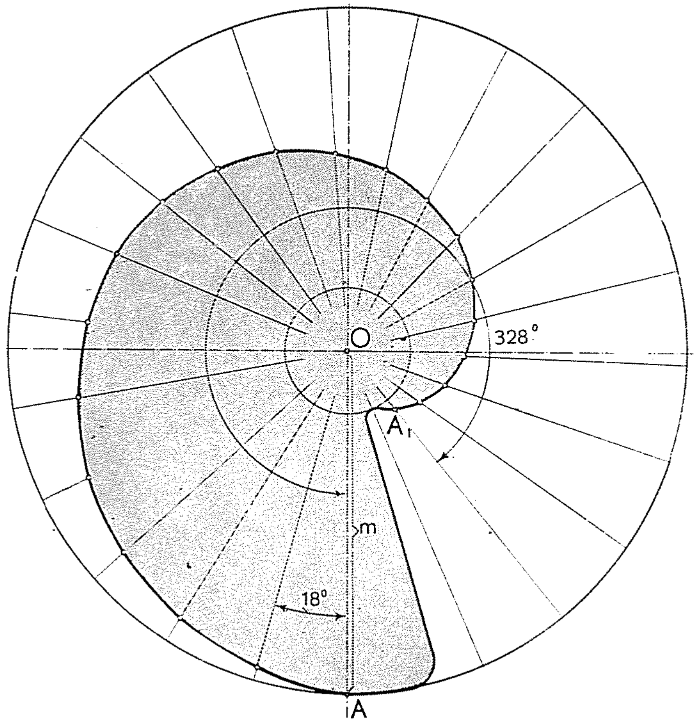

Fig. 13. Shape of the extended curve

The gear segment 13 is rotated by a wheel 21 over a worm drive 17 and a shaft 14 through a bevel gear transmission 15. With the insertion of a gear 12, the gear segment 13 rotates a drum 11 provided with suitable graduations. The shaft 14 of the wheel 21 rotates the drum 11 in such a manner that each revolution corresponds to one graduation. On the threaded part 16 of the shaft 14 the nut 19, secured against rotation, is displaced, and its nose butts against the fixed stop 18 at the end of its path. The delicate gear transmission of the reading device is protected in this way. A pivot 9 of the forked end of a lever 10 of the double-armed, lever-shaped gear segment 13 actuates a rod 8, whose upper two-piece end portion 5 serves to rotate the mirror 1 by means of lever 3 about shaft 2. Backlash is eliminated by the aid of spring 7, attached to lever 3. Exact adjustment of the rod length, that is, of the mirror position, is effected by sleeve nut 6.

