ABOUT THE BALANCING OF HALF-WAVE PUSH-PULL MAGNETIC AMPLIFIERS

By

A. FRIGYES

Department of Special Electrical Machines and Automation, Polytechnical University, Budapest

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Nomenclature

A_c	core area of the magnetic material
B_s	flux density at saturation
I_A	current in the output winding of reactor "A"
I_B	current in the output winding of reactor "B"
I_{eff}	r.m.s. value of the maximum allowable current in the output winding of a reactor
I_e	bias current in a circuit balanced with biased rectifier
I_0	output current
N_0	number of turns of the output winding of a reactor
P_0	output power in the basic circuit
P_{sym}	output power in the push-pull circuit
$P_{sym(t)}$	instantaneous value of the output power in the push-pull circuit
$R_A=R_B=R$	ballast resistances
R_c	resistance of the output winding of a reactor
R_0	load resistance
R_i	resistance of the input circuit
U	supply voltage
U_A and U_B	voltage drops across the ballast resistances
U_c	induced voltage in a reactor
U_{cA} and U_{cB}	induced voltages in reactors "A" and "B", respectively
U_0	output voltage
U_i	input voltage

Explanation of indices

av	average value
m	peak value
max	maximum allowable average value in case of some control ranges or circuit arrangements.

Currents and voltages as functions of time are marked with small letters, their average and peak values with capital letters.

On building half-wave magnetic amplifiers of symmetrical output one of the problems arising is how to carry out the balancing of the amplifier elements connected in push-pull in order to keep the response time within 1 cycle, while at the same time obtaining the maximum output for a given core.

In our present investigations the basic circuitry of Ramey will be considered as a basis of comparison. The maximum output of an amplifier built with a given core and winding data in the basic circuit will be determined

Then the ratio of the attainable maximum output to the output of the basic circuitry will be determined when building a symmetrical output circuit from two cores with the same winding data.

In order to elucidate the problem let us start from the basic circuit shown in Fig. 1. Let us suppose that the hysteresis loop of the core is square shaped (Fig. 2.). The characteristic of the basic circuit is shown in Fig. 3 if the input signal is of alternating voltage.

The half-wave magnetic amplifiers are generally used as preamplifiers in control circuits. Their task is to control a further amplifier stage (*e. g.* magnetic amplifier, rotating amplifier, signal converter etc.). The control of the next stage needs a certain value of current and voltage. It is known, however, that the controlability of the above-mentioned elements is usually determined

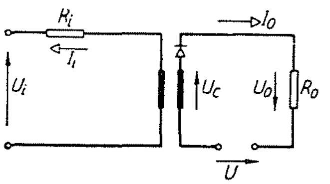

Fig. 1

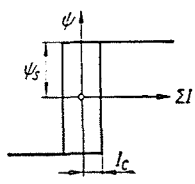

Fig. 2

by the product of some mean value of the current and voltage because *e. g.* in case of servo-elements to be controlled by the change of the magnetic flux the current and voltage necessary for the control change is in reverse ratio if the number of the exciting turns is changed.

One factor of the valuation of an amplifier is therefore the product of some mean value of current and voltage obtainable from it: the output power. In view of the fact that the effect controlling the next stage generally depends on the linear average value of the output current and voltage, therefore, it is expedient to define the output power as the product of the linear average values of the output current and voltage. The output so defined therefore differs from the power, in the strict physical sense, appearing at the output terminals.

The output of an amplifier changes during control. In this article by the word "output", always the maximum output arising in the course of control, will be meant.

As an amplifier of basic circuit shown in Fig. 1 is completely open in case of $U_i = 0$, the product of the average value of the current flowing through the output resistance and of the average value of the voltage across it, has in this case, to be determined.

The sinusoidal voltage on the core winding cannot be higher than that changing the core flux from the one point of saturation to the other. If the average value of the voltage is denoted $U_{c\,\mathrm{av}}$ then

$$U_{c\,\mathrm{av}} = 4 f N_0 A_c B_s \tag{1}$$

The supply voltage to be used cannot be higher than that. In order to secure a good utilisation of the core for the average value U_{av} of the supply voltage

$$U_{\mathrm{av}} = U_{c\,\mathrm{av}} \tag{2}$$

is to be chosen. Accordingly the supply voltage to be used is determined by the geometric dimensions of the core and the number of turns of the output winding.

The maximum average value of the output current in case of $U_i = 0$ is

$$I_{0\,\mathrm{av}} = \frac{1}{2} \frac{U_{\mathrm{av}}}{R_c + R_0} \tag{3}$$

The factor $^1/_2$ is due to half-wave rectification. From equation (3) the output power is

$$P_0 = U_{c\,\mathrm{av}} \cdot I_{0\,\mathrm{av}} = I_{0\,\mathrm{av}}^2 R_0 = \frac{1}{4} U_{\mathrm{av}}^2 \frac{R_0}{(R_c + R_0)^2} \tag{4}$$

P_0 has its maximum possible value if

$$R_c = R_0$$

in which case the current is

$$I_{0\,\mathrm{av\,max}} = \frac{1}{4} \frac{U_{\mathrm{av}}}{R_c} = \frac{1}{4} \frac{U_{c\,\mathrm{av}}}{R_c} \tag{5}$$

The output power is

$$P_{0\,\mathrm{max}} = \frac{1}{16} \frac{U_{\mathrm{av}}^2}{R_c} = \frac{1}{16} \frac{U_{c\,\mathrm{av}}^2}{R_c} \tag{6}$$

For the output given by equation (6) only cores of smaller size can be matched. In case of larger cores the current given by equation (5) causes excessive heat in the output windings of the amplifier. In this case the amplifier cannot be matched for the maximum output but the maximum allowable current should be determined from the heat dissipating capacity of the windings.

