SOME QUESTIONS ON REACTIVE POWER AND REACTIVE CONSUMPTION

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According to the flowing of energy in alternating current systems we calculate with three different sorts of power.

The (active, real) power (p, P, Fig. 1) is a derivative of energy:

$$\frac{dW}{dt} = p = iu \quad \text{and} \quad P = \frac{1}{T}\int_0^T p\,dt \tag{1}$$

T being the duration of one cycle of the current or of the voltage.

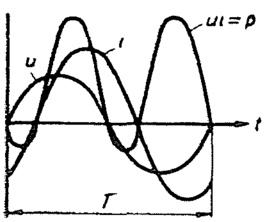

Fig. 1

The apparent power S is defined as rms value:

$$S = IU \text{ (} I \text{ and } U \text{ being rms values)} \tag{2}$$

The reactive power Q is defined by the equation

$$P^2 + Q^2 = S^2 = (IU)^2 \tag{3}$$

It should be mentioned here: the apparent power S as a physical quantity does not exist in the electrical circuit — it is a fiction.

As to the reactive power: first we may examine it in the exceptional case when there is no real power at all in the circuit (the case of ideal condensers or of absolutely linear coils of self-induction, both without losses). The energy of a condenser or of a coil respectively is expressed (Fig. 2) by:

$$W_{0,c} = (U_{max})^2 \frac{C}{2} \quad \text{and} \quad W_{0,s} = (I_{max})^2 \frac{L}{2}$$

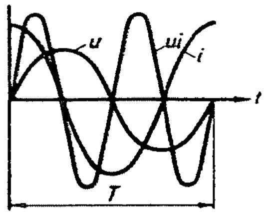

Fig. 2

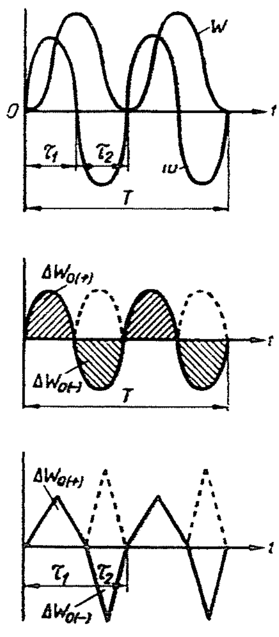

Fig. 3

further, the energy of the circuit for a multiple of a cycle of the voltage or of the current ($t = kT$) must be zero (Fig. 3):

$$\sum_{t=0}^{kT} \Delta W_{0,c} = 0 \quad \text{and} \quad \sum_{t=0}^{kT} \Delta W_{0,s} = 0 \tag{3a}$$

because the numerical values of the increase ($\Delta W_{0(+)}$) and of the decrease ($\Delta W_{0(-)}$) of energy by definition are equal. This shows that equations similar to equation (1) cannot be used for the definition of the reactive power Q.

As is generally known the reactive power Q (Fig. 3) is defined by

$$Q = \frac{\pi}{2} \frac{\sum |\Delta W_0|}{\sum \Delta t} = \frac{\pi}{2} \frac{2 W_0}{\tau_1 + \tau_2} \tag{3b}$$

τ_1 being the time during which the energy of the condenser (or of the coil) from zero increases to its maximum value, whereas during τ_2 its energy decreases to zero again. Or, if during one second *the maximum energy* W_0 does not change:

$$Q = \frac{\pi}{2} 4 f W_0 = \omega W_0 \tag{3c}$$

The reactive power Q in this equation has nothing to do with real power or real energy just as the factor $\pi/2$ shows the equation is an arbitrary definition, it is merely a fiction.

Equations (3b) and (3c) are also applicable in general cases when in the circuit real power is also existing.

The above-mentioned maximum energy of a condenser or of a coil may also be defined by the "reactive power function" q:

$$W_0 = \left| \int_0^{\tau_1} q \, dt \right| \quad \text{being} \quad \left| \int_0^{\tau_1} q \, dt \right| = \left| \int_{\tau_1}^{\tau_2} q \, dt \right| = \ldots \tag{3d}$$

Often the function q is mentioned as "the instantaneous value of the reactive power" which of course is not correct.

The definite integral of the function q for one cycle of the current or of the voltage must of course be zero :

$$\int_0^T q \, dt = 0 \tag{3e}$$

It is to be seen that the value Q of the reactive power is not expressed by the definite integral of the function q, contrariwise to the active (real) power (p, P).

