STATISTICAL QUALITY CONTROL USING AN ANALOGUE COMPUTER*

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1. Introduction

One of the fundamental features of the increasing productivity of the rapidly developing industries is the manufacture of large series. The manufacture of large quantities of a product raise new problems of quality control; for lack of time, instruments and personnel frequently renders it uneconomical for each item to be examined. There may also be a certain time lag between production and control, in which case faults discovered by control cannot, with timely intervention, avoid their repetition.

It was to solve these problems that quality control by sampling was developed on the bases of mathematical statistics. The characteristics of a sample containing the requisite number of items, made to approximate the characteristics of the entire manufactured series with any degree of accuracy desired, while obviating the necessity for an inordinately large number of measurements.

Data obtained from samples, taken during manufacturing, enable production processes to be controlled, for the tendencies of the measured data clearly indicate changes in raw materials, wear of tools etc. The evaluation of the samples is, however, frequently a lengthy process. It is, therefore, worth while to find electric analogies for the statistical relationships and to construct an analogue computer.

2. Statistical relationships [1], [2]

For large series it may be presumed that the characteristic of the product being examined is of normal (Gaussian) distribution. Let the average value of the distribution be m and its standard deviation σ. How can these be determined without measuring the entire series?

* Rewarded by the Hungarian Academy of Sciences.

Let the value of the characteristic measured on the i-th member of a sample containing n items be x_i, in which case the average value is

$$\bar{x} = \frac{1}{n} \sum_{i=}^{n} x_i$$

This does not, in general, agree with the average of the large series but very probably falls between the limits

$$m \pm \frac{3\sigma}{\sqrt{n}} .$$

The square of the empirical deviation* of the sample is

$$s^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2 .$$

It is probable that the standard deviation of the sample will be smaller than that of the entire series, since items on the limits of the original distribution only rarely occur in the sample. The probable value of s and σ are therefore related by

$$s = c_n \sigma,$$

where $c_n < 1$ (*e. g.* for $n = 10$, $c_n = 0.9227$). Evidently s also varies around σ and the extent of this variation as a function of n — also as c_n — is contained in tables. If a sample of many items is used ($n > 15$), then $c_n \to 1$ and the variation of s is very small.

The average absolute deviation of the members of the sample

$$d = \frac{1}{n} \sum_{i=1}^{n} |x_i - \bar{x}|$$

and the range

$$R = x_{\max} - x_{\min}$$

are also generally defined.

For normal distributions there are very probable relationships between the expected values of d and R and σ**. It follows, that if the range as well

* The above expression for empirical deviation is used for simplicity of operation, noting that the quantity $\sqrt{\frac{n}{n-1}}\, s$, generally defined as "deviation" hardly differs from s, where n is sufficiently large.

** The relationships and tables of the coefficients used in these may be found in the literature [1], [2].

as the average and empirical deviations are found, any departure of the distribution from normalcy can be established.

All four sample characteristics ($\bar{x}$, s, d, R) may be determined without numerical calculation using an analogue computer and conclusions on the true average and standard deviation may, as has been shown, then be drawn from the samples.

3. Operating principles of the computer [3]

The block diagramm of the analogue computer is shown in Fig. 1.

The items ($M_1 \ldots M_n$) of the sample are simultaneously placed into the instrument which incorporates n measuring heads. The values ($x_1 \ldots x_n$) of the characteristic to be measured are proportionately transformed into alter-

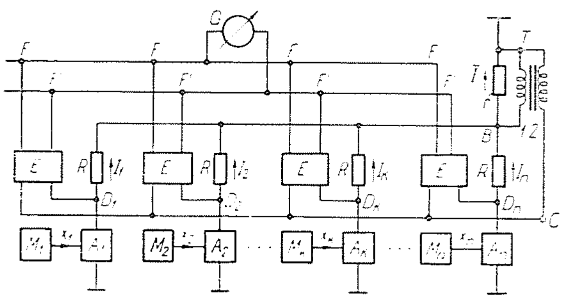

Fig. 1. Block diagram of analogue computer

nating currents $I_1 \ldots I_n$ by the transducers A. The currents $I_1 \ldots I_n$ flow through a resistance R each and are then connected in junction B. The current through the resistance r is therefore

$$I = \sum_{i=1}^{n} I_i = C_1 \sum_{i=1}^{n} x_i ,$$

which is related to the average current $\bar{I}$ by $I = n\,\bar{I}$. An ammeter in series, or a voltmeter across the resistance will therefore, if properly calibrated, give a direct reading of the average value of the characteristic to be measured.

To determine the standard deviation, absolute deviation and range, signals proportional to the differences $I_i - \bar{I}$ have to be produced. The primary windings of a transformer T with a ratio of 1:2 are therefore connected across the resistance r. If the value of r is chosen to be $\frac{R}{n}$ then the average value of the voltage drop on the resistances R due to I_i is the same as that on r due to

$n\bar{I}$. Transforming the latter up by a ratio of 1: 2, point C will be equipotential with the point D_k, through which a current of just $\bar{I}$ is flowing. If the current I_i differs from the average, the voltage between C and D_i is proportionate to $I_i - \bar{I}$.