The heat caused in the winding depends on the r.m.s. value of the current. Let us denote the r.m.s. value of the maximum allowable current I_{eff}. Since a half-rectified current having a form factor of $\pi/2$ flows through the power windings, thus the corresponding average value is $2/\pi \cdot I_{eff}$.

If

$$\frac{1}{4}\frac{U_{av}}{R_c} > \frac{2}{\pi} I_{eff} \tag{7}$$

then

$$R_0 > R_c$$

is to be chosen so that

$$\frac{1}{2}\frac{U_{av}}{R_0 + R_c} = \frac{1}{2}\frac{U_{c\,av}}{R_0 + R_c} = \frac{2}{\pi} I_{eff} \tag{8}$$

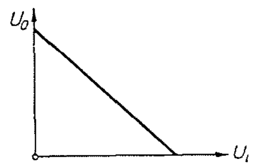

Fig. 3

Hence

$$R_0 = \frac{\pi}{4}\frac{U_{c\,av}}{I_{eff}} - R_c \tag{9}$$

The output power is

$$P_{0\,max} = U_{0\,av} \cdot I_{0\,av} = I_{0\,av}^2 R_0 = \frac{U_{c\,av} I_{eff}}{\pi} - \frac{4}{\pi^2} I_{eff}^2 R_c \tag{10}$$

It can easily be seen that in the case of limit of the unidentity (7) the equation (10) leads to equation (6).

In the following the output power of the symmetrical circuit will be separately investigated, if the load is matched for the maximum output first (case I) and then for the maximum allowable current in the output winding (case II). The symmetrical circuit is supposed to be built up from cores which in the basic circuit can each be loaded with $P_{0\,max}$.

In Fig. 4 the three possible methods for balancing are given. In Fig. *a* and *b* the balancing is done by means of passive elements — the resistances R_A and R_B — and by means of the biased rectifiers Rc_3 and Rc_4, respectively. Fig. *c* shows a bridge circuit where no passive balancing elements are necessary. It will be shown that from the point of view of maximum output in case I all the three circuits are equivalent, in case II, however, under similar circumstances, from the bridge circuit lower output can be obtained than from the other two circuits.

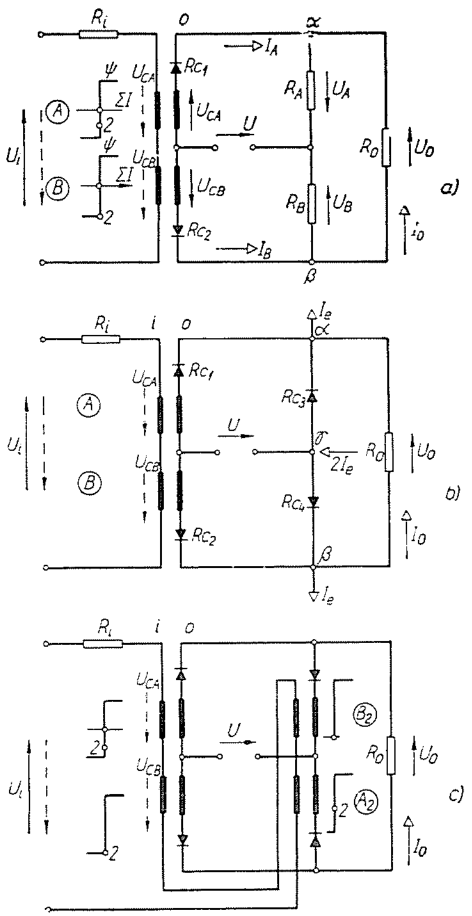

Fig. 4

A) Balancing by means of resistance

The operation method can be followed in Fig. 4*a*. In the operating half-cycle the direction of the alternating voltage *U* is marked with the full line arrow. If at the beginning of the operating half-cycle any of the reactors is unsaturated, as a result of the voltage drop on it from *U* the flux of core „A" gets into the upper, that of core "B" into the lower point of saturation still in the operating half-cycle (provided, of course, that the voltage U is high enough to deliver the voltage-time integral necessary to saturate the core). Therefore, it is certain that at the end of the operating half-cycle both cores are in a saturated condition.

In the next half-cycle the rectifiers in the circuit O are blocking. Therefore the change of flux in the cores can be influenced only by U_i. Depending on the direction of U_i in the resetting half-cycle either the flux of core "A", or that of core "B" will be reset. Let us suppose that U_i is an alternating voltage and of such a phase that its direction in the resetting half-cycle corresponds to the dotted line arrow. Theoretically voltage U_i is divided into voltages across the resistance R_i and reactors "A" and "B", on the latters the voltage drops being U_{cA} and U_{cB} whose directions are indicated by the dotted arrow. Therefore the flux of core "A" changes in a downward direction and will thus be reset. The flux of core "B" — however — cannot be changed by the voltage of indicated direction because it is in the lower point of saturation. Consequently voltage cannot appear on reactor "B" ($u_{cB} = 0$) thus the voltage is divided between the resistance R_i and reactor "A". At the end of the resetting half-cycle the flux levels of each core are indicated with the points 2 on the saturation curve in Fig. 4*a*. In the next operating half-cycle a voltage having a direction marked with the full line arrow would again appear on the reactors. A voltage with such a direction, however, can appear only on reactor "A" and not on reactor "B", on account of the flux of the latter being in the lower point of saturation. Consequently current i_A is zero in a part of the operating half-cycle — until the flux of core "A" gets into the upper point of saturation, — then, after the core "A" has been saturated circuit O of core "A" fires. On the other hand circuit O of core "B" is open during the whole operating half-cycle and the current i_B changes according to voltage U and the resistances of the circuit. As the upper and lower part of circuit O is symmetrical the voltage U_A and U_B have the same instantaneous values after the firing and thus voltage $u_0 = u_B - u_A$ appearing between point α and β is then zero.