The values P, Q and S are connected by the equations :

$$P = \lambda S \text{ and } Q = \varkappa S \;\; (\lambda = \text{power factor}) \tag{3f}$$

where $\lambda^2 + \varkappa^2 = 1$ therefore $\lambda \leqq 1$ and $\varkappa \leqq 1$
further

$$Q = \frac{\varkappa}{\lambda} P$$

For the definition of the function q of the reactive power we first of all transform the function p of the active power to p^* which should have only positive values and its average value should not differ from $P = \lambda I U$. We find

$$p^* = \lambda \sqrt{2}\, I \sin \omega t \sqrt{2}\, U \sin \omega t = 2 \lambda I U \sin^2 \omega t$$

Indeed the average value of p^* is

$$P^* = \frac{1}{T} \int_0^T p^* \, dt = \frac{2 \lambda}{T} I U \int_0^T \sin^2 \omega t \, dt = \lambda I U = P$$

Next we form the function q^* of the reactive power as the difference of both the real power functions p and p^* (Fig. 4) :

$$q^* = p - p^* \tag{3g}$$

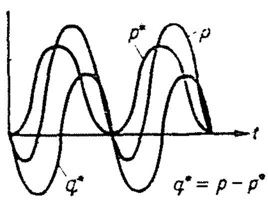

Fig. 4

This fulfils the requirements of equation (3e) :

$$\int_0^T q^* \, dt = \int_0^T (p - p^*) \, dt = T (P - P) = 0$$

We made no conditions concerning the current and the voltage ; their curves are generally not formed pure sine, they may contain higher harmonic components too, therefore the power p and the reactive power function q^* may also have higher harmonic components.

If the value $\varkappa = \sqrt{1 - \lambda^2}$ is known, another function q^{**} may be formed :

$$q^{**} = \varkappa \, [\sqrt{2}\, I \sin (\omega t - \pi/_2)] \, [\sqrt{2}\, U \sin \omega t] =$$
$$= - \varkappa \, 2 \, I \, U \cos \omega t \sin \omega t \tag{3h}$$

This also fulfils the requirement (3e) as

$$\int_0^T q^{**} \, dt = - \varkappa \, 2 \, I \, U \int_0^T \cos \omega t \sin \omega t \, dt = 0$$

The function q^{**} is of pure sine form, the absolute values $|Y|$ of its extreme values (co-ordinated to $t = T/8$, $3T/8$, $5T/8$ and $7T/8$) give the effective value Q of the reactive power:

$$|Y| = 2\varkappa IU \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} = \varkappa IU = Q$$

and the absolute value of the function's average value for a quarter of a cycle is

$$\left|\frac{4}{T}\int_0^{T/4} q^{**}\,dt\right| = \left|\frac{4}{T}\varkappa 2\,IU\int_0^{T/4}\cos\omega t\sin\omega t\,dt\right|$$

$$= \frac{4}{T}\varkappa 2\,IU\frac{1}{\pi}\frac{T}{4} = \frac{2}{\pi}\varkappa IU = \frac{2}{\pi}Q$$

Of course the average value of the function for one cycle is zero.

The functions q^* and q^{**} should be compared.

If we succeed in substracting in the load-circuit the reactive power q^* from the real power p, in this way we obtain the new real power function p^*. The latter has positive values only and in it the rms value of the current is diminished from its original value I to λI, so practically annulling any phase difference between current and voltage. *So the function q^* may be named a compensating one*, however, the value Q cannot be designated by it.

The application of the reactive power function q^{**} to the above described purpose leads only to an approximate compensation as

$$p - q^{**} \leqq p^*$$

but in the function q^{**} the characteristic value Q of the reactive power can be pointed out precisely.

Both the functions q^* and q^{**} are expressed and bound to reality by rms values of current and of voltage, q^{**} being of pure sine form, q^* in general having higher harmonic components too. *Neither of the reactive power functions for an immediate measurement is accessible.* Which of them should be chosen for practical use can be decided only after a closer discussion of the circumstances in three phase circles.

The task of the measurement of the three sorts of power leads to further conclusions.

As is generally known the real power can be properly measured by means of electrodynamical or electrostatical wattmeters. The function p may be recorded by an electrodynamical oscillograph.

