THEORETICAL METHODS CONCERNED WITH THE ASYNCHRONOUS OPERATION OF TURBO-GENERATORS

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Introduction

Reviewing the fundamental books discussing the transient phenomena and behaviour of the synchronous machines [2, 3, 14, 23, 16, 18, 22] it may be stated that for most of the transient phenomena a clear explanation could be found. Examining, however, the material elaborated, it becomes conspicuous that the theoretical results refer mostly to machines operating with a constant angular speed, or speed of rotation. Even if there is an obvious change (*e. g.* at asynchronous strating, or synchronizing), the angular speed is frequently assumed to be constant at least at a certain initial period, as this simplifies the discussion of the problem.

A variable speed leads namely to a nonlinear differential equation system. The difficulties in solving nonlinear differential equations are explanations for the fact the cases of the synchronous machines with variable speeds having not been dealt with in details, but lately [15, 33, 36, etc.], though raising the problem may look back upon a past of many decades [*e. g.* 13, 32] and almost in every transient phenomenon there must be a change in the speed. To avoid the nonlinear differential equations, when discussing the cases with variable speeds, the starting point is many times the prescribed motion of the rotor, *e. g.* a change of small amplitude, a harmonic angular swing [30, 4, 7, 22, 23], or a constant angular acceleration [26 etc.] is assumed and for these cases are the changes in the torque determined. Nevertheless, in the practice just the torque may be assumed to be known and the quantity searched for is the change in the angle, or in the slip describing the swing or other relative motion of the rotor.

The present paper discusses also a nonlinear problem on the whole, the asynchronous operation of turbo-generators. Though with an absolutely symmetrical rotor the slip would be — similar to that of the induction motors or generators — constant, in the turbo-generators realized up to now, even in those having a wholly cylindrical rotor, an asymmetry is caused by the field coil and this is the reason why a periodical change in the slip is arising.

The asynchronous operation is generally treated with in the books on the transient phenomena of synchronous machines, but only by assuming a *constant slip* [*e. g.* 3, 14, 23]. Even in the book discussing the asynchronous operation in a most detailed way [34] no treatment of the variable slip may be found.

So here may be also revealed the procedure outlined generally in the foregoing. Although it is evident that because of the asymmetry in the rotor the slip has to change, to avoid nonlinear differential equations, the slip is assumed to be constant and on the basis of the prescribed rotor motion taken in this way a priori, the torque is determined, and in the latter, as a consequence of the asymmetry, an average and pulsating component will arise, though in the reality the situation is the contrary: the torque is nearly constant, and the slip is changing. Here also the procedure so often applied in the engineering practice is adopted, not using a way of computation giving approximately true picture of the reality, but one supplying simple results. Author of the present paper followed also this way in his former works [6, 8, 9].

Studying the oscillograms relative to the asynchronous operation of turbo-generators, the question arose, if it is possible to elaborate a relatively simple method of engineering for determining first of all the variable slip, further the stator current and the reactive and apparent power.

The essence of the problem is, whether no simple method could be found for a simple solution of the nonlinear differential equation system describing the phenomena.

Naturally, the *step-by-step* method — similarly to the case of computing the dynamic, or transient stability — could be effective, but this is, though being an engineering method, quite a laborious and lengthy procedure for solving differential equations of second degree. Though adoption of the analog and digital computers could remove the difficulties involved in the step-by-step method — to-day this seems to be the most practicable way of solving the problems described by nonlinear differential equations — even if a great number of computers were available, the other *analytic methods* must be esteemed high as from theoretical as from practical point of view, especially if they give the solution in a closed form, as well as the simple *graphical constructions*, even if the results are less accurate.

Before giving an outline about the subjects to be dealt with, some remarks must be emphasized:

The differential equation of the problem analogous to that of the asynchronous operation of turbo-generators has been established decades ago [13] when studying the synchronization of synchronous machines. (To solve this problem, the differential analyzer method has been suggested.) Consequently, present treatise offers no new ideas when applying the above differential equation for the investigation of the variable slip.

On the other hand, no new ideas are suggested when showing the possibility to solve the above differential equation in a way differing from the step-by-step method either. In the second part of an engineering mathematical book [12] dealing with nonlinearities [17] published in 1942 the respective differential equation figures on page 210 in the following form:

$$\frac{d^2 \theta}{d \tau^2} + k(1 - b \cos 2\theta) \frac{d\theta}{d\tau} + r \sin 2\theta + \sin \theta = T$$

The left-side first term characterises the torque originating from the inertia (and the angular acceleration), the fourth term represents the synchronous torque, while the third one the reluctance torque. In the second term the slip is given by the differential quotient of the angle, its coefficient is not constant containing, however, the cosine function of the double angle too.

The book mentioned above [17] supplies after all the solution of the nonlinear differential equation by neglecting the term sin Θ (this corresponds just to the asynchronous operation of the de-excited machine). As a method of solution it adopts essentially the expansion in series and calculates the time functions of the angle, as well as the slip in form of an *infinite series*. Consequently this paper does not vindicate the claim of suggesting as first the calculation of the slip and the change in angle, respectively, in function of the time for asynchronous operation.

The solution mentioned above gives a clear picture about the advantages and disadvantages of the series expansion when solving the nonlinear differential equations. Although several methods of solution are known, the final result has generally a form of infinite series and a result in closed form (containing elementary functions of finite number), or a solution of higher order (containing at least tabulated functions) may be expected only exceptionally. Anyway, the solution obtained in form of an infinite series has undoubtedly the advantage of establishing the quantities searched for as an explicite function of the time. From an engineering viewpoint the series solution is especially advantageous if the series converges quickly, resulting a practically adequate accuracy when not considering, but some of the members. In any case, the terms of final number do not give, but an approximate value being perhaps near to the accurate one. It must be also considered, that already at the start, when having established the differential equation of the problem, some neglections were necessary. If the determination of the coefficients of the series is laborious (being the calculation of the coefficient next in turn always more troublesome, than the previous one), then engineering practice is satisfied with fewer members to the detriment of accuracy, or transforming the initial differential equation, looks for a faster converging solution. In searching for the solution methods of the nonlinear differential equations, comprehension of the physical phenomena and preliminary estimation of the presumable characteristic of the solution (periodic, aperiodic, etc.) plays an important part. Opposite to the problems described by the linear differential equations, nonlinear cases demand always a certain individual treatment.

Consequently, the method followed in the present study cannot be expected to be of *general validity* in solving the nonlinear differential equations. The method exposed in this paper wishes to give new ideas *merely at the field of the asynchronous operation of turbo-generators.*

References [34, 35], but also home experiences [6, 7, 8, 9, 10] show the medium slip being very small: a magnitude of a thousandth (a tenth per cent). This renders reasonable the simplification of assuming an approximately constant torque and turbine power, respectively (also the solution exposed above considers a constant torque), hence the change in the speed of rotation remains within the insensitivity zone of the turbine regulator. In addition, not only the medium slip is small, but also the rates of change in the periodic slip are "slow", consequently, the first differential quotients may be neglected against the slip itself (and also against the current), that is, the term containing the inertia constant, as well as the additional torques being created as a consequence of the slip *change* may be left out of consideration.