The higher is the absolute value of the voltage U_i of a given polarity the higher is the degree to which the flux of core "A" is reset and consequently the higher is the phase angle at which core "A" fires in the operating half-cycle. It can be seen that by increasing the voltage U_i the average of the output voltage increases.

In case of $U_i = 0$ the fluxes of neither core will be reset and thus the output voltage is always zero. Finally when U_i changes its polarity, the flux of core "B" will be reset in the control half-cycle, the flux of core "A" — however — remaining all the time in the upper point of saturation. Thus the average of the output voltage changes its polarity as opposed to the previous case.

The output impedance will be matched to the output terminals α and β. In order to determine the output power let us suppose that during the resetting half-cycle preceding the operating half-cycle under investigation the flux of core "A" has been reset as compared to the upper point of saturation, while the flux of core "B" is in the lower point of saturation. As no voltage can appear on reactor "B" the symbol of its winding has been omitted in Fig. 5, and only

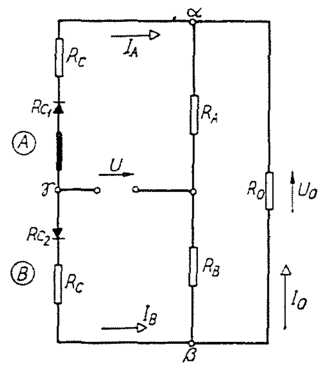

Fig. 5

its resistance R_c is indicated. The forward resistance of the rectifier being incorporated in the resistance of the winding, under these conditions the voltage drop on the rectifier during the operating half-cycle was taken into consideration and it can be omitted in the circuit. Fig. 6 has been drawn

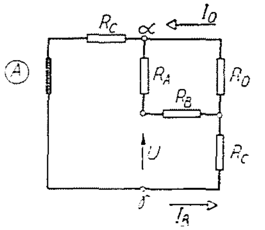

Fig. 6

accordingly. It can easily be seen that Fig. 6 is the same as Fig. 5 excepting the arrangement of the circuit elements. This equivalent circuit applies to a section of the operating half-cycle preceding the firing of core "A". In the circuit, even during this time, flows a current I_B which returns to the terminal marked with the point of the arrow of the voltage source, partly through resistance R_B, partly through resistances R_0 and R_A connected in series. The output current is the component I_0 flowing through output resistance R_0. This component causes a voltage drop of I_0R_A across resistance R_A.

On reactor "A" does not therefore appear voltage U as in case of the basic circuit, but a voltage reduced by the drop I_0R_A across resistance R_A.

Consequently the flux of core "A" changes at a lower rate toward the upper point of saturation and owing to this fact the firing occurs later. The higher current I_0 is or $\int i_0 R_A dt$ voltage-time integral, the later does core "A" fire. The average value of the output current increases with the increase of the firing angle, thus the described phenomenon actually causes an inherent positive feedback.

Fig. 7 shows the voltage across ballast resistances R_A and R_B at an intermediate state of control. The output voltage appearing across the resistance R_0 is proportional to the section lines between the curves.

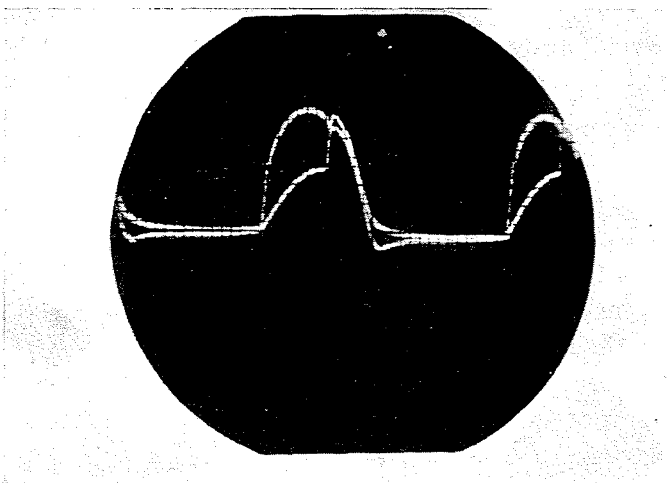

Fig. 7

The operation within 1 cycle is based on the fact that at the end of each operating half-cycle the cores arrive at the border of saturated and unsaturated state independently of the previous state of control. Consequently the "past" of the cores previous to the beginning of the control half-cycle can have no influence on their subsequent "fate".