The apparent power S as a physical quantity does not exist in the electric circuit and so cannot be measured. However, we can generate and measure a real power P_s numerically equal to the apparent power: we must only insert (Fig. 5) an electrodynamical comparator and so transform current J_1 of the circle to current I_1 synchronous to voltage U (J_1 is generated by voltage U through the regulated resistance R). The balance of the comparator shows the equality of the currents :

$$I = J_1$$

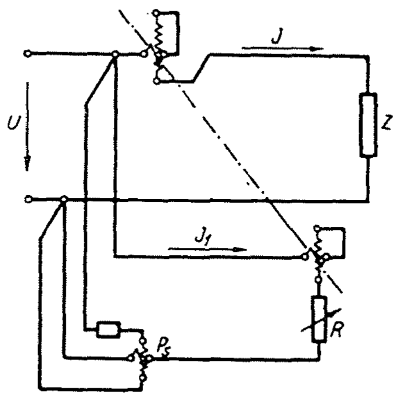

Fig. 5

in which case a wattmeter fed by U and J_1 measures the value

$$C\,\alpha = P_s' = J_1\,U = IU = S$$

Similar way must be chosen for measuring the reactive power Q: we have to generate and measure a real power numerically equal to the reactive power Q, because our wattmeters are able to measure real power only.

This task becomes a very simple one in the exceptional (nearly imaginary) case, if both current and voltage are of pure sine form :

$$i = \sqrt{2}\,I \sin(\omega t + \varphi) \quad \text{and} \quad u = \sqrt{2}\,U \sin \omega t$$

in which case the instantaneous and the average values of the power are

$$p = iu = 2\,IU \sin(\omega t + \varphi) \sin \omega t \quad \text{and} \quad P = IU \cos \varphi$$

therefore

$$\lambda = \cos \varphi \quad \text{and} \quad \varkappa = \sin \varphi$$

and in this exceptional case the value Q of the reactive power is

$$Q = \sin\varphi \, IU = IU \cos(\varphi - \pi/_2) \tag{3i}$$

further the real power P_q sought for the above purpose will be described by the function

$$p_q = \sqrt{2}\, I \sin(\omega t + \varphi - \pi/_2)\, \sqrt{2}\, U \sin\omega t \tag{3j}$$

In this case the real power P_q is numerically equal to the value Q of the reactive power to be measured. P_q might be generated by means of the actual current and voltage of the circle if, according to their mutual phase position we turn the current by 90 degrees in lagging or the voltage by 90 degrees in leading direction. It is not possible to imply any physical meaning into this turning of the phase angle, it only serves to make a calculation in an electromagnetic manner. However if the direction of this turning is consequently uniform, the sign of the so generated real power is the opposite in the case of inductive load to that of capacitive load.

If current and voltage are of pure sine form the "reactive power function" is found :

$$q = \sin\varphi\, [\sqrt{2}\, I \sin(\omega t + \pi/_2)]\, [\sqrt{2}\, U \sin\omega t] =$$

$$= \sin\varphi\, 2\, IU \cos\omega t \sin\omega t \tag{3k}$$

In this case the equations (3g) and (3h) describe the same function as (3k) which may be demonstrated by substituting the values of the current and of the voltage into equation (3g).

The 90°-turning might be arranged in the circle of the current coil or of the voltage coil of the wattmeter by combining ohmic resistors with inductive reactances (HUMMEL, GÖRGES) or with capacitors (JÓBA). Of course the result depends on the frequency. Should the vector of the current be turned by $90° - \delta$ instead of by 90°, the relative error is

$$h_q = \frac{\sin(\varphi + \delta) - \sin\varphi}{\sin\varphi} = \cos\delta + \operatorname{ctg}\varphi \sin\delta - 1$$

and the full error is

$$H_q = h_q\, IU \sin\varphi = IU(\sin\varphi\cos\delta + \cos\varphi\sin\delta - \sin\varphi)$$

If the angle δ is very small ($\delta \leq 4°$), $\cos\delta \cong 1$, and the absolute value of the error is

$$H_q \cong IU \cos\varphi \sin\delta$$

The result of the 90°-turning is right when current and voltage are of pure sine form. *The described phase turning connections shall be applied in general cases too,* if current and voltage have higher harmonic components. Concerning the base-harmonic, the phase turning method leads to correct results, but there may failures occur concerning the reactive power of the higher harmonics (this errors though be less than the double of the apparent powers referred to).