Hence after all only the second member containing the slip and the third member characteristic of the reluctance torque remains on the left side of the aforesaid nonlinear differential equation. That is, the nonlinear differential equation is reduced with respect to the angular change to a first order one. Owing to the fact, however, that no time function figures at the right side, as the torque may be taken for constant, the differential equation belongs to a well-known class, that of the so-called *separable differential equations*. Nevertheless, separable differential equations may be solved by a simple integration, and at the very most the question arises, if there exists at all a primitive function and if the solution may be expressed by well-known functions. Author followed the method outlined already in some previous works [10, 19, 20] and succeeded in giving a closed form solution of the slip and the angular change for the most simple case. It is true, that opposite to the series expansion, the solution does not contain the time in an explicite form, nevertheless, in the mathematics and in the engineering practice the parametric establishment of the functions is just as much accepted, as the explicite form. Namely the solution mentioned give the time, as well as the slip in function of the angle, consequently it expresses the relation between slip and time by aid of the angle parameter. Although the solution — taking into account the above neglections — is only a first approximation, the result expressed in a *closed form* is advantageous as from theoretical, as from practical point of view.

Present serial treatise makes an important advance on the way commenced: it develops the procedure outlined to a method, and making use of the possibilities given by the method suggested, examines in details, on the general solutions of closed form, the influence of the different machine constants (damping factor, reluctance factor) upon the time function of the slip, the angle, the reactive power, etc.

The differential equation of second order mentioned many times in the foregoing, and also the simplified differential equation of first order originates from the essential simplification of the problem, namely — as we shall see — through substituting the admittance diagrams assumed to be circular, by their initial tangents, *i. e.* by the so-called primitive lines.

The method suggested permits to examine by analytic procedure other, substantially more complicated starting conditions too. Namely, the *approximation* of the admittance diagram of any (*i. e.* not of circular) form *by a generally situated straight line*, or *rendering it linear by sections*, or its *approximation by a parabola*. Finally — considering the rotor being of solid iron — the *graphical method* based on the procedure suggested will be elaborated taking in account the admittance diagrams of arbitrary form. Thereby the time function of the slip and the relative angle, as well as that of the stator current, the reactive and the apparent power may comparatively simply be determined.

The study finally compares the theoretical results with the oscillograms plotted in course of the experimental test of the asynchronous operation to state the justifiability of the neglections made and the efficiency of the method suggested.

Asynchronous operation of the synchronous machines is not autotelic, but permits, as it is generally recognized, to substitute the electrically damaged exciter for a reserve one during service, rendering thereby practically possible to increase the safety of the co-operative power system. Present paper does not discuss the possibilities, conditions and limitations of the asynchronous operation, as this field has been handled in details by the previously published studies and articles, as from the viewpoint of the turbogenerators, as from that of the network [34, 35, 6, 8, 9, 10, 11, 19, 20, 21].

1. Assumptions, conventions, fundamental equations

Before coming to the discussion of the subject: the methods for determining the variable slip in the steady-state asynchronous operation, a short summary will be given about the assumptions, and conventions applied in the theoretical study of the synchronous machines, and the fundamental relations will be indicated. The material summarized in this chapter presents merely an introduction for the further investigations.

1.1. *Assumptions and conventions*

In the fundamental works describing the transient behaviour of the synchronous machine [29, 30, 3, 14, 2, 23] the starting assumption is an idealized two-pole synchronous machine [27, 14, 1]. Present treatise accepts the usual simplifications, respectively agreements too. When using, however, the per unit system, we prefer to introduce the synchronous angular speed ω_0 in the expression, in order the equations should represent better the physical picture. Naturally, at any step $\omega_0 = 1$ may be substituted too, then simultaneously the time must be understood instead of seconds in radians.

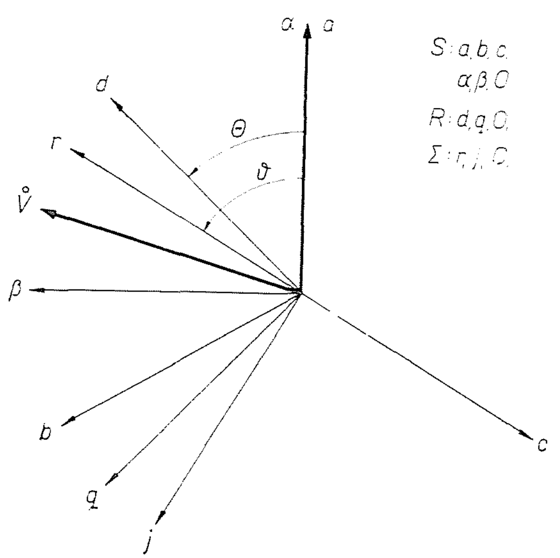

Fig. 1—1

By introducing the angular speed, respectively the angular frequency ω_0, the form of voltage equations is valid for the quantities expressed both in the per unit system and in defined (volt, amper, ohm, etc.) one.

In addition to the generally accepted assumptions and conventions, treatise makes the following agreements:

All of the coils are assumed to be of right-side turn. Therefore the *positive direction* of the flux linkage, the voltage and the current are chosen to be the same [24].

The positive power hence corresponds to the power consumed, taken from the network. Similarly, the positive torque means a driving motor torque.

On the contrary, the negative power corresponds to the power supplied into the network and the negative torque represents a braking generator torque.

The sense of rotation of the rotor is counterclockwise. The magnetic axis of the stator phase windings is reached by the rotor in a sequence, *a*, *b*, *c*, hence the phase *b* leads phase a by 120 (electric) degrees, the phase *c* lags it by 120 (electric) degrees in the space. The positive direction of the quadrature axis *q* advances by 90 electric degrees the positive direction of the direct axis *d* (See Fig. 1—1).

Instead of the Laplace transform the study applies the more simple operational calculus $\left(p \text{ is the differential operator } \frac{d}{dt}\right)$, but the unit-functions are omitted.

The symbols applied here are of general use in literature. The complex vectors constant in time are marked by an upper line, *e. g.* $\overline{v}$ (its conjugate is $\hat{v}$), while *e. g.* $\mathring{v}$ means a complex vector, variable in time (its conjugate is $\ddot{\mathring{v}}$).

1.2. *Co-ordinate transforms*

In the study three kinds of co-ordinate systems occur (Fig. 1—1): *a)* the co-ordinate system *S*, fixed to the stator, being either a three-phase system, with axes *a*, *b*, *c*, placed at 120° from each other, or a two-phase system, with axes α, β being at right angle to each other; *b)* the co-ordinate system *R*, fixed to the rotor with the two-phase axes *d*, *q* ; *c)* the synchronously revolving co-ordinate system Σ with the real *r* and the imaginary *j* axis. The arbitrary vector $\mathring{v}$ ($v = u$ or ψ or *i*) may be written in the different co-ordinate systems by aid of the instantaneous values as follows:

$$\mathring{v}_S = \frac{2}{3}(v_a + \bar{a}\, v_b + \bar{a}^2 v_c) = v_\alpha + j\, v_\beta$$

$$\mathring{v}_R = v_d + j\, v_q \tag{1—1a}$$

$$\mathring{v}_\Sigma = v_r + j\, v_j$$

where

$$\bar{a} = -\frac{1}{2} + j\frac{\sqrt{3}}{2} = e^{j2\pi/3}$$

$$\bar{a}^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2} = e^{-j2\pi/3} \tag{1—1b}$$

as in balanced operation

$$v_0 = \frac{1}{3}(v_a + v_b + v_c) \tag{1—1c}$$

Let be (Fig. 1—1):

$$(a, d) \sphericalangle = \theta = \theta_0 + \omega t \tag{1—2a}$$

and

$$(a, r) \sphericalangle = \vartheta = \vartheta_0 + \omega_0 t \tag{1—2b}$$

so

$$(d, r) \sphericalangle = \vartheta - \theta = \vartheta_0 - \theta_0 + s\,\omega_0 t \tag{1—2c}$$

then the co-ordinate transforms are according to Fig. 1—1:

$$\begin{aligned} & \qquad\qquad \dot{v}_S = e^{j\theta}\, \dot{v}_R \qquad\qquad \dot{v}_S = e^{j\vartheta}\, \dot{v}_\Sigma \\ & \dot{v}_R = e^{-j\theta}\, v_S \qquad\qquad\qquad\qquad \dot{v}_R = e^{j(\vartheta-\theta)}\, \dot{v}_\Sigma \\ & \dot{v}_\Sigma = e^{-j\vartheta}\, \dot{v}_S \quad \dot{v}_\Sigma = e^{j(\theta-\vartheta)}\, \dot{v}_R \end{aligned} \tag{1—3}$$