To determine the standard deviation these voltages are applied to rectifiers E with quadratic characteristics, the output current of the i-th of which is

$$I_{Ei} = \beta U_i^2 = \beta R^2 (I_i - \bar{I})^2 = C_2 (x_i - \bar{x})^2 ,$$

a function of the deviation from the average, where β is a coefficient due to the characteristic of the detector. Connecting the currents from the detectors E in the junctions F and F' the Deprez instrument G will measure the current

$$\sum_{i=1}^{n} I_{Ei} = \beta R^2 \sum_{i=1}^{n} (I_i - \bar{I})^2 ,$$

which is proportional to the variance.

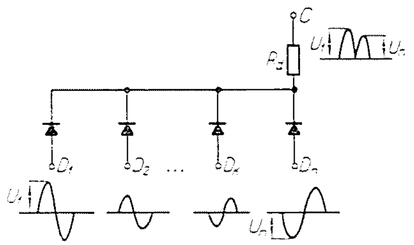

Fig. 2. Range measuring circuit

The measurement of the value of the average absolute deviation is thus straightforward: detectors with linear instead of quadratic characteristics are used, so that the sum of the rectified currents is n times the average absolute deviation.

A simple addition to the circuit of Fig. 1. will also allow the apparatus to be used for measurements of the range. In Fig. 2 (retaining the symbols of Fig. 1) diodes are connected between points D_1, D_2 ... D_n and point C through a common load resistance R_d. For easy presentation the items to be measured have in the figure been arranged so that the item deviating most in the positive direction is connected to D_1, while that with the greatest negative deviation is at D_n. The phases of the voltages U_i developing on the various points with respect to C are dependent on the directions of the deviations, for if $I_i > \bar{I}$, the difference voltage is in phase with I_i and *vice versa*. This has also been shown on the figure. Obviously, if R_d is very much greater than the forward resistance of the diode, then in the first half period only the diode connected to point D_1 will conduct; if it conducts, all the rest will have backward potentials applied to them. Similarly, in the other half-period only the diode of D_n will conduct. The load resistance will therefore develop a signal consisting of half sine curves of different amplitudes. If this signal is connected to a phase-sensitive amplifier the maximum peak-to-peak voltage measured at the output will be proportional to the range.

4. Quadratic detector [4], [5]

It may be seen from the foregoing, that the computer may, for the larger part, be assembled using conventional circuitry, except for the quadratic rectifiers used to measure the variance.

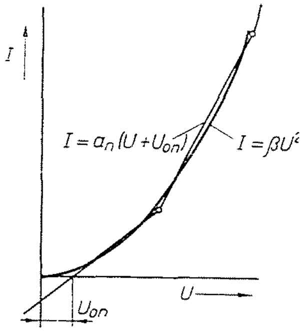

Fig. 3. Approximation to a quadratic parabola

Exact quadratic characteristics are possessed by instruments utilizing the heating or electrodynamic effects of currents. Neither is suited to the present purpose, due to their large power consumption and in the case of the former, the fact that the effect of the temperature of the environment is not negligible. An exact quadratic characteristic may also be obtained by means of an instrument with a bilinear characteristic, based on the Hall effect, but this method is still not feasible. The quadratic portions of the characteristics of electronic valves and semiconductors are too short and not stable.

A quadratic parabola may be approximated at will by a polyangular curve (Fig. 3) produced by means of a network consisting of biased diodes and resistances (Fig. 4). On the initial portion of the curve only diode 1 conducts and the gradient of the curve is determined by the resistance R_1 connected in series with it. At greater voltages diode 2, then diode 3 will begin to conduct, connecting resistances R_2 and R_3 in parallel with R_1. Since the diodes only function as switching elements, precision and stability depend nearly exclusively on the resistances.

The squaring circuit used in the apparatus should be simple, with few components since — due to the large number of units to be incorporated — their congruence may thus be more easily attained. Fig. 5., being an enlarged part of Fig. 3, will serve to design the network. It is stipulated that the relative deviation of the polyangular curve from the parabola may nowhere be greater than h.

The equation of the parabola is

$$I = \beta\, U^2,$$

that of the straight line is

$$I' = a_n(U + U_{on}),$$

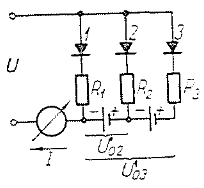

Fig. 4. A circuit to produce a polyangular characteristic

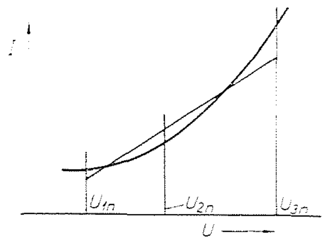

Fig. 5. A straight line section to approximate the parabola

where a_n is the tangent gradient of the n-th straight section and U_{on} is the shift potential of the n-th section. The maximum permitted deviation is