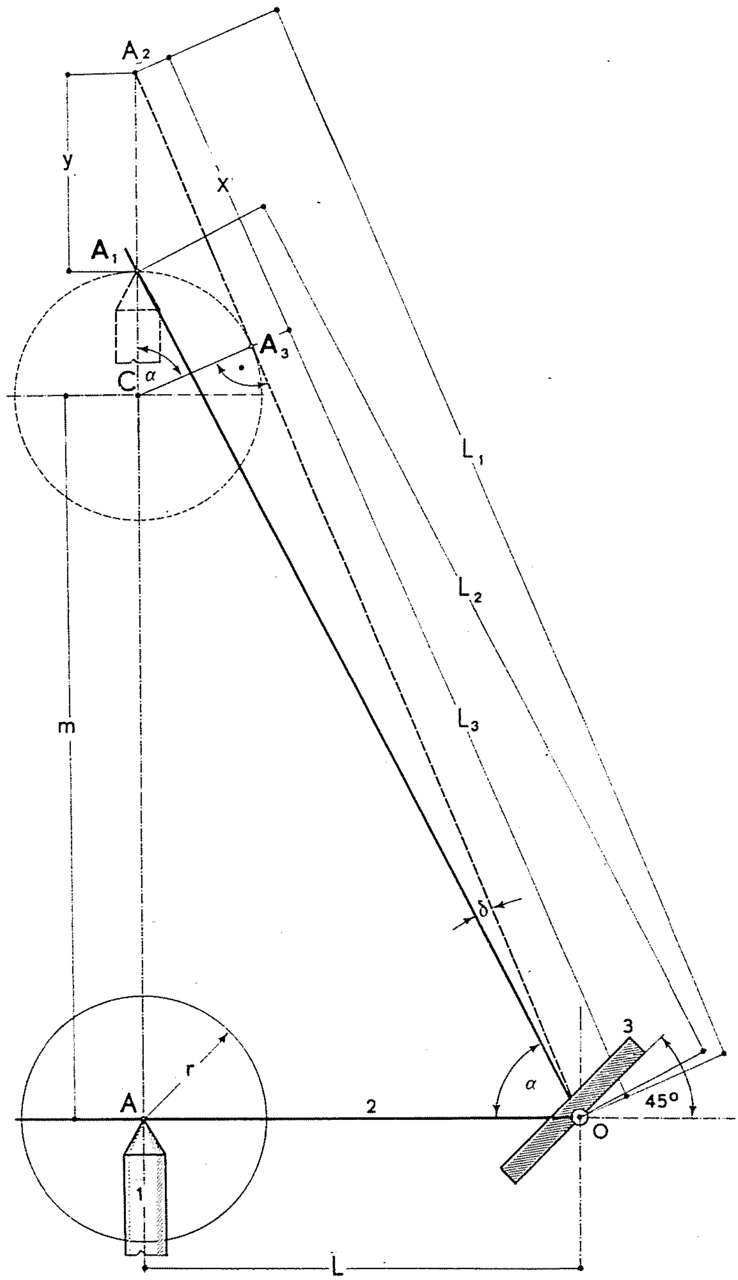

Fig. 14. Operating principle of pointed and roller-ended micrometer leading screws

When the sighting of object points situated at different altitudes, such as indoor reading of instrument graduations is intended, instead of angle measurements, one can resort to an extremely simple and inexpensive adjusting arrangement (Fig. 16), which for this purpose can readily replace the above-described costly equipment.

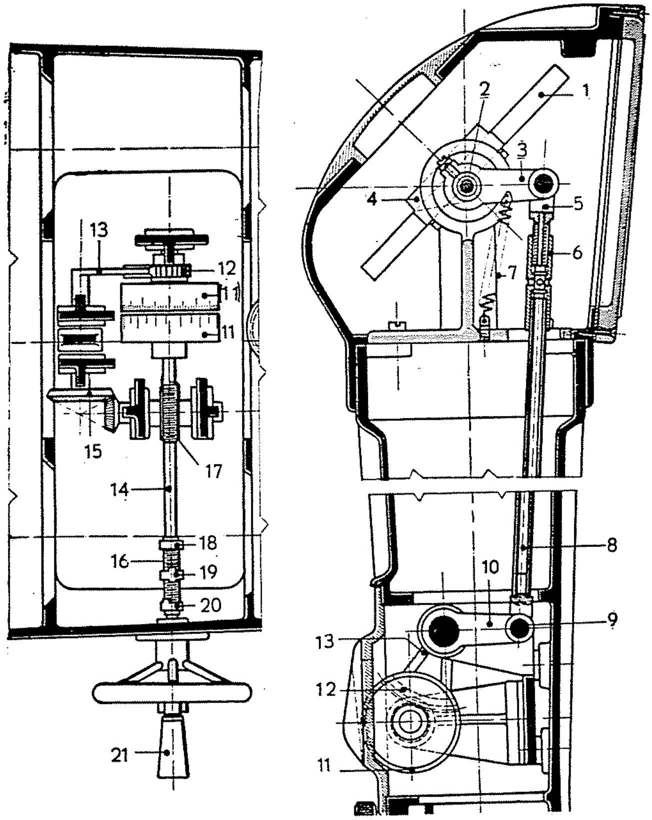

Fig. 15. Mirror shifting by parallel geare drive. The drum bears linear graduations

Head prism 1 rotates in ball bearings. A thin, twisted steel wire 2 connects the external mantle of the prism mounted to theadjusting equipment 3. In addition to the said role, this latter also serves to limit the rotation of the prism, as was described in connection with Fig. 15.