1.3. Co-ordinate transform of the stator voltage equations

The three (or two) stator voltage equations may be established in the co-ordinate system S (a, b, c) in form of a single complex vector equation, as follows:

$$\dot{u}_S = p\, \dot{\psi}_S + r\, \dot{i}_S \tag{1—4}$$

As the mutual inductance between the stator and the rotor windings is a periodic function of the angle formed by the magnetic axes, to avoid differential equations with periodic coefficients, it is practicable to change over from co-ordinates S (a, b, c) to co-ordinates R (d, q).

Applying the co-ordinate transform equations, as well as Heavyside's shifting theorem, the stator voltage equations are from Eq. (1—4) in the co-ordinate system R (d, q):

$$\dot{u}_R = (p + j\,\omega)\, \dot{\psi}_R + r\, \dot{i}_R \tag{1—5a}$$

Separating the real and imaginary terms [29, 3, 1, 2, 5]:

$$\begin{aligned} u_d &= p\, \psi_d - \omega\, \psi_q + r\, i_d \\ u_q &= p\, \psi_q + \omega\, \psi_d + r\, i_q \end{aligned} \tag{1—5b}$$

Besides the induced (transformatory) voltage components also generated (rotary) voltage components arise, hence equations are now somewhat more complicated, but have the great advantage, however, of being linear differential equations, with constant coefficients, if the rotor angular speed is constant [29, 3, 1, 2, 5].

Finally (similar to the foregoing) the voltage equation of the stator is in complex vector notation with respect to the synchronously revolving Σ (r, j) co-ordinate system

$$\mathring{u}_\Sigma = (p + j\,\omega_0)\,\mathring{\psi}_\Sigma + r\,\mathring{i}_\Sigma \tag{1--6}$$

14. *Operational inductances*

The relation between the flux-linkage components and the current components in the direct and quadrature axis, respectively, may be expressed as follows [3, 28, 29, 5]:

$$\begin{aligned} \omega_0\,\psi_d &= \omega_0\,l_d(p)\,i_d = x_d(p)\,i_d \\ \omega_0\,\psi_q &= \omega_0\,l_q(p)\,i_q = x_q(p)\,i_q \end{aligned} \tag{1--7}$$

where $l_d(p)$, $l_q(p)$ and $x_d(p), x_q(p)$ are the so-called operational inductances, respectively, operational impedances.

Assuming on the rotor in addition to the field only a single direct-axis damping circuit and a single quadrature-axis damping circuit, the operational impedances may be written in the following form:

$$x_d(p) = x_d\,\frac{(pT''_d + 1)(pT'_d + 1)}{(pT''_{d0} + 1)(pT'_{d0} + 1)}$$

and

$$x_q(p) = x_q\,\frac{pT''_q + 1}{pT''_{q0} + 1} \tag{1--8}$$

Though the two amortisseur circuits do not replace the solid rotor, for the sake of simplicity, this is our starting assumption. Expanding in partial fractions, the operational impedances may be expressed as follows [23, 25, 38]:

$$\begin{aligned} x_d(p) &= x_d - (x_d - x'_{d0})\,\frac{pT'_{d0}}{pT'_{d0} + 1} - (x'_{d0} - x''_d)\,\frac{pT''_{d0}}{pT''_{d0} + 1} \\ x_q(p) &= x_q - (x_q - x''_q)\,\frac{pT''_{q0}}{pT''_{q0} + 1} \end{aligned} \tag{1--9}$$

while their reciprocals have the following form:

$$\frac{1}{x_d(p)} = \frac{1}{x_d} + \left(\frac{1}{x'_d} - \frac{1}{x''_d}\right)\frac{pT'_d}{pT'_d+1} + \left(\frac{1}{x''_d} - \frac{1}{x'_d}\right)\frac{pT''_d}{pT''_d+1}$$
$$\frac{1}{x_q(p)} = \frac{1}{x_q} + \left(\frac{1}{x''_q} - \frac{1}{x_q}\right)\frac{pT''_q}{pT''_q+1} \tag{1—10}$$

1.5. *Voltage equations of the stator*

With regard to Eq. (1—7), the voltage equations (1—5*b*) may be written as follows:

$$\begin{aligned} u_d &= [p\, l_d(p) + r]\, i_d - \omega\, l_q(p)\, i_q \\ u_q &= \omega\, l_d(p)\, i_d + [p\, l_q(p) + r]\, i_q \end{aligned} \tag{1—11a}$$

respectively

$$\begin{aligned} u_d &= \left[\frac{p}{\omega_0} x_d(p) + r\right] i_d - \frac{\omega}{\omega_0} x_q(p)\, i_q \\ u_q &= \frac{\omega}{\omega_0} x_d(p)\, i_d + \left[\frac{p}{\omega_0} x_q(p) + r\right] i_q \end{aligned} \tag{1—11b}$$

finally, taking into account the relation $\omega = (1—s)\omega_0$:

$$\begin{aligned} u_d &= \left[\frac{p}{\omega_0} x_d(p) + r\right] i_d - (1-s)\, x_q(p)\, i_q \\ u_q &= (1-s)\, x_d(p)\, i_d + \left[\frac{p}{\omega_0} x_q(p) + r\right] i_q \end{aligned} \tag{1—11c}$$

In possession of the voltages, the currents may be determined:

$$\begin{aligned} i_d &= \frac{[p\, l_q(p) + r]\, u_d + \omega\, l_q(p)\, u_q}{[p\, l_d(p) + r]\,[p\, l_q(p) + r] + \omega^2 l_d(p)\, l_q(p)} \\ i_q &= \frac{-\omega\, l_d(p)\, u_d + [p\, l_d(p) + r]\, u_q}{[p\, l_d(p) + r]\,[p\, l_q(p) + r] + \omega^2 l_d(p)\, l_q(p)} \end{aligned} \tag{1—12}$$

Taking into consideration the expressions (1—8) of the operational inductances, the relations (1—12) get a quite complicated form. Neglecting, however, the stator resistance, considerable simplifications may be achieved:

$$\begin{aligned} i_d &= \frac{p\, u_d + \omega\, u_q}{(p^2 + \omega^2)\, l_d(p)} \\ i_q &= \frac{-\omega\, u_d + p\, u_q}{(p^2 + \omega^2)\, l_q(p)} \end{aligned} \tag{1—13}$$

1.6. *Torque equations*

The shaft of the synchronous generator is influenced by three torques: the mechanical driving torque of the turbine m_m, the electromagnetic (generally braking) torque m_e and the torque arising at angular acceleration or deceleration (resulting from the inertia) m_i. The resultant of the three torques is zero:

$$m_m + m_e + m_i = 0 \tag{1—14}$$

The electromagnetic torque may be written consecutively in the stator S, rotor R and synchronous Σ co-ordinate system — as it is known [*e. g.* 3] — in the following form:

$$\begin{aligned} m_e &= \psi_\alpha i_\beta - \psi_\beta i_\alpha \\ m_e &= \psi_d i_q - \psi_q i_d \\ m_e &= \psi_r i_j - \psi_j i_r \end{aligned} \tag{1—15}$$

The torque originating from the change in angular speed because of the inertia may be written as follows [*e. g.* 25, 23]:

$$m_i = -p T_m \frac{\omega}{\omega_0} = p T_m s \tag{1—16}$$

where T_m is the so-called mechanical time constant: it is twice the value of the kinetic energy stored in the revolving masses at synchronous speed related to the rated apparent power of the generator. If a constant torque, corresponding to the rated apparent power, affects the generator shaft, within this time T_m would the rotor accelerate from the rest up to the synchronous speed. It must be noted, that in the technical literature instead of T_m sometimes the symbol $2H$ may be found [23, 2, etc.], where H is the so-called inertia constant.