$$I - I' = \pm hI.$$

The ends U_{1n}, U_{3n} of the approximating line (both ends are beneath the parabola) may be found from the equation

$$a_n\,(U_{1n,3n} + U_{on}) = (1 - h)\,\beta\, U^2_{1n,3n}$$

while the position of the point of maximum distance between the chord and the arc, whose abscissa is U_{2n}, may be calculated from the inequality

$$a_n\,(U_{2n} + U_{on}) - \beta\, U^2_{2n} \leqq h\beta U^2_{2n}\,.$$

If a relative deviation of just h is to be permitted at U_{2n}, then the deviation will necessarily be smaller elsewhere and the inequality becomes a mixed second order equation with only one physically real root. This is obviously the coincident pair of roots of the rearranged equation.

Thus

$$(1+h)\,\beta U_{2n}^2 - a_n U_{2n} - a_n U_{on} = 0$$

and

$$a_n^2 + 4\,(1+h)\,\beta a_n U_{on} = 0\,,$$

whence

$$U_{2n} = \frac{a_n}{2\,\beta\,(1+h)}$$

and

$$U_{on} = -\,\frac{a_n}{4\,\beta\,(1+h)}$$

The positions of the end points are

$$U_{1n,3n} = \frac{a_n \pm \sqrt{a_n^2 + 4\,(1-h)\,\beta\, a_n U_{on}}}{2\beta\,(1-h)}$$

Using the relation obtained for U_{on}

$$U_{1n,3n} = \frac{a_n \pm \sqrt{a_n^2 - \frac{1-h}{1+h}\, a_n^2}}{2\,(1-h)\,\beta}$$

whence

$$U_{1n} = \frac{a_n}{2\,\beta\,(1-h)}\left[1 - \sqrt{\frac{2\,h}{1+h}}\right]$$

and

$$U_{3n} = \frac{a_n}{2\,\beta\,(1-h)}\left[1 + \sqrt{\frac{2\,h}{1+h}}\right]$$

If $h = 10\%$ is to be permitted,

$$U_{1n} = \frac{a_n}{1.8\,\beta}\,0.576 = 0.32\,\frac{a_n}{\beta}$$

$$U_{2n} = \frac{a_n}{2.2\,\beta} = 0.45\,\frac{a_n}{\beta}$$

$$U_{3n} = \frac{a_n}{1.8\,\beta}\,1.424 = 0.79\,\frac{a_n}{\beta}$$

$$U_{on} = -\,\frac{a_n}{4.4\,\beta} = -\,0.227\,\frac{a_n}{\beta}$$

The ratio U_{3n}/U_{1n} determines the range of a single straight section:

$$\frac{U_{3n}}{U_{1n}} = \frac{U_{3n}}{U_{3(n-1)}} = \frac{0{,}79}{0{,}32} \cong 2{,}5 \, .$$

If, therefore, the abscissae of the breaks are determined by an exponential function of shape 2.5^n, the approximation will be better everywhere by 10%. This would indicate, that a polyangular line with an infinite number of sections is required to approximate a parabola originating at zero. In practice, exploiting the curved characteristic of the first diode, the first break is put between 100—150 mV.

The values of the resistances to be used are plainly

$$\frac{1}{R_1} = a_1$$

$$\frac{1}{R_2} = a_2 - \frac{1}{R_1}$$

$$\vdots$$

$$\frac{1}{R_n} = a_n - a_{n-1} \, .$$

5. Factors influencing precision

The measurement of the average is reduced to a plain measurement of alternating voltage. It is the simplest to use a Deprez instrument with a rectifier for this purpose, which — depending on the class of the instrument — enables precisions of 1—3% to be attained.

An attributive error arises if the alternating voltage to be measured contains harmonics. Harmonics may be generated in the power supply or, more probably, in the possibly non-linear measuring transducers. The second harmonic will probably have the highest amplitude and superimposed on the fundamental harmonic, will once be added to it, next subtracted, thus sharpening one peak and flattening the next of the resultant waveform. A peak rectifying Deprez instrument containing a single diode will then give completely false readings, but if there is a peak-to-peak (two-way) rectification, the effect of the second harmonic may be eliminated. If, however, several harmonics are present it is best to use a selective valve voltmeter tuned to the fundamental harmonic.

The work of the variance meter can be influenced the most by errors in forming averages. If the common resistance r is not exactly an n-th part of R, or if the ratio of the transformer is not exactly 1:2, then voltages not exactly corresponding to the average value will be subtracted from the potentials of the measuring points (D_i of Fig. 1). The error thus caused may easily be estimated.

