THE CHARACTERISTICS OF HOMOGENEOUS NEUTRON AMPLIFIERS

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1. Introduction

In a former publication [1] we examined some questions connected with the amplification factor and calculation of multiplier systems functioning as a neutron amplifier. In the present work numerical results are published on the amplification factor and costs of homogeneous amplifiers of simple construction. The meaning of the symbols used later, see [1].

Of the amplification factors defined in several ways, two will be emphasized as especially characterizing ones. The one is the *thermal amplification factor* :

$$K_T = \frac{1}{Q}\int_V \Sigma_a(\mathbf{r})\,\Phi(\mathbf{r})\,\mathrm{d}V = \frac{p}{k-1}K. \tag{1}$$

The other is the *fission amplification factor* :

$$K_F = \frac{1}{Q}\int_V \nu_0\,\Sigma_f(\mathbf{r})\,\Phi(\mathbf{r})\,\mathrm{d}V = \frac{k}{k-1}K \tag{2}$$

where K is the *normalized amplification factor*.

Further the concept of the *degree of criticality* was introduced:

$$s^3 = \frac{M}{M_0} = \frac{V}{V_0}. \tag{3}$$

Another, though less usable, characteristic of the state of criticality is negative reactivity:

$$-\varrho = \frac{1-k_{\mathrm{eff}}}{k_{\mathrm{eff}}}, \quad k_{\mathrm{eff}} = k\frac{e^{-B^2\tau}}{1+B^2L^2} \approx \frac{k}{1+B^2M^2}. \tag{4}$$

A further characteristic of the amplifier is its *cost* (Z), what is in essence equal to the cost of the fissionable material consumed and eventually of the moderator. Cost is a better parameter than the quantity of the material consumed, because in this way the influence of *e. g.* enriched uranium and of heavy water can be examined.

By a prescribed degree of criticality, several designs with different amplification factors and costs, in function of material composition and geometry, were arrived at. The best characteristic curve of the amplifier plots just the amplification factor in function of necessary costs. On comparing two amplifiers, evidently that one is regarded more advantageous, which ensures a higher amplification factor by equal costs or by which lower costs are necessary to obtain the same amplification factor.

It was stated that in the near-critical state of the system the various approximative calculation methods produce results not differring greatly. In the followings, therefore, calculations will be carried out on the basis of the one or two-group approximation. To make results more easy to survey, further neglections are generally applied.

It was supposed that the value of τ and of the diffusion coefficients D_T and D_F is not influenced by the dilution of fuel. Namely calculations would be complicated by taking into account this effect, too, and results are influenced by small corrections only negligibly.

It should be kept in mind, that the results of the one or two-group calculation can only be regarded as informative values. If the moderator is water, the error is particularly high. Consequently our results, as listed in the following, are qualitatively right, but their numerical value should be corrected by measurements.

2. Bare spherical amplifier

In the cited paper, the distribution of neutron flux in the homogeneous, bare spherical system, with the source being in the centre, was calculated.

For the normalized amplification factor the value

$$K = \frac{\varkappa R}{\sin \varkappa R} - 1. \tag{1}$$

It is characterized by the fact of growing infinite, if $\varkappa R = \pi$, that is, if the radius of the system is equal with the

$$R_0 = \frac{\pi}{\varkappa} = \pi \frac{M}{\sqrt{k-1}} \tag{2}$$

critical radius. If the system is in a near-critical state, so

$$R = s R_0 = (1 - \varepsilon) R_0, \quad \varepsilon \ll 1. \tag{3}$$

$$K = \frac{(1-\varepsilon)\pi}{\sin \varepsilon \pi} - 1 \approx \frac{1 - 2\varepsilon}{\varepsilon}. \tag{4}$$

In case of $\varepsilon < 0{,}08$ ($s > 0{,}92$, $s^3 > 0{,}78$), the approximation means an error of less than 1 p. C. In the critical state the limit value of the εK expression is:

$$G = \lim_{\varepsilon \to 0} \varepsilon K = 1. \tag{5}$$

It is known that the prescription of the degree of criticality means a determined negative reactivity. In one-group approximation, according to [1 (5.11)] approximately

$$K = \frac{k-1}{k} \frac{2}{-\varrho} - \frac{1}{2}, \tag{6}$$

where the first member is much greater than the second. In the critical condition:

$$G^* = \lim_{\varrho \to 0} (-\varrho K) = 2 \frac{k-1}{k}. \tag{7}$$

The expression for the fission amplification factor in case of a small negative reactivity:

$$K_F = \frac{k}{k-1} K \approx \frac{2}{-\varrho}. \tag{8}$$

In connection with expression (8) the following should be noted. In the literature the following line of thought is often met with. Let us place into a system, having an effective multiplication factor k_{eff}, a neutron source of Q intensity. At the end of the first neutron cycle $k_{\text{eff}}Q$ fast neutrons are produced. During the second neutron cycle, beside the Q source neutrons and the multiplied $k_{\text{eff}}Q$ neutrons, the k_{eff}-fold of the previously multiplied $k_{\text{eff}}Q$ neutrons, *i. e.* $k_{\text{eff}}^2 Q$ neutrons are produced, and so on. The limit value of the number of neutrons in the system is:

$$Q_F = Q + k_{\text{eff}} Q + k_{\text{eff}}^2 Q + \ldots = \frac{Q}{1 - k_{\text{eff}}} \approx Q \frac{1}{-\varrho}, \tag{9}$$

if $k_{\text{eff}} < 1$.