Let us examine the equivalent circuit in Fig. 6 and let us substitute according to Thevenin's law an equivalent generator for that part of the circuit which is to the right of points $a - \gamma$ (Fig. 8). The voltage U' of this is the voltage appearing on the points $a - \gamma$ of the separated part of the circuit caused by voltage U, and R'_0 is the resultant resistance of the separated circuit to be measured at terminals $a - \gamma$.

$$u' = u - i_0 R_A \tag{11}$$

Based on reasons of symmetry the resistances R_A and R_B must be equal:

$$R_A = R_B = R$$

Taking this into consideration the current i_0 can be expressed as follows:

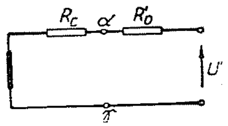

Fig. 8

$$i_0 = u \frac{R}{R_c(2R + R_0) + R^2 + RR_0} \tag{12}$$

Substituting this in equation (11)

$$u' = u \cdot \frac{R_c(2R + R_0) + RR_0}{R_c(2R + R_0) + R^2 + RR_0} \tag{13}$$

It can easily be seen from Fig. 8 that prior to the firing the magnetic state of the core and the reaction of the output circuit on the input circuit are described by the same equations as in case of the basic amplifier circuits. The core "A" fires when the $\int u'dt$ voltage-time area equals the reset flux.

If the maximum obtainable output from the symmetrical circuit is to be determined then only that case is to be investigated when core "A" is blocking all through the operating half-cycle. The condition of this of course is that in the preceding resetting half-cycle the flux should be reset with a value of $2\,\Psi_s$. The voltage U' is to be chosen so that its half-wave changes the flux of the core with just the value of $2\,\Psi_s$. In other words U' must be a sinusoidal voltage for the average of which

$$U'_{\text{av}} = U_{c\,\text{av}} \tag{13a}$$

holds.

Considering equation (11) it can be seen that the supply voltage U can exceed U' and thus it can be higher than in the case of a basic amplifier circuit built with the same core and winding data. The voltage increase depends on R_0 and i_0 the latter being dependent on the U itself, besides the other circuit constants. Therefore, it must essentially be determined that in order to attain the maximum output

$$P_{\text{sym max}} = I_{0\,\text{av}}^2 R_0 \tag{14}$$

obtainable in case of a given core at the simultaneous fulfilling of the condition $U'_{\text{av}} = U_{c\,\text{av}}$, which values the freely selectable R_1, R_0 and U are to have.

According to the results derived in App. I $P_{\text{sym max}}$ has its extreme value both in case of I and II at infinite ballast resistance R and supply voltage U. In case of smaller cores (case I) U and R must approach infinite on the condition that

$$\lim \frac{U_{\text{av}}}{R} = \frac{U_{c\,\text{av}}}{4\,R_c} \tag{15}$$

where $U_{c\,\text{av}}$ is the highest voltage the core is just capable of absorbing [see equation (1)]. At the same time it must be fulfilled that

$$R_0 = 2\,R_c \tag{16}$$

as a consequence of which — also according to the equations given in App. I — in case of full control

$$P_{\text{sym max}} = \frac{1}{32} \frac{U_{c\,\text{av}}}{R_c} \tag{17}$$

output is obtained on the output resistance. Since at full control core "A" is blocking through the whole duration of the operating half-cycle the current flowing through both ballast resistances is carried by the winding of core "B". Because for $R \to \infty$ the other resistances can be neglected while determining the value of the current, therefore the instantaneous value of the current in the winding of reactor "B" is

$$i_B = 2 \lim \frac{u}{R} \tag{18a}$$

The r.m.s. value of this current is

$$I_B = \frac{1}{2} \cdot 2 \lim \frac{U_m}{R} = \lim \frac{U_m}{R} = \frac{\pi}{2} \lim \frac{U_{\text{av}}}{R} \tag{18b}$$

(On account of the half-wave rectification the peak factor is 2 instead of $\sqrt{2}$.) If I_B is higher than I_{eff} corresponding to the maximum allowable temperature rise we have case II (larger cores). In this case

$$\lim \frac{U_{\text{av}}}{R} = \frac{2}{\pi} I_{\text{eff}} \tag{19}$$

must be substituted instead of equation (15) and of course

$$\lim \frac{U_{\text{av}}}{R} < \frac{U_{c\,\text{av}}}{4 R_c} \tag{20}$$

holds true.

In case II the output has its maximum if

$$R_0 = \frac{\pi}{2} \frac{U_{c\,\text{av}}}{I_{\text{eff}}} - 2 R_c \tag{21}$$

and its value is

$$P_{\text{sym max}} = \frac{U_{c\,\text{av}} \cdot I_{\text{eff}}}{2\pi} - \frac{2}{\pi^2} I_{\text{eff}}^2 R_c . \tag{22}$$

It can easily be seen that at the limit when current I_B calculated from equation (18b) equals current I_{ff} allowable for heating considerations case I and II are equivalent.

Namely, if in equations (21) and (22) according to equations (18b) and (15)

$$I_{\text{eff}} = \frac{\pi}{2} \lim \frac{U_{\text{av}}}{R} = \frac{\pi}{2} \frac{U_{c\,\text{av}}}{4R_c}$$

is substituted for I_{eff}, equations (16) and (17) are obtained. By comparing the equations (6), (7) also (10) and (22) it can be seen that from the push-pull circuit half as much output can be obtained than from the basic circuit built with the same core and winding data. The maximum theoretically obtainable output for $R \to \infty$ can be well approached if $R = 6-8\ R_c$ is chosen.