2. Fundamental equations of the asynchronous operation

In the following the fundamental relations of the asynchronous operation will be discussed. Though theoretically the consideration of the stator resistance would not involve difficulties, formula would get quite complicated and lengthy, rendering thereby heavier the perspicuity. As it was underlined in *clause 1.5*, neglection of the stator resistance results in considerable simplifications. As when discussing the steady-state conditions, the stator resistance of small value does not influence, but slightly the results, the present study supplies the formula only for this more simple case.

2.1. *Fundamental equations of the synchronous operation by assuming a constant slip*

If the rotor runs with a constant slip s, the angle to be measured between the rotor and the revolving field changes linearly. Let us denote by δ the angle of the rotor direct axis as compared with the vector $\mathring{\psi}$ of the synchronously revolving flux linkage at an arbitrary instant (Fig. 2—1); then evidently

$$\delta = \delta_0 - s\,\omega_0 t \tag{2—1}$$

as $-s\,\omega_0$ is the relative angular speed of the rotor with respect to the revolving field. It must be noted, that in an asynchronous motor operation the slip s

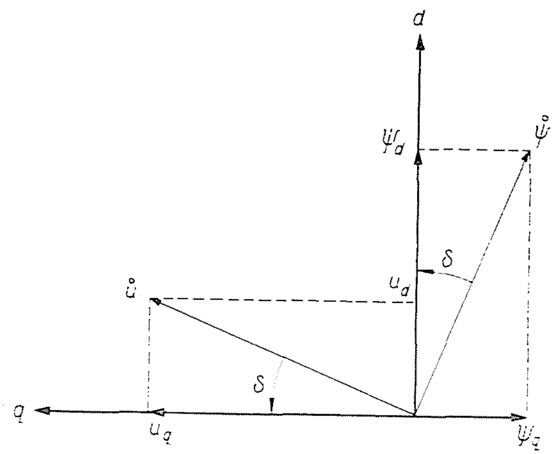

Fig. 2—1

is positive, consequently the angle δ is decreasing; on the contrary, in asynchronous generator operation, the slip s is negative, hence the angle δ is increasing.

In the co-ordinate system R (d, q) of the rotor the vector of the flux linkage is

$$\mathring{\psi}_R = \psi_d + j\,\psi_q = \psi e^{-j\delta} \tag{2—2}$$

consequently the co-ordinates of the flux linkage are

$$\begin{aligned} \psi_d &= \psi \cos\delta \\ \psi_q &= -\,\psi \sin\delta \end{aligned} \tag{2—3}$$

The voltage equation in system R, according to (1—5a), by substituting $\omega = (1 - s)\,\omega_0$

$$\mathring{u}_R = [p + j\,(1 - s)\,\omega_0]\,\mathring{\psi}_R + r\,\mathring{i}_R$$

which may be expressed on the basis of $r = 0$ and Eqs. (2—1), (2—2) for the steady state (by substituting the relation $p = -j\,s\,\omega_0$) :

$$\mathring{u}_R = [j\,s\,\omega_0 + j\,(1 - s)\,\omega_0]\,\mathring{\psi}_R$$

Taking also the relation (2—2) into account

$$\mathring{u}_R = u_d + j\,u_q = j\,\omega_0\,\mathring{\psi}_R = j\,\omega_0\,\psi\,e^{-j\delta} = j\,u\,e^{-j\delta} \tag{2—4}$$

where

$$u = \omega_0\,\psi \qquad \text{respectively} \qquad \psi = \frac{u}{\omega_0} \tag{2—5}$$

Accordingly, the vector of the terminal voltage leads by 90° the vector of the flux linkage and so the angle δ is at the same time the relative angle of the quadrature axis q with respect to the voltage vector $\mathring{u}$ (Fig. 2—1).

On the basis of Eq. (2—4), the co-ordinates of the voltage vector are in system R:

$$\begin{aligned} u_d &= \omega_0\,\psi \sin\delta = u \sin\delta \\ u_q &= \omega_0\,\psi \cos\delta = u \cos\delta \end{aligned} \tag{2—6}$$

As demonstrated in formula (2—1), the direct-axis and quadrature-axis components of the voltage and the flux linkage are harmonic functions of the slip frequency. Therefore the symbolic method, well-known from the alternating currents theory, may be applied for the arbitrary

$$v = u, \psi, i$$

variables

$$\begin{aligned} v_d &= \operatorname{Re} \mathring{V}_d \\ v_q &= \operatorname{Re} \mathring{V}_q \end{aligned} \tag{2—7}$$

So according to (2—6) e. g.:

$$\begin{aligned} \mathring{U}_d &= j\,u\,e^{-j\delta} \\ \mathring{U}_q &= u\,e^{-j\delta} \end{aligned} \tag{2—8}$$

and considering both Eq. (2—3) and Eq. (2—6)

$$\mathring{\Psi}_d = \psi\, e^{-j\delta} = \frac{j\,u}{j\,\omega_0}\, e^{-j\delta} = \frac{1}{j\,\omega_0}\, \hat{U}_d$$
$$\mathring{\Psi}_q = -\,j\,\psi\, e^{-j\delta} = \frac{u}{j\,\omega_0}\, e^{-j\delta} = \frac{1}{j\,\omega_0}\, \hat{U}_q \tag{2—9}$$

On the basis of relations (1—7)

$$i_d = \frac{\psi_d}{l_d(p)} = \frac{\omega_0\,\psi_d}{x_d(p)}$$
$$i_q = \frac{\psi_q}{l_q(p)} = \frac{\omega_0\,\psi_q}{x_q(p)} \tag{2—10}$$

hence taking into account (2—9), and substituting besides $p = j\,s\,\omega_0$ for the steady state:

$$\mathring{I}_d = \frac{\mathring{\Psi}_d}{\bar{l}_d(j\,s\,\omega_0)} = \frac{\omega_0\,\mathring{\Psi}_d}{\bar{x}_d(j\,s\,\omega_0)} = \frac{\hat{U}_d}{j\,\bar{x}_d(j\,s\,\omega_0)}$$
$$\mathring{I}_q = \frac{\mathring{\Psi}_q}{\bar{l}_q(j\,s\,\omega_0)} = \frac{\omega_0\,\mathring{\Psi}_q}{\bar{x}_q(j\,s\,\omega_0)} = \frac{\hat{U}_q}{j\,\bar{x}_q(j\,s\,\omega_0)} \tag{2—11}$$

