THE NEW DIGITAL COMPUTER OF THE POLYTECHNICAL UNIVERSITY BUDAPEST

By

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I. Historical background

The idea of devising a digital computer first came to head in 1955. It was two years earlier in 1953 that instruction of switching technics was taken up in the curriculum of the Chair of Wired Telecommunication of the Polytechnical University, Budapest, and soon the need was greatly felt for having some sort of an equipment on hand, suitable for the practical demonstration of switching operations. Consequently, the principles to be embodied by this new design had to be mainly *didactic* ones. In addition it was thought appropriate that the new computer should be so designed as to allow other chairs of the University to refer any particular mathematical problems of their own sphere to the computer for handling.

Before taking up actual design work a number of points had to be made clear. The Hungarian Academy of Sciences offered its help to the promoters. For both capacity and dimensions the computer had to be designed with due regard to the limitations set by the amount at disposal. Moreover, the promoters of the digital computer had also to bear in mind that it was the Academy itself that had already commissioned its own Institute of Cybernetic Research to construct a computer. Substantial assistance had been extended to this Institute, mainly in the form of informative Soviet matter. Here the guiding principle was that the computer should be large enough to satisfy the country's need for calculations, mainly in the sphere of economics, for many years to come. With these considerations in mind the decision was thought appropriate, that in case of the University's computer, the components produced in Hungary in bulk should be given preference. At that time manufacture of electronic apparatus and assemblies had not yet been taken up by the Hungarian industry, so that even for the equipment of the Institute of Cybernetic Research the majority of assemblies and components had to be brought from the Soviet. On the other hand, production of electromagnetic relays for telephone exchange equipment was at that time already established since several decades. Undoubtedly, computers may be built up of any two-valued elements such as relays are. However, by resorting to this expedient, the designers of the computer had from the very outset reconcile themselves to a lower speed of operation inherent to relay type equipment, inasmuch as a computer incorporating electromagnetic relays only would work much slower than one composed of electronic units. Still when it was remembered that the computer would mainly serve purposes of instruction, and would be called upon for no special works, then in the opinion of the designers lower speed meant no particular disadvantage.

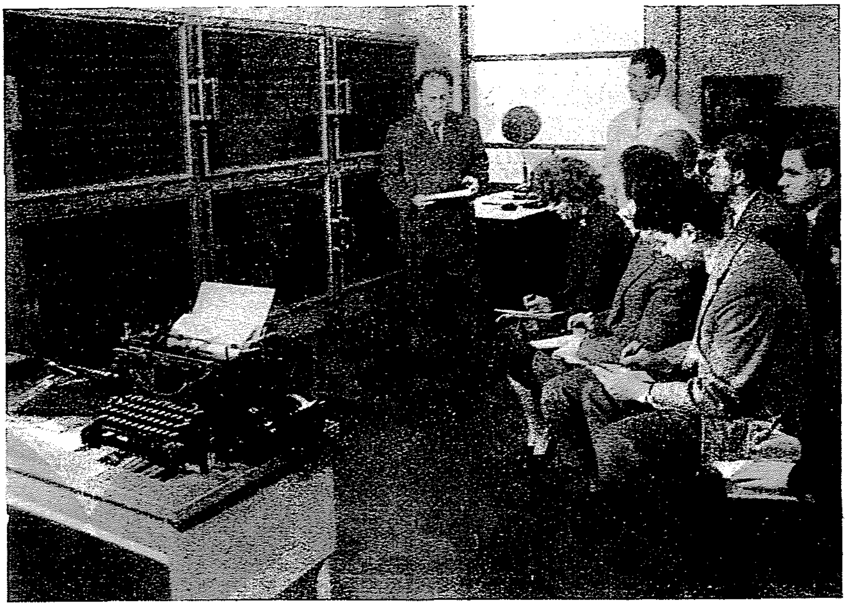

Fig. 1. Overall view of the computer

The use of relays of home manufacture, as a matter of course, entailed that the circuit had also to be developed at home. Although foreign circuit diagrams were also accessible, still their use was prohibited by the fact that these circuits were based on relays of altogether different operational characteristics. Also the designers were fully aware of the fact that circuit diagrams by themselves meant little or nothing in the line of assistance without their associated specifications. As for relays of home manufacture, since each type of relay had its own specific time of operation assigned to it, the rather convenient decision was taken to adopt the cheapest type, *i. e.* Type "R", of relays for the circuits of the digital computer.

