LINEAR APPROXIMATION OF ADMITTANCE DIAGRAMS FOR THE THEORETICAL EXAMINATION OF TURBO-GENERATORS IN ASYNCHRONOUS OPERATION

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In the first part [1] of this study a general theoretical method was suggested for the examination of the turbogenerators in asynchronous operation. The present second part has the aim to show, how the general method can be applied in the most simple cases: when the real admittance diagrams are approximated with straight lines. (To emphasize the close connection with the first part of the study and for the sake of simplicity, the numbers of formulas and chapters are continued).

4. Primitive linear approximation

If each of the admittance diagrams $\bar{y}_d$ and $\bar{y}_q$ are approximated by a straight line parallel with the (negative) real axis and starting from the (negative) imaginary axis, (*i. e.* the point belonging to $s = 0$ coincides with the imaginary axis), introduction of the following notations seems to be practicable (Fig. 4—1):

$$\begin{aligned} \bar{y}_q &= g_q(s) - jb_q(s) = s\,k_q - jb_q\,, \\ \bar{y}_d &= g_d(s) - jb_d(s) = s\,k_d - jb_d\,. \end{aligned} \tag{4—1}$$

where each value k_q, k_d, b_q, b_d figuring at the right side now means a constant, independent of the slip.

The real and imaginary component of the resultant admittance, on the basis of Eqs. (3—1)* and (3—10), considering Eq. (4—1) is:

$$g = s\,k_S + s\,k_D \cos 2\delta - b_D \sin 2\delta\,, \tag{4—2}$$

$$b = b_S - b_D \cos 2\delta - s\,k_D \sin 2\delta\,, \tag{4—3}$$

where

$$\begin{aligned} k_S &= \frac{1}{2}(k_q + k_d)\,, \qquad & b_S &= \frac{1}{2}(b_q + b_d)\,, \\ k_D &= \frac{1}{2}(k_q - k_d)\,, \qquad & b_D &= \frac{1}{2}(b_q - b_d)\,. \end{aligned} \tag{4—4}$$

* The formulas (and clauses) whose number begins with 1-, 2-, or 3- may be found in the first part [1].

By the way, with the approximations applied in *clause 2.3* for the admittances — comparing the form relative to small slips of relations (2—33) and the expressions (4—1) — the relations between the constants b_q, b_d, k_q, k_d and the well-known machine constants are as follows:

$$b_q = \frac{1}{x_q}, \qquad b_d = \frac{1}{x_d}$$

$$k_q = \left(\frac{1}{x''_q} - \frac{1}{x_q}\right) \omega_0 T''_q, \quad k_d = \left(\frac{1}{x''_d} - \frac{1}{x'_d}\right) \omega_0 T''_d + \left(\frac{1}{x'_d} - \frac{1}{x_d}\right) \omega_0 T'_d.$$

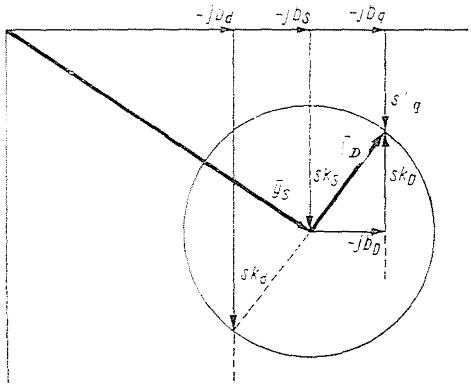


Fig. 4—1

If the approximation of the solid iron by one direct-axis and one quadrature-axis damping coil is not accepted, then the real admittance diagrams, determined by measurements or other analytical methods, has to be approximated by the respective primitive straight lines, with suitable choice of the constants k_q, k_d, b_q, b_d at the same time.

Before getting to the most simple application of the general method outlined in *chapter 3*, it must again be emphasized, that Eq. (4—2) is nothing else, but the differential equation presented in the Introduction [1], neglecting, however, the term of second order in the latter (not speaking from the fourth member, of course). The solutions obtained with the method suggested, permit us to get a better physical picture of the asynchronous operation. Just in order to demonstrate the effectiveness of this method, the study deals with the differential equation (4—2) in some more details, examining the effect of the different parameters on the solution, *i. e.* on the course of the slip-time, angle-time functions. (A similar general and detailed investigation with a numerical, or a series expansion method would be laborious and tedious.)

Before discussing the most general case of the primitive approximation adopted in the present chapter, three more simple, special cases will be dealt with.

4.1. Congruent, primitive straight lines of approximation

If the direct- and quadrature-axis admittance diagrams are identical, then the straight lines of their approximation are also congruent, consequently

$$b_q = b_d \quad \text{and} \quad k_q = k_d ,$$

so

$$b_D = 0 \quad \text{and} \quad k_D = 0 .$$

i. e. the resultant admittance diagram is reduced to a point.

Eq. (4—2) now becomes very simple

$$g = s\, k_S .$$

Its solution is:

$$s = \frac{g}{k_S} = s_0 = \text{const.} \tag{4—5}$$

consequently the slip does not change. It must be noted, that in asynchronous *generator* operation both $g < 0$ and $s_0 < 0$. On the other hand, taking Eq. (3—5) as a basis

$$\omega_0 t = -\int_{\delta_0}^{\delta} \frac{d\delta}{s_0} = \frac{\delta_0 - \delta}{s_0}$$

in other terms

$$-\, s_0\, \omega_0\, t = \delta - \delta_0 \tag{4—6}$$

or with the initial condition $\delta_0 = 0$:

$$-\, s_0\, \omega_0\, t = \delta. \tag{4—7}$$

Therefore, the angle increases linearly in function of time. The whole period T of a complete relative rotor rotation may be calculated from Eq. (4—7) with the substitution $\delta = 2\pi$:

$$\omega_0 T = -\frac{2\pi}{s_0} . \tag{4—8}$$

This clause, (besides introducing some notations and notions) again shows that in case of a symmetrical rotor, the slip is constant. Naturally, according to (4—3) also the reactive current (and the apparent power, too) are constant.

4.2. *Primitive straight lines of approximation starting from a common point*

If the primitive straight lines of approximation start from a common point, but their points belonging to the same slip do not coincide, then

$$b_q = b_d\,, \qquad b_D = 0\,,$$

i. e. there is no reluctance effect.

Now from Eq. (4—2)

$$s(\delta) = \frac{g}{k_S + k_D \cos 2\delta}\,. \tag{4—9}$$

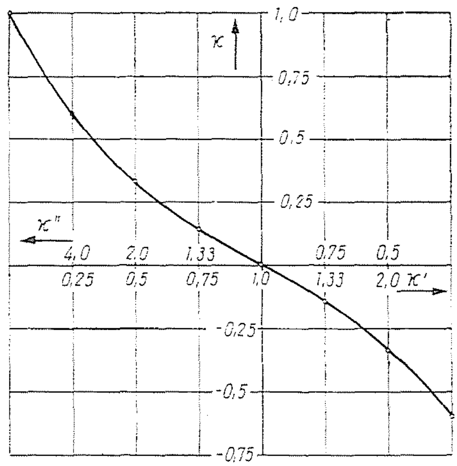


Fig. 4—2

Let us introduce by notation

$$\varkappa = -\frac{k_D}{k_S} = \frac{k_d - k_q}{k_d + k_q} \tag{4—10}$$

the notion of the so-called damping factor. The relations between the factor $\varkappa$ and the ratio $\varkappa' = k_q/k_d$ and $\varkappa'' = k_d/k_q$, respectively are shown in Fig. 4—2. When $0 < \varkappa' < 1$, then $0 < \varkappa < 1$ and when $0 < \varkappa'' < 1$, then $-1 < \varkappa < 0$.

Considering Eqs. (4—10) and (4—5), now Eq. (4—9) assumes the following form

$$s(\delta) = \frac{s_0}{1 - \varkappa \cos 2\delta}\,. \tag{4—11}$$

In the present case the fundamental integral (3—5) is

$$\omega_0 t = -\int_{\delta_0}^{\delta} \frac{1 - \varkappa \cos 2\delta}{s_0} \, d\delta .$$

From this relation the time as a function of the angle may be expressed in a closed form:

$$- s_0 \omega_0 t = \delta - \delta_0 - \frac{\varkappa}{2} (\sin 2\delta - \sin 2\delta_0) \qquad (4-12)$$

or with the initial condition $\delta_0 = 0$:

$$- s_0 \omega_0 t = \delta - \frac{\varkappa}{2} \sin 2\delta . \qquad (4-13)$$

Substituting the value $\delta = 2\pi$ into the right side of formula (4—13), it may be stated, that the period is given by formula (4—8) also in this case.

By formulae (4—11) and (4—13), the slip-time function $s(t)$ searched for is given in a parametric form through angle δ. The angle-time function $\delta(t)$ wanted is given, on the other hand, by the inverse of relation (4—13). (These two functions were previously calculated for a case [2, 3, 4].)

The case discussed in the present clause is the most simple one among the cases leading to the variable slip and really forms the cardinal point of the problem dealt with in this study.

4.3. *Parallel primitive straight lines of approximation starting from different points*

This is the other most simple case already resulting in a variable slip. There is a reluctance effect, nevertheless,

$$k_q = k_d , \qquad k_D = 0 ,$$

i. e. the points belonging to identical slips are at the same distance from the imaginary axis.

Now from Eq. (4—2):

$$s(\delta) = \frac{g + b_D \sin 2\delta}{k_S} . \qquad (4-14)$$

Let us introduce by the symbol

$$\beta = -\frac{b_D}{g} \tag{4—15}$$

the notion of the so-called reluctance factor. It must be noted, that for the usual synchronous generator and for asynchronous generator operation always $\beta \geqq 0$. On the basis of Eqs. (4—15) and (4—5), from Eq. (4—14):

$$s(\delta) = s_0(1 - \beta \sin 2\delta). \tag{4—16}$$

The fundamental integral (3—5) is now:

$$\omega_0 t = -\int_{\delta_0}^{\delta} \frac{1}{1 - \beta \sin 2\delta}\, d\delta .$$

The time in function of angle may also be expressed from this in a closed form. The integral tables [*e. g.* 5, 6, 7] give the solution in the form of two indefinite integrals. The first form [5] after substituting the limits:

$$-s_0 \omega_0 t = \frac{1}{\sqrt{1-\beta^2}} \left\{ \operatorname{arc\,tg}\left[\sqrt{\frac{1+\beta}{1-\beta}}\, \operatorname{tg}\left(\delta - \frac{\pi}{4}\right)\right] - \operatorname{arc\,tg}\left[\sqrt{\frac{1+\beta}{1-\beta}}\, \operatorname{tg}\left(\delta_0 - \frac{\pi}{4}\right)\right]\right\}^{*}. \tag{4—17}$$

While the second form [6, 7]:

$$-s_0 \omega_0 t = \frac{1}{\sqrt{1-\beta^2}} \left\{ \operatorname{arc\,tg} \frac{\operatorname{tg}\delta - \beta}{\sqrt{1-\beta^2}} - \operatorname{arc\,tg} \frac{\operatorname{tg}\delta_0 - \beta}{\sqrt{1-\beta^2}} \right\}. \tag{4—18}$$

After all, both expressions (4—17) and (4—18) may be transformed with some algebraic arrangements to the form

$$-s_0 \omega_0 t = \frac{1}{\sqrt{1-\beta^2}} \operatorname{arc\,tg} \frac{\sqrt{1-\beta^2}\,(\operatorname{tg}\delta - \operatorname{tg}\delta_0)}{1 + \operatorname{tg}\delta \operatorname{tg}\delta_0 - \beta\,(\operatorname{tg}\delta + \operatorname{tg}\delta_0)}. \tag{4—19}$$

With the initial condition $\delta_0 = 0$ all three expressions may become more simple

$$-\sqrt{1-\beta^2}\, s_0 \omega_0 t = \operatorname{arc\,tg}\left[\sqrt{\frac{1+\beta}{1+\beta}}\, \operatorname{tg}\left(\delta - \frac{\pi}{4}\right)\right] + \operatorname{arc\,tg}\sqrt{\frac{1+\beta}{1-\beta}} \tag{4—20}$$

* In the mathematical and technical literature sometimes $\tan^{-1}$ is used instead of arc tg.

and

$$-\sqrt{1-\beta^2}\, s_0\, \omega_0 t = \operatorname{arc\,tg} \frac{\operatorname{tg}\delta - \beta}{\sqrt{1-\beta^2}} + \operatorname{arc\,tg} \frac{\beta}{\sqrt{1-\beta^2}} \tag{4—21}$$

finally

$$-\sqrt{1-\beta^2}\, s_0 \omega_0 t = \operatorname{arc\,tg} \frac{\sqrt{1-\beta^2}\operatorname{tg}\delta}{1-\beta\operatorname{tg}\delta}\,. \tag{4—22}$$

By the way, the inverse function may now be as an exception expressed also in an explicite form, *e. g.* from (4—22):

$$\delta = \operatorname{arc\,tg} \frac{\operatorname{tg}(-\sqrt{1-\beta^2}\, s_0\, \omega_0 t)}{\sqrt{1-\beta^2} + \beta \operatorname{tg}(-\sqrt{1-\beta^2}\, s_0\, \omega_0 t)}\,.$$

Substituting the value $\delta = 2\pi$ into the right side of expressions (4—20), (4—21), or (4—22), the following formula may be obtained for the period:

$$\omega_0 T = -\frac{2\pi}{\sqrt{1-\beta^2}\, s_0}\,. \tag{4—23}$$

The value of the medium slip is consequently

$$s_m = s_0 \sqrt{1-\beta^2}. \tag{4—24}$$

Should the value of β not be too large, there is but a slight difference between s_m and s_0.