Using the other way (8) means a number of neutrons which is just the double of those obtained by the above reasoning. This is, however, incorrect, because the multiplication of the number of neutrons during an effective neutron cycle is given by the k_{eff} effective multiplication factor, if neutrons are produced by fission, that is the distribution of source intensity is determined by neutron distribution. If, however, as in the present case, there are extraneous sources, then not the k_{eff} value characterizes their multiplication. The number of fast neutrons produced at the end of the first neutron cycle is $k_1 Q$, where k_1 is a function of the effective multiplication factor and naturally of

the position of the source, too. The $k_1 Q$ neutrons are again multiplied at the end of the second neutron cycle, what is characterized by multiplication factor k_2, for the calculation of which we should have to know neutron distribution, immediately before the second fission act. The same can be said of the succeeding processes, consequently the limit value of the number of produced fast neutrons is:

$$Q_F = Q + k_1 Q + k_1 k_2 Q + k_1 k_2 k_3 Q + \ldots \tag{10}$$

Neutron distribution as a limit value is not influenced by the position of the extraneous source, consequently

$$\lim_{n \to \infty} k_n = k_{\text{eff}}. \tag{11}$$

Practically, this situation already arises after a few neutron cycles. Let us suppose that $k_n = k_{\text{eff}}$, if $n \geqq 3$, so

$$Q_F = Q \left[1 + k_1 + \frac{k_1 k_2}{1 - k_{\text{eff}}} \right]. \tag{12}$$

From expression (9) the conclusion is drawn that in the case of $k_{\text{eff}} \to 1$ the inverse value of the C counting rate, that can be measured proportionally with the Q_F number of neutrons, approximates zero. This is true, however, according to (12), in the general case too, as

$$\frac{1}{Q_F} = \frac{1}{Q} \frac{1 - k_{\text{eff}}}{(1 + k_1)(1 - k_{\text{eff}}) + k_1 k_2} \to 0, \quad \text{if} \quad k_{\text{eff}} \to 1. \tag{13}$$

It is evident that this limit value is independent of the number of neutron cycles after which neutron distribution becomes independent of the position of the source.

3. Bare cylindrical amplifier

Let us examine the influence of the geometry of the equipment on the expression of the amplification factor. For this purpose the amplification factor is determined for a cylindrical arrangement the (extrapolated) radius and half-height of which are R and H, respectively, and the point-like neutron source is at the centre. By one-group approximation the following expression is obtained for normalized amplification factor:

$$K = 2 \sum_{n=1}^{\infty} \frac{1 - \cos \varkappa_n H}{\xi_n J_1(\xi_n) [1 - (\xi_n / \varkappa R)^2] \cos \varkappa_n H}, \tag{1}$$

where J_0 and J_1 are the Bessel functions of the first kind and of the zero and first order, respectively. $J_0(\xi_n) = 0$, further

$$\varkappa_n^2 = \varkappa^2 - \left(\frac{\xi_n}{R}\right)^2 = \frac{k-1}{M^2} - \left(\frac{\xi_n}{R}\right)^2. \tag{2}$$

The radius R is regarded as a given value because in this way the boundary condition is fulfilled. The critical half-height pertaining to the given radius is:

$$H_0 = \frac{\pi}{2\varkappa_1} = \frac{\pi}{2}\frac{R}{\sqrt{(\varkappa R)^2 - \xi_1^2}}, \quad R > \frac{\xi_1}{\varkappa}. \tag{3}$$

In case of a given material composition, the volume and simultaneously the costs have minimum values, if

$$R = R_m = \sqrt{\frac{3}{2}}\frac{\xi_1}{\varkappa} = \frac{2{,}94}{\varkappa}, \quad H_0 = H_{0m} = \frac{\sqrt{3}\,\pi}{2\varkappa} = \frac{2{,}72}{\varkappa}, \tag{4}$$

as is known from reactor technics.

If the amplifier is in near-critical condition, so the higher members of the infinite series under (1) beside the first member can be neglected. The radius can be written in the following form: $R = \lambda R_m$ and so:

$$K = \frac{1{,}6}{1-(2/3\,\lambda^2)}\,\frac{1-\sin(1-s^3)\,\pi/2}{\sin(1-s^3)\,\pi/2} \approx \frac{1{,}02}{1-(2/3\,\lambda^2)}\,\frac{1}{1-s^3}. \tag{5}$$

In the case of minimum volume $\lambda = 1$, further $1 - s^3 \approx 3\,\varepsilon$, therefore

$$K_m \approx \frac{1{,}02}{\varepsilon}, \tag{6}$$

which is 2 p. C. higher than expression (2.4) deduced for a sphere. If $\lambda < 1$, so the value for K will be higher.

By expressing amplification factor with reactivity, in linear approximation

$$K = \frac{k-1}{k}\,\frac{2{,}04}{-\varrho} + 1{,}02\,\frac{1}{1-(2/3\,\lambda^2)}. \tag{7}$$

Somewhat different expressions are obtained for maximum volume, if negative reactivity is regarded as given at the outset.