Let the value of the characteristic to be measured again be x_i, the correct average be $\bar{x}$ and the erroneous average be $(1 + \varepsilon)\,\bar{x}$. The variance computed from the erroneous average is

$$s_h^2 = \frac{1}{n} \sum_{i=1}^{n} [x_i - (1 + \varepsilon)\,\bar{x}]^2 = \frac{1}{n} \left\{ \sum_{i=1}^{n} x_i^2 - \bar{x} \sum_{i=1}^{n} \left[2x_i\,(1+\varepsilon) - \bar{x}\,(1+\varepsilon)^2\right] \right\}$$

The correct variance is

$$s^2 = \frac{1}{n} \sum_{i=1}^{n} [x_i - \bar{x}]^2 = \frac{1}{n} \left\{ \sum_{i=1}^{n} x_i^2 - \bar{x} \sum_{i=1}^{n} [2x_i - \bar{x}] \right\}$$

The difference between the two is

$$s_h^2 - s^2 = -\frac{\bar{x}}{n} \sum_{i=1}^{n} \left\{2x_i\,\varepsilon - \bar{x}\left[(1 + \varepsilon)^2 - 1\right]\right\}$$

Using the expressions

$$\sum_{i=1}^{n} x_i = n\bar{x}$$

$$(1 + \varepsilon)^2 - 1 = \varepsilon(2 + \varepsilon)$$

and substituting

$$s_h^2 - s^2 = -\,2\,\bar{x}^2\,\varepsilon + \bar{x}^2\varepsilon(2 + \varepsilon) = \bar{x}^2\,\varepsilon^2$$

The error is always positive, irrespective of the sign of the error ε, so that the erroneously measured variance is always greater than the correct value. If for instance $\varepsilon = 1\%$ and the relative deviation with respect to the mean value is $s/\bar{x} = 10\%$, the relative error of the relative variance is

$$\frac{s_h^2 - s^2}{\bar{x}^2} : \frac{s^2}{\bar{x}^2} = \frac{s_h^2 - s^2}{s^2} = \frac{\bar{x}^2\,\varepsilon^2}{s^2} = 10^{-2}\,,$$

but if $s/\bar{x} = 5\%$ it is $4 \cdot 10^{-2}$. The averaging resistance should therefore be set with special care for samples with narrow distribution patterns.

It has been pointed out, that squaring circuits can by simple methods only be made approximately quadratic. The variance meter is therefore not absolutely precise. The method of design presented secures an error ceiling of 10%. An examination of Fig. 3 shows that the sign of the error changes along the function and it may therefore be expected that if several points of the polyangular curve are simultaneously used for detection then the errors will, to a certain extent, compensate each other. This condition is automatically fulfilled, for the characteristic to be measured differs from the average by different amounts for each item of the sample, so that different difference signals are fed to each of the detectors.

For a more accurate calculation of the error, it is presumed that the difference signals are continuously and normally distributed, with a mean value of zero. The frequency function is then simply

$$f(z) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{z^2}{2\sigma^2}\right)$$

when z is a non-dimensional probability variable proportional to the difference signal (voltage). The variance of this distribution may be obtained if the square of the deviation from the mean be multiplied by the frequency function and integrated between infinite limits:

$$s^2 = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x^2 \exp\left(-\frac{z^2}{2\sigma^2}\right) dz .$$

Introducing the new variable $w = z/\sigma\sqrt{2}$ and partially integrating:

$$s^2 = \frac{2\sigma^2}{\sqrt{\pi}} \int_{-\infty}^{\infty} w \cdot w e^{-w^2} dw = \frac{2\sigma^2}{\sqrt{\pi}} \left\{ \left[-w \frac{e^{-w^2}}{2} \right]_{-\infty}^{\infty} + \int_{-\infty}^{\infty} \frac{e^{-w^2}}{2} dw \right\} = \sigma^2 .$$

as the first term is zero and the second $\sqrt{\pi}/2$. It may be observed that it was necessary to multiply the frequency function by $y = z^2$, the dimensionless form of the ideal detector characteristic.

If a polyangular characteristic is used instead of a parabola, then the mutually relative positions of the breaks and some characteristic point of the distribution have to be determined. Let the polyangular characteristic in our case consist of three sections, and let the upper limit of the uppermost section (-10% deviation; see Fig. 3) be related to point $3\,\sigma$ of the distribution. The abscissae of the breaks are, according to Chap. 4. (again using the dimensionless variable z)

$$U_{3n} \div z_{33} = 3\sigma$$

$$U_{3(n-1)} \div z_{32} = \frac{3}{2.5}\,\sigma = 1.2\,\sigma$$

$$U_{3(n-2)} \div z_{31} = \frac{3}{6.25}\,\sigma = 0.48\,\sigma.$$

The coefficients a_n are

$$a_3 = \frac{z_{33}}{0.79} = \frac{3\sigma}{0.79} = 3.8\,\sigma$$

$$a_2 = \frac{3.8}{2.5}\,\sigma = 1.52\,\sigma$$

$$a_1 = \frac{0.9(0.48)^2\,\sigma^2}{0.48} = 0.43\,\sigma.$$

In calculating a_3 and a_2 use was made of the expression $\beta = 1$ and a_1 was determined so that the first straight section should connect the origin with a point 10% under the parabola at $z_{31} = 0{,}48\,\sigma$. The approximation to the lower stretch of the parabola is thus worse than was actually the case when making use of the curved characteristic of the diode.