Circuit design work was entrusted to a single person, who did the work in his spare time. Consequently, this part of the work could not be completed

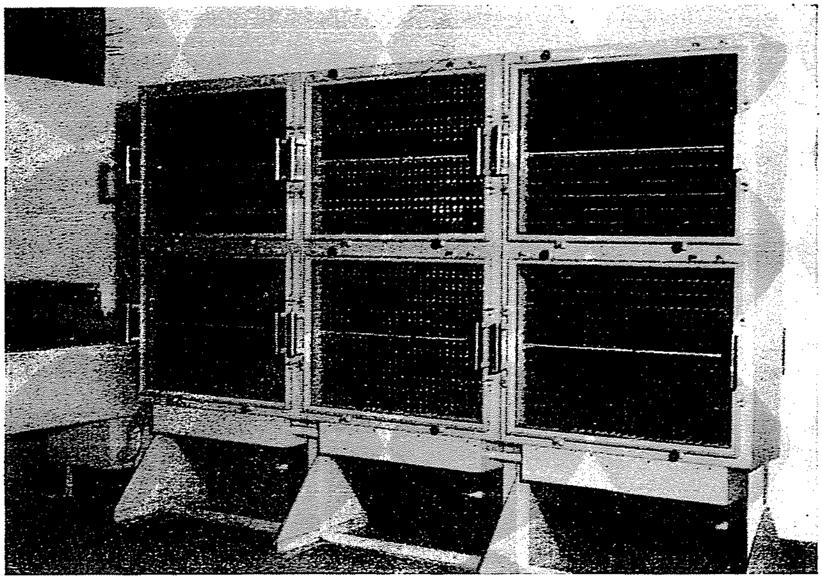

Fig. 2. The three relay cabinets

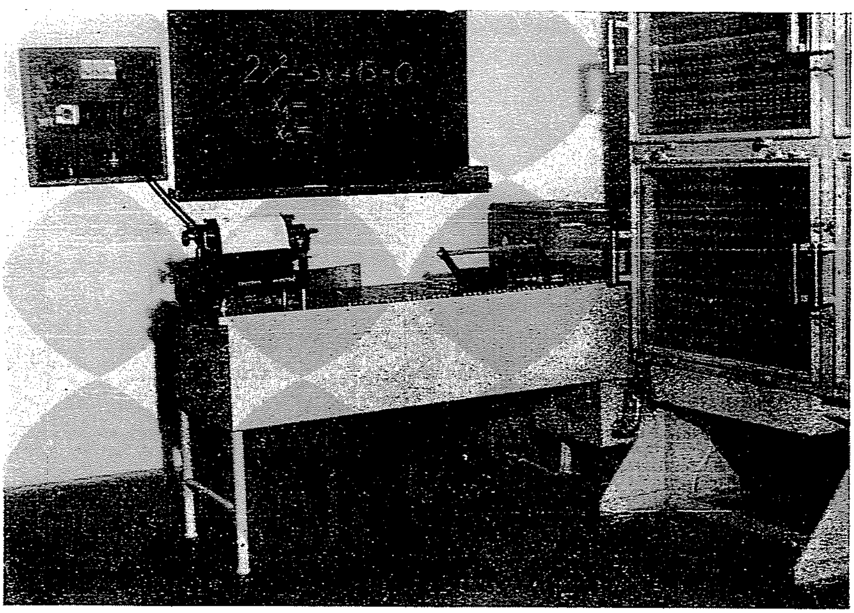

Fig. 3. Control desk of the computer

earlier than the spring of 1957. Assembly, cabling, and soldering were finished in the beginning of 1958, while electrical tests required a subsequent period of about a year. These comparatively long delays in the execution of the project