4. The effect of reflector

The calculation of reflected systems is generally a difficult task, because the one-group approximation is very rough. In the following the simplest case is examined: a sphere of radius R is surrounded by a reflector of infinite thick-

ness, the material of which is identical with that of the moderator in the active zone.

By calculating with the two-group approximation, with the usual neglection ($\mu^2 \ll \nu^2$, $\nu R \gg 1$), the normalized amplification factor will be:

$$K = \frac{\mu R + u}{\sin \mu R + u \cos \mu R} - 1 , \tag{1}$$

where

$$u = \mu L_0 \frac{1 + \nu \sqrt{\tau} + \beta_0 \sqrt{\tau}/L_0}{1 + \nu \sqrt{\tau} + \beta_0} , \tag{2}$$

$$\beta_0 = \frac{\alpha}{\beta} \frac{1 + \beta/\gamma}{1 - \alpha/\gamma} (1 + \nu L_0) . \tag{3}$$

The critical dimension from (1):

$$R_0 = \frac{\pi}{\mu} - \frac{1}{\mu} \operatorname{arc\,tg} u = \frac{\pi}{\mu} - T , \tag{4}$$

where the value T is the reflector savings. The limit of the $(1-s)$ K expression is:

$$G = \lim_{\varepsilon \to 0} \varepsilon K = \frac{1}{\sqrt{1 + u^2}} \left| 1 + \frac{u}{\pi - \operatorname{arc\,tg} u} \right| . \tag{5}$$

The correlation between the degree of criticality and negative reactivity is hardly influenced by the reflector.

The limit value for expression $(-\varrho K)$ is:

$$G^* = \lim_{\varrho \to 0} (-\varrho K) = 2 \frac{k - 1}{k} \left| 1 - \frac{1}{\pi} \operatorname{arc\,tg} u \right| . \tag{6}$$

The limit value of fission amplification is proportional to $(-1/\varrho)$.

5. Material characteristics

To illustrate the above and following calculations, let us examine the variation of the amplification factor and of costs in the following case:

1. Fuel is uranium, enriched to 20 p. C., the world price of which is $3{,}22 \cdot 10^3$ dollars/kg. Fuel is diluted in the moderator in the form of uranyl sulfate (UO_2SO_4).

2. The moderator is common water.

The ratio of fuel and moderator is characterized by dilution g:

$$g = \frac{N_0}{N_U} , \tag{1}$$

where N_0 and N_U is the number of water and uranium atoms, respectively, in 1 cm^3.

3. If no other data are given, the actual active volume is 90 p. C. of the critical volume, *i. e.* $s^3 = 0{,}9$ ($s = 0{,}9655$).

4. For fuel diluted in heavy water we give no results. According to our examinations not published here: heavy water is less advantageous than common water, both in respect to amplification factor and costs. In special arrangements, however, also heavy water is applied. The world price of heavy water is about 60 dollars/kg.

5. The cost of the source is left out of calculations, because it is much lower than the other costs.

6. Some numerical results

In the enclosed diagrams the results of our calculations are listed. The following arrangements were examined (letter markings in accordance with those on the various curves):

a) Bare sphere.

b) Bare cylinder, $R = R_m$.

c) Bare cylinder, $R = 0{,}9\ R_m$.

d) Bare cylinder, $R = 0{,}85\ R_m$.

e) Sphere with a reflector of infinite dimensions, where R_m is the radius ensuring minimum volume.

On Figs. 1 and 2 the critical radius of the spherical system and the critical dimensions of the cylindrical arrangement are plotted in function of dilution. On Figs. 3 and 4 amplification factors K_T and K_F, respectively, are plotted in function of dilution in case of $s^3 = 0{,}9$. The curves for cases a) and b) are practically identical. On Fig. 5 the costs of uranium are shown. Costs have a minimum value in the range $g = 60-80$.

Amplification factors depend on the degree of criticality s only through the normalized amplification factor. The $\varphi(s)$ curves shown on Fig. 6 are identical with K in the case of a bare sphere, while for the case of a cylinder

$$\varphi(s) = 3\left(1 - \frac{2}{3\lambda^2}\right) K(s). \tag{1}$$

With the help of the diagram, amplification factors shown on Figs. 4—5 can be easily calculated for other degrees of criticality. Costs are simply in proportion with the s^3 value.

With the help of Figs. 3—5 the best characteristic of the amplifier can be determined: amplification factor in function of costs. This is shown for the examined cases on Figs. 7 and 8. On the basis of these the following statements can be made. Relatively small amplification factors ($K_T < 200$, $K_F < 300$)

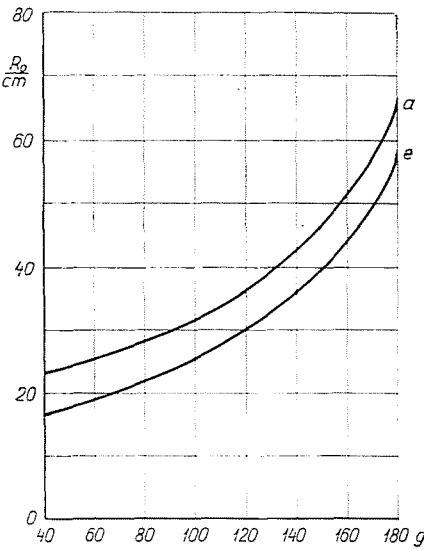

Fig. 1. Critical radius of the bare and reflected sphere in function of dilution. Curve "a": Bare sphere. Curve "e": Sphere with infinite reflector