4.4. The general case of the primitive approximation

In the general case (Fig. 4—1) $b_q \neq b_d$ and $k_q \neq k_d$, consequently $b_D \neq 0$ and $k_D \neq 0$. From relation (4—2) the slip may be expressed as follows:

$$s(\delta) = \frac{g + b_D \sin 2\delta}{k_S + k_D \cos 2\delta}. \tag{4—25}$$

Considering Eqs. (4—5), (4—10) and (4—15):

$$s(\delta) = s_0 \frac{1-\beta\sin 2\delta}{1-\varkappa\cos 2\delta}. \tag{4—26}$$

The fundamental integral (3—5) is now:

$$\omega_0 t = -\frac{1}{s_0}\int\limits_{\delta_0}^{\delta} \frac{1-\varkappa\cos 2\delta}{1-\beta\sin 2\delta}\, d\delta\,.$$

The right-side integral may be calculated in a closed form in this case, too:

$$- s_0 \omega_0 t = \frac{1}{\sqrt{1-\beta^2}} \operatorname{arc\,tg} \frac{\sqrt{1-\beta^2}\,(\operatorname{tg}\delta - \operatorname{tg}\delta_0)}{1+\operatorname{tg}\delta \operatorname{tg}\delta_0 - \beta\,(\operatorname{tg}\delta + \operatorname{tg}\delta_0)} + \\ + \frac{\varkappa}{2\beta} \ln \frac{1-\beta \sin 2\delta}{1-\beta \sin 2\delta_0} \, . \tag{4—27}$$

If the initial condition is $\delta_0 = 0$, then

$$- \sqrt{1-\beta^2}\, s_0 \omega_0 t = \operatorname{arc\,tg} \frac{\sqrt{1-\beta^2}\, \operatorname{tg}\delta}{1-\beta \operatorname{tg}\delta} + \varkappa \frac{\sqrt{1-\beta^2}}{2\beta} \ln (1-\beta \sin 2\delta). \tag{4—28}$$

So by Eqs. (4—26) and (4—28), the desired slip-time function is given in a parametric form through angle δ. On the other hand, the inversion of (4—28) supplies the wanted angle-time function $\delta(t)$.

Comparing formulae (4—13) and (4—28), the influence of the reluctance effect (β) is clearly demonstrated.

Substituting the value $\delta = 2\pi$ into the right side of expression (4—28), we obtain for the value of the period and the medium slip, again the expressions given by (4—23) and (4—24), respectively.

4.5. The effect of the parameters $\varkappa$ and β on the angle-time function and slip-time function

The relatively simple solution obtained in the former clause permits to examine in details the influence of the two parameters β and $\varkappa$ competent at primitive approximation.

Let us introduce the notion of the relative slip

$$\sigma = \frac{s}{s_0} \tag{4—29}$$

as well as that of the reduced time

$$\tau = - \sqrt{1-\beta^2}\, s_0 \omega_0 t = - s_m \omega_0 t \tag{4—30}$$

which is measured — as may be seen — always in radians (it is, properly speaking, of angular dimension and will be called "time" merely for the sake of brevity, to distinguish it from angle δ).

By the aid of Eqs. (4—29) and (4—30), Eqs. (4—26) and (4—28) may be written as follows

$$\sigma(\delta) = \frac{1 - \beta \sin 2\delta}{1 - \varkappa \cos 2\delta} \tag{4—31}$$

and

$$\tau(\delta) = \operatorname{arc\,tg} \frac{\sqrt{1-\beta^2}\operatorname{tg}\delta}{1 - \beta \operatorname{tg}\delta} + \varkappa \frac{\sqrt{1-\beta^2}}{2\beta} \ln(1 - \beta \sin 2\delta). \tag{4—32}$$

In the series of figures 4—3 to 4—6 the course of the slip-angle functions $\sigma(\delta)$ calculated on the basis of formula (4—31) will be illustrated for values $\beta = 0$; 0.2; 0.4; 0.6, while $\varkappa = -0.6$; -0.4; -0.2; 0.0; 0.2; 0.4; 0.6. If $\beta = 0$, there is no reluctance effect and the course of the relative slip, in function of the angular displacement δ is symmetrical. If $\varkappa > 0$ the maximum slip occurs at $\delta = 180°$, in case of $\varkappa < 0$, however, at $\delta = 90°$. Parameter β increasing, the slip curve always becomes more asymmetric.

For the values $\delta = 45°$, 135°, 225°, etc., the curves relative to $\varkappa > 0$ are in a mirror symmetry with the curves referring to $\varkappa < 0$. As the displacement of the rotor takes place in the range $0° \leq \delta \leq 180°$, in the same way as in the range $180° \leq \delta \leq 360°$, for the de-excited machine, here and also further on, it is sufficient to restrict ourselves to the first range, this in itself determining the whole course of the periodic functions.

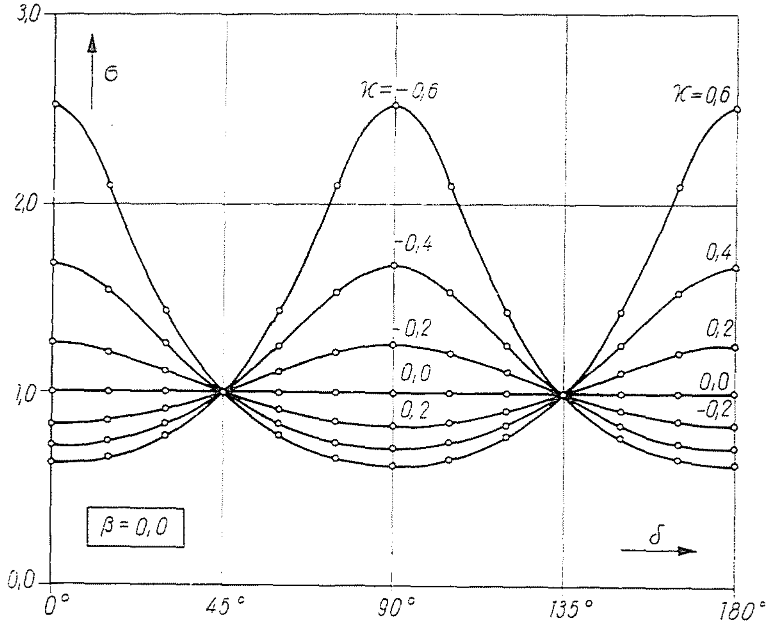


Fig. 4—3

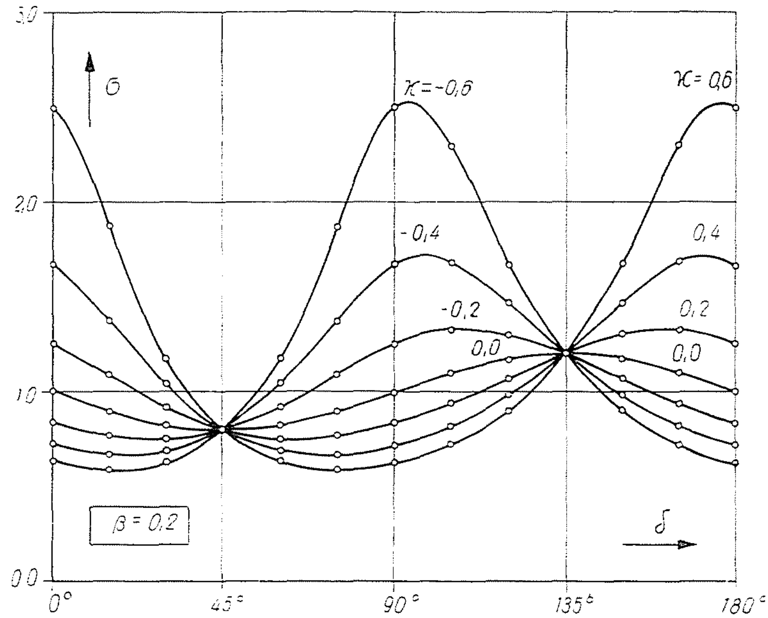
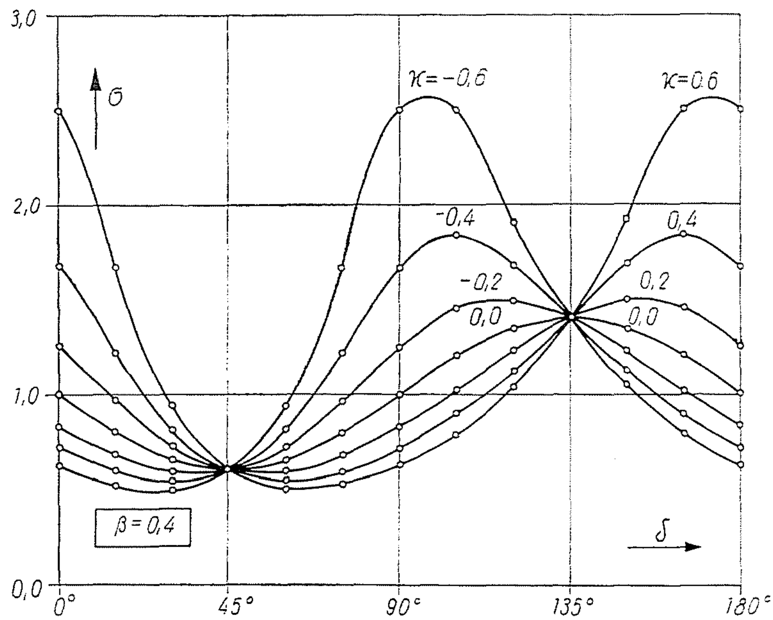

3,0
2,0
1,0
0,0
σ
κ=-0,6
κ=0,6
-0,4
0,4
-0,2
0,2
0,0
0,0
β=0,2
δ
0°
45°
90°
135°
180°

Fig. 4—4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σ
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-0,4
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0,0
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β=0,4
δ
0°
45°
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135°
180°

Fig. 4—5

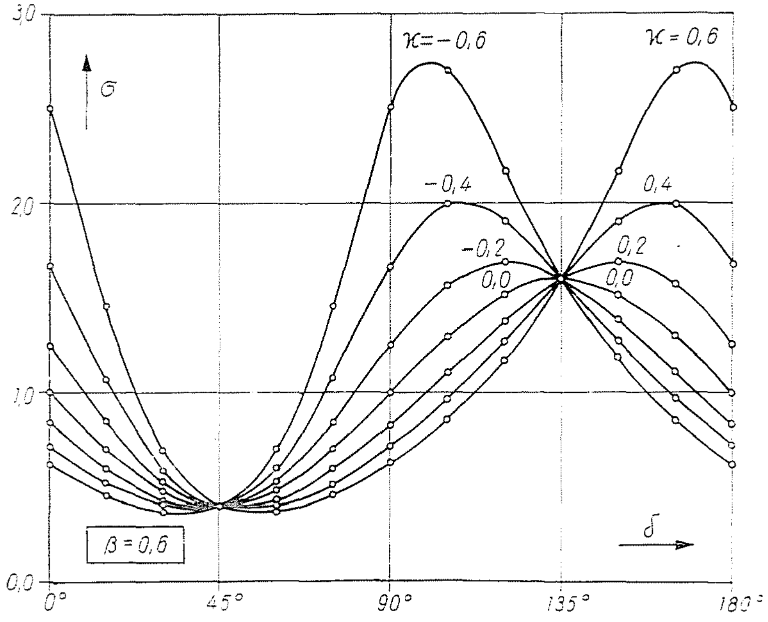

3,0
2,0
1,0
0,0
σ
ϰ=−0,6
ϰ = 0,6
−0,4
0,4
−0,2
0,2
0,0
0,0
β = 0,6
δ
0°
45°
90°
135°
180°

Fig. 4—6

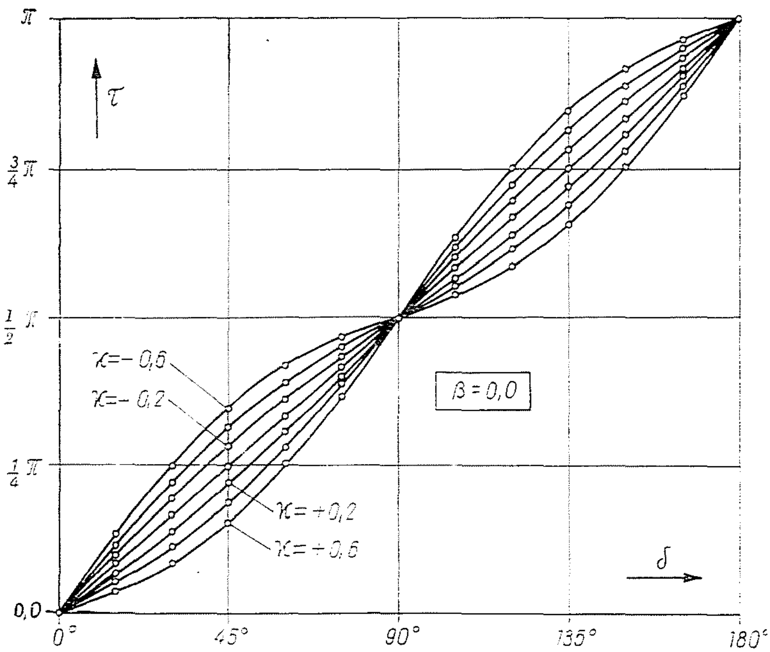
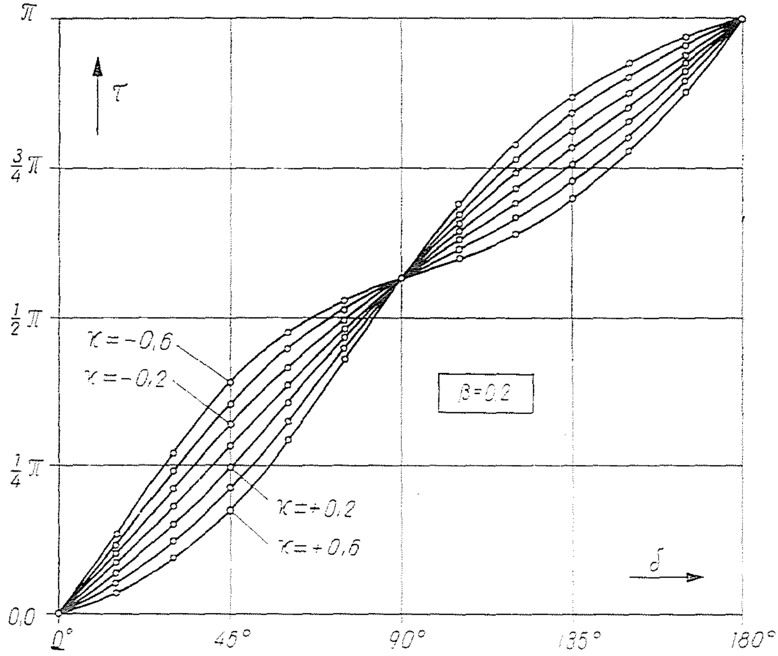