Here the very interesting experiment of TRÖGER should be mentioned, who sought and found relations between the "swinging power" P_{sw} (energy swinging to and fro), real power P and reactive power Q. For pure sine form of current and voltage he found

$$P_{sw} = \frac{2}{\pi} P \text{ ev } |\varphi| \quad \text{and} \quad Q = P \frac{\lambda}{\varkappa} = P \operatorname{tg} \varphi$$

so on the base of the measurement of P and P_{sw} may ev $|\varphi|$, $\operatorname{tg} \varphi$ and Q be calculated. However, this method does not bring immediate results, so at present it only serves laboratory purposes.

In a three-phase circle in the phases 1, 2 and 3 the instantaneous values of current and voltage are

$$i_1 \quad i_2 \quad i_3 \,; \qquad u_1 \quad u_2 \quad u_3$$

The instantaneous values of the real power are

$$p_1 = i_1 u_1 \,; \quad p_2 = i_2 u_2 \,; \quad p_3 = i_3 u_3$$

and their average values are

$$P_1 = \lambda_1 I_1 U_1 \qquad P_2 = \lambda_2 I_2 U_2 \qquad P_3 = \lambda_3 I_3 U_3$$

The power of the whole three phase circle is

$$p = p_1 + p_2 + p_3 \text{ and the average } P = P_1 + P_2 + P_3$$

The apparent power is to be determined in each phase :

$$S_1 = I_1 U_1 \qquad S_2 = I_2 U_2 \qquad S_3 = I_3 U_3$$

The apparent power of the three-phase system cannot be defined by adding the values found in each phase of the circle.

The reactive power in each phase is defined by

$$Q_1 = \varkappa_1 I_1 U_1 = \varkappa_1 S_1 \qquad Q_2 = \varkappa_2 S_2 \qquad Q_3 = \varkappa_3 S_3$$

In general cases the reactive power of the three-phase system cannot be expressed by adding Q_1 and Q_2 to Q_3.

In the exceptional case of ideal symmetry of supply and load, all the phase-quantities are equal :
the voltages $U_1 = U_2 = U_3$, the currents $I_1 = I_2 = I_3$, the power factors $\lambda_1 = \lambda_2 = \lambda_3 = \lambda$ and $\varkappa_1 = \varkappa_2 = \varkappa_3 = \varkappa$ therefore

$$P_1 = P_2 = P_3 = P_{ph}$$

$$Q_1 = Q_2 = Q_3 = Q_{ph} \text{ and } S_1 = S_2 = S_3 = S_{ph}$$

thus the power in the three-phase circle is

$$P = P_1 + P_2 + P_3 = 3\, P_{ph}$$

further based on equation (3) :

$$P_{ph}^2 + Q_{ph}^2 = S_{ph}^2 \text{ and } (3\, P_{ph})^2 + (3\, Q_{ph})^2 = (3\, S_{ph})^2$$

So, in this exceptional case we get numerically correct result, if we consider the algebraic sum of the apparent powers in each phase as the apparent power of the three phase circle. We may apply the same method regarding the reactive power of the three-phase circle.

The real power in a three-phase circle might be properly measured by wattmeters the result being independent of the frequency, of the wave-shape and of the symmetry or asymmetry of supply and load.

The apparent power S in each phase of a three-phase circle may be measured by using comparators as described earlier.

The reactive power Q could be measured in each phase by the 90° phase turning, according to equation (3j). However, if the supply system is ideally symmetrical, one of the vectors of voltages in delta-connection is always perpendicular to one of the vectors of voltages in star-connection, and this circumstance may also be used for the 90°-phase turning. — Further, if supply system and load system are symmetrical and there is no zero line, the reactive power can also be measured by two wattmeters (similar to the Aron-connection), if the power indicated by one of the two instruments is taken into account with changed sign :

$$Q = \sqrt{3}\,(P''' - P')$$

If the 90°-turning is arranged in each phase by means of specially connected impedances, the result depends on the frequency, but not on the symmetry of the supply nor of the load. If the phase turning is carried out by the above-mentioned delta star connection, the result does not depend on changes of frequency, nor on the asymmetry of the load, but is influenced by the asymmetry of the supply system. — The measurement with two wattmeters can only be correct if both supply and load are ideally symmetrical.

Our considerations should also be extended to the territory of consumption.

The electrical energy W which flows through a circle can be properly measured by an induction type meter according to equation

$$W = \int_{t_1}^{t_2} p \, dt = \int_{t_1}^{t_2} i \, u \, dt \tag{4}$$

The apparent consumption W_s (according to the apparent power S) should be measured according to equation

$$W_s = \sum_{t_1}^{t_2} S \Delta t \tag{5}$$

e.g. by an electrodynamical meter combined to an electrodynamical comparator, as described. — This task in three-phase systems is solved, within acceptable error limits, by the "trivector", this specially constructed three-phase induction-type meter.