A physical meaning can be attributed to lim u/R in equation (18a) on the following consideration: The currents caused by voltage u flow through one of the resistances R. In case of $R \to \infty$ the other resistances in the circuit have no influence on the currents. If either of the cores or both are open the supply voltage appears on the ballast resistances apart from a small differential part. Based on this in the circuit of Fig. 9 for the Y composed of the branches of the supply voltage and the ballast resistances two current generators each supplying a current of

$$i_e = \frac{u}{R}$$

can be substituted. From the afore-said it could be assumed that at point γ of Fig. 5a current $2\ i_e$ is forced on a circuit, the half of which flows away at point α and the other half at point β. According to this view prior to the firing the upper (a) circuit of Fig. 9 is valid: the current of an instantaneous value of $2i_e$ flowing in at point γ can only flow through the lower core because core

"A" is unsaturated. From this current $2i_e$ the amount of i_e flows away at point β the remaining i_e flowing away through resistance R_0 at point a. The

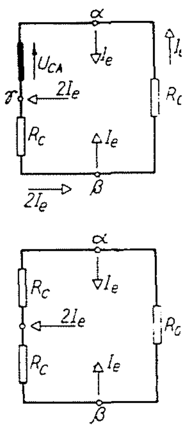

Fig. 9

instantaneous value of the voltag u_{cA} changing the flux of core "A" can be determined by means of the 2nd Kirchhoff's law applied to the only loop in the figure.

$$u_{cA} = 2i_e R_c + i_e R_0 = i_e (2R_c + R_0) \tag{23}$$

Consequently the blocked core is saturated by the resultant of the voltage drops across the resistances R_c and R_0 due to the current forced through them.

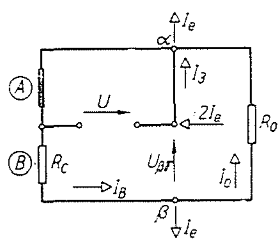

Fig. 10

u_{cA} and $2 R_c$ are fixed values in equation (23). From the same equation the current can be determined. It is the current in resistance R_0 being connected to a voltage source of internal voltage u_{cA} and internal resistance $2 R_c$. From this latter, however, the maximum output is obtained in case of $R_0 = 2 R_c$; *i.e.* when the internal and external resistances are equal. This corresponds to equation (16).

After the firing the current distribution is according to the lower (b) circuit in Fig. 10. Owing to the perfect symmetry the current in resistance R_0 is zero.

B) Balancing by means of biased rectifiers

According to Fig. 4b, in this circuit rectifiers are used in place of the ballast resistances. The rectifiers are biased from a current generator of infinite internal resistance. The current generator not indicated in the figure supplies the current $2i_e$ in the circuit at point γ. Divided into two equal parts this current flows back at point α and β into the current generator.

The rectifiers are assumed to be ideal; *i. e.* they constitute a short circuit in the forward direction and a break in the reverse direction. Let us suppose that in the resetting half-cycle preceding the operating half-cycle to be investigated the flux of core "A" has been reset, while that of core "B" has remained at the lower point of saturation. Thus at the beginning of the operating half-cycle the reactor "A" is blocking and the reactor "B" is opening. It can easily be seen that the condition of operation is, that prior to the firing a part of the current I_B of reactor "B" is to flow through resistance R_0 until core "A" also fires. Therefore prior to the firing Rc_4 must block otherwise R_0 would be short-circuited by the rectifer and thus no voltage could appear across it. On the other hand Rc_3 must conduct so as to make a closed circuit for the current in R_0. If the afore-mentioned conditions are given there is a break (between points β and γ) in the branch of rectifier Rc_4 while there is a short-circuit (between point α and γ) in the branch of rectifier Rc_3. Fig. 10 was drawn accordingly. Core "B" is in saturated condition, therefore, the winding resistance R_c was drawn in its place. Until it fires the negligibly small magnetizing current flows through reactor "A", consequently the circuit of reactor "A" may be assumed to be broken, with the remark, that on account of the conductance of rectifiers Rc_1 and Rc_3 (see Fig. 4b) the total supply voltage appears on reactor "A". Consequently — in opposition to the balancing with ballast resistance — the average of the supply voltage is to be chosen as in case of the basic circuit (see equation 6):

$$U_{av} = U_{c\,av} \tag{24}$$

A break was assumed in place of the blocking rectifier Rc_4 and the reverse voltage $U_{\beta\gamma}$ was indicated. As all the other rectifiers conduct short-circuits have been put in their place.

As long as the above described conditions exist the circuit is linear and the usual methods for calculating linear circuits can be applied. It is evident that the condition of Rc_3 being conducting and Rc_4 being blocking is the solution of the circuit in Fig. 10 for the instantaneous values

$$u_{\beta\gamma} > 0 \tag{25}$$

and

$$i_3 > 0 \tag{26}$$

presents itself. The following equations can be set up:

$$i_e = i_3 + i_0 \tag{27a}$$

$$i_e = i_B - i_0 \tag{27b}$$

$$u = i_B R_c + i_0 R_0 \tag{27c}$$

As the solutions of the above equations for i_0, i_B and i_3 the following are obtained:

$$i_0 = \frac{u - i_e R_c}{R_c + R_0} \tag{28a}$$

$$i_B = \frac{u + i_e R_0}{R_c + R_0} \tag{28b}$$

$$i_3 = i_e \frac{2R_c + R_0}{R_c + R_0} - \frac{u}{R_c + R_0} \tag{28c}$$

$$\mu_{\beta\gamma} = u - i_B R_c = u \frac{R_0}{R_0 + R_c} - i_e R_c \frac{R_0}{R_0 + R_c} \tag{28d}$$