The variance can now be determined in three parts, corresponding to the three straight sections:

$$s_1^2 = \frac{2}{\sigma\sqrt{2\pi}} \int_0^{z_{31}} a_1 z \exp\left(-\frac{z^2}{2\sigma^2}\right) dz = \frac{4\sigma a_1}{\sqrt{2\pi}} \int_0^{w_1} w e^{-w^2}\, dw =$$

$$= \frac{4\sigma a_1}{\sqrt{2\pi}} \left[\frac{e^{-w^2}}{2}\right]_0^{w_1} = \frac{2\sigma a_1}{\sqrt{2\pi}} \left[1 - \exp\left(-\frac{z_{31}^2}{2\sigma^2}\right)\right]$$

$$s_2^2 = \frac{2}{\sigma\sqrt{2\pi}} \int_{z_{31}}^{z_{32}} [a_2(z - z_{31}) + a_1 z_{31}] \exp\left(-\frac{z^2}{2\sigma^2}\right) dz =$$

$$= \frac{2\sigma a_1}{\sqrt{2\pi}} \left[\exp\left(-\frac{z_{31}^2}{2\sigma^2}\right) - \exp\left(-\frac{z_{32}^2}{2\sigma^2}\right)\right] +$$

$$+ \frac{2(a_1 - a_2) z_{31}}{\sigma\sqrt{2\pi}} \int_{z_{31}}^{z_{32}} \exp\left(-\frac{z^2}{2\sigma^2}\right) dz$$

$$s_3^2 = \frac{2}{\sigma\sqrt{2\pi}} \int_{z_{32}}^{\infty} \left[a_3 (z - z_{32}) + a_1 z_{31} + a_2 (z_{32} - z_{31}) \right] \exp\left(-\frac{z^2}{2\sigma^2}\right) dz =$$

$$= \frac{2\sigma a_3}{\sqrt{2\pi}} \exp\left(\frac{z_{32}^2}{2\sigma^2}\right) +$$

$$+ \frac{2(a_2 - a_3) z_{32} + 2(a_1 - a_2) z_{31}}{\sigma\sqrt{2\pi}} \int_{z_{32}}^{\infty} \exp\left(-\frac{z^2}{2\sigma^2}\right) dz .$$

The integral forming the second term of the results cannot be solved in its closed form, but its value may be found in tables.

Substituting $z_{33} = 3\sigma$ etc. values in these expressions,

$$s^2 = s_1^2 + s_2^2 + s_3^2 = (0.038 + 0.279 + 0.723)\, \sigma^2 = 1.040\, \sigma^2$$

so that the error of the variance is reduced to 4%, which means an error of 2% in the standard deviation.

It is not worth while improving the calculations still further, because the distribution of the sample is — due to the small number of items — only approximately normal. The most significant divergence between the calculated and the actual data is caused by the not continuous distribution of the sample.

6. Measurements of the averages and standard deviations of the mutual conductances of electronic valves

The block diagramm of Fig. 1 shows that the apparatus may be simply assembled if the transformation of the quantity to be measured does not require a special transducer. Thus, if the grid of an electronic valve has an alternating voltage applied, the alternating current in the anode circuit will be proportional to the control voltage and the mutual conductance. The design of an apparatus to measure average mutual conductances and standard deviations may thus be attempted.

Fig. 6 shows how n electronic valves may, together with the resistances R in their anode circuits, be connected in parallel, with all the anode currents leading through resistance r. The voltage drop on the latter is used, partly to measure the average, and partly, after being boosted by a transformer, to be connected in opposition to the potentials on the anodes. The voltage drops thus obtained are connected to the quadratic detectors, whose outputs are summed.

Anode circuit resistances may not be chosen to be as large as desired. If the resistance r is considered as being composed of n resistances of $n\, r = R$

magnitude connected in parallel, then obviously there will be 2 R in the anode circuit of each valve. The alternating voltage in the anode circuit may be calculated from the expression

$$U_a = g_m U_r \frac{2R}{1 + 2R/R_i} = U_r \frac{g_m(2R)}{1 + g_m(2R)/\mu}$$

where U_r is the amplitude of the control voltage at, say, 800 $c/s.$, g_m is the mutual conductance of the valve, R_i is its internal resistance and μ its gain