The reactive consumption W_q is defined by the equation

$$W_q = \sum_{t_1}^{t_2} Q \Delta t \tag{6}$$

and can be measured as the definite integral in time of a real power P_q numerically equal to Q. For this purpose we use the induction-type meters in the 90°-phase turning connection, as described above.

Here let us bring to mind that apparent power S and reactive power Q are based on rms values of current and voltage, they can be strictly interpreted only for whole cycles of voltage, so Δt in equations (5) and (6) may be a multiple of the time of one cycle only :

$$\Delta t = k \, T$$

k being an integer number.

Out of equation (3) follows :

$$(P^2 + Q^2) \Delta t = S^2 \Delta t$$

or, in summation

$$\sum_{t_1}^{t_2} P^2 \Delta t + \sum_{t_1}^{t_2} Q^2 \Delta t = \sum_{t_1}^{t_2} S^2 \Delta t \tag{6a}$$

but this equation does not connect the squares of the consumptions. For those is only valid :

$$\left(\sum_{t_1}^{t_2}{}' P \Delta t\right)^2 + \left(\sum_{t_1}^{t_2}{}' Q \Delta t\right)^2 \neq \left(\sum_{t_1}^{t_2}{}' S \Delta t\right)^2$$

or

$$W^2 + W_q^2 \neq W_s^2 \tag{6b}$$

and if, $W = \Lambda W_s$ and $W_q = K W_s$, so

$$\Lambda^2 + K^2 \neq 1. \tag{6c}$$

The expression (6b) has an energetical base : the apparent power S can have positive values only and their summation in time always means the summation of positive values. The real power P can change its sign, so that energy W might be expressed by summation in time of positive and negative parts — the circumstance being possible for the reactive power Q and reactive consumption W_q. So it may happen that for a time interval the apparent consumption W_s amounts to a considerable value, but the sum of energy W and of reactive consumption W_q may be small or even zero :

$$\sum_{t_1}^{t_2}{}' S \Delta t \gg 0 \quad \text{but} \quad \sum_{t_1}^{t_2}{}' P \Delta t \approx 0 \quad \text{and} \quad \sum_{t_1}^{t_2}{}' Q \Delta t \approx 0$$

Thus in general cases

$$W^2 + W_q^2 < W_s^2 \text{ and } \Lambda^2 + K^2 < 1 \tag{6d}$$

where Λ and K are average values valid for the time interval of the measurement.

A relation between the squares of the consumptions similar to equation (3) could only be correct, if the values of consumptions would be defined as rms values out of the squares of the values of the powers, in this form :

$$\begin{aligned} A_p &= (t_2 - t_1) \sqrt{\frac{\sum_{t_1}^{t_2}{}' P^2 \Delta t}{t_2 - t_1}} \\ A_q &= (t_2 - t_1) \sqrt{\frac{\sum_{t_1}^{t_2}{}' Q^2 \Delta t}{t_2 - t_1}} \\ A_s &= (t_2 - t_1) \sqrt{\frac{\sum_{t_1}^{t_2}{}' S^2 \Delta t}{t_2 - t_1}} \end{aligned} \tag{6e}$$

but just the expression A_p for energy is physically false.

With the values given in (6e) we may write :

$$A_p^2 + A_q^2 = A_s \tag{6f}$$

we may use also the writing

$$A_p = \Lambda_A A_s \quad A_q = K_A A_s \quad A_q : A_p = K_A : \Lambda_A \tag{6g}$$

here Λ_A and K_A are average values connected by the equation

$$\Lambda_A^2 + K_A^2 = 1 \tag{6h}$$

In the exceptional case when current and voltage are of pure sine form, the above-defined values A of the consumptions may be connected by the equations

$$A_p = A_s (\cos \varphi)_{\text{aver}} \text{ and } A_q = A_s (\sin \varphi)_{\text{aver}}$$

therefore

$$A_q : A_p = (\operatorname{tg} \varphi)_{\text{aver}}$$

It is not impossible to generate and measure the above-defined average values A_p, A_q and A_s of the consumptions, by transforming the angular velocity of the disk of the induction type meter to a proportional voltage (*e.g.* by some impulse method used in tele-measuring) and by integrating in time the square of this voltage using a meter adequate for this purpose. However, this method is complicated and very expensive with the only result that the functions defined in (6e) could be determined, the equation (6f) would be valide with the sure knowledge that the value A_p found for the energy is false.