Introducing the symbols

$$\bar{y}_d = \bar{y}_d(j\,s\,\omega_0) = \frac{1}{j\,\bar{x}_d(j\,s\,\omega_0)} = \frac{1}{j\,\omega_0\,\bar{l}_d(j\,s\,\omega_0)}$$
$$\bar{y}_q = \bar{y}_q(j\,s\,\omega_0) = \frac{1}{j\,\bar{x}_q(j\,s\,\omega_0)} = \frac{1}{j\,\omega_0\,\bar{l}_q(j\,s\,\omega_0)} \tag{2—12}$$

(2—11) may be written after all in the following form:

$$\mathring{I}_d = \bar{y}_d\,\hat{U}_d = j\,u\,e^{-j\delta}\,\bar{y}_d$$
$$\mathring{I}_q = \bar{y}_q\,\hat{U}_q = u\,e^{-j\delta}\,\bar{y}_q \tag{2—13}$$

With due regard to the current vector (2—7), expressed in the rotor coordinate system R

$$\mathring{i}_R = i_d + j\,i_q = \mathrm{Re}\,\mathring{I}_d + j\,\mathrm{Re}\,\mathring{I}_q = \frac{1}{2}\,(\mathring{I}_d + \overset{*}{I}_d) + j\,\frac{1}{2}\,(\mathring{I}_q + \overset{*}{I}_q) \tag{2—14}$$

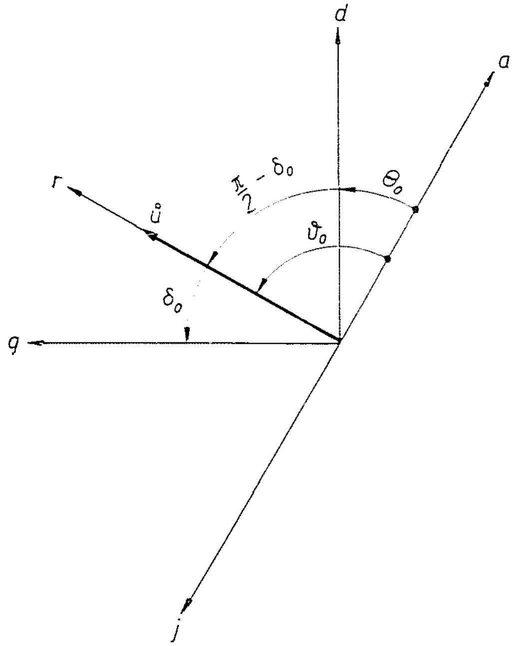
d
a
r
q
j
$\mathring{u}$
$\frac{\pi}{2}-\delta_0$
Θ_0
ϑ_0
δ_0

Fig. 2—2

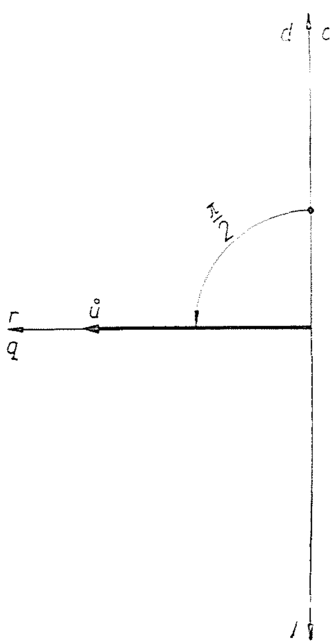
d
c
r
q
$\mathring{u}$
$\frac{\pi}{2}$

Fig. 2—3

which becomes, after substituting Eq. (2—13)

$$\hat{i}_R = u\left[j\,\frac{\bar{y}_d+\bar{y}_q}{2}\,e^{-j\delta} - j\,\frac{\hat{y}_d-\hat{y}_q}{2}\,e^{j\delta}\right]$$

that is

$$\hat{i}_R = j\,u\,e^{-j\delta}\left[\frac{\bar{y}_d+\bar{y}_q}{2} - \frac{\hat{y}_d-\hat{y}_q}{2}\,e^{j2\delta}\right] \tag{2—15}$$

and considering Eq. (2—4)

$$\hat{i}_R = \hat{u}_R\left[\frac{\bar{y}_d+\bar{y}_q}{2} - \frac{\hat{y}_d-\hat{y}_q}{2}\,e^{j2\delta}\right] \tag{2—16}$$

Transforming into the synchronously revolving co-ordinate system Σ by aid of Eq. (1—3) using the relation

$$\mathring{v}_\Sigma = e^{j(\theta-\vartheta)}\,\mathring{v}_R$$

with the initial conditions according to

$$\vartheta_0 = \theta_0 + \frac{\pi}{2} - \delta_0$$

(Fig. 2—2), which may be realized most naturally by choosing

$$\delta_0 = 0\,,\quad \theta_0 = 0 \quad \text{and} \quad \vartheta_0 = \frac{\pi}{2}$$

(Fig. 2—3), that is, the initial situation of the rotor corresponds to the synchronous no-load position, we obtain after all for the voltage and the current vectors the following relations:

$$\hat{u}_\Sigma = u \tag{2—17}$$

$$\hat{i}_\Sigma = u\left[\frac{1}{2}(\bar{y}_q+\bar{y}_d) + \frac{1}{2}(\hat{y}_q-\hat{y}_d)\,e^{j2\delta}\right] \tag{2—18}$$

In addition the complex vector of apparent power is:

$$\mathring{S} = \overset{*}{\hat{u}}\,\hat{i} = u^2\left[\frac{1}{2}(\bar{y}_q+\bar{y}_d) + \frac{1}{2}(\hat{y}_q-\hat{y}_d)\,e^{j2\delta}\right] \tag{2—19}$$

and finally the torque:

$$m = \frac{u^2}{\omega_0} Re\left[\frac{1}{2}(\bar{y}_q + \bar{y}_d) + \frac{1}{2}(\hat{y}_q - \hat{y}_d)\, e^{j2\delta}\right] \tag{2—20}$$

The above four relations express the variation in the four main quantities, characterizing the turbo-generator working with constant slip in asynchronous operation. According to the first relation the terminal voltage is constant, consequently the voltage vector is in the synchronous co-ordinate system of constant magnitude and position. As per the second relation, the stator current fluctuates between a certain maximum and minimum value with double slip-frequency. (It must be noted that relation (2—18) gives of course only the envelope curve of the current alternating with the frequency of the mains.) The current vector describes in the synchronously revolving co-ordinate system a circle of excentric position. The apparent power changes similarly to the current. The torque fluctuates also between a certain minimum and maximum value.

It is worth to mention, that the time course of both the current and the apparent power, as well as that of the torque is characterized by the resultant admittance vector

$$\dot{y} = \frac{1}{2}(\bar{y}_q + \bar{y}_d) + \frac{1}{2}(\hat{y}_q - \hat{y}_d)\, e^{j2\delta}\,. \tag{2—21}$$

Let us examine in some more details the geometrical locus of the resultant admittance given by relation (2—21).

2.2. The resultant admittance diagram

The course of the current, power and torque mentioned in the previous *clause 2.1* may be perspicuously represented by the admittance diagram (Fig. 2—4). The voltage vector u is directed towards the real axis of vertical position. For the negative slips characteristic of the asynchronous generator operation, both the admittance diagram $\bar{y}_d = \bar{y}_d(j\,s\,\omega_0)$ referring to the direct axis, and that $\bar{y}_q = \bar{y}_q(j\,s\,\omega_0)$ referring to the quadrature axis are situated below the negative imaginary axis. Choosing for an arbitrary s slip, but of the same magnitude the terminal points of the vectors $\bar{y}_d$ respectively $\bar{y}_q$ on the corresponding admittance diagrams, the arithmetical mean vector

$$\bar{y}_S = \frac{1}{2}(\bar{y}_q + \bar{y}_d) \tag{2—22}$$

as well as the half difference vector

$$\bar{y}_D = \frac{1}{2}(\bar{y}_q - \bar{y}_d) \tag{2—23}$$

may easily be determined. Constructing a circle with the absolute value of the latter as a radius around the terminal point of the former as a centre, we obtain the geometrical locus of the resultant admittance diagram expressed in formula (2—21). The resultant admittance diagram $\mathring{y}$ is a circle diagram, the centre of the circle being determined by the terminal point of vector $\bar{y}_S$.