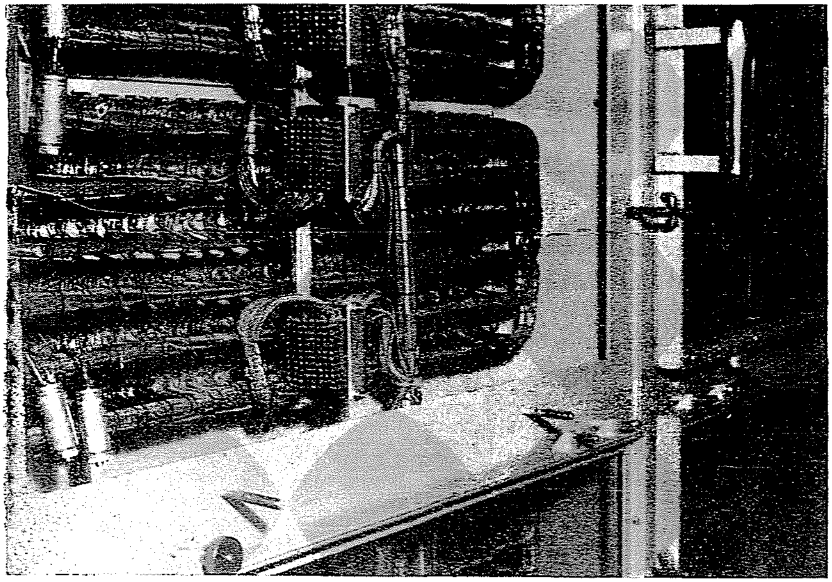

Fig. 4. Details of cabling

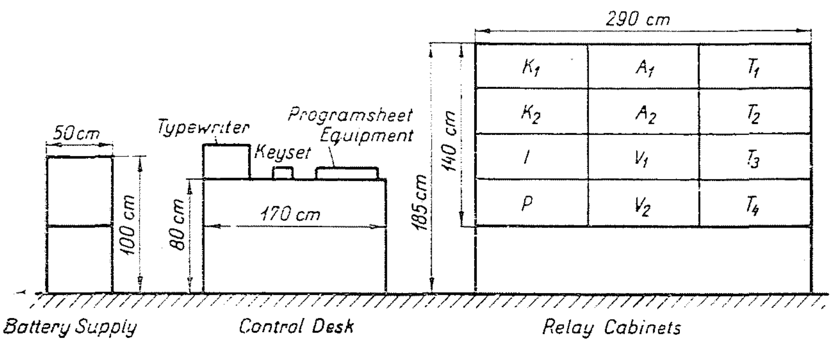

Fig. 5. Layout diagram of the equipment

were due mainly to the circumstance that only single persons who were actually working on the computer, and even these only in their spare time.

Nevertheless, by the end of 1958 the computer could be completed, and in the current, 1959, year it was already used for demonstration purposes, moreover a number of calculating problems were already referred to it.

Since this was the first electrical computer of the University, it was given the code designation MESZ I. An overall view of the computer may be seen on the attached photograph Fig. 1. Other details are displayed in Figs. 2, 3, and 4, while Fig. 5 shows the schematic layout diagram of the computer with the main dimensions in the drawing.

II. Principal features of the equipment

In principle, Equipment MESZ I is *a programme controlled automatic digital computer built up of electromagnetic relays and operating on the binary system.* After the basic data have been introduced the equipment performs all operations in a wholly automatic manner without any external aid, and causes the results to be typed out by a conventional office typewriter fitted with actuating magnets. In principle, MESZ I is capable of performing any operation otherwise assigned to large electronic computers. However, owing to the restricted capacity of its memory units, for practical purposes an upperlimit had to be set to the volume of tasks which could be entrusted to the computer.