Considering equations (25) and (26) the condition of operation is

$$i_e \frac{2R_c + R_0}{R_c + R_0} - \frac{u}{R_c + R_0} > 0 \tag{29}$$

and

$$u \frac{R_0}{R_0 + R_c} - i_e R_c \frac{R_0}{R_0 + R_c} > 0 \tag{30}$$

be fulfilled. Consequently

$$\frac{u}{2R_c + R_0} < i_e < \frac{u}{R_c} \tag{31}$$

By setting the value of the biasing current between these limits the condition of the above described operation can be assured. As the unidentity refers to instantaneous values the value of i_e in the operating half-cycle is to be chosen within the zone marked with lines (see Fig. 11), in such a way, that the output power should be a maximum. Based on equation (28a) the instantaneous value of the output power is

$$P_{sym(t)} = i_0^2 R_0 = \frac{(u - i_e R_c)^2 R_0}{(R_c + R_0)^2} \tag{32}$$

In case of fixed R_0 the value of $P_{sym(t)}$ is maximum if i_e has the smallest value given by unidentity (31):

$$i_e = \frac{u}{2R_c + R_0} \tag{32a}$$

This corresponds to the case when rectifier Rc_3 has just been blocking ($i_3 = 0$) but the reverse voltage has not yet appeared on it. Then

$$i_e = i_0$$

and thus

$$P_{sym(t)} = i_0^2 R_0 = i_c^2 R_0 = u^2 \frac{R_0}{(2R_c + R_0)^2} \tag{33}$$

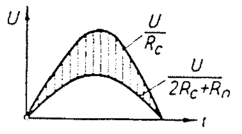

Fig. 11

With varying R_0 the above expression obtains its maximum when

$$R_0 = 2R_c \tag{34}$$

This maximum is

$$P_{sym(t)max} = \frac{u^2}{8R_c} \tag{35}$$

(It is to be noted that this circuit also operates with a smaller bias than given by equation (32*a*). Then rectifier Rc_3 is stressed in the reverse direction and $i_e = i_0$ as the limit of the above discussed. In this case according to equation (32) the output power is decreased as compared to the above given maximum.)

Equation (32*a*) can only hold true for every instantaneous value if i_e varies sinusoidally in the operating half-cycle; *i. e.*

$$i_c = I_{cm} \sin \omega t$$

where

$$I_{em} = \frac{U_m}{2R_c + R_0} = \frac{U_m}{4R_c} = \frac{\pi}{2} \frac{U_{c\,av}}{4R_c}$$

In the resetting half-cycle $i_e = 0$ may be true. If it is desired to turn from the instantaneous value of the output power given by the expression (35) to the average value of the same, then keeping in mind that the average of U is

$U_{av}/2$ (this average being effective for the output power) and substituting this for u in equation (35) and also taking (24) into consideration

$$P_{\text{sym max}} = \frac{U_{av}^2}{32 R_c} = \frac{U_{c\,av}^2}{32\, R_c} \tag{36}$$

This expression is the same as the one (17) derived for the case of balancing with ballast resistances.

C) Bridge circuit

In a bridge circuit amplifier elements are also used in place of ballast resistances and biased rectifiers (see Fig. 4c). The firing of the individual cores is to be controlled in such a way that reactor "A_1" fire at the same time as reactor "A_2" and reactor "B_1" as reactor "B_2". Furthermore reactors "A_1" and "A_2" are to fire at the upper point of saturation and reactors "B_1" and "B_2" at the lower one. In the circuit arrangement of the control windings given in Fig. 4c these conditions are fulfilled, provided the reactors in the opposing branches of the bridge are equal magnetically as well as electrically. Equality can most suitably be ensured by using a common core and winding for the opposing circuit elements.

For the sake of comparison let us assume that cores of the same dimensions and windings with the same number of turns are used as in the circuits given earlier, with the difference, that the output windings are divided into two halves. One half each is used as an output winding of the amplifier elements in the opposing branches of the bridge. Thus the same amount of active material is necessary in the bridge circuit as in the two other circuits. In order to obtain the same utilization of the winding area the same total number of turns and the same wire size are also assumed in the bridge circuit. The resistance of the output winding of a core was denoted earlier R_c. Now the resistance of a half-coil is $R_c/2$. In the basic circuit the highest average value of the sinusoidal voltage that the core can absorb is $U_{c\,av}$. On account of the half number of turns and the same core dimensions in the bridge circuit a half-coil can absorb the half-wave of a sinusoidal voltage having an average value of $U_{c\,av}/2$.

As in the earlier cases only the operating half-cycle is now to be investigated. Let us start out from the moment when the flux-level of core "A" has been reset in the preceding resetting half-cycle as compared to the upper point of saturation (point marked 2 in the magnetizing characteristic drawn beside cores "A_1" and "A_2" in Fig. 4c) while the flux of core "B" is at the lower point of saturation. The corresponding equivalent circuit is given in Fig. 12. On account of core "B" being saturated no voltages can be induced in coils "B_1" and "B_2", consequently the coils are indicated only by their resistance $R_c/2$. In coils "B_1" and "B_2" current starts to flow at the beginning of the operating half-cycle, and it is sinusoidal all through. This current is output current i_0 flowing in resistance R_0 until core "A" fires. The following voltages are on the half-coils of core "A": Voltage on half-coil "A_1" between the points δ and α is

$$u_{cA1} = i_0 \frac{R_c}{2} + i_0 R_0 \tag{37a}$$

That on half-coil "A_2" between the points β and γ is

$$u_{cA2} = i_0 R_0 + i_0 \frac{R_c}{2} \tag{37b}$$

Consequently

$$u_{cA1} = u_{cA2} = u_{cA} \tag{38}$$

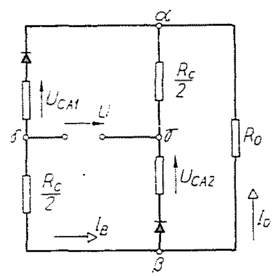

Fig. 12

U_{cA} cannot be higher than that determined by the voltage-time integral of the half-wave which can be absorbed by each half-coil of core "A". In case I the extreme value of the output power is to be sought for in the condition

$$\frac{\omega}{\pi} \int_0^{\pi/\omega} u_{cA}\, dt = \frac{U_{c\,av}}{2} \tag{39}$$