π
¾π
½π
¼π
0,0
τ
ϰ=−0,6
ϰ=−0,2
ϰ=+0,2
ϰ=+0,6
β = 0,0
δ
0°
45°
90°
135°
180°

Fig. 4—7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τ
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$\frac{1}{2}\pi$
$\frac{1}{4}\pi$
0,0
ϰ = −0,6
ϰ = −0,2
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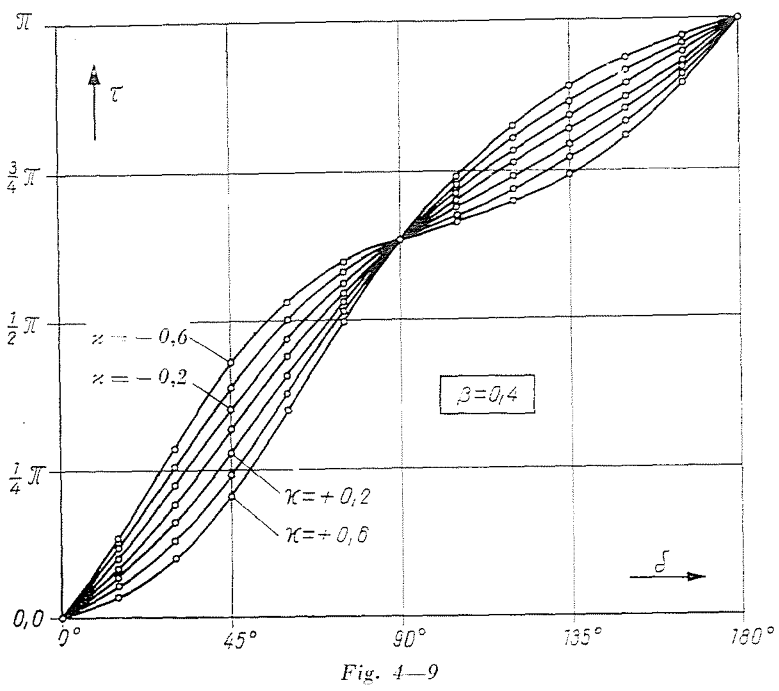
ϰ = +0,6
β = 0,2
δ
0°
45°
90°
135°
180°

Fig. 4—8


π
τ
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$\frac{1}{4}\pi$
0,0
ϰ = −0,6
ϰ = −0,2
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δ
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90°
135°
180°

Fig. 4—9

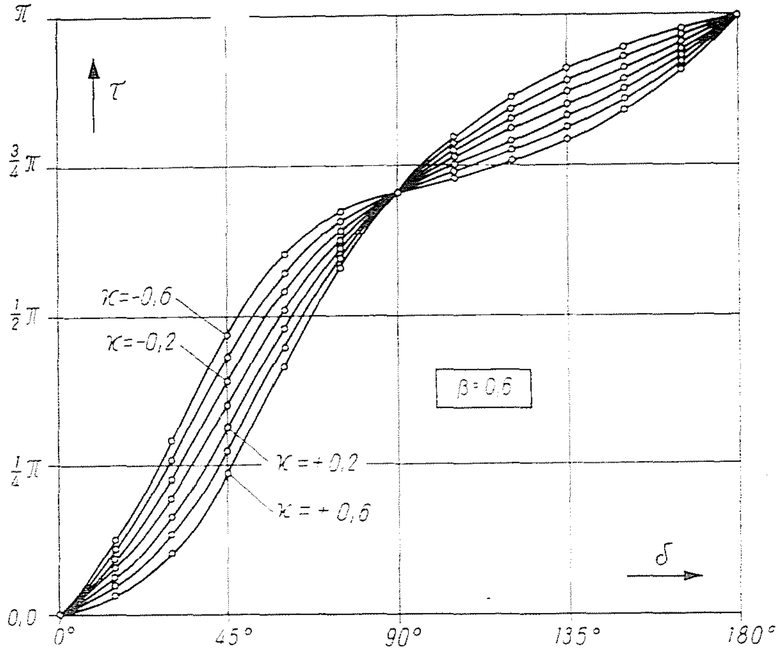

Fig. 4—10

The series of figures 4—7 to 4—10 gives information about the course of the time-angle functions $\tau(\delta)$ calculated by formula (4—32) for the values β and $\varkappa$ enumerated in the foregoing. The greater the reluctance effect, the greater the difference between the time necessary for making the first and the second quarter turn and the more asymmetric the course of the curves with any given value $\varkappa$ is.

Naturally, the curves provide at the same time the demanded inverse angle-time functions $\delta(\tau)$, too. The series of figures 4—11 to 4—14 illustrates the course of the required slip-time functions $\sigma(\tau)$ and clearly shows the influence of the different parameters β and $\varkappa$. When calculating the curves, the values $\delta = 0°, 15°, 30° \ldots 180°$ are substituted one by one into formulae (4—31) and (4—32), then on the basis of the determined values σ and τ, belonging to each angle the function $\sigma(\tau)$ could be plotted point by point. Here also, similarly to the former figures, the points corresponding to the above-mentioned values of the angle are marked by small circles. Consequently, on Figs. 4—11 ... 4—14 the series of points of the different curves $\sigma(\tau)$ belonging to the same angle and marked by circles, illustrate at the same time the course of the functions $\delta(\tau)$, too.

The construction of the series of figures 4—11 ... 4—14 was based on the same values β and $\varkappa$ as before.