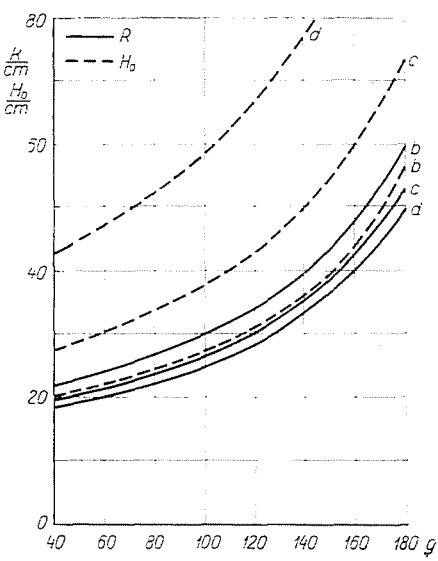

Fig. 2. Critical radius and critical half-height of the bare cylinder in function of dilution. Curve "b": $R = R_m$. Curve "c": $R = 0{,}9\ R_m$. Curve "d": $R = 0{,}85\ R_m$

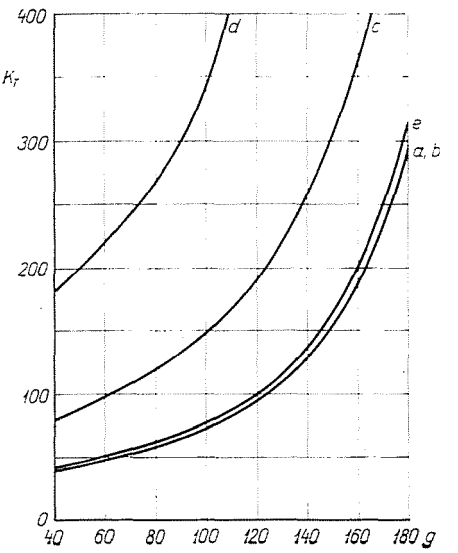

Fig. 3. K_T thermal amplification factor in function of dilution

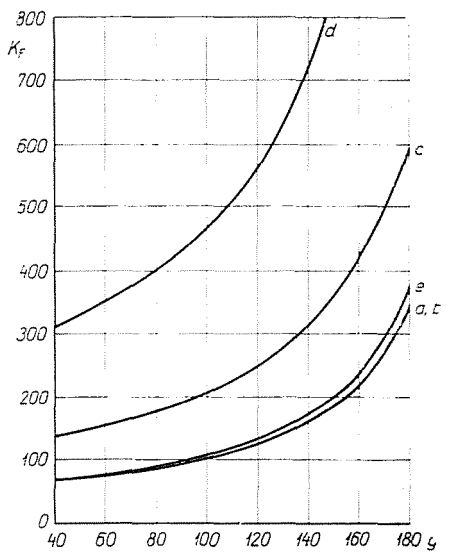

Fig. 4. K_F fast amplification factor in function of dilution

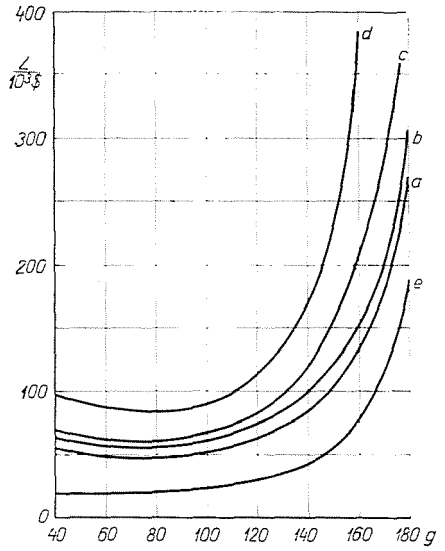

Fig. 5. Costs Z of consumed uranium in function of dilution

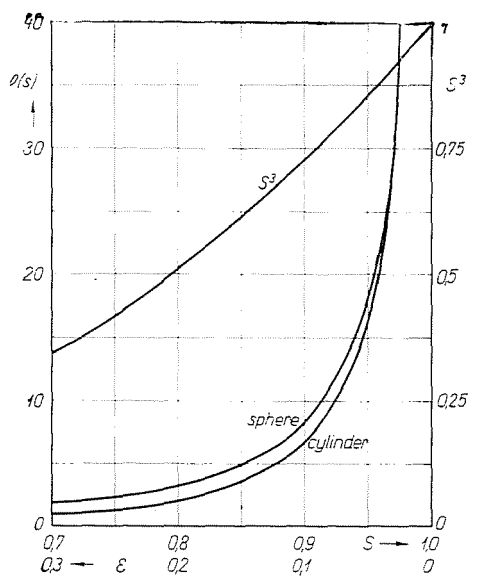

Fig. 6. The function $\varphi(s)$ which characterizes the dependence of the amplification factor of the degree of criticality