The complete equipment is assembled of three parts, *viz.*:

a) The control desk.

b) The calculator unit, composed of three cabinets fitted with relays.

c) The rectifier unit for power supply.

a) *The control desk*

The control desk accommodates the *programme reading device.* This device is substantially a field of contacts, on which certain groups of combinations of contacts can be earthed according to a pre-arranged pattern of perforations punched in a chart, made of insulating material. To each of the calculations individual perforated charts have been assigned with the *instructions* the equipment has to execute in order to solve a specific problem. Thus *e.g.* a mixed quadratic equation has such a chart associated with it, which, when inserted into the contact field pre-determines a set of operations the computer will actually *have to perform in a legitimate sequence.* Then at the end of this set of operations the two radices of the equation will be obtained as results, and typed out on the typewriter.

The basic data are fed into the computer by means of a set of keys. In addition to the keys associated with the ten digits of the decimal system, each key has been provided to indicate the negative sign, the decimal point, and the end of the digit.

Mounted on the control desk there is a conventional typewriter as used in offices. This typewriter records both the data introduced into the computer and the results obtained from the process of calculation, on a blank sheet of paper. Actuating magnets have been fitted under the keys required for the communication of information and results. The magnets themselves are actuated by the calculator itself in a pre-determined sequence.

b) *The calculator*

This unit incorporates close to *2000 relays* distributed over three cabinets. All relays are of *uniform design, i. e.* type "R" relays as manufactured by the Beloiannis Telecommunication Factory are used throughout. As for contact combinations and windings the close to 2000 relays include ten varieties only. The relays are mounted on twelve panels of uniform design. This number of panels is given by the pre-determined functions assigned to each individual panel.

c) *Mains rectifier unit*

The *mains rectifier unit supplies* the d. c. power required for the operation of the equipment, *i. e.* 60 volts, the power consumption on the d. c. side being 600 to 800 watts. For signalling purposes conventional switchboard lamps as known in telephone switchboards are used. These too are lighted with 60 volts d. c.

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Each type of computer has a characteristic system of commands of its own allotted to it. Computer MESZ I is a *single-address equipment, i. e. a single command indicates the address of a single memory unit only.* Operations with two or more digits demand two or more commands, as the case may be. As regards design a single-address equipment is by far simpler than any of the frequently occurring double-address computers.

A single command having a row of twelve contact points associated with it on the programme chart, containing *two informations, viz.*:

1. *the type of operation to be performed*, and
2. *the memory unit in which the digit allotted to the operation is stored.*

These two data are recorded on the programme chart in the form of a binary code. Since the elements are of the two-valued type, out of the twelve points of a row with combinations of five

$$2^5 = 32$$

different operations may be performed, while with the combinations of seven

$$2^7 = 128$$

memory units may be marked out.

The maximum number of commands that may be registered on a programme chart is 45. When for the solution of a specific problem, commands in excess of this number are required, then depending on the actual number of commands, two or even more programme charts will have to be drawn up. As the computer is capable of storing all data in a manner independent of the programme charts, the charts may be exchanged while calculations are in progress.

Equipment MESZ I operates *on the binary system*. The basic data are introduced into the computer *on the decimal system*, and then translated automatically into a binary code. Re-conversion of the binary results to the decimal system automatically takes place in the same way. The results are then typed out in the decimal system.

As regards the number of memory units, the equipment in its actual design is still rather modest. Since owing to the limitations set to the dimensions the designers had from the very outset to waive the idea of using electromagnetic and electronic components in conjunction, relays having been employed also for the storage of information. One of the three cabinets of the computer was earmarked in its entirety for information storage, so that there are twelve memory units for digit storage, while a definite number of fixed memory units were assigned to storing certain numbers of particular importance, a few frequently recurring fractions, furthermore the values of π and e, and of certain logarithms. All these are, of course, stored in the binary system.