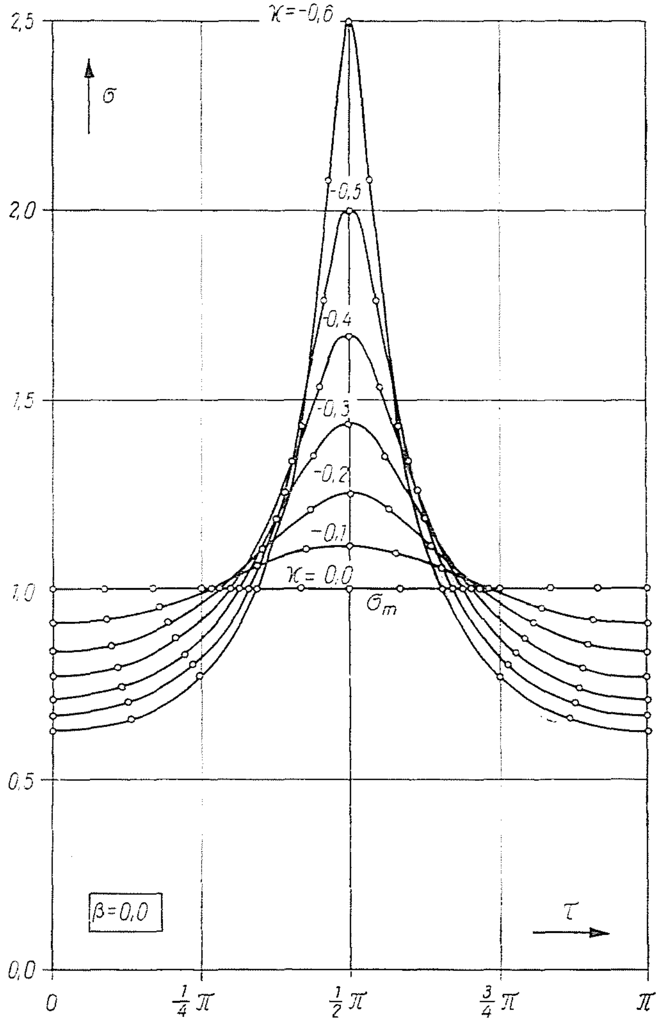

Fig. 4—11a

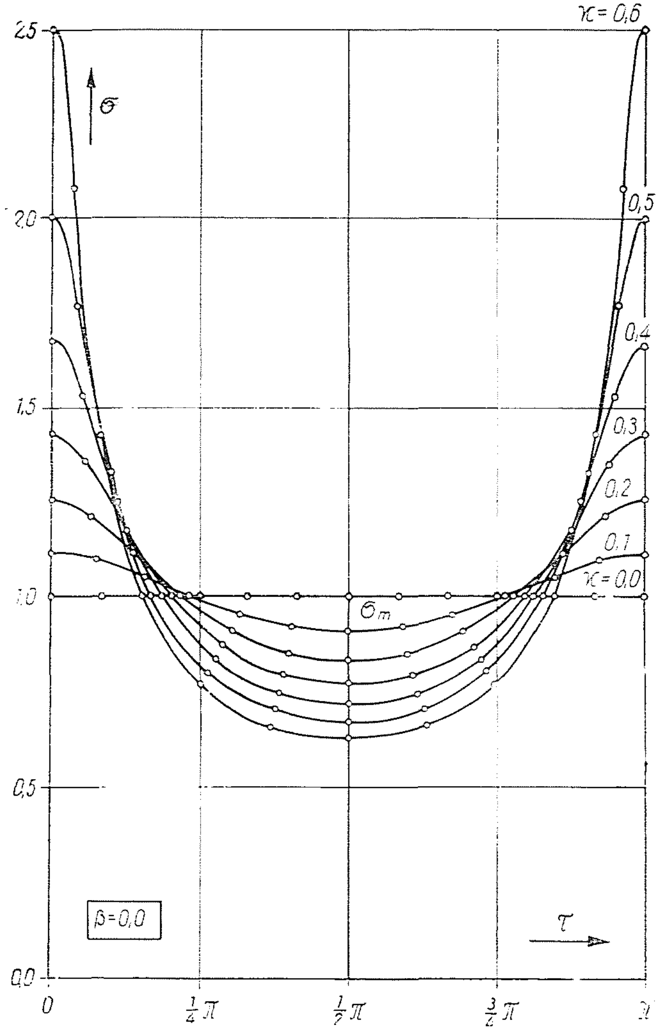

Fig. 4--11b

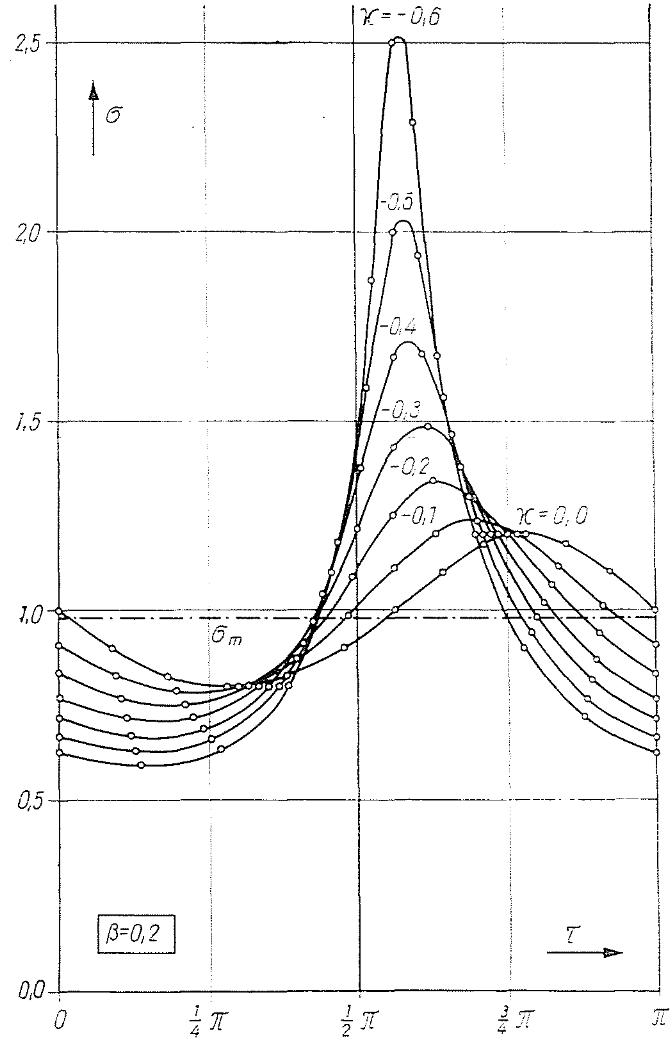

Fig. 4—12a

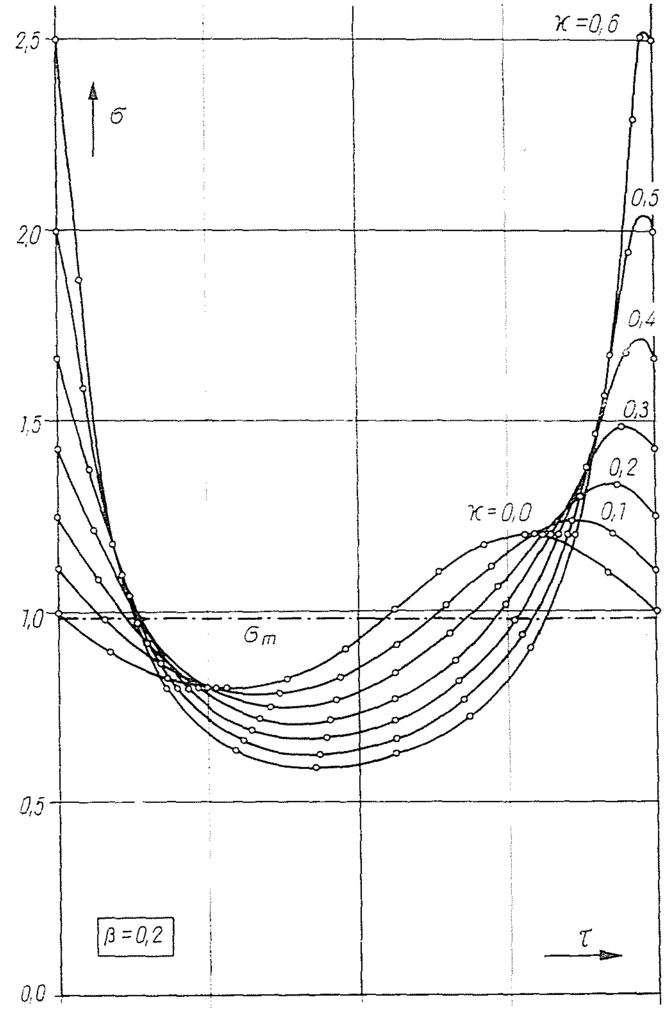

Fig. 4—12b

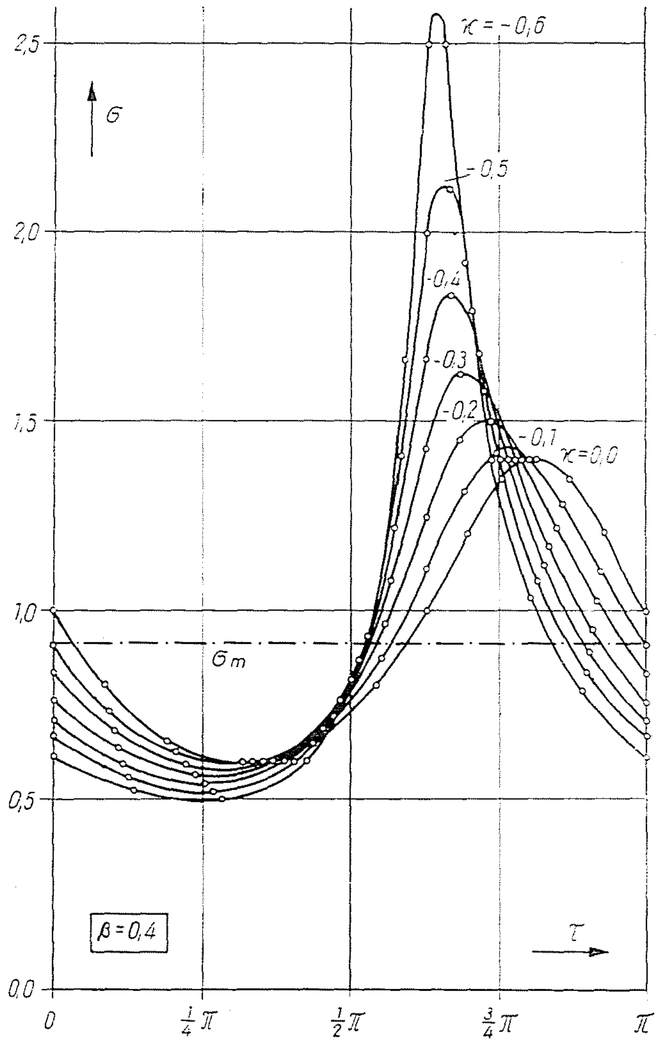

Fig. 4—13a

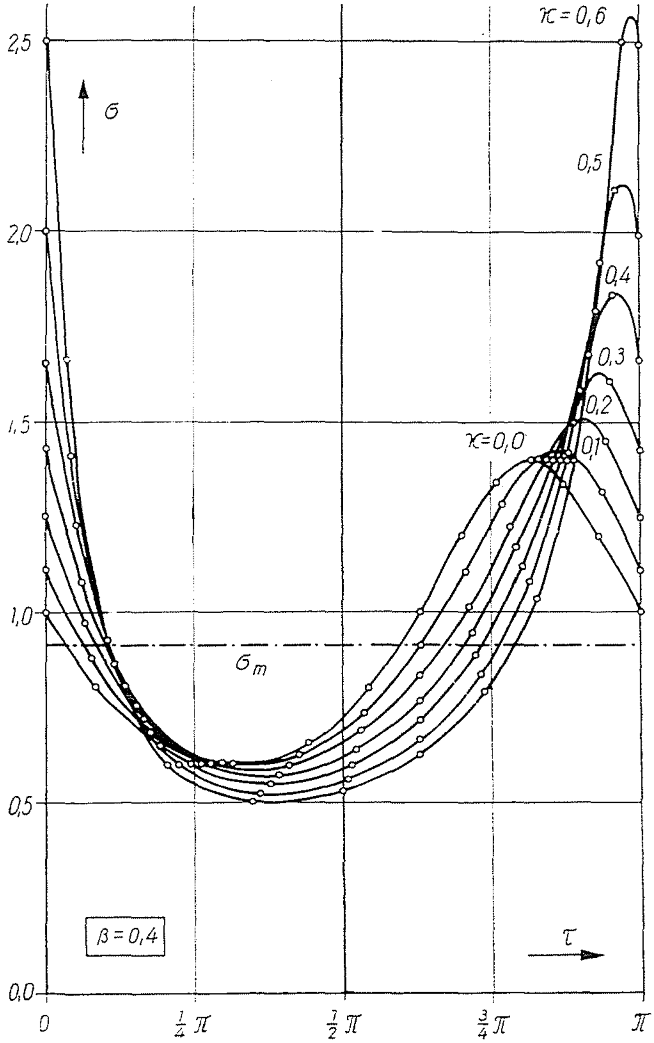

Fig. 4—13b

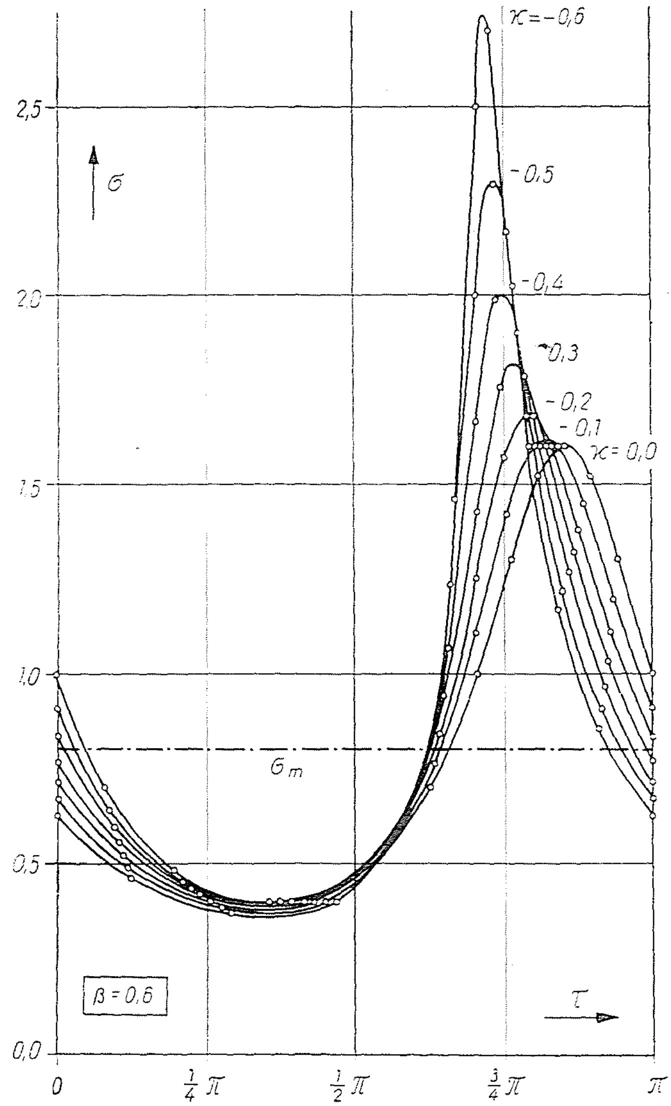

Fig. 4—14a

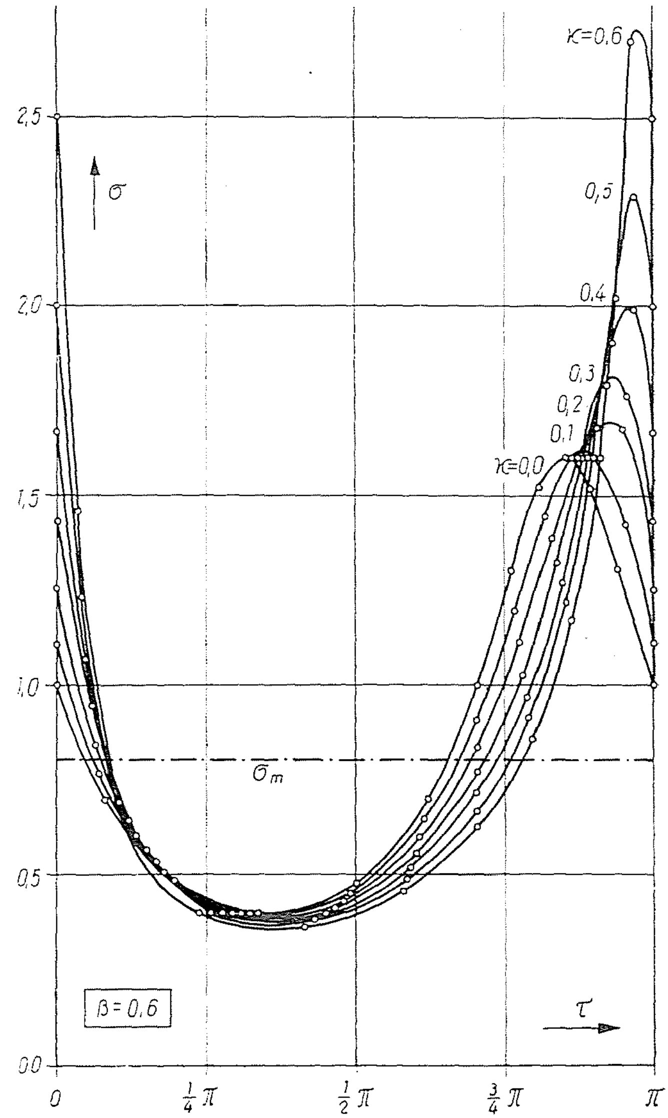

Fig. 4—14b

4.6. *The extreme and medium values of the slip*

Fig. 4—15 summarizes the maximum σ_{max}, the minimum σ_{min} and the medium σ_m values of the slip for the range $0 \leq \beta \leq 0.6$; $-0.6 \leq \varkappa \leq 0.6$ of the parameters. Among the three set of curves, for calculating the sets illustrating the two extreme values, firts of all the relation (4—31) has to be differentiated with respect to the angle:

$$\frac{d\sigma}{d\delta} = \frac{-2\beta\cos 2\delta\,(1-\varkappa\cos 2\delta) - 2\varkappa\sin 2\delta\,(1-\beta\sin 2\delta)}{(1-\varkappa\cos 2\delta)^2} . \tag{4—33}$$

Equalizing the derivative with zero, the following equation may be obtained for the critical angle δ_k belonging to the extreme values:

$$\beta\cos 2\delta_k + \varkappa\sin 2\delta_k = \beta\varkappa . \tag{4—34}$$

Introducing the notation $\alpha = \operatorname{arc\,tg}\frac{\varkappa}{\beta}$, the above transcendent equation may easily be solved:

$$\cos(2\delta_k - \alpha) = \frac{\beta\varkappa}{\sqrt{\beta^2+\varkappa^2}} \tag{4—35}$$

and

$$\delta_k = \frac{1}{2}\arccos\frac{\beta\varkappa}{\sqrt{\beta^2+\varkappa^2}} + \frac{1}{2}\operatorname{arc\,tg}\frac{\varkappa}{\beta} . \tag{4—36}$$

Substituting the critical angle δ_k into Eq. (4—31), the values of σ_{max} and σ_{min}, respectively, may be determined (and from Eq. (4—32) also the critical time τ for which the extreme values arise, may be calculated).

Finally the medium slip on the basis of Eq. (4—24) may be obtained from the following relation

$$\sigma_m = \sqrt{1-\beta^2} . \tag{4—37}$$

Neither the value of σ_{max}, nor that of σ_{min} and σ_m depends on the sign of $\varkappa$, consequently Fig. 4—15 is valid equally for $\varkappa \gtrless 0$.

4.7. *Change in the reactive power*

The reactive power (as well as the reactive current) may be judged by the aid of susceptance b *(see clause 2.2)*. Consequently, for determining their course in time, the functions $\delta(t)$ and $s(t)$ must be substituted into expression (4—3).