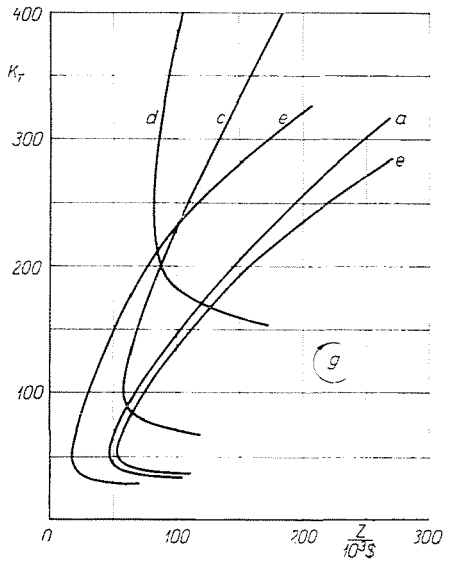

Fig. 7. Thermal amplification factor in function of costs

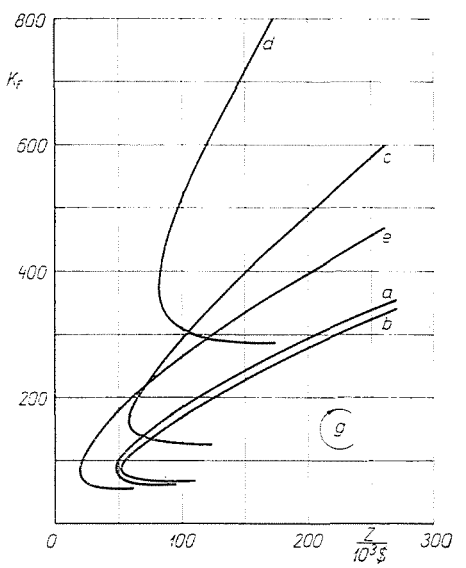

Fig. 8. Fast amplification factor in function of costs

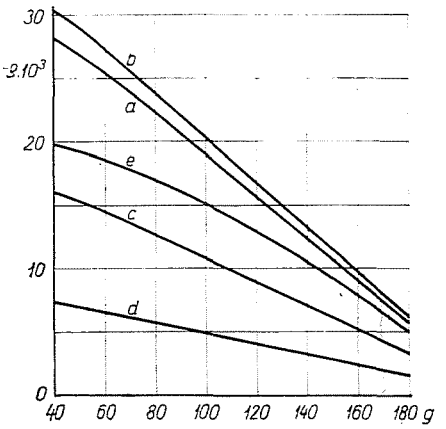

Fig. 9. Negative reactivity in function of dilution

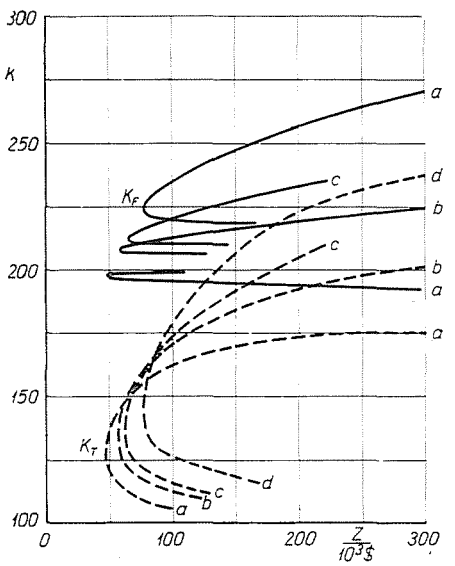

Fig. 10. The characteristics of neutron amplifiers in case of constant negative reactivity ($\varrho = -0{,}01$)

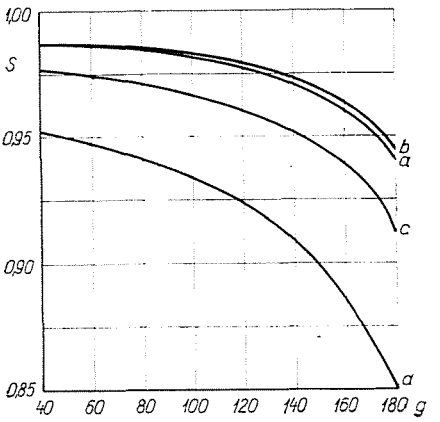

Fig. 11. Degree of criticality (s) pertaining to a negative reactivity of $\varrho = -0{,}01$, in function of dilution (g)

can best be achieved by a reflected spherical amplifier. Further, the bare spherical amplifier is somewhat better than the bare cylindrical one, as costs pertaining to a given amplification factor are steadily growing in the said order. Amplification factors higher than the mentioned ones are advisably realized rather by longer cylinders than by a sphere for which dilution differs from the value ensuring minimum costs. In general, the solution ensuring minimum costs can always be found for any given amplification factor.

On Fig. 9 negative reactivity in function of dilution is shown. On Fig. 10 the characteristic of the amplifiers is shown for the case, when $\varrho = -0{,}01$, a constant. The characteristics are similar to those on Figs. 7 and 8, where the degree of criticality was constant. On Fig. 11 the values of s are still plotted in function of g, if $\varrho = -0{,}01$.

7. The application of an inner reflector

In the following the question will be examined as to whether the amplification factor pertaining to a given cost can be increased by the application of special arrangements. The more or less evident result should, however, be mentioned, in advance, that only systems with a cost higher than the minimum can in this way be made. The amplification factors of these, however, are higher than in the case when increased costs are brought about by altering the dilution pertaining to the minimum cost.