Corresponding to decimal numbers of eight digits the computer operates with binaries of 27 digits. In the memory units the digits are represented by *floating binary points*, the relays determining the order of magnitude giving the first "unit" of the number expressed in the form of different powers of two.

On assembling and cabling the panels the conventional methods, as used for the bays of telephone exchange equipment, were applied throughout. Between both the panels and the cabinets terminal strips were inserted, mainly to facilitate cabling, soldering, and failure tracing.

In its present design the computer contains parts and components representing an overall value of roughly one hundred thousand florins. *Labour* invested in the construction of the equipment expressed in the number of hours spent, adds up to the following figures :

Development	1500 hours
Engineering and draughting	1200 ,,
Installation, cabling, soldering	1500 ,,
Electrical testing	1400 ,,
Sundry work in shops	400 ,,
Grand total of labour	6000 hours

III. General principles of operation

A description of the method of operation of the computer is given here on hand of the block schematic in Fig. 6.

The three assemblies accommodated on the control desk have been given designations as distinct from those of the panels of the relay cabinets. The three assemblies are as follows :

1. the "carry-in" keyboard,
2. the typewriter, and
3. the programme chart assembly.

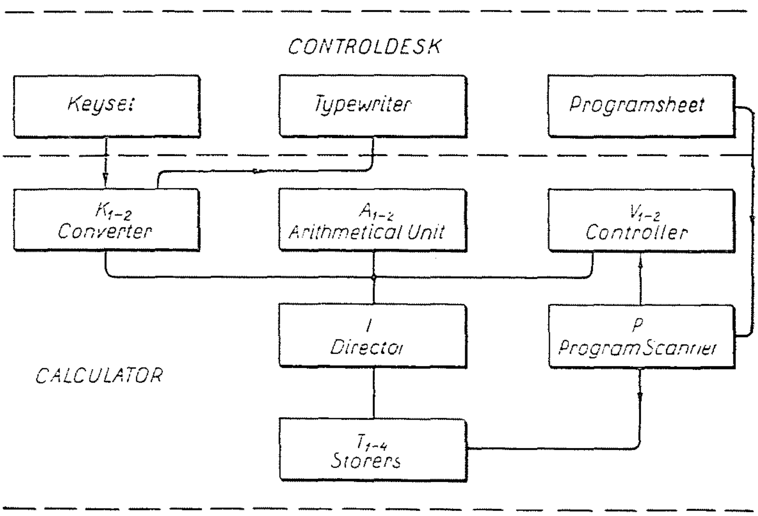

Fig. 6. Block schematic of the equipment

The twelve panels in the three relay cabinets have been designated with a capital letter each, *i. e.* the abbreviated symbolism expressing their respective functions :

K_{1-2} Converter (translator) units
A_{1-2} Arithmetic units
V_{1-2} Control units
I Director unit
P Programme reader unit
T_{1-4} Memory (storage) units

The constants of any problem are introduced into the equipment by means of a set of keys, and are then received in the converter unit. The end of the digits of a number is indicated by momentarily depressing key "Carry-in".

In response to the depression of this key circuit K_{1-2} translates the decimal number fed into the equipment into a number of the binary system, and advances this binary number over I to one of the memory units.

For conversion the known *method of halving-doubling* has been adopted. Panel K_1 is equipped with two halving, panel K_2 with two doubling circuits. The first halving circuit of K_1 receives the integers, the first doubling circuit of K_2 the decimal fractions. At signal "Carry-in" the halving circuits begin to halve, while the doubling circuits begin to double, among themselves. At halving odd numbers the residual units are passed on by K_1 to the register of circuit V_1, in the binary code. Similarly at doubling performed by K_2 the resultant integer "units" represent the values of the decimals in a binary code. Halving and doubling continue as long as the number to be converted has been exhausted, *i. e.* when K_1 and K_2 become vacated, or else the operation is stopped automatically when 27 doublings have been completed, this being the capacity of the memory units. (On the halving side a number of eight decimal digits will become exhausted anyway when 27 halvings have taken place.) The units produced by K_2 are advanced to the register of V_2.