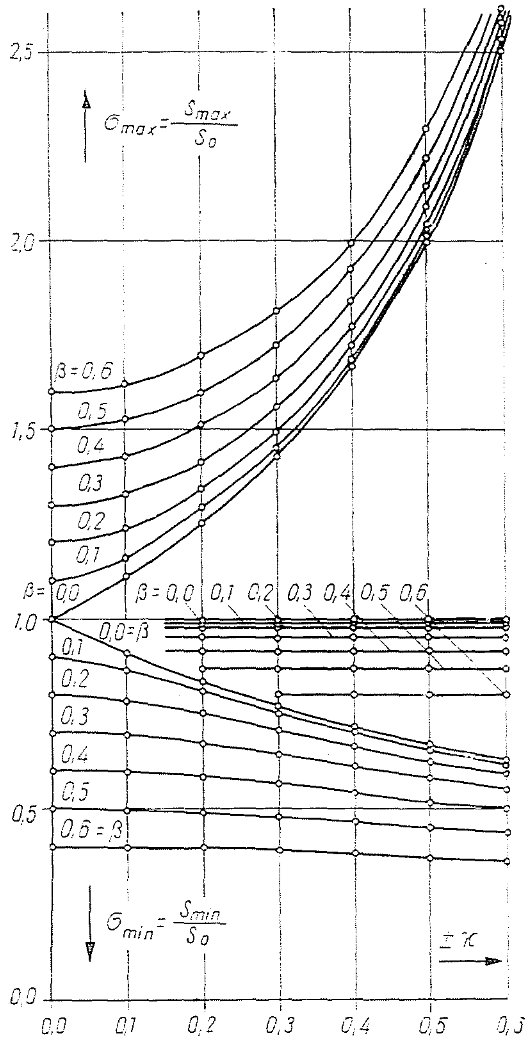
2,5
2,0
1,5
1,0
0,5
0,0
$\sigma_{max} = \frac{S_{max}}{S_0}$
$\beta = 0,6$
0,5
0,4
0,3
0,2
0,1
$\beta =$ 0,0
$\beta = 0,0$ 0,1 0,2 0,3 0,4 0,5 0,6
$0,0 = \beta$
0,1
0,2
0,3
0,4
0,5
$0,6 = \beta$
$\sigma_{min} = \frac{S_{min}}{S_0}$
$\pm \varkappa$
0,0 0,1 0,2 0,3 0,4 0,5

Fig. 4—15

To derive more general relations, let us divide throughout, this expression by $-g$ and so from (4—3):

$$\xi(\delta) = \xi_S + \Delta\xi(\delta), \qquad (4-38)$$

where

$$\xi = -\frac{b}{g} > 0, \qquad \xi_S = -\frac{b_S}{g} > 0, \qquad (4-39)$$

further, considering Eqs. (4—5), (4—10), (4—15) and (4—29)

$$\Delta\xi(\delta) = -\beta \cos 2\delta - \varkappa\, \sigma(\delta) \sin 2\delta. \qquad (4-40)$$

The last expression may be written, considering Eq. (4—31), also as follows:

$$\Delta\xi(\delta) = -\frac{\beta \cos 2\delta + \varkappa \sin 2\delta - \beta \varkappa}{1 - \varkappa \cos 2\delta}. \qquad (4-41)$$

As shown in expression (4—38), the reactive power, the reactive current and the susceptance may be traced back to function ξ consisting of a constant term ξ_S and a variable one $\Delta\xi$. The variable term $\Delta\xi$ is worthy of a detailed examination.

On the basis of relations $\Delta\xi(\delta)$ and $\delta(\tau)$ already obtained, we have after all the function $\Delta\xi(\tau)$ characteristic of the time course of the change in the reactive current, or power.

The series of figures 4—16 to 4—19 illustrates the course of functions $\Delta\xi(\tau)$ for values $\beta = 0.0;\ 0.2;\ 0.4;\ 0.6;$ and $\varkappa = 0;\ 0.1;\ 0.2;\ 0.3;\ 0.4;\ 0.5;\ 0.6.$

Comparison of expressions (4—41) and (4—34) proves, that where $\Delta\xi(\delta) = 0$, that is $\Delta\xi(\tau) = 0$, there $\sigma(\delta)$ and $\sigma(\tau)$ assume extreme values (at $\delta = \delta_k$ and $\tau = \tau_k$). In other words: in case of primitive approximation, when the change in the reactive power (or current) is zero, the slip has then an extreme value.

4.8. Mean value of the reactive power

The question arises, if the mean value ξ_m of function ξ deviates at all, and if so, to what an extent from the constant value ξ_S. Answer to this may be given by calculating the mean value ξ_m:

$$\xi_m = \frac{1}{\omega_0 T} \int_0^{\omega_0 T} \xi(\omega_0 t)\, d\,\omega_0 t = \frac{1}{\omega_0 T} \int_0^{2\pi} \xi\big(\omega_0 t(\delta)\big)\ \frac{d\,\omega_0 t(\delta)}{d\delta}\, d\delta\,. \qquad (4-42)$$

Considering Eqs. (3—4) and (4—23), moreover (4—29), after all

$$\xi_m = \frac{s_0 \sqrt{1-\beta^2}}{2\pi} \int_0^{2\pi} \frac{\xi(\delta)}{s(\delta)}\, d\delta = \frac{\sqrt{1-\beta^2}}{2\pi} \int_0^{2\pi} \frac{\xi(\delta)}{\sigma(\delta)}\, d\delta\,. \qquad (4-43)$$

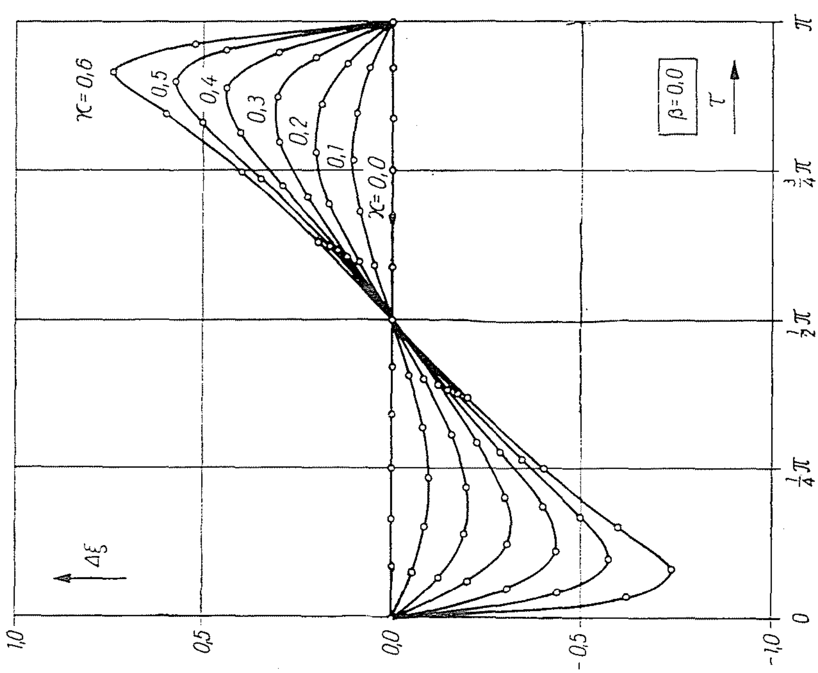

Fig. 4—16

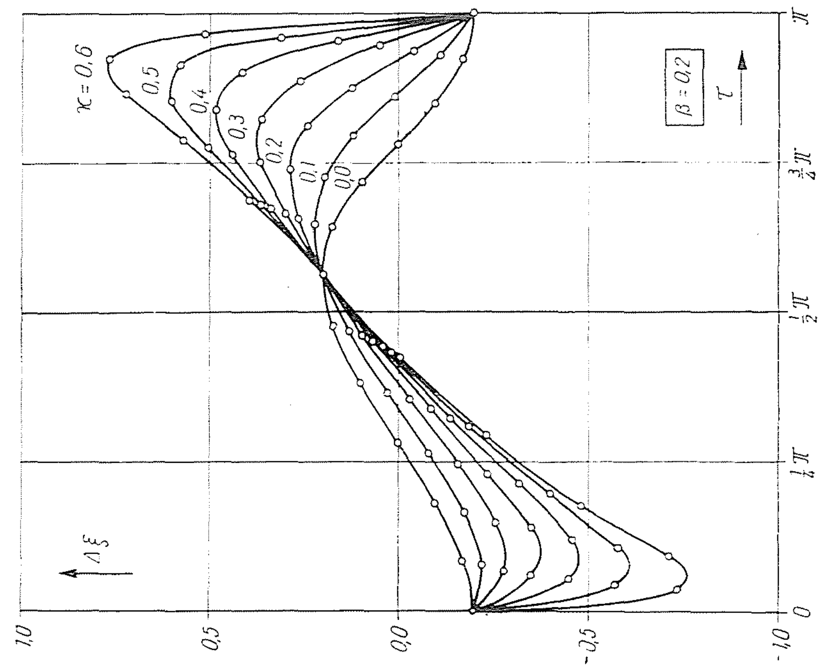

Fig. 4—17

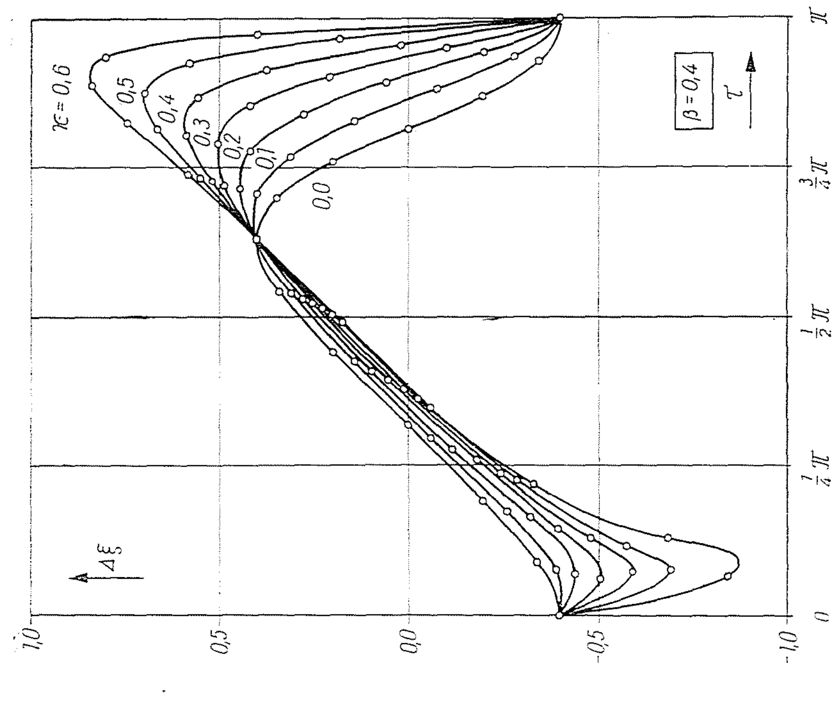

Fig. 4—18

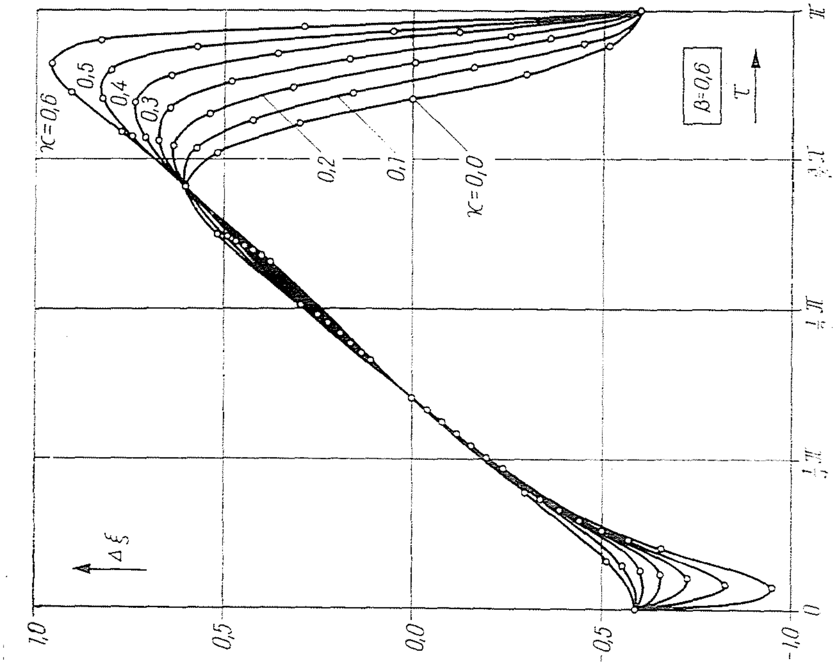

Fig. 4—19

As the mean value of a constant equals itself: $\xi_{Sm} = \xi_S$ it is sufficient to calculate the mean value of $\Delta\xi$ by the aid of Eq. (4—43). Let us start from expression (4—41): considering Eq. (4—31), the integrand is

$$\frac{\Delta\xi(\delta)}{\sigma(\delta)} = -\frac{\beta\cos 2\delta + \varkappa\sin 2\delta - \beta\varkappa}{1 - \beta\sin 2\delta}. \qquad (4—44)$$

Instead of calculating the integral, it is more practicable to expand in series, first the denominator (*see e. g.* [7, 8]),

$$\frac{1}{1-\beta\sin 2\delta} = \frac{2}{\sqrt{1-\beta^2}}\left[\frac{1}{2} - \left(\frac{1-\sqrt{1-\beta^2}}{\beta}\right)\sin 2\delta - \left(\frac{1-\sqrt{1-\beta^2}}{\beta}\right)^2\cos 4\delta + - \ldots\right] \qquad (4—45)$$

and then to consider the basic relations

$$\int_0^{2\pi} \begin{matrix}\cos\mu t\cos\nu t\\ \sin\mu t\sin\nu t\end{matrix}\, dt \begin{cases} = 0 & \text{if } \mu \neq \nu \\ = \pi & \mu = \nu \end{cases} \qquad (4—46)$$

$$\int_0^{2\pi} \cos\mu t\sin\nu t\, dt = 0\,.$$

So finally

$$\Delta\xi_m = \frac{\sqrt{1-\beta^2}}{2\pi}\int_0^{2\pi}\frac{\Delta\xi(\delta)}{\sigma(\delta)}\,d\delta = \frac{\sqrt{1-\beta^2}-(1-\beta^2)}{\beta}\varkappa\,. \qquad (4—47)$$

(Naturally, starting from expression (4—40) gives the same result.)