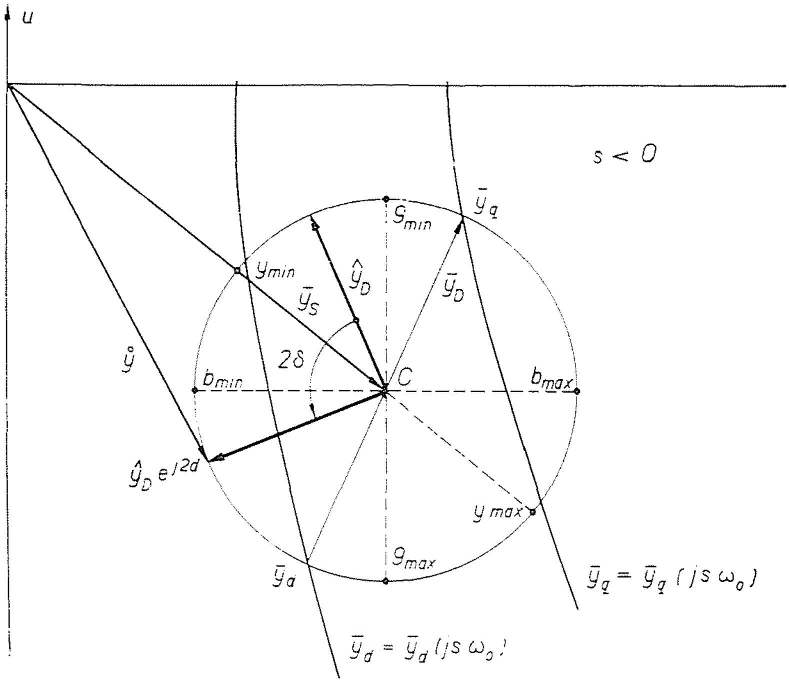

Fig. 2—4

The initial position of the radius vector (at $t = 0$) is indicated by vector

$$\hat{y}_D\, e^{j2\delta_0}$$

If the initial time is chosen so as $\delta_0 = 0$, *i.e.* at the starting instant the rotor is in a situation just corresponding to that of the synchronous no-load operation, the initial position of the radius vector can be expressed by

$$\hat{y}_D = \frac{1}{2}(\hat{y}_q - \hat{y}_d)$$

i.e. it is the conjugate (that is the mirage) vector of $\bar{y}_D$ referred to the vertical real axis (Fig. 2—4). When the rotor leads by an angle δ with respect to the initial (no-load) position mentioned above, then the radius vector displaces by an angle 2δ with respect to its initial position (consequently, while the rotor covers a full turn, the radius vector revolves two times in the counter-clockwise, positive sense of rotation). Hence at an arbitrary instant the resultant admittance vector is (Fig. 2—4):

$$\mathring{y} = \bar{y}_S + \hat{y}_D\, e^{j2\delta} \tag{2—24}$$

Of course, in case of a constant slip, the instantaneous value of the current, the apparent power and also that of the torque may easily be read from the admittance vector diagram $\mathring{y}$ (Fig. 2—4) for any given time.

The maximum and minimum current (or apparent power) are characterized by points $g_{\max}$ and $g_{\min}$ respectively, where

$$y_{\max} = y_S + y_D\,, \qquad y_{\min} = y_S - y_D\,.$$

On the other hand, the maximum and minimum torque are characterized by points $g_{\max}$ and $g_{\min}$ respectively, and the maximum and minimum active currents are given by the same two points.

Similarly, the maximum and minimum reactive currents are characterized by $b_{\max}$ and $b_{\min}$ respectively.

It may be noted here, that the admittance $\mathring{y}$ is expressed by the conductance g and susceptance b in the following form:

$$\mathring{y} = g - jb \tag{2—25}$$

Similarly

$$\begin{aligned} \bar{y}_S &= g_S - jb_S \\ \bar{y}_D &= g_D - jb_D \end{aligned} \tag{2—26}$$

and

$$\begin{aligned} \bar{y}_d &= g_d - jb_d \\ \bar{y}_q &= g_q - jb_q \end{aligned} \tag{2—27}$$

consequently the relations (2—22) and (2—23) are not valid only for the admittances, but also for the conductances and susceptances separately too, hence

$$g_S = \frac{1}{2}(g_q + g_d) \qquad b_S = \frac{1}{2}(b_q + b_d) \tag{2—22'}$$

$$g_D = \frac{1}{2}(g_q - g_d) \qquad b_D = \frac{1}{2}(b_q - b_d) \tag{2—23'}$$

It must be mentioned that with the chosen symbols $b > 0$ in all cases, while in asynchronous motor operation $g > 0$ and in asynchronous generator operation $g < 0$ (similarly to the slip s).

By reason of notations (2—25)... (2—27), Eq.(2—24) gets the following form:

$$\mathring{y} = g - jb = (g_S - jb_S) + (g_D + jb_D)\, e^{j2\delta} \tag{2—28}$$

that is

$$g = g_S + g_D \cos 2\delta - b_D \sin 2\delta \tag{2—29}$$

$$b = b_S - b_D \cos 2\delta - g_D \sin 2\delta \tag{2—30}$$

Summarizing, it may be stated that assuming a constant slip, by formula (2—28) the course of the total current and of the apparent power, by formula (2—29), however, the course of the active power and of the torque, while by formula (2—30) the course of the reactive current and of the reactive power may be obtained. Naturally, to get a current, the admittance or its components must be multiplied by voltage u, to get a power they have to be multiplied by the voltage square u^2, and to get a torque, they must be multiplied by the expression u^2/ω_0 too. An especially simple relation may be obtained in the per unit system ($\omega_0 = 1$), if the voltage is just the rated one, *i.e.* it is of unit value: $u = 1$. In this case, namely, the numerical value of the admittance y indicates simultaneously the current and the apparent power too, the conductance g gives also the active current, the active power and the torque, while the susceptance b supplies also the reactive current and the reactive power.