In Appendix 5 the block schematic of conversion is shown, and the operation is illustrated by numerical examples.

The number thus converted into a binary expression now advances to the memory unit over units A_{1-2}, where when necessary it is rounded off before being stored. As a matter of fact the majority of numbers having a portion of decimal fractions may have 27 binary digits after translation, and, consequently, when such numbers have a portion of integers, then in the binary code there may be more than 27 digits, whereas the memory units were designed for a storage capacity of only 27 digits. Rounding off is performed in such a way that unit A adds unity to the value of position 28. This addition of unity will be ineffective whenever there is zero in the 28th position. On the other hand, unity is added to the number of 27 binary digits whenever there is unity in the 28th position. The binary number thus rounded off is then transmitted over the I director to the memory unit.

In the memory unit the binary number is stored in two parts. As a first stage the memory unit will store the possible number in the binary code of 27 digits, and then the memory unit will be informed by I of the order of magnitude of the number, *i.e.* the value of the first "unit" expressed as a power of two. Each memory unit is equipped with six elements for storing the order of magnitude (*i. e.* in principle the order of magnitude of any number in the binary code may actuate $2^6 = 64$ position values, however, actually only 54 values).

The translator (converter) then advances the digits simultaneously with the "carry-in" to the typewriter in order to fix on paper the starting data, *i. e.* the constants.

The final results of calculation, or any intermediate information, may then be "carried out" on the equipment. Since any number to be carried out on the computer is expressed in a binary code, the results will first have to be converted into the decimal system. This conversion takes place in the same translating circuits as have taken part in the conversion to the binary code.

Panel P is essentially an electrical sequence switch, activating the instructions *stored simultaneously in the programme chart* by discrete steps. After the starting data are introduced into the equipment, key "Start of calculation" has to be operated for a moment. On depressing this key circuit P carries out the first instruction of the programme chart. As has already been explained, this programme chart contains two informations, *viz.* the type of operation the computer has to perform, and, secondly, the storing place of the digit to be used for the operation. The instruction for operation is forwarded to control circuit V, which in turn *signals to all other circuits* the action they have to take in order to perform the operation. The programme chart further specifies the memory unit to be connected for the purpose of the operation to be performed.

Panel I constitutes a link between the memory units and the acting circuits K_{1-2}, A_{1-2}, and V_{1-2}. While the digits are stored with floating binary points in the memory units, the binary points are allotted a fixed point in the three acting circuits. The director is set by the order of magnitude. This director is essentially a switch having 27 branches and 54 positions built up of relays, and controlled on the binary system. The number stored in the memory unit and formed of a maximum of 27 digits passes over this director to the acting circuits, where it occupies a position corresponding to its real value on both sides of the binary point. Circuit I is a two-way circuit whose function is to take care of the transmission of the digits to the memory units in a correct manner.

The arithmetical unit of fixed binary points has a specific importance from *didactic* considerations. During tuition the operation of the calculator may be slowed down by means of a set of special keys to any desired speed, so that *partial results may at any time be visualized*, and also the momentary condition of control observed.

The function of control circuit V is, after collecting the operation instructions, to set all circuits in a condition suitable for the performance of the operation indicated. The functions of circuit V are decisively determined by the circumstance that the equipment is of the *singleaddress* type.

Arithmetical unit A is by itself capable of performing *addition only*. Substraction is in principle a *complementary addition*, and for operations including a set of additions, such as multiplication, division, extraction of roots, circuit V assumes the functions also of the *sequence switch*. Thus *e.g.* prior to multiplication the factor is passed on to circuit V, and in turn during multiplication the circuit advances the multiplicand as many times from the memory unit in the direction of A as there are "units" in the factor. Naturally, the circuit will in every case have in mind the order of magnitude of the multiplicand, and that of the unit actually multiplying. At multiplication it is circuit V which actually takes care of the addition of the different orders of magnitude, and at division of the subtraction of the orders of magnitude of the dividend and divisor. This will then determine the binary position value of the quotient. Extraction of roots is essentially a division, and only the value of the divisor changes, dependent on the value of the root already determined.