According to those derived in App. II the output power has an extreme value if

$$R_0 = R_c/2 \tag{40}$$

and the average value of the supply voltage is

$$U_{av} = \frac{3}{2} U_{c\,av} \tag{41}$$

Then the extreme value of the output power is

$$P_{\text{sym max}} = \frac{1}{32} \frac{U_{c\,\text{av}}^2}{R_c} \tag{42}$$

According to Fig. 12 the instantaneous value of the current flowing in the output winding, prior to the firing of core "A" is

$$i_B = \frac{u}{R_c + R_0} \tag{43a}$$

When matching in accordance with equation (40)

$$i_B = \frac{2}{3} \frac{u}{R_c} \tag{43b}$$

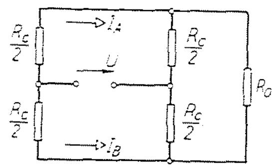

Fig. 13

After firing both cores are open and the corresponding equivalent circuit now valid is shown in Fig. 13. According to this a current of instantaneous value

$$i_A = i_B = \frac{u}{R_c} \tag{44}$$

flows in both branches. This current is 3/2 times as high as that which could be obtained from equation (43b) valid prior to the firing. The heat-dissipating capacity of the amplifier elements is to be checked in the most unfavourable case when the circuit given in Fig. 13 is valid during the whole operating half-cycle; *i. e.* in case of zero control. Then the average value of the current in the output winding of either core is

$$I_{A\,\text{av}} = I_{B\,\text{av}} = \frac{1}{2} \frac{U_{\text{av}}}{R_c} \tag{45}$$

As earlier, the factor 1/2 is due to the fact that the average of the supply voltage was calculated for the whole cycle (as in case of full-wave rectification) although current flows in the output winding only in every other half-cycle. As owing to the half-wave rectification the form factor is $\pi/2$ — according to equation (45) the r.m.s. value of the current in each reactor is

$$I_A = I_B = \frac{\pi}{4} \frac{U_{\text{av}}}{R_c}$$

or — taking equation (41) into consideration —

$$I_A = I_B = \frac{3\pi}{8} \frac{U_{c\,\text{av}}}{R_c} \tag{46}$$

It can be seen that the r.m.s. value of the current taken into account from the point of view of heating is now three times as high as in the case of the two other circuits. Consequently the matching for maximum output has to be given up in case of cores smaller than those to be used in the other two circuits. For the same reason the output obtainable from the bridge circuit is reduced, in case of larger cores (case II), when the matching of the output resistance is made for the largest allowable current in the output winding, even in case of the other two circuits.

This drawback of the bridge circuits is reduced by the circuit suggested by LUFCY, SCHMID and BARNHART in which the cores are provided with a third set of windings. The role of these windings is to reset the flux-level of the cores even in the case of zero control, thereby preventing the short-circuiting of the supply voltage through the output windings'own resistances as shown in Fig. 13.

Appendix I

Based on equations (12) and (13) it holds true that

$$i_0 = u' \frac{R}{R_c (2R + R_0) + RR_0} \tag{47}$$

It is expedient to turn to average values.

It must be taken into consideration that the average of the output current determined for the whole cycle is denoted $I_{0\,\text{av}}$, at the same time keeping in mind, that the instantaneous values of the output current are zero in the second half-cyle. On the other hand, according to equation (13a) U'_{av} means the average of the fully rectified sinusoidal voltage. Therefore, when turning to average values on the right side of equation (47) the factor 1/2 is to be written:

$$I_{0\,\mathrm{av}} = \frac{1}{2} U'_{\mathrm{av}} \frac{R}{R_c(2R + R_0) + RR_0} \tag{48}$$

By substituting this into equation (14) and taking equation (13*a*) into consideration, we obtain

$$P_{\mathrm{sym}} = \frac{U^2_{c\,\mathrm{av}}}{4} \frac{R^2 R_0}{[R_c(2R + R_0) + RR_0]^2} \tag{49}$$

In this expression $U_{c\,\mathrm{av}}$ is a constant given by the dimensions of the core and the number of turns of the output windings according to equation (1). First, let us determine the value of R_0 at which P_{sym} has a maximum, while R is kept constant.