The first special arrangement examined is the following: To ensure a better neutron economy, the source is not placed immediately into the active zone, but is surrounded by a moderator layer (Fig. 12). Consequently, fast source neutrons arrive into the active zone more or less slowed down, whereby the average resonance escape probability is increased.

Calculation is carried out for the case when there is no outside reflector (on Fig. 1. $c = 0$). For calculation purposes dimension a is given various values and characteristics plotted with the help of this parameter. If common water is used for both solvent and moderator, then it is evident that dimension of the inside reflector should not be more than some cm-s, because in case of an

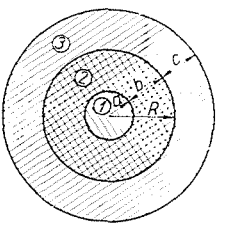

Fig. 12. Spherical amplifier with inner reflector. 1 and 3: moderator, 2: active zone

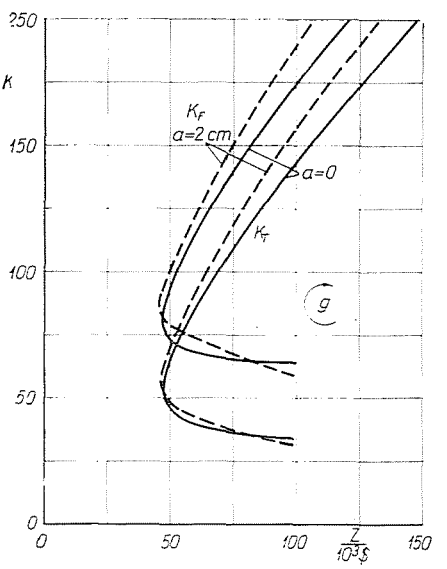

Fig. 13. Influence of the inner reflector on the characteristics

excessively great dimension thermal neutrons are intensively absorbed by that layer. Our previous examinations, in turn, have shown that the use of heavy water means a considerable cost increase.

It is evident from the charasteristics (Fig. 13), that conditions are improved by using a 1—2 cm thick reflector only to a very small extent. However, because of technological difficulties the application of this method is not worth while.

8. Two-layer arrangement

The value of both the thermal and the fission amplification factor can be increased by increasing the dimensions of the active zone. As the degree of criticality is prescribed, the dimension of the active zone can only be increased if its critical dimension is somehow also increased. One possible way of increasing the critical dimension is the arrangement of absorbents in the fissionable material. By a spherical arrangement *e. g.* a sphere surface can be formed of a thin cadmium layer which divides the active zone into two parts (Fig. 14). It was supposed during calculations that thermal neutrons are completely absorbed by the absorbent layer, however, it is of no influence at all on fast neutrons.

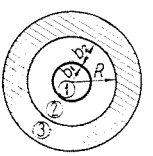

Fig. 14. Two-layer arrangement
1 and 2: active zone, 3: reflector, between 1 and 2: absorbent layer

The arrangement of the absorbent layer naturally does not increase the amplification factor unequivocally as the distortion of the flux has a diminishing effect. Beyond this, increasing the dimensions of the active zone increases costs, too. As a last result, only detailed examination can decide, whether there is an optimum b_1 dimension and g dilution which ensures more advantageous conditions by a one layer arrangement.

Calculations were carried out for the unreflected case. The following relationship for the normalized amplification factor was obtained:

$$K = \frac{\beta_0 \mu b_1 \sin \mu b_2 + \mu b_1 + \mu b_2}{\beta_0 \sin \mu b_1 \sin \mu b_2 + \sin \mu (b_1 + b_2)} - 1 , \tag{1}$$

$$\beta_0 = 2 \frac{\nu}{\mu} \frac{\beta}{\alpha} . \tag{2}$$

If dimension b_1 (or b_2) is given, critical dimension b_{20} (or b_{10}) can be calculated from the following equation:

$$- \operatorname{tg} \mu b_{20} = \frac{\operatorname{tg} \mu b_1}{\beta_0 \operatorname{tg} \mu b_1 + 1} = u . \tag{3}$$

It should be noted that relationship (1) arose under the supposition that $\nu b_1 \gg 1$ and $\nu b_2 \gg 1$. If b_1 or b_2 are very small this supposition is not fulfilled. If $b_1 = 0$, the absorbent layer disappears. If in turn $b_2 = 0$, thus the absorbent

layer is moved to the boundary surface of the system. In both cases the solution has to change into relationship

$$K = \frac{\mu b}{\sin \mu b} - 1 \tag{4}$$

deduced directly for the bare sphere. By examining expression (1) we can see that in case of $b_1 = 0$ and $b_2 = 0$, respectively, it is actually transformed into

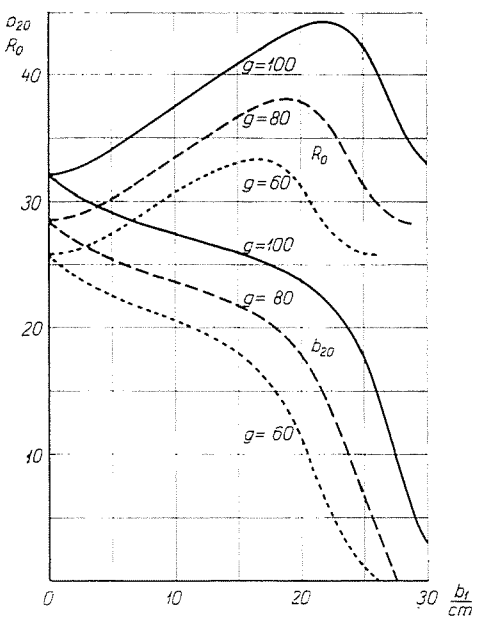

Fig. 15. Critical dimensions of the two-layer arrangement in the case of constant dilution

equation (4). The expression given under (1) can therefore be regarded as a good approximation for both low and high values of νb_1 and νb_2.