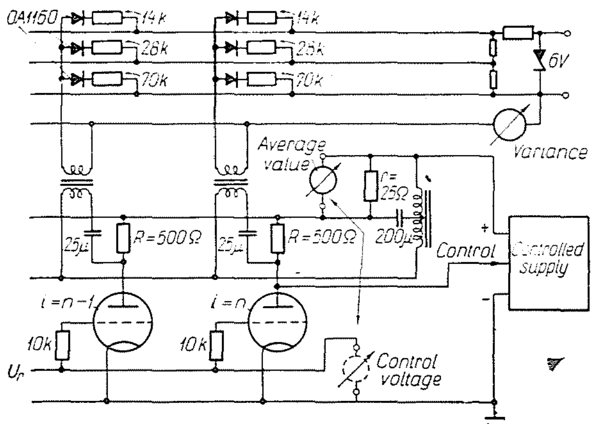

Fig. 6. Circuit diagram of apparatus to examine electronic valves

factor. The second term of the denominator is due to anode reaction, causing a relative error in measurement of

$$\frac{1}{1 + g_m(2R)/\mu} - 1 \approx -\frac{g_m(2R)}{\mu} .$$

In the case of triodes, $\mu = 20 \ldots 100$, if 5% inaccuracy is to be permitted, $g_m(2R)$ may be 1 at the most. If it is also considered, that U_r may generally not be more than 1 V, a value of $U_{a\,\max} \approx 1$ V is obtained. Of this, one half is developed on r, which is sufficient for measurements of the average but not enough to work the quadratic detectors measuring the variance.

The input voltages to the detectors may be increased in two ways. One method is to connect an amplifier before each detector, which might well be transistorized. This is a fairly expensive solution. The voltage drop on the resistance 2 R may, however, also be increased without undue interference by anode reaction, if we see to it that a minimal alternating voltage is developed on the *anode* with respect to the *cathode*. The circuit shown in Fig. 6 makes provision for this by the proper design of the stabiliser of the power supply.

If the control voltage of the stabilizer is taken straight from the anode terminal (D_n), stabilisation takes place with respect to this point and the supply voltage will not be stable at the point marked +, but will incorporate an alternating component just large enough to ensure a minimum alternating voltage on the anode terminal with respect to earth.

The experimental valve measuring apparatus whose circuit is shown in Fig. 6 was constructed at the Department of Electronic Valves of the Technical University of Budapest. The apparatus can accomodate 10 twin triodes

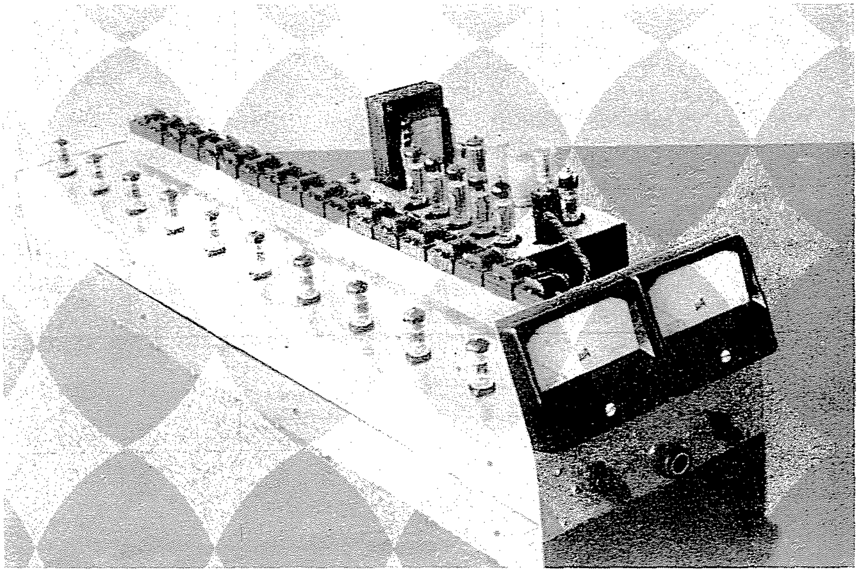

Fig. 7. Photograph of apparatus for examining electronic valves

(ECC 82 or ECC 85) and so computes averages and variances, for a system of $n = 20$ valves. The working point is set by means of the adjustable voltages of the stabilized power supply. The control voltage to the grids is supplied by an external A. F. generator. To ensure precise measurement, the control voltage is measured with the same instrument as that used for the alternating anode voltage. Grid circuit resistances have been included to prevent ultra-frequency swinging and anode circuit transformers and capacitors to achieve D. C. separation of the variance meter network. The diodes of the quadratic detectors were biased by voltages stabilized with a Zener-diode. A photograph of the apparatus may be seen in Fig. 7. The unit on the left of the photo accomodates the valves to be measured. To their right, are the anode circuit transformers. The instrument panel mounts the average indicator Deprez instrument on the left, the variance meter on the right. The special stabiliser that was described is on the right hand side of the photo.

10 ECC 82 and 10 ECC 85 type valves were examined with the apparatus. Column 1 of Table I contains the symbols for the 20 systems of the 10 ECC 85 twin triodes. Column 2 has the individually measured mutual conductances of the valves (each the mean of 2 readings), and column 3 shows the deviations from the average. Column 4 contains the difference voltages measured between points C and D_i of the apparatus (Fig. 1) in mV (each the mean of 8 readings). A 1% difference in mutual conductance (= 0,066 mA/V) here corresponds to 26 mV. The calculated average and the relative deviation calculated from the data of columns 3 and 4 are shown at the bottom of the table. The instruments in the apparatus then showed $\bar{g}_m = 6{,}45$ mA/V and $\frac{s}{\bar{g}_m} = 10{,}4\ \%$, which is a very good approximation to the calculated values. All readings were taken at the working point stated in the catalogue.