There is one case when the mean values of the consumptions calculated from sqares ("A") are equal to the consumptions calculated on linear base, namely, when during the whole measurement all the powers P, Q and S are unaltered :

$$P = \text{const} \quad Q = \text{const} \quad S = \text{const}$$

when therefore

$$\lambda = \text{const} = \Lambda \text{ and } \varkappa = \text{const} = K$$

in this case

$$W = A_p \qquad W_q = A_q \qquad W_s = A_s$$

and so in this case

$$\text{exceptionally!} \quad W^2 + W_p^2 = W_s^2$$

The more the powers are changing, the more will the application of the last equation of the squares be unjustified — leading to intolerable errors in practical cases.

Our considerations may be summarized as follows.

1. Contrary to the measurement of the (real) power in a circuit by means of wattmeters, *the measurement of the reactive power* according to its definition *may depend:*

on the frequency,

on the wave-shape of current and voltage,

in three-phase circuits it may depend also on the symmetry or asymmetry of supply and load.

2. It was stated that the addition of the squares of the powers by definition leads to correct results, such an addition of consumptions W, W_q and W_s is not permitted, it may lead to rough errors.

3. It is important to establish that concerning the powers P, Q and S we can define and reasonably use the quantity "power factor" λ (respectively $\cos\varphi$) and its complementary $\varkappa$ (respectively $\sin\varphi$), concerning the consumptions we are not able to define an "average power factor", so the best is not to speak of the "average $\cos\varphi$" nor of its complementary the "average $\sin\varphi$" and so we do not even wish to measure them.

We have to consider what are the advantages of the knowledge of apparent and reactive power and of apparent and reactive consumption for the electrotechnical practice.

Out of the value of the apparent power we may calculate how many times $(1/\lambda)$ is the current larger than it would be in the optimal case (when $\lambda = 1$) at the same real power P and voltage U. From this we can reliably judge the "superfluous" overload of the conductors which could be calculated as

$$\frac{I^2 - I_{\text{opt}}^2}{I_{\text{opt}}^2} = \left(\frac{1}{\lambda}\right)^2 - 1 = \frac{\varkappa^2}{\lambda^2}$$

The knowledge of the apparent power is also important because the electrical machines and transformers are constructed according to the apparent power — so its value characterizes the actual load demanded from the machines of the power station.

On base of the value and character ("sign") of the reactive power, we can judge the excitation of the alternators in a power station and about the thermic state of their rotors.

The knowledge of the apparent and of the reactive consumption enables us to get a good estimation of the average development of the above-described phenomena during the measurement — this determines the importance of the measurement. Whereas the overload of the conductors causes excess of the losses, the overexcitation of a turbogenerator diminishes by its thermic effect the duration of life of the alternator's rotor — this being its most compactly constructed part, and also the mostly sensitive against overheating.

All we have stated according to energy-consumption, to apparent consumption and to the reactive consumption precautions us concerning these measurements. We have to measure the reactive consumption by means of two reactive-meters connected in opposition, fitted with reversal preventing devices, so that one of them measures the inductive reactive consumption, the other the capacitive one. The same method should be applied for the measurement of energy, if the reversal of the energy-flow is probable, or if it is only possible.

It is obvious that the measurement of the reactive consumption is connected with considerably more sources of error than the measurement of the energy. This circumstance was also appreciated in the international standardizations, as the error limits of the reactive meters are nearly the double of those of the induction-type energy meters. And so it is fair; reactive consumption should be measured mostly by large industrial consumers and these should be prepared technically to safely avoid the consumption-limits as are fixed by tariff.

The tariff-limits at present concerning the reactive consumption are graded according to values of the "average power factor". As this cannot be defined, nor measured, it seems to be equitable to substitute it by well-fixed values of the ratio of reactive consumption and of consumed energy, in accordance with the possibilities of the measurement.

Summary

Dealing with the definitions of apparent and reactive power and consumption the author shows that they do not exist as physical quantities, they are only fictitious, so their very important values cannot be measured but by substitution. He shows that the "power factor" may be defined for powers only, it is impossible to define it as an average value in time according to consumptions. — He deals with several sources of error in connection with the measurement of reactive power and consumption and points out the practical importance of the knowledge of apparent and reactive power and consumption — touches their role according to tariff.

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