The expression for G can again be formed:

$$G = \frac{\beta_0 \mu b_1 u + (\mu b_1 + \operatorname{arc\,tg} u)\sqrt{1+u^2}}{[-\beta_0 \sin \mu b_1 - \cos \mu b_1 + u \sin \mu b_1][\mu b_1 - \operatorname{arc\,tg} u]}. \tag{5}$$

In the case of $b_1 \to 0$, $u \to (-0)$; while in the case of $\mu b_1 \to \pi$, $u \to (+0)$. In both cases $G \to 1$.

On Fig. 15 critical dimensions are plotted in function of b_1 for uranyl sulfate diluted in water. For the various curves the $g = N_0/N_U$ dilution value is given. Only the $g = 60, 80, 100$ dilution values were examined. It is evident from Fig. 15 that the radius of the active zone has a maximum in the case

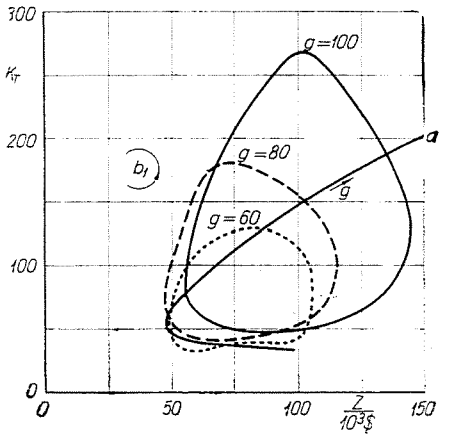

Fig. 16. Characteristics of the two-layer arrangement for the thermal amplification factor

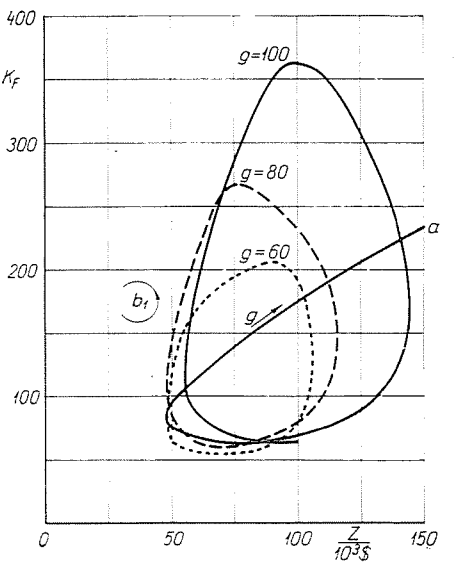

Fig. 17. Characteristics of the two-layer arrangement for the fission amplification factor

of a precisely determined b_1 dimension. Values b_1 and b_2 can be exchanged on the figure — as evident from (3).

This exchangeability, however, is not valid for the amplification factor as well. In the followings dimension b_1 will be regarded as an independent variable, this ensuring a higher amplification factor.

On Figs. 16 and 17 the characteristics of the thermal and fission amplification factor are plotted for three g values. As the cases $b_1 = 0$ and $b_2 = 0$ mean the same physical arrangements, closed curves are obtained. But only

the left half of these is of interest. This curve section pertains to high b_1 values as can be read from Fig. 18. For the sake of comparison the corresponding characteristic of the bare spherical system is also shown on the diagrams. It can be read of the diagram that the maximum amplification factor of the two-layer arrangement is about the double of that of the one-layer arrangement, costs being identical. The characteristic of the amplifier is, therefore, advantageously influenced by this method, its effect is sensible, however, only with costs higher than minimum.

This method can be further developed by increasing the number of layers on the one hand and by the use of reflector on the other hand.

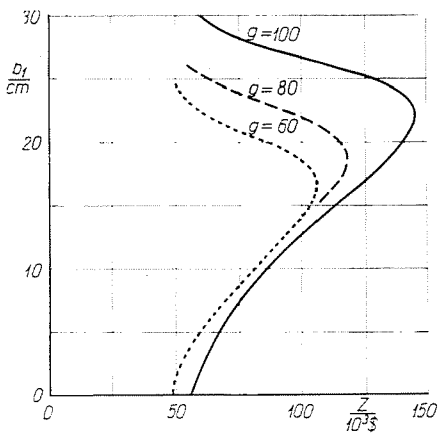

Fig. 18. Correlation between costs and dimension b_1 in the two-layer arrangement

9. Convergatron

The principle of convergatron was raised, in a short paper [2], where only slab arrangements were discussed and along lines different from those of the present paper. The basic principle of the convergatron can be formulated as follows: neutrons leaving a neutron amplifier may act as a high-intensity neutron source for the next amplifier. By connecting such units in series, the amplification factor as related to the original source intensity can be increased at will. At the same time this means that neutron flux and power will also be higher and higher in the stages following one another. Actually, the two-layer system discussed previously can also be regarded as such an arrangement. However, this method cannot be applied forthwith, because it is not enough that the various stages are subcritical but the complete system should also be in a subcritical condition. The problem can be formulated in other words, too: neutrons pass, not only from the first stage into the second, but they are "fed back" from the second system into the first one. If this effect

can be prevented, the various stages can be examined separately, which means a considerable easing in calculation technique.