2.3. *The average and pulsating torque*

The torque expressed in relation (2—20) may be resolved into two components: the first is the *average* torque

$$m_m = \frac{u^2}{\omega_0} \operatorname{Re}\, (\bar{y}_q + \bar{y}_d) \tag{2—31}$$

while the second the *pulsating* torque

$$m_p = \frac{u^2}{\omega_0} \operatorname{Re} \frac{1}{2} (\hat{y}_q - \hat{y}_d)\, e^{j2\delta} \tag{2—32}$$

As on the basis of Eqs. (2—12) and (1—10), after substituting $p = j s \omega_0$:

$$\bar{y}_d = \bar{y}_d(j s \omega_0) = \frac{1}{jx_d} + \left(\frac{1}{jx'_d} - \frac{1}{jx_d}\right) \frac{j s \omega_0 T'_d}{j s \omega_0 T'_d + 1} + \left(\frac{1}{jx''_d} - \frac{1}{jx'_d}\right) \frac{j s \omega_0 T''_d}{j s \omega_0 T''_d + 1}$$

and

$$\bar{y}_q = \bar{y}_q(j s \omega_0) = \frac{1}{jx_q} + \left(\frac{1}{jx''_q} - \frac{1}{jx_q}\right) \frac{j s \omega_0 T''_q}{j s \omega_0 T''_q + 1} \tag{2—33}$$

therefore the expression of the average torque is

$$m_m = \frac{u^2}{2\omega_0}\left[\left(\frac{1}{x_d''}-\frac{1}{x_d'}\right)\frac{s\,\omega_0 T_d''}{1+(s\,\omega_0 T_d'')^2}+\left(\frac{1}{x_d'}-\frac{1}{x_d}\right)\frac{s\,\omega_0 T_d'}{1+(s\,\omega_0 T_d')^2}+\right.$$

$$\left.+\left(\frac{1}{x_q''}-\frac{1}{x_q}\right)\frac{s\,\omega_0 T_q''}{1+(s\,\omega_0 T_q'')^2}\right] \qquad (2\text{—}34)$$

while that of the pulsating torque

$$m_p = \frac{u^2}{2\omega_0}\left\{\left[\left(\frac{1}{x_q''}-\frac{1}{x_q}\right)\frac{s\,\omega_0 T_q''}{1+(s\,\omega_0 T_q'')^2}-\right.\right.$$

$$\left.-\left(\frac{1}{x_d''}-\frac{1}{x_d'}\right)\frac{s\,\omega_0 T_d''}{1+(s\,\omega_0 T_d'')^2}-\left(\frac{1}{x_d'}-\frac{1}{x_d}\right)\frac{s\,\omega_0 T_d'}{1+(s\,\omega_0 T_d')^2}\right]\cos 2\delta-$$

$$-\left[\left(\frac{1}{x_q''}-\frac{1}{x_q}\right)\frac{(s\,\omega_0 T_q'')^2}{1+(s\,\omega_0 T_q'')^2}+\frac{1}{x_q}- \qquad (2\text{—}35)\right.$$

$$\left.\left.-\left(\frac{1}{x_d''}-\frac{1}{x_d'}\right)\frac{(s\,\omega_0 T_d'')^2}{1+(s\,\omega_0 T_d'')^2}-\left(\frac{1}{x_d'}-\frac{1}{x_d}\right)\frac{s\,\omega_0 T_d'}{1+(s\,\omega_0 T_d')^2}-\frac{1}{x_d}\right]\sin 2\delta\right\}$$

where according to Eq. (2—1) $\delta = \delta_0 - s\,\omega_0 t$.

In case of a constant slip, the time integral of the pulsating torque taken for a half slip period is of course zero.

Should the slip equal zero: $s = 0$, then the average torque component disappears, while the pulsating torque supplies the so-called reluctance torque:

$$[m_p]_{s=0} = m_r = \frac{u^2}{2\omega_0}\left(\frac{1}{x_q}-\frac{1}{x_d}\right)\sin 2\delta_0 \qquad (2\text{—}36)$$

Should the slip but slightly differ from zero, so that the terms containing the square of the slip may be neglected, the whole torque may be expressed as follows:

$$m = \frac{u^2}{2\omega_0}\left\{\left[\left(\frac{1}{x_d''}-\frac{1}{x_d'}\right)T_d''+\left(\frac{1}{x_d'}-\frac{1}{x_d}\right)T_d'+\left(\frac{1}{x_q''}-\frac{1}{x_q}\right)T_q''\right]-\right.$$

$$\left.-\left[\left(\frac{1}{x_d''}-\frac{1}{x_d'}\right)T_d''+\left(\frac{1}{x_d'}-\frac{1}{x_d}\right)T_d'-\left(\frac{1}{x_q''}-\frac{1}{x_q}\right)T_q''\right]\cos 2\delta\right\}s\,\omega_0+$$

$$+\frac{u^2}{2\omega_0}\left(\frac{1}{x_d}-\frac{1}{x_q}\right)\sin 2\delta \qquad (2\text{—}37)$$

It will be observed, that subsituting instead of δ the value of θ and instead of $s\,\omega_0$ the differential quotient of the angle $d\,\theta/d\tau$, we get the second and third term of the differential equation figuring in the introduction. This means, that in the above differential equation instead of the whole expression of the asynchronous torque, *its approximate expression valid only for small slips* is figuring.

The expression of the whole torque supplied by Eqs. (2—34) and (2—35) is quite complicated in spite of the many simplifications involved. The least admissible expedient seems to be the replacement of the solid iron by one direct-axis and one quadrature-axis damping coil, respectively, when according to Eq. (2—33) the quadrature-axis admittance diagram $\bar{y}_q$ describes a circle, while the direct-axis admittance diagram $\bar{y}_d$ is of double squirrel-cage character, which may be approximated by two circles [18]. The admittance diagrams $\bar{y}_q$ and $\bar{y}_d$ taking in consideration to a certain extent the effect of the solid iron mass, would have a much more complicated form [34]. This problem is not dealt with in the study, our task being the approximation of the real admittance diagrams by simple curves (straight lines, parabolic curves) in order to determine the variable slip. In the following the subject matter will be just the problem, what kind of approximations are sufficient and necessary to obtain the expression of the variable slip in a closed form. Consequently, formulae (2—33) may be regarded as a first approximation.

Though assumption of a constant slip leads to incorrect results — *e. g.* the torque varies between wide limits, or the angular displacement of the rotor is uniform — it means an important step in passing over to the case of the variable slip.

3. The method suggested for determining the variable slip

Present chapter expounds the method for determining the variable slip: after the starting assumptions the essence of the method suggested is discussed, and the further tasks are indicated.

3.1. Simplifying assumptions

Foreign [34, 35] and home measurements [6, 8, 10, 11] have shown alike, that during asynchronous operation the slip of turbo-generators is small, being of a thousandth (*i.e.* a tenth per cent) order of magnitude. Consequently, the turbine regulator having a zone of insensitivity of approximately $\pm$ 0,1 per cent does not perceive at all, or only slightly the change in the speed.

The above circumstance inspires the first supposition of the method suggested: during asynchronous operation the turbine power remains approximately constant. As the speed differs but slightly from the synchronous one, and variates insignificantly, also the torque may be regarded as constant.

Starting from the measurement experiences — *i.e.* from the small and relatively slow change in the speed — also the second assumption seems to be right: no considerable error arises, when neglecting the effect of the inertia torques. *E.g.* with respect to the medium slip $s_m = 0.001$, assuming, exaggerating, a maximum rate of change in slip variation $(ds/dt)_{\max} = 0.001/\text{sec}$, the maximum value of the arising torque according to formula (1—16) with a too high mechanical time constant $T_m = 20$ sec, is as follows

$$(m_i)_{\max} = 20 \ \text{sec} \cdot 0.001/\text{sec} = 0.02$$

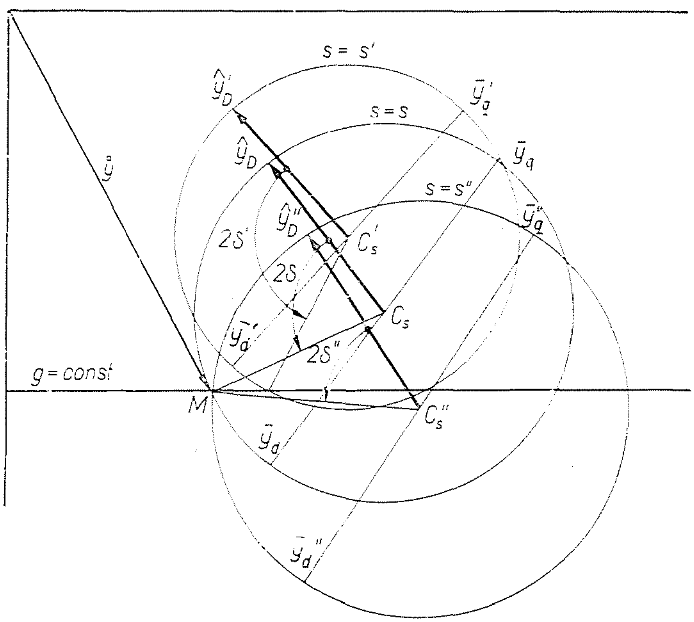

Fig. 3—1

As at the same time the driving torque of the turbine is at least

$$m_m = 0.4$$

neglection of the torque m_i introduces an error of at most 5 per cent.