If the lever engages the end of the leading screw directly, this end part is usually rounded off with a certain radius (Fig. 17). In this case, however, the lever swinging about the point 0 rolls along the arc and engages the spherical surface of radius r at C_2. Peak C of the leading screw on the other hand is shifted to position C_1, thus giving rise to an error h. Accordingly, the nominal

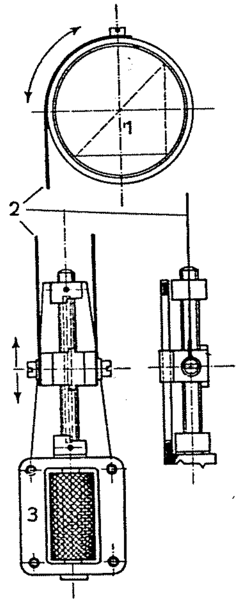

Fig. 16. Prism shifting by a simple wire control arrangement

drum reading will not correspond to the axial displacement of the leading screw The derivation of the error yields

$$h = \frac{r[k^2 - ry + y^2 + k\sqrt{k^2 - 2ry + y^2}]}{k^2 + [r - y]^2}$$

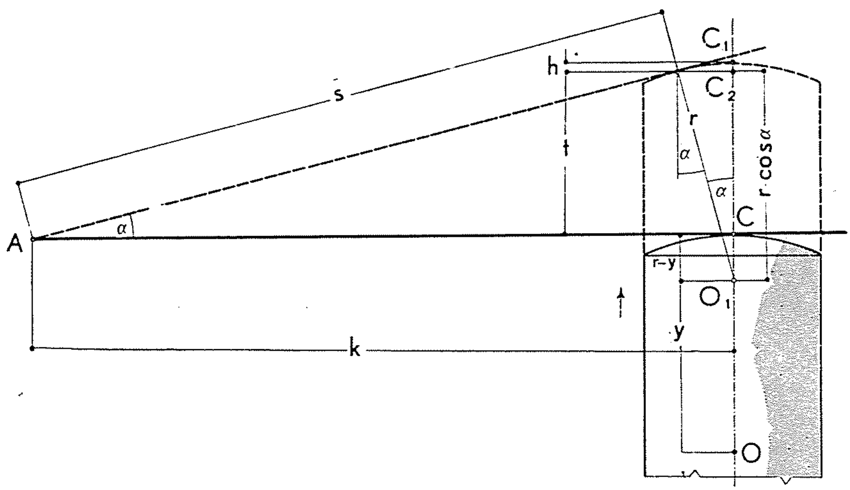

Fig. 17. Phenomena arising upon lever control, with the leading screw rounded off at a certain radius

In an equation of the second degree, a radial of positive sign has a meaning only if the result is positive. To ascertain this, practice has proven k to be smaller than r, which in turn is smaller than y. Accordingly, one can introduce appropriate neglections, as a result of which the error is

$$h = \frac{ry^2}{2\,k^2}$$

For practical purposes this means that the shorter r is, the smaller the axial displacement of the screw is and the longer the lever is, the smaller will the error be.

Summary

The paper is concerned with micrometer rotating arrangements for mirrors and prisms incorporated into instruments employed for angle measurement, with due regard to the optical, kinematical and precision mechanical correlations. The wire control arrangements devised and put into practice by the author at the Institute, of which he is the leader, proved to be of good use during a period of considerable length. The paper further specifies the limiting device which when locked in time will protect the measuring equipment against injuries. Finally, the paper describes the effect of pointed, rounded and roller-type leading screws.

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