As now $U_{c\,\mathrm{av}}$ and R remains unchanged the solution is where the function

$$f(R_0) = \frac{R_0}{[R_c(2R + R_0) + RR_0]^2} \tag{50}$$

has its extreme value. According to the usual method of determining extreme values the solution of equation

$$\frac{\partial f(R_0)}{\partial R_0} = 0$$

is

$$R_0 = \frac{2RR_c}{R + R_c} \tag{51}$$

If the above expression is substituted into equations (49) and (13) for the power

$$P_{\mathrm{sym}} = \frac{U^2_{c\,\mathrm{av}}}{32 R_c} \frac{R}{R + R_c} \tag{52}$$

is obtained. The supply voltage is

$$U_{\mathrm{av}} = U_{c\,\mathrm{av}}\left[1 + \frac{R}{4R_c}\right] \tag{53}$$

According to equation (52) P_{sym} increases with the quotient R/R_0; *i. e.* the output power monotonously increases with R and attains its maximum in infinity. The theoretically obtainable maximum of P_{sym} if $R \to \infty$ is

$$P_{sym\,max} = \frac{U_{c\,av}^2}{32 R_c} \tag{54}$$

In case II when the output power is limited by the r.m.s. value of the current in the reactor, the condition of proper matching can be determined in the following way: If the r.m.s. value of the current flowing in the unsaturated reactor is I_{eff}, then owing to the half-wave rectification its average value is $2/\pi\, I_{eff}$.

When flowing through the output winding of reactor "B" the current will cause a voltage-drop having an average value of $2/\pi I_{eff}$ across the resistance R_c. On the other hand, a voltage higher than $U_{c\,av}/2$ must not appear on the unsaturated reactor. On the unsaturated reactor appears in fact the sum of the voltage-drops across resistances R_c and R_0. Thus the following equation has to be fulfilled:

$$\frac{2}{\pi} I_{eff} R_c + I_{0\,av} R_0 = \frac{U_{c\,av}}{2} \tag{55}$$

As the right side of the above equation and the first member of its left side are fixed values, the second member of the right side — this being the output voltage itself — is also determined by the above equation:

$$U_{0\,av} = I_{0\,av} R_0 = \frac{U_{c\,av}}{2} - \frac{2}{\pi} I_{eff} R_c \tag{56}$$

As according to equation (56) the output voltage has a value fixed by the given voltage $U_{c\,av}$ on the reactor and by the current I_{eff} allowed to flow in the output winding, the output power can only be increased by increasing the output current. It can easily be seen that in case of finite R and R_0 values less than a half of the current in reactor "B" will flow through the output resistance. In case of $R \to \infty$ the average of the output current is just half of the average current $2/\pi\, I_{eff}$ flowing in the output winding of reactor "B":

$$I_{0\,av} = \frac{1}{\pi} I_{eff} \tag{57}$$

In this case the output power is

$$P_{sym\,max} = U_{0\,av} \cdot I_{0\,av} = \frac{U_{c\,av} I_{eff}}{2\pi} - \frac{2}{\pi^2} I_{eff}^2 R_e \tag{58}$$

The output resistance is to be chosen so that the output voltage-drop $U_{0\,av}$ caused by current $I_{0\,av}$ across it, should be as high as determined by equation (56):

$$R_0 = \frac{U_{0\,av}}{I_{0\,av}} = \frac{\pi}{2}\frac{U_{c\,av}}{I_{eff}} - 2\,R_c \tag{59}$$

Appendix II

According to the equivalent circuit given in Fig. 12 the instantaneous value of the output current i_0 is

$$i_0 = \frac{u}{R_c + R_0} \tag{60}$$

Correspondingly the instantaneous value of the output power is

$$P_{0(t)} = u^2 \frac{R_0}{(R_c + R_0)^2} \tag{61}$$

But, considering the fact that the extreme value of the above expression is to be determined at a constant voltage $u_c/2$ appearing on a half-coil, let us substitute in the above expression u_c for u according to the following equation:

$$i_0\left(\frac{R_c}{2} + R_0\right) = \frac{u}{R_c + R_0}\left(\frac{R_c}{2} + R_0\right) = \frac{u_c}{2} \tag{62}$$

Hence

$$u\,\frac{R_c + 2\,R_0}{R_c + R_0} = u_c; \quad u = u_c\,\frac{R_c + R_0}{R_c + 2\,R_0} \tag{63}$$

Substituting (63) into (61) we obtain

$$P_{0(t)} = u_c^2\,\frac{R_0}{(R_c + 2\,R_0)^2}$$

With changing R_0 the above expression has its extreme value in case of

$$R_c = 2\,R_0 \tag{64}$$

By substituting this into equation (63) for the supply voltage

$$u = \frac{3}{4}\,u_c \tag{65}$$

is obtained. Or turning to average values

$$U_{av} = \frac{3}{4} U_{c\,av} \tag{66}$$

Now based on

$$P_{sym\,max} = I_{0\,av}^2 R_0 \tag{67}$$

let us determine the maximum of the output power.

As

$$I_{0\,av} = \frac{1}{2} \frac{U_{av}}{R + R_0} \tag{68}$$

after substituting (64) and (66) we obtain

$$I_{0\,av} = \frac{1}{4} \frac{U_{c\,av}}{R_c}$$

Hence

$$P_{sym\,max} = I_{0\,av}^2 R_0 = \frac{1}{16} \frac{U_{c\,av}^2}{R_c^2} \cdot \frac{R_c}{2} = \frac{1}{32} \frac{U_{c\,av}^2}{R_c} \tag{69}$$

Summary

One of the problems arising in connection with the construction of half-wave push-pull amplifiers concerns the possibility of achieving the maximum possible output of a core of given size. The author investigating the three theoretical possibilities of balancing, shows that in case of properly choosing the circuit constants the circuits are generally equivalent. If the circuit constants are selected according to the method shown in the paper, half of the output is to be achieved in case of any of the three circuits than in that of a basic circuit amplifier built with the same core.

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A. FRIGYES, Budapest, XI., Egry József u. 18., Hungary