Finally, the low value of the slip itself and of its variation in time renders the third assumption adopted plausible: in the calculations the static asynchronous torque-slip characteristic may be applied, neglecting therefore the effect of the change in the slip upon the torque curve.

These three assumptions are supported also by the periodic course of the slip, namely the error made at the angular acceleration is compensated by that arising at the angular deceleration.

Adoption of the three above simplifying assumptions permits us to apply further the resultant admittance diagram derived in *chapter 2*. Nevertheless, in the following, as the incorrect supposition of the slip being constant falls out, the starting assumption being the invariability of the active power (torque) and the active current, instead of a single admittance circle diagram C plotted for a certain constant slip (Fig. 2—4), now, on the one hand, a set of circle diagrams plotted for different slips must be traced, and, on the other hand, the terminal point of the resultant admittance vector has to move along the straight line $g =$ const., being parallel with the imaginary axis (Fig. 3—1).

3.2. *The suggested method*

In case of a variable slip and a constant torque the terminal points of the resultant admittance vectors are determined by the intersection of the straight line $g =$ const. and of the circles C_s plotted for different slips s (Fig. 3—1). On the plane of complex quantities, to each intersection belong at least three values: the conductance g taken as constant, the slip s belonging to the respective circle C_s, and the angle 2δ respectively δ to be determined from the initial $\hat{y}_D$ and momentary $\hat{y}_D e^{j2\delta}$ position of the radius vector of circle C_s in question (Fig. 3—1). (Moreover the admittance $\dot{y}$ and the susceptance b may be read.)

Owing to the fact that the equation of circles is given by formula (2—28), the relation between the values s and δ determined by the intersection of one of the circles and the straight line $g =$ const. is indicated by Eq. (2—29), having now the following form

$$g = g_S(s) + g_D(s) \cos 2\delta - b_D(s) \sin 2\delta \tag{3—1}$$

as, on the one hand, each circle C_s belonging to slip s has a different centre vector and radius vector, consequently $g_S(s), g_D(s), b_D(s)$ etc. are now functions of the slip s and, on the other hand, the resultant conductance g being constant, is independent of the slip.

As the relative angular speed is the differential quotient of the angle δ with respect to the time:

$$\frac{d\delta}{dt} = - s\,\omega_0 \tag{3—2}$$

the relation (3—1) is the differential equation of motion of the rotor. The dependent variable is the angle δ, the independent variable the time t. As it may be seen, the relation (3—1) means a nonlinear differential equation given in implicite form.

The essence of the suggested method of solution is as follows:

Assume the slip s may be expressed from Eq. (3—1) as a function of the angle δ:

$$s = s(\delta) \tag{3—3}$$

Then Eq. (3—2) may be written as follows:

$$\frac{d\delta}{dt} = -s(\delta)\,\omega_0 \tag{3—4}$$

Should the angular value be $\delta = \delta_0$ at the initial time $t = 0$, then on the basis of Eq. (3—4):

$$-\int\limits_{\delta_0}^{\delta} \frac{d\delta}{s(\delta)} = \int\limits_0^t \omega_0\,dt = \omega_0\,t \tag{3—5}$$

Consequently, if the left-side integral exists, the time t is obtained as a function of angle δ:

$$t = t(\delta) \tag{3—6}$$

In conclusion, Eqs. (3—3) and (3—6) give indirectly the relation between the required dependent variable s and the independent variable t as functions of parameter δ. Accordingly, though in a parametric form, we succeeded in determining by $s(\delta)$ and $t(\delta)$ the desired relation

$$s = s(t) \tag{3—7}$$

Finally, with the aid of the inverse function of (3—6), the relation

$$\delta = \delta(t) \tag{3—8}$$

searched for is obtained too.

Substituting the relations (3—7) and (3—8) into expressions (2—28) and (2—29) respectively, which may be written similarly to (3—1) as follows

$$\mathring{y} = \big(g_S(s) - j\,b_S(s)\big) + \big(g_D(s) + j\,b_D(s)\big)\,e^{j2\delta} \tag{3—9}$$

$$b = b_S(s) - b_D(s)\cos 2\delta - g_D(s)\sin 2\delta \tag{3—10}$$

the admittance vector

$$\mathring{y} = \mathring{y}(t) \tag{3—11}$$

as well as its absolute value

$$y = y(t) \qquad (3\text{—}11')$$

and the susceptance

$$b = b(t) \qquad (3\text{—}12)$$

are produced after all as functions of time. Consequently, the course of the apparent power and the stator current, as well as of the reactive power and reactive current may be obtained in function of the time.

3.3. Versions of the method suggested

Usefulness of the method exposed in *clause 3.2* depends substantially upon two circumstances: whether we succeed in expressing the relation (3—2) in an explicite form from Eq. (3—1) and whether the integral figuring at the left-side of Eq. (3—5) can be calculated, namely in a possibly closed form.

Consequently, study regards as its task in the following just to look for simple approximations assuring success in both steps and at the same time not leading to unacceptable results.

Application of the form of the admittances expressed in Eq. (2—33) must evidently be avoided, as it would create an equation of too high degree in the relation (3—1). An even smaller hope might be for adopting the more complicated admittance expressions considering more exactly the effect of the solid iron. (Naturally by the use of digital computers, the two possibilities now refused may be of question too.)

On the contrary, approximating the direct-axis and quadrature-axis admittance diagrams by straight lines, or by quadratic parabolas, the success of the first step is assured, as for the slip s only an equation of first — or second degree must be solved. The integrability remains, however, an undecided problem.

On the other hand, if the direct-axis and quadrature-axis admittance diagrams were as a result of other measurements available — plotting of the admittance diagram on the basis of measurements is, but lately dealt with [*e.g.* 31] — then the intersection assembly of the set of circles C_s with the straight line $g =$ const, *i.e.* the values s and δ belonging to each other may be established, and then the integral figuring on the left-side of equation (3—5) may be determined by planimetering, by graphic, or numeric integration. The other steps included in *clause 3.2* can be realized without difficulty.

In this way the field of the further examinations may be outlined as follows:

a) Approximation by straight lines of the direct-axis and quadrature-axis admittance diagrams (linear approximation).

b) Approximation of the above admittance diagrams by parabolas (quadratic approximation).

c) Graphical construction.

Summary

Present paper — being the first part of a series consisting of several articles — makes a review of the methods applied till now for the theoreticale xamination of the turbo-generators in asynchronous operation and suggests a relatively simple method for the solution of the essentially nonlinear problem. The exposition and application of the method suggested will be discussed in the articles to be published later. For the preparation of this task, the fundamental relations serving as the starting point may be found here.

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