After all

$$\xi_m = \xi_S + \frac{\sqrt{1-\beta^2}-(1-\beta^2)}{\beta}\varkappa\,. \qquad (4—48)$$

Multiplying the above relation (4—48) by $(-g)$, and considering formula (4—15)

$$b_m = b_S + b_D\frac{\sqrt{1-\beta^2}-(1-\beta^2)}{\beta^2}\varkappa\,. \qquad (4—49)$$

Hence, for small values, (*e. g.* $\beta < 0.5$):

$$\frac{\sqrt{1-\beta^2}-(1-\beta^2)}{\beta} \approx \frac{1}{2}\beta; \quad \frac{\sqrt{1-\beta^2}-(1-\beta^2)}{\beta^2} \approx \frac{1}{2},$$

consequently

$$\xi_m \approx \xi_S + \frac{1}{2}\beta\varkappa \qquad (4—48')$$

and

$$b_m \approx b_S + b_D\frac{\varkappa}{2}\,. \qquad (4—49')$$

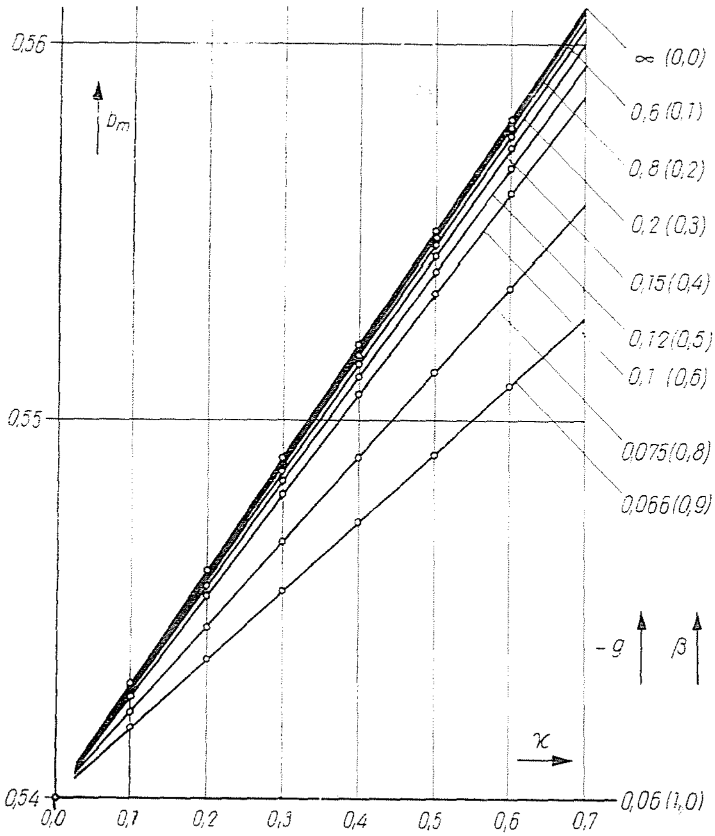

Fig. 4—20

Finally, Fig. 4—20 shows as an example, assuming the values $b_S = 0.54$, $b_D = 0.06$ the mean value b_m in function of $\varkappa$ with different values g (*i. e.* different loads, torques). Fig. 4—20 proves, that there is practically no difference between b_m and b_S.

4.9. The time course of the stator current (or the apparent power)

The stator current and the apparent power may be judged on the basis of the resultant admittance $\mathring{y}$ or its absolute value y (*see clause 2.2*). For determining their time course, the function $\delta(t)$ must be substituted into expression

$$y(\delta) = \sqrt{g^2 + b^2(\delta)}\,. \qquad (4\text{—}50)$$

Introducing the notation

$$\eta = -\frac{y}{g} > 0 \qquad (4\text{—}51)$$

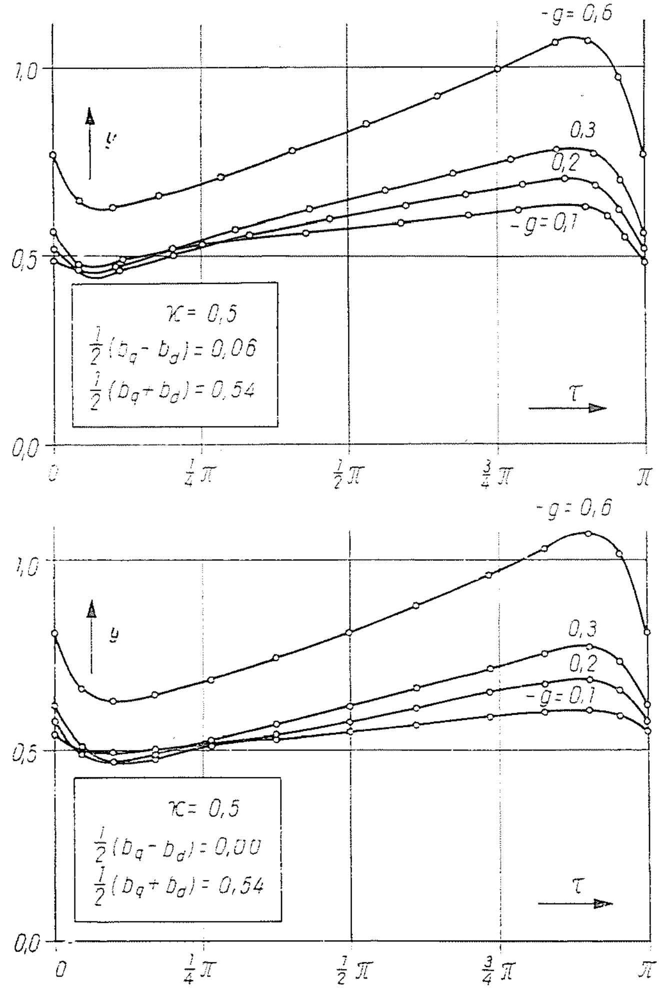

Fig. 4—21

and taking into account formula (4—38), from relation (4—50)

$$\eta(\delta) = \sqrt{1 + \xi_S^2 + 2\,\xi_S\,\Delta\xi(\delta) + \Delta\xi^2(\delta)}\,. \qquad (4-52)$$

From the above expression (4—52), substituting function $\delta(\tau)$, the desired function $\eta(\tau)$, characteristic of the stator current and of the apparent power may be obtained. As in addition to β and $\varkappa$, ξ_S means a new, third parameter, the general course of function $\eta(\tau)$ could be illustrated unfortunately only by a high number of figures. Therefore, instead of doing this, merely two special cases will be presented.

As an example, by adopting the values $b_S = 0.54$, $\varkappa = 0.5$ and $-g = 0.1$; 0.2; 0.3; 0.6, corresponding to different loads, assuming first $b_D = 0{,}06$ and then $b_D = 0.0$, in Fig. 4—21 instead of $\eta(\tau)$ the course of $y(\tau)$ may be seen at once. The curves obtained are a proof of the reluctance effect being hardly effective, especially in case of greater loads (when $-g$ is higher).

On the other hand, comparing the curves of Fig. 4—21 with the oscillograms obtained by measurements *e.g.* [2, 3, 4], it may be stated that the calculated curves are qualitatively of similar course as the measured ones. Consequently, the primitive linear approximation, though in a simplified form, gives a correct picture of the physical processes occurring in asynchronous operation.

Extending the conclusions, given at the end of *clause 4.6*, it may be proved that with primitive approximation, when the slip (s or σ) attains an extreme value at $\tau = \tau_k$ (and $\Delta\xi = 0$), then according to (4—52) just

$$\eta(\tau_k) = \sqrt{1 + \xi_S^2}$$

i. e. (4—53)

$$y(\tau_k) = \sqrt{g^2 + b_S^2}.$$

4.10. Root-mean-square value of the stator current (or the apparent power)

For heating the root-mean-square value of the current is competent. In the present case the root-mean-square value

$$y_{\text{eff}} = \sqrt{\frac{1}{\omega_0 T} \int_0^{\omega_0 T} y^2(\omega_0 t)\, dt} \tag{4—54}$$

has to be calculated. Therefore first of all the function η^2 will be integrated, according to (4—52) and (4—40)

$$\begin{aligned} \eta^2(\delta) = 1 + \xi_S^2 - 2\xi_S(\beta\cos 2\delta + \varkappa\,\sigma(\delta)\sin 2\delta) + \\ + \beta^2\cos^2 2\delta + \beta\varkappa\,\sigma(\delta)\sin 4\delta + \varkappa^2\sigma^2(\delta)\sin^2 2\delta. \end{aligned} \tag{4—55}$$

Applying the method described in *clause 4.7* to function η^2, the integration may be realized in the six steps shown in Table 4—1.

Table 4—1

a) $$\frac{1}{\omega_0 T}\int_0^{\omega_0 T} (1 + \xi_S^2)\, d\,\omega_0 t = 1 + \xi_S^2$$

b) $$\frac{\sqrt{1-\beta^2}}{2\pi}\int_0^{2\pi} \frac{-2\xi_S\beta\cos 2\delta}{\sigma(\delta)}\, d\delta = 2\xi_S\varkappa\,\frac{\sqrt{1-\beta^2} - (1-\beta^2)}{\beta}$$

c) $$\frac{\sqrt{1-\beta^2}}{2\pi}\int_0^{2\pi} 2\xi_S\varkappa\sin 2\delta \cdot d\delta = 0$$

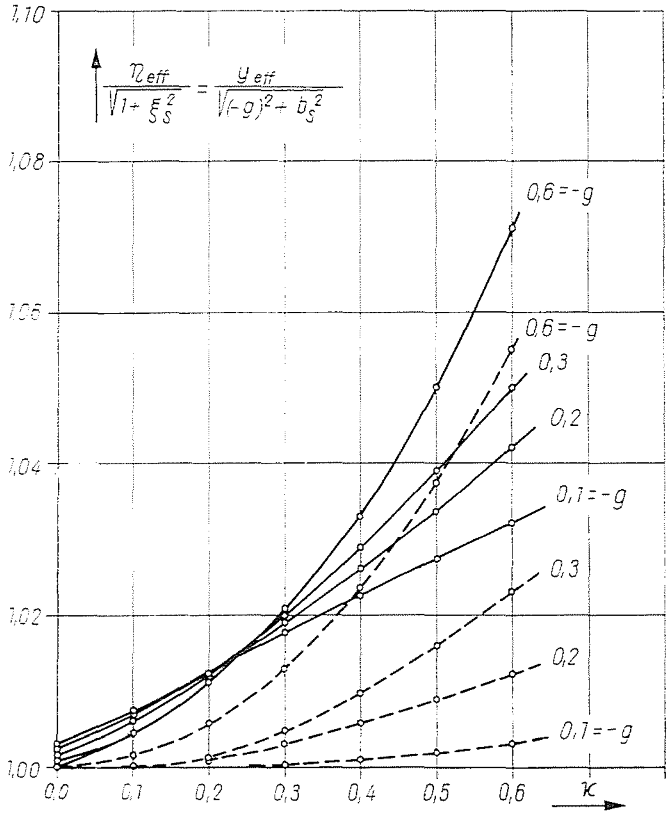

Fig. 4—22

d)

$$\frac{\sqrt{1-\beta^2}}{2\pi}\int_0^{2\pi}\left(\frac{\beta^2}{2}+\frac{\beta^2}{2}\cos 4\delta\right)\frac{1}{\sigma(\delta)}\,d\delta =$$

$$=\frac{1}{\pi}\int_0^{2\pi}\left(\frac{\beta^2}{2}-\frac{3\beta^2\varkappa}{4}\cos 2\delta+\frac{\beta^2}{2}\cos 4\delta-\frac{\beta^2\varkappa}{4}\cos 6\delta\right)\times$$

$$\times\left[\frac{1}{2}+\left(\frac{1-\sqrt{1-\beta^2}}{\beta}\right)\sin 2\delta-\left(\frac{1-\sqrt{1-\beta^2}}{\beta}\right)^2\cos 4\delta-+\dots\right]d\delta =$$

$$=\sqrt{1-\beta^2}-(1-\beta^2)$$

e)

$$\frac{\sqrt{1-\beta^2}}{2\pi}\int_0^{2\pi}\beta\varkappa\sin 4\delta\cdot d\delta = 0$$

f)

$$\frac{\sqrt{1-\beta^2}}{2\pi}\int_0^{2\pi}\varkappa^2\,\sigma(\delta)\sin^2 2\delta\cdot d\delta =$$

$$= \frac{\varkappa^2\sqrt{1-\beta^2}}{\pi\sqrt{1-\varkappa^2}} \int_0^{2\pi} \left(\frac{1}{2} - \frac{1}{2}\cos 4\delta - \frac{3\beta}{4}\sin 2\delta + \frac{\beta}{4}\sin 6\delta\right) \times$$

$$\times \left[\frac{1}{2} + \left(\frac{1-\sqrt{1-\varkappa^2}}{\varkappa}\right)\cos 2\delta + \left(\frac{1-\sqrt{1-\varkappa^2}}{\varkappa}\right)^2 \cos 4\delta + \ldots\right] d\delta =$$

$$= \sqrt{1-\beta^2}\,(1-\sqrt{1-\varkappa^2})$$

Thus, after taking into account (4—55) and Table 4—1:

$$\eta_{\text{eff}} = \sqrt{\begin{array}{c} 1 + \xi_S^2 - 2\xi_S\varkappa\,\dfrac{\sqrt{1-\beta^2}-(1-\beta^2)}{\beta} + \\ + \sqrt{1-\beta^2}-(1-\beta^2) + \sqrt{1-\beta^2}\,(1-\sqrt{1-\varkappa^2}) \end{array}} \tag{4—56}$$

As a consequence of the three $\beta, \varkappa, \xi_S$ parameters, again only a special case will be dealt with. In Fig. 4—22, referring to values $b_S = 0.54$, as well as to $b_D = 0.06$ (full line) and to $b_D = 0.0$ (dotted line), further on to $-g = 0.1; 0.2; 0.3; 0.6$, first of all, to illustrate the effect of $\varkappa$ in function of the latter, the relative admittance

$$\frac{\eta_{\text{eff}}}{(\eta_{\text{eff}})_{\beta,\varkappa=0}} = \frac{y_{\text{eff}}}{(y_{\text{eff}})_{\beta,\varkappa=0}}$$

may be seen. In analogy to Fig. 4—22, assuming

$$\eta_{\text{eff}} \approx \sqrt{1+\xi_S^2} \qquad y_{\text{eff}} \approx \sqrt{g^2+b_S^2}$$

no too great error occurs.