According to [2] the task can be solved as follows (Fig. 19): One stage consists of three parts: an active zone, a thermal absorbent surrounding the active zone, and a moderator zone. From the active zone only fast neutrons pass into the moderator. A part of the neutrons is returned into the active zone, another part is absorbed in the moderator, the remaining neutrons pass into the 2. active zone, where a chain reaction is started. Essentially all the neutrons leaving the 2. active zone arrive to the boundary of the 1. zone slowed down, if the dimension a of the moderator is suitably chosen. At the boundary, neutrons are absorbed by the absorbent layer, consequently there is practically no feed-back. A similar process takes place in the following stages.

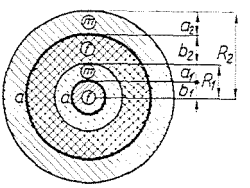

Fig. 19. Convergatron arrangement. *f)* fuel, *m)* moderator, *a)* absorbent layer

The thickness of the moderator should be as high as possible, on the one hand, so as to prevent feed-back in any case. On the other hand, moderator dimensions cannot be increased excessively, because otherwise direct coupling is decreased on account of absorption. Dimension a should fulfill the following unequation:

$$\sqrt{\tau_0} \ll a \ll L_0. \tag{1}$$

From which follows that only such materials can be used as moderator, for which $\sqrt{\tau_0} \ll L_0$. The only such moderator is heavy water ($\sqrt{\tau_0} = 11$ cm, $L_0 = 123$ cm). According to relationship (1) the thickness of the moderator layer was chosen at 40 cm.

Calculation procedure was the following: As a first step, the reflector surrounding the active zone was regarded as infinite. As the second step, the system 1. moderator — 2. active zone — 2. infinite moderator was calculated. In this case at position $r = b_1$, $\Phi = 0$, while the values for Ψ and J_F are identical with results of the preceding calculation. The last mentioned two conditions mean a redundancy, but differences obtained with the help of the two conditions can be neglected by suitably choosing dimensions. The calculation of the third and higher stages can be carried out by simply changing symbols.

To reduce dimensions and costs, water was chosen as moderator in the active zone, however, this makes the fulfilment of boundary conditions more

complicated. To make results more easy to survey, many mathematical approximations were used.

For the normalized amplification factor and critical dimensions we get rather complicated expressions. They are not given here because of lack of space. The numerical results are given on Fig. 20, where the lower curves represent the characteristics of a simple reflected amplifier, while the upper ones that of a two-stage convergatron. It can be seen that to achieve amplification factors higher than 1000 (by $s^3 = 0{,}9$) the use of the convergatron is more economical. At the same time, even the less expensive convergatron costs

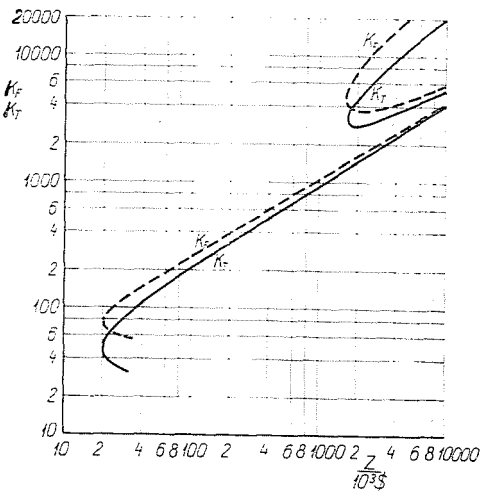

Fig. 20. Comparison of the characteristics of the convergatron and of the reflected spherical amplifier

more than one million dollars, while the amplification factor is still so low that by this arrangement *e. g.* no energy can be produced. By increasing the number of stages both amplification factor and costs rapidly increase.

The amplification factor either of the convergatron or of the other arrangements can be raised by increasing the degree of criticality, as in the near critical condition K is proportional to $1/\varepsilon$. If in place of $s^3 = 0{,}9$ the value of $s^3 = 0{,}99$ is allowed, so costs rise only 10 p. C., while the amplification factor is increased 2,15 fold. On the other hand a system being on the border of criticality is to be equipped with protecting devices similar to a critical reactor. Consequently, every advantage of the subcritical system is lost thereby.

10. Summary

Along principles laid down in paper [1] the data and characteristics of various simple homogeneous neutron amplifiers were calculated: amplification factor in function of costs. For every arrangement there is a dilution value ensuring minimum costs. From certain points of view this can be regarded as an optimum condition. Amplification factor can be

increased partly by suitably chosen geometry, partly by using more-stage systems. By these methods, however, minimum costs rise in respect to the spherical or cylindrical arrangement, consequently their use is generally not practical.

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3. Reactor Handbook: Engineering: U.S.A. Geneva, 1955.
More detailed literature see in [1].

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