Table I

	g_m (mA/V)	Δg_m (mA/V)	ΔU (mV)
1/a	6.45	—0.15	— 30
b	6.95	+0.35	+110
2/a	8.45	+1.85	+620
b	6.1	—0.5	—430
3/a	6.85	+0.25	+ 60
b	6.6	0	0
4/a	7.4	+0.8	+240
b	7.0	+0.4	+150
5/a	6.35	—0.25	—120
b	7.15	+0.55	+230
6/a	5.4	—1.2	—530
b	6.35	—0.25	—100
7/a	6.05	—0.55	—180
b	5.6	—1.0	—430
8/a	7.2	+0.6	+270
b	7.45	+0.85	+350
9/a	5.75	—0.85	—370
b	6.5	—0.1	— 50
10/a	6.3	—0.3	—150
b	6.6	0	+ 50

$$\bar{g}_m = 6{,}6 \text{ mA/V} \quad s_{gm} = 0{,}698 \text{ mA/V} \quad s_{\Delta U} = 283 \text{ mV}$$

$$\frac{s_{gm}}{\bar{g}_m} = 10{,}6\ \% \quad \frac{s_{\Delta U}}{\bar{U}} = \frac{283}{2600} = 10{,}9\ \%$$

The sum of the figures of column 4 is not zero, but — 310 mV. This means that the averaging resistance was not properly set, because — 310:20 = = — 15.5 mV were added to the voltages of each of the measuring points, corresponding to an error of over 0.5% in setting. In the case considered, this did not noticeably influence the variance value obtained.

Similar measurements were also carried out with type ECC 82 valves, where the following is a brief summary of the results obtained:

Average mutual conductance according to individual measurements	1.92 mA/V
Indicated mutual conductance	1.85 mA/V
Calculated variance according to individual measurements	11.6%
Calculated variance according to voltage differences	14.1%
Indicated variance	13.1%

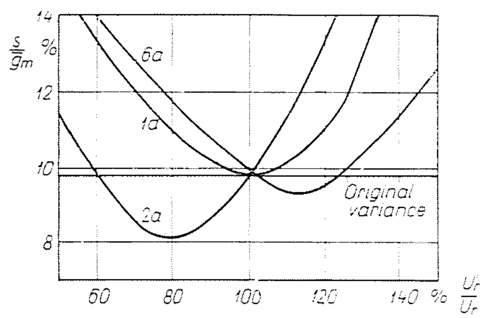

Fig. 8. Change of variance for changes of the mutual conductance of one valve

It was noted that the mutual conductance of one of the valves considerably varied with time, which explains the differences of the variance values that were measured at different times. The mutual conductance of this same valve was, incidentally, some — 50% different from the mean, thus considerably increasing the calculated variance, while not increasing the measured variance to a similar extent, as the valve was well outside the normal functioning range of the quadratic detector. After changing this valve, the variance decreased to below 9%.

To control the variance meter the following experiment was carried out: The grid of one of the triodes was disconnected from the rest and a control voltage of variable amplitude, coherent with the common control signal was applied to it. Changes in the control voltage changed the apparent mutual conductances of the valve, thus altering the average mutual conductance and especially the variance. The readings are shown in Fig. 8.

Valve 1*a* is of nearly average mutual conductance. Correspondingly, at 100% control voltage the variance reading does not change with respect to its original value, but a decrease or increase in control voltage increases the variance. The mutual conductance of valve 2*a* is a great deal over the average. If about 80% control voltage is applied its apparent mutual conductance attains the average and the variance considerably decreases. A similar result can also be attained with valve 6*a* if the control voltage is increased to 115%.

Interesting results are shown in Table II. The average and the variance of mutual conductivity were measured for different values of heater voltage. It was found that while variance for normal heater voltage was acceptable, it decreased somewhat during overheating and increased considerably by underheating.

Table II

U_f (V)	$\bar{g}_m$ (mA/V)	$s/\bar{g}_m$ %
5.0	3.85	20.5
5.5	5.8	12.5
6.3	6.55	10.4
7.0	7.1	9.0

7. Applications

Apparatus for measuring averages, variances and ranges cannot only be used for measuring the mutual conductances of electronic valves. The direct current through electronic valves at the working point may, for instance, also be controlled after having, of course, first been converted to alternating current. The simplest way of doing this is periodically to cut the valve off.

A 1:1 ratio square wave voltage is connected to the grid, whose positive peak value is the bias corresponding to the working point to be measured (this may be kept at a constant value with a fixing diode), while the negative peak completely cuts the valves off. The current through the common anode load will then have a peak-to-peak amplitude of

$$nI_0 = \sum_{i=1}^{n} I_{0i}$$

where I_{0i} is the working point current of the individual valve. The average measuring instrument is calibrated in working point current, with due attention to the wave-shape factor of the square wave signal.