5. Generalized linear approximations

The primitive linear approximation discussed in the previous *chapter 4* may seem quite special. Undoubtedly, it has the great advantage, however, that the phenomena taking place do not depend but only on two parameters (β and $\varkappa$), permitting a deep inspection into the physics of the variable slip and the influence of the individual parameters, as it was shown, may easily be illustrated with sets of curves.

Present chapter discusses the more general cases of the linear approximation. On the one hand, each of the direct- and quadrature-axis admittance diagrams will be approximated by an oblique straight line, starting from an arbitrary point and, on the other hand, also method of the piecewise-linear approximation will be introduced.

While the previous chapter 4 starting from the most simple cases turned to the more complicated ones, now *chapter 5* starts from the most general case, later turning to some special ones.

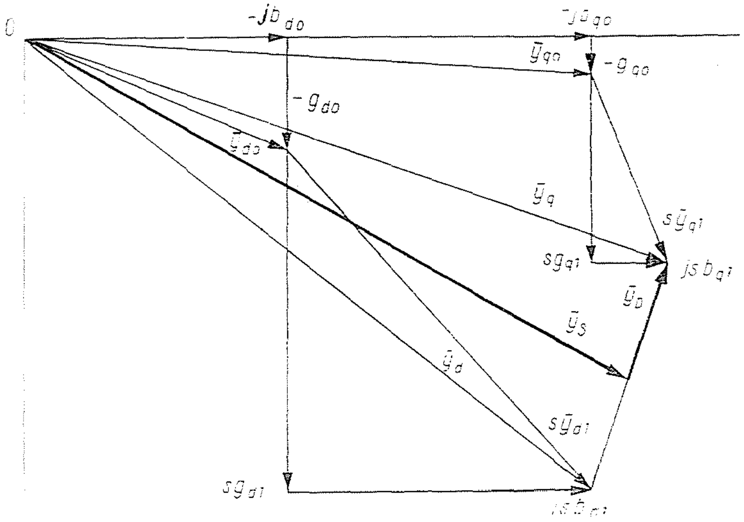

Fig. 5—1

5.1. *The general case of the oblique straight line approximation*

For the case the direct-axis and quadrature-axis admittance diagrams, respectively, are approximated by an oblique straight line of general position, conditions are shown in Fig. 5—1.

The two vectors of the starting points are

$$\bar{y}_{d0} = -g_{d0} - jb_{d0}, \qquad \bar{y}_{q0} = -g_{q0} - jb_{q0}, \tag{5—1}$$

while the two directional vectors

$$\bar{y}_{d1} = g_{d1} + jb_{d1}, \qquad \bar{y}_{q1} = g_{q1} + jb_{q1} \tag{5—2}$$

(where, according to Fig. 5—1, all constants at the right side

$$g_{d0}, g_{q0}, g_{d1}, g_{q1}, b_{d0}, b_{q0}, b_{d1}, b_{q1} > 0).$$

So on the basis of Eqs. (5—1) and (5—2), the vectors approximating the direct-axis and the quadrature-axis admittance diagrams, respectively, (Fig. 5—1):

$$\bar{y}_d = \bar{y}_{d0} + s\bar{y}_{d1} = (-g_{d0} + sg_{d1}) - j(b_{d0} - s\,b_{d1}), \qquad \bar{y}_q = \bar{y}_{q0} + s\bar{y}_{q1} = (-g_{q0} + sg_{q1}) - j(b_{q0} - s\,b_{q1}), \tag{5—3}$$

i. e. by

$$\bar{y}_d = g_d(s) - jb_d(s),$$
$$\bar{y}_q = g_q(s) - jb_q(s),$$

in the present case

$$g_d(s) = -g_{d0} + s\,g_{d1},$$
$$g_q(s) = -g_{q0} + s\,g_{q1},$$

and (5—4)

$$b_d(s) = \quad b_{d0} - s\,b_{d1},$$
$$b_q(s) = \quad b_{q0} - s\,b_{q1}.$$

As may be seen, at the right side merely general linear expressions figure, provided none of the coefficients equal zero. It must be noted, that to avoid the indexes, *chapter 4* adopted the more simple $b_{d0} = b_d$, $b_{q0} = b_q$ and $g_{d1} = k_d$, $g_{q1} = k_q$ symbols.

Considering Eqs. (5—3), the centre vector

$$\bar{y}_S = \frac{1}{2}(\bar{y}_q + \bar{y}_d)$$

and the difference vector

$$\bar{y}_D = \frac{1}{2}(\bar{y}_q - \bar{y}_d)$$

as well as its reflection about to the real axis, *i. e.* the initial radius vector

$$\hat{y}_D = \frac{1}{2}(\hat{y}_q - \hat{y}_d)$$

necessary for constructing the resultant admittance diagram

$$y = \bar{y}_S + \hat{y}_D\,e^{j2\delta}$$

may be seen in Fig. 5—1.

The components of these vectors in the present case, taking into account (5—4), are

$$g_S(s) = -\frac{1}{2}(g_{q0} + g_{d0}) + \frac{1}{2}(g_{q1} + g_{d1})\,s = -g_{S0} + s\,g_{S1},$$

$$b_S(s) = \quad \frac{1}{2}(b_{q0} + b_{d0}) - \frac{1}{2}(b_{q1} + b_{d1})\,s = \quad b_{S0} - s\,b_{S1},$$

and (5—5)

$$g_D(s) = -\frac{1}{2}(g_{q0} - g_{d0}) + \frac{1}{2}(g_{q1} + g_{d1})\,s = -g_{D0} + s\,g_{D1},$$

$$b_D(s) = \quad \frac{1}{2}(b_{q0} - b_{d0}) - \frac{1}{2}(b_{q1} - b_{d1})\,s = \quad b_{D0} - s\,b_{D1}.$$

Substitution of the above expressions (5—5) into relations (3—9), (3—1) and (3—10) now yields

$$\hat{y} = (-g_{S0} + s\,g_{S1}) - j(b_{S0} - s\,b_{S1}) + \\ + \left[(-g_{D0} + s\,g_{D1}) + j(b_{D0} - s\,b_{D1})\right] e^{j2\delta} \tag{5—6}$$

and

$$g = (-g_{S0} + s\,g_{S1}) + (-g_{D0} + s\,g_{D1}) \cos 2\delta - (b_{D0} - s\,b_{D1}) \sin 2\delta\,, \tag{5—7}$$

$$b = (b_{S0} - s\,b_{S1}) - (b_{D0} - s\,b_{D1}) \cos 2\delta - (g_{D0} + s\,g_{D1}) \sin 2\delta\,. \tag{5—8}$$

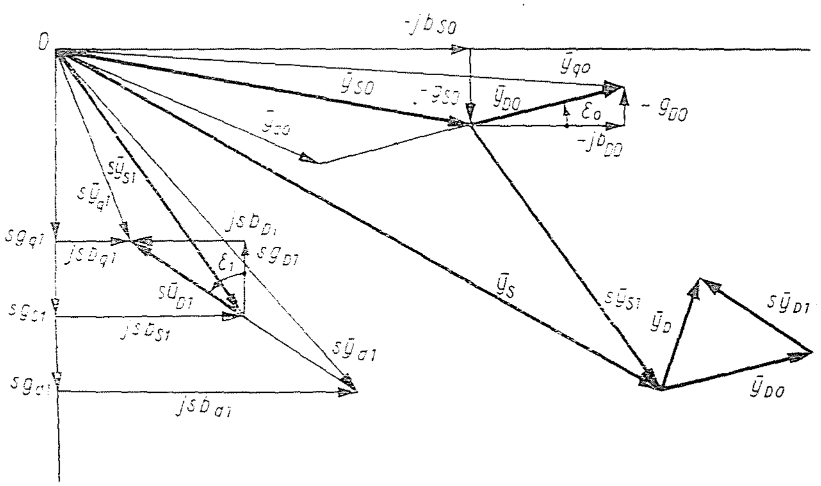

Fig. 5—2

It must be noted, that (5—6) may also have the following form

$$\hat{y} = (\bar{y}_{S0} + s\,\bar{y}_{S1}) + (\hat{y}_{D0} + s\,\hat{y}_{D1}) e^{j2\delta} \tag{5—9}$$

where

$$\bar{y}_{S0} = -\,g_{S0} - jb_{S0}\,,$$

$$\bar{y}_{D0} = -\,g_{D0} - jb_{D0}\,,$$

and

$$s\,\bar{y}_{S1} = s\,g_{S1} + j\,s\,b_{S1}\,,$$

$$s\,\bar{y}_{D1} = s\,g_{D1} + j\,s\,b_{D1}\,. \tag{5—10}$$

In Fig. 5—2 (giving the same result as Fig. 5—1) the vectors

$$\bar{y}_S = \bar{y}_{S0} + s\,\bar{y}_{S1}$$

and

$$\bar{y}_D = \bar{y}_{D0} + s\,\bar{y}_{D1}$$

has been constructed on the basis of relations (5—10).

For applying the method suggested in *chapter 3*, first of all the slip must be expressed from Eq. (5—7):

$$s(\delta) = \frac{g + g_{S0} + g_{D0}\cos 2\delta + b_{D0}\sin 2\delta}{g_{S1} + g_{D1}\cos 2\delta + b_{D1}\sin 2\delta}. \qquad (5-11)$$

As follows from formula (5—11), the slip now depends instead of four constants upon seven ones. To reduce the number of parameters, introduction of some symbols seems to be practical. Let it be:

$$g + g_{S0} = g'. \qquad (5-12)$$

where $g' < 0$, if $g < 0$ and $|g| > |g_{S0}|$.
Further

$$\frac{g_{D0}}{g'} = \alpha_0. \qquad (5-13)$$

where $\alpha_0 \lesseqgtr 0$, i $g_{d0} \lesseqgtr g_{q0}$ and $g' < 0$.
Further

$$-\frac{b_{D0}}{g'} = \beta_0. \qquad (5-14)$$

where $\beta_0 > 0$, if $b_{q0} > b_{d0}$ and $g' < 0$.
And besides

$$-\frac{g_{D1}}{g_{S1}} = \varkappa_1, \qquad (5-15)$$

where $\varkappa_1 \lesseqgtr 0$, if $g_{d1} \lesseqgtr g_{q1}$.
And

$$-\frac{b_{D1}}{g_{S1}} = \lambda_1, \qquad (5-16)$$

where $\lambda_1 > 0$, if $b_{d1} > b_{q1}$.
Finally

$$\frac{g'}{g_{S1}} = s_0'. \qquad (5-17)$$

where $s_0' < 0$, if $g' < 0$.

By notations (5—9) ... (5—14), Eq. (5—8) may be written as follows

$$s(\delta) = s_0' \frac{1 + \alpha_0 \cos 2\delta - \beta_0 \sin 2\delta}{1 - \varkappa_1 \cos 2\delta - \lambda_1 \sin 2\delta}. \qquad (5-18)$$

Let it be

$$\varepsilon_0 = \operatorname{arc\,tg} \frac{\alpha_0}{\beta_0} \qquad (5-19)$$

and

$$\beta' = \sqrt{\beta_0^2 + \alpha_0^2}, \tag{5—19}$$

further

$$\varepsilon_1 = \text{arc tg} \frac{\lambda_1}{\varkappa_1}$$

and (5—20)

$$\varkappa' = \sqrt{\varkappa_1^2 + \lambda_1^2}.$$

Hence, considering notations (5—13) ... (5—16), the first equation of (5—19) and (5—20) may also be written in the following form

$$\varepsilon_0 = \text{arc tg} \left(- \frac{g_{D0}}{b_{D0}} \right)$$

and (5—21)

$$\varepsilon_1 = \text{arc tg} \frac{b_{D1}}{g_{D1}} .$$

On this principle the interpretation of angles ε_1 and ε_0 may be seen in Fig. 5—2. ε_1 is the angular displacement of vector $s\,\bar{y}_{D1}$ as compared with the positive real axis, while ε_0 is the angular displacement of vector $\bar{y}_{D0}$ with respect to the negative imaginary axis. It must be noted, that in case of $g_{d1} > g_{q1}$ if $b_{d1} \lesseqgtr b_{q1}$ then $\varepsilon_1 \lesseqgtr 0$, while in case of $b_{q0} > b_{d0}$ if $g_{d0} \lesseqgtr g_{q0}$ then $\varepsilon_0 \lesseqgtr 0$.

Taking into account relations (5—19) and (5—20), (5—18) may have the following form

$$s(\delta) = s_0' \frac{1 - \beta' \sin (2\delta - \varepsilon_0)}{1 - \varkappa' \cos (2\delta - \varepsilon_1)} . \tag{5—22}$$

Let us introduce a new variable for the angular displacement of the rotor, let it be

$$\delta' = \delta_0 - \frac{\varepsilon_0}{2} , \tag{5—23}$$

then evidently

$$2\delta - \varepsilon_0 = 2\delta' \tag{5—24}$$

and

$$2\delta - \varepsilon_1 = 2\delta' - (\varepsilon_1 - \varepsilon_0) = 2\delta' - \varepsilon,$$

where $\varepsilon = \varepsilon_1 - \varepsilon_0$. So lastly from (5—22):

$$s'(\delta') = s_0' \frac{1 - \beta' \sin 2\delta'}{1 - \varkappa' \cos (2\delta' - \varepsilon)} . \tag{5—25}$$

Expression (5—25) seems to be quite similar to (4—26). Essential difference is not introduced, but by angle ε figuring in the denominator.

The fundamental integral now with the initial condition $\delta_0' = 0$, *i. e.* in case of $\delta_0 = \frac{\varepsilon_0}{2}$:

$$\omega_0 t = -\frac{1}{s_0'} \int\limits_0^{\delta'} \frac{1 - \varkappa' \cos(2\delta' - \varepsilon)}{1 - \beta' \sin 2\delta'} \, d\delta',$$

i. e. after a trigonometrical transformation and rearrangement

$$-s_0' \omega_0 t = \int\limits_0^{\delta'} \frac{\varkappa'}{\beta'} \sin \varepsilon \cdot d\delta' +$$

$$+ \int\limits_0^{\delta'} \frac{1 - \frac{\varkappa'}{\beta'} \sin \varepsilon}{1 - \beta' \sin 2\delta'} \, d\delta' + \int\limits_0^{\delta'} \frac{-\frac{\varkappa' \cos \varepsilon}{2\beta'} 2\beta' \cos 2\delta'}{1 - \beta' \sin 2\delta} \, d\delta',$$

consequently

$$-s_0' \omega_0 t = \left(\frac{\varkappa'}{\beta'} \sin \varepsilon\right) \delta' + \frac{1 - \frac{\varkappa'}{\beta'} \sin \varepsilon}{\sqrt{1 - \beta'^2}} \operatorname{arc\,tg} \frac{\sqrt{1 - \beta'^2} \operatorname{tg} \delta'}{1 - \beta' \operatorname{tg} \delta'} + \frac{\varkappa' \cos \varepsilon}{2\beta'} \ln(1 - \beta' \sin 2\delta'). \tag{5—26}$$

The above formula (5—26) is a generalization of formula (4—28); the similarity and disagreement between the two formulae is obvious.

Naturally, by substituting the relation (5—23) into function $t(\delta')$ determined by formula (5—26), also the function $t(\delta)$ may be determined, if required.

By formulae (5—25) and (5—26) the wanted slip-time function $s(t)$ is given in a parametric form through angle δ' for the case of the oblique straight line approximation; further the inversion of (5—26) supplies the wanted angle-time relation $\delta'(t)$. Substituting the latter functions into expressions (5—6) and (5—8), respectively, considering Eq. (5—23). the functions $\dot{y}(t)$ and $b(t)$, respectively, may be obtained.

Finally, substitution of the value $\delta' = 2\pi$ into the right side of (5—26) yields the relation

$$\omega_0 T = \frac{2\pi}{s_0'} \, \frac{1 - (1 - \sqrt{1 - \beta'^2}) \frac{\varkappa'}{\beta'} \sin \varepsilon}{\sqrt{1 - \beta'^2}} \tag{5—27}$$

for the period and

$$s'_m = s'_0 \frac{\sqrt{1-\beta'^2}}{1-(1-\sqrt{1-\beta'^2})\,\frac{\varkappa'}{\beta'}\sin\varepsilon} \qquad (5-28)$$

for the medium slip.

5.2. *Some special cases*

In the following only some special cases will be discussed, when in formulae (5—25) and (5—26), respectively, an essential structural simplification may be achieved.

a) In the most important special case $\varepsilon = 0$, *i. e.* $\varepsilon_1 = \varepsilon_0$. This means (Fig. 5—2) the vectors $\bar{y}_{D0}$ and $s\,\bar{y}_{D1}$ being perpendicular to one another, $s\,\bar{y}_{D1}$ leading, namely, with respect to vector $\bar{y}_{D0}$. Then from Eq. (5—25)

$$s'(\delta') = s'_0 \frac{1-\beta'\sin 2\delta'}{1-\varkappa'\cos 2\delta'} \qquad (5-29)$$

and from Eq. (5—26)

$$-s'_0\,\omega_0\,t = \frac{1}{\sqrt{1-\beta'^2}}\,\mathrm{arc\,tg}\,\frac{\sqrt{1-\beta'^2}\,\mathrm{tg}\,\delta'}{1-\beta'\,\mathrm{tg}\,\delta'} + \frac{\varkappa'}{2\beta'}\ln(1-\beta'\sin 2\delta'). \qquad (5-30)$$

Consequently, with formulae (4—26) and (4—28) structurally identical expressions arise. In this case all the results and diagrams of the primitive approximation described in *chapter 4* may directly be applied.

b) If $\varepsilon = \pm\pi$ that is $\varepsilon_1 - \varepsilon_0 = \pm\pi$, so vector $s\,\bar{y}_{D1}$ is lagging by 90° with respect to vector $\bar{y}_{D0}$, then

$$s'(\delta') = s'_0 \frac{1-\beta'\,\mathrm{sing}\,2\delta'}{1+\varkappa'\cos 2\delta'} \qquad (5-31)$$

and

$$-s'_0\,\omega_0\,\mathrm{t} = \frac{1}{\sqrt{1-\beta'^2}}\,\mathrm{arc\,tg}\,\frac{\sqrt{1-\beta'^2}\,\mathrm{tg}\,\delta'}{1-\beta'\,\mathrm{tg}\,\delta'} - \frac{\varkappa'}{2\beta'}\ln(1-\beta'\sin 2\delta'). \qquad (5-32)$$

From this it becomes clear, that all results and diagrams of the primitive approximation described in *chapter 4* may further be adopted, merely case $\varkappa' > 0$ must correspond to case $\varkappa < 0$.

c) The case

$$\varepsilon = \varepsilon_1 - \varepsilon_0 = +\frac{\pi}{2}, \qquad \varkappa' = \beta'$$

is remarkable, too. Then Eqs. (5—25) and (5—26) become very simple

$$s'(\delta') = s_0' \tag{5—33}$$

and

$$-s_0'\omega_0 t = \delta', \tag{5—34}$$

that is, the slip is constant and the angle increases uniformly.

This occurs evidently if $\bar{y}_D = 0$, *i. e.* the circular diagram of the resultant admittance vector is reduced to a point. As condition $\varepsilon_1 = \varepsilon_0 + \frac{\pi}{2}$ means the two components $\bar{y}_{D0}$ and $s\,\bar{y}_{D1}$ of $\bar{y}_D$ being of opposite direction, condition $\varkappa' = \beta'$ signifies the absolute value of the two components being equal.

The condition $\bar{y}_D = 0$ takes place either at any arbitrary slip s, or at a certain critical slip s_k. In the first case the straight lines of admittances $\bar{y}_d$ and $\bar{y}_q$ coincide, and for any arbitrary value $g =$ const. the slip is always constant. (This occurs, if the rotor is symmetrical.) In the second case the straight lines of admittances $\bar{y}_q$ and $\bar{y}_d$ intersect each other at a certain point and just at the critical slip s_k. The straight line $g =$ const. pass just through the point of intersection.

d) If

$$\varepsilon = \varepsilon_1 - \varepsilon_0 = +\frac{\pi}{2}, \qquad \varkappa' \neq \beta',$$

so $\bar{y}_{D0}$ and $s\,\bar{y}_{D1}$ are of opposite direction, but for all slips $\bar{y}_D \neq 0$.

Then from Eq. (5—25):

$$s'(\delta') = s_0' \frac{1-\beta'\sin 2\delta'}{1-\varkappa'\sin 2\delta'} \tag{5—35}$$

and from Eq. (5—26)

$$-s_0'\omega_0 t = \frac{\varkappa'}{\beta'}\delta' + \frac{1-\dfrac{\varkappa'}{\beta'}}{\sqrt{1-\beta'^2}} \operatorname{arc\,tg} \frac{\sqrt{1-\beta'^2}\,\operatorname{tg}\delta'}{1-\beta'\operatorname{tg}\delta'}. \tag{5—36}$$

e) If $\bar{y}_{D0} = 0$, *i. e.* the two, not congruent, straight lines start from the same point, then $a_0 = 0$, $\beta_0 = 0$ and $\beta' = 0$, consequently from (5—25)

$$s'(\delta') = \frac{s_0'}{1-\varkappa'\cos(2\delta'-\varepsilon)} \tag{5—37}$$

Accordingly, substituting

$$2\delta' - \varepsilon = 2\delta''$$

yields

$$s''(\delta'') = \frac{s_0'}{1 - \varkappa' \cos 2\delta''}. \tag{5—38}$$

The structure of formula (5—38) is in full agreement with that of formula (4—11). In the following we have to proceed further according to *clause 4.2.*

f) If $\bar{y}_{D1} = 0$ (*i. e.* the vectors $\bar{y}_D$ and $\bar{y}_{L0}$ are always of identical direction and magnitude, $\bar{y}_D = \bar{y}_{D0}$ for any slip), that is $g_{D1} = 0$ and $b_{D1} = 0$ so $\varkappa_1 = 0$, $\lambda_1 = 0$ and $\varkappa' = 0$, then from (5—25):

$$s'(\delta') = s_0'(1 - \beta' \sin 2\delta'). \tag{5—39}$$

As the structure of formulae (5—39) and (4—16) is identical, we have to proceed further on as in *clause 4.3.*

5.3. *Piecewise-linear approximation*

If the direct-axis, or quadrature-axis admittance diagram (or both) are considerably curved, or the distance between the adjacent points of each curve essentially changes with the slip, the admittance diagram cannot be approximated by a single straight line, but it must be approximated with two, or more straight sections. Each straight section is starting from an arbitrary point, where already the slip $s \neq 0$, but $s = s_i$ and is ending at a point belonging to a certain value $s = s_e$.

Accordingly, introducing the symbol $\Delta s = s - s_i$ a relation similar to Eq. (5—6) may be established for the range $s_i \leqq s \leqq s_e$ *i. e.* for the range $0 \leqq \Delta s < s_e - s_i$:

$$\begin{aligned} y = (- g_{S0} + g_{S1} \Delta s) - j(b_{S0} - b_{S1} \Delta s) + \\ + \left[(- g_{D0} + g_{D1} \Delta s) + j(b_{D0} - b_{D1} \Delta s)\right] e^{j2\delta}. \end{aligned} \tag{5—40}$$

Introducing the notations

$$\begin{aligned} g_{S0} + s_i g_{S1} &= g_{S0}', \\ g_{D0} + s_i g_{D1} &= g_{D0}', \\ b_{S0} + s_i b_{S1} &= b_{S0}', \\ b_{D0} + s_i b_{D1} &= b_{D0}', \end{aligned} \tag{5—41}$$

(5—40) may also have the following form:

$$\begin{aligned} y = (- g_{S0}' + s g_{S1}) - j(b_{S0}' - s\, b_{S1}) + \\ + \left[(- g_{D0}' + s g_{D1}) + j(b_{D0}' - s\, b_{D1})\right] e^{j2\delta}, \end{aligned} \tag{5—42}$$

or resolving into components:

$$g = (-g'_{S0} + s g_{S1}) + (-g'_{D0} - s g_{D1}) \cos 2\delta - (b'_{D0} - s b_{D1}) \quad \sin 2\delta \, , \quad (5-43)$$

$$b = \; (b'_{S0} \; - \; s b_{S1}) \; - (b'_{D0} - \; s b_{D1}) \; \cos \; 2\delta \; - (-g'_{D0} + s g_{D1}) \sin 2\delta \, . \quad (5-44)$$

As it follows from the aforesaid, the expressions (5—42), (5—43), (5—44) have exactly the same form as (5—6), (5—7), (5—8). Consequently we succeeded in tracing back the formulae of the piecewise-linear approximation to those of the approximation by oblique straight lines, so the procedure described in *clause 5.1* may be repeated step-by-step in the following (merely at starting Eq. (5—41) must be taken into account).

Approximating by several sections, the final slip s_e of the previous section naturally means the initial slip s_i of the consecutive section. As for each section the numerical value of the constants figuring in the formulae have to be determined repeatedly, the calculating work increases with the number of the sections.

A comparison of the results obtained with the piecewise-linear approximation and the data of measurements will be found for a certain case later on (in *clause 8.5* of *chapter 8*).

Summary

This paper presents the result of applying the general theoretical method suggested in the foregoing [1] in connection with the asynchronous operation of turbo-generators for the most simple case, when the direct-axis and quadrature-axis admittance diagram is approximated by one or more straight lines.

In case of the generalized linear approximation the slip-angle function may be calculated by Eq. (5—22), or (5—25), the time-angle function, however, by Eq. (5—26). The same formulas may be adopted for the piecewise-linear approximation, too.

In case of the primitive linear approximation, the slip-angle function may be determined by Eq. (4—26), while the time-angle function by Eq. (4—28). For the latter case sets of curves are constructed to demonstrate the effect of the competent parameters.

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