The measurement of variances can also take place by the method described for sine waves, but care must be taken to secure adequate impulse transfer through the transformers.

The apparatus described in Chap. 3 and shown on Fig. 1 may be used without transducers in all cases where the quantity to be measured is a current, or proportional to a current, *e. g.*

$$G = \frac{I}{U}$$

in the case of a conductance. Measurements may thus be made of the output conductances, mutual conductances and even the current amplification of transistors. Similarly the current

$$I = U j \omega C$$

through a condenser is proportional to the capacity and the apparatus is thus suited for the control of mass produced condensers.

In measuring resistances or parameters resembling resistances allowance has to be made for certain errors. Let n resistances of values $R_1, R_2 \ldots R_n$ be connected in parallel. The apparatus will measure the parallel resultant proportional to the total current flow

$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

Writing the components as $R_i = \overline{R} + \Delta R_i$

$$\frac{1}{R_e} = \frac{1}{\overline{R} + \Delta R_1} + \frac{1}{\overline{R} + \Delta R_2} + \ldots + \frac{1}{\overline{R} + \Delta R_n} = \frac{1}{\overline{R}} \sum_{i=1}^{n} \frac{1}{1 + \Delta R_i/\overline{R}}$$

which may be developed into a binomial series according to the second term of the denominator, *i. e.* the relative difference. Terminating the series with the second term we obtain

$$\frac{1}{R_e} = \frac{1}{\overline{R}} \left[n - \left(\frac{\Delta R_1}{\overline{R}} + \frac{\Delta R_2}{\overline{R}} + \ldots + \frac{\Delta R_n}{\overline{R}} \right) + \left(\frac{\Delta R_1}{\overline{R}} \right)^2 + \right.$$
$$\left. + \left(\frac{\Delta R_2}{\overline{R}} \right)^2 + \ldots + \left(\frac{\Delta R_n}{\overline{R}} \right)^2 \right]$$

The definition of the mean tells us that

$$\sum_{i=1}^{n} \frac{\Delta R_i}{\overline{R}} = 0$$

and using

$$V^2 = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{\Delta R_1}{\overline{R}}\right)^2 = \frac{1}{\overline{R}^2} s^2$$

(V being the variation coefficient or relative variance) whence, eliminating the quadratic terms

$$\frac{1}{R_e} = \frac{n}{\overline{R}} + \frac{1}{\overline{R}} n V^2$$

The current through the measuring resistance is proportional to $\frac{1}{R_e}$ and the average value thus obtained is

$$\overline{R}' = nR_e = \overline{R} \frac{1}{1+V^2}$$

which is less than it should be. If, for instance, $\pm$ 20% resistances are being examined which all fall within the given tolerance limits then, presuming normal distribution, the value of V is obtained from the condition

$$\pm \frac{3s}{\overline{R}} = \pm 3V = \pm 20\,\%$$

as it is very probable that all resistance values will be within three times the variance limits in the positive and negative directions. Then V = 6.7% and

$$\overline{R}' = \overline{R} \frac{1}{1 + 45 \cdot 10^{-4}} \approx \overline{R} \,.$$

The difference is insignificant. It may similarly be proved that though conductances are measured instead of resistances, the variance of resistances is obtained with small errors.

In conclusion, it should be pointed out, that the computer described in this paper is primarily suited to the evaluation of electric parameters or in cases where simple transducers can be used.

The possibility of using a pneumatic analogue of this apparatus to measure lengths in engineering should be investigated, making use of the analogies between differences of pressure and voltage and of volume velocity and current.

Acknowledgement

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Summary.

Quality control in modern mass production is usually made by sampling. The statistical evaluation of the measurements, however, frequently requires much time. The Department of Electronic Valves of the Technical University of Budapest is, therefore, developing a computer which by means of electric analogies instantaneously determines the average value, variance, range or any other statistical property of the characteristic to be controlled, from a sample consisting of numerous items. Transducers convert the values of the characteristics to proportionate alternating currents. The apparatus sums the currents to present the average value and by squaring the differences between the average and the individual values and summing the squares presents the variance, both by direct reading. The model that was built is for controlling the mutual conductances of electronic valves.

Literature

1. Statisztikai minőségellenőrzés. Szerkesztette Vincze István (Statistical Quality Control. Edited by I. Vincze). Közgazdasági és Jogi Könyvkiadó, Budapest 1958.
2. Schaafsma, A. H., Willemse, F. G.: Moderne Qualitätskontrolle. Philips Technische Bibliothek, 1955.
3. Hungarian patent regd. N° Ao-216.
4. Burt, E. G. C., Lange, O. H.: Function generators based on linear interpolation with applications to analogue computing. Proc. I. E. E. **102**, 856 (1955).
5. Wahrmann, C. G.: A true RMS instrument. Brüel — Kjaer Technical Review 1958. N° 3, p : 9.

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