To sum up, the function of unit V is to cause the four rules of arithmetic to be performed in accordance with the instructions received. When any one instruction has been carried out, then V passes a signal on to P, in response to which P activates the set of instructions next in order of sequence on the programme chart. Consequently, instructions will issue to V to perform another set of operations. This cycle will then be repeated until all instructions on the programme chart are exhausted.

The result or results are in like manner "carried out" on the computer as the execution of a command. Unit V receives a signal indicating the memory unit from which the number stored there should be "carried out" to the translator. The number may in certain cases be carried out directly from A, or V itself as the case may be. Now K translates the binary codes into decimal numbers in a way that integers are doubled and fractions halved.

In addition to adding circuit A may perform comparisons, too. The circuit is capable of ascertaining which of two numbers is the larger, either in absolute values, or by virtue of their respective signs. While calculation is progressing, the circuit will be able to change its operation so as to comply with the conditions of such a comparison : the circuit may leap to any one row of the programme charts, both forwards and backwards, it may skip certain operations which as the result of comparison appear to be redundant, or unnecessary, while on the other hand it will be able to reiterate a set of instructions, either in the pre-determined number of cycles, or as long as a certain specified value, e.g. a value lower than a pre-determined error value, was obtained after the reiterative process. The operation of a computer consists, to the major extent, of a set of operations reiterated in cycles of a definite pattern. Consequently the programme chart will have to be laid down in such a way that at the corresponding moment of the progress of calculation the computer will be able to make a choice of the alternatives as the function of partial results, on hand of such a comparison.

E. g. on hand of formula

$$X_{1,2} = -\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

the two roots of a mixed quadratic equation may be obtained accordingly, as under the root there stands

$$\left(\frac{b}{2a}\right)^2 \gtrless \frac{c}{a}$$

Consequently, when the discriminant is a real number, the computer causes the values of X_1 and X_2 to be typed out, on the other hand when there is a negative under the root, the machine will cause the real and imaginary terms of the result to be typed out separately, in the form

$$X_{1,2} = -\frac{b}{2a} \pm \sqrt{\frac{c}{a} - \left(\frac{b}{2a}\right)^2}\, j$$

An example of cyclic operation is offered by the solution of mixed cubic equations, further the calculation of logarithms as explained in Appendices 2 and 3, respectively.

The contact field of the programme chart has sixty rows each, formed of twelve contacts. However, since in addition to instructions for operations other indications, too, may be required, further since the programme chart may be used also for storing information related to the specific problem, the number of instructions assignable to a programme chart had to be limited to 45. In the event that calculation of a specific problem requires two programme charts, or even more, then the charts may be exchanged in a definite order, however, without the necessity of once again carrying in the starting data.

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So far the computer has operated in a fairly convincing manner. After certain errors of principle, initial faults in design and assembly could be eliminated from the equipment, it appeared that the computer could operate with tolerable safety for any length of time. It was found that when the equipment had been laid off for a few days, errors would interlope in the calculations, *e. g.* owing to corrosions and dusting. However, generally after a problem or two were calculated the equipment returned to normalcy, and continued in this state. Even when the equipment failed to resume operations in a flawless manner, the presence of apparatus failures could be easily detected, on the appearance of gross errors. Failure tracing was made easy in first order by making use of the delayed action features incorporated in the equipment. *Routine* examples had been worked out involving the shifting of numbers to and fro between the units inside the computer, when use was being made of each of the memory units. At the completion of each step the partial results were typed out. It was found that preventive maintenance and failure tracing were made considerably easier by applying this method.

A number of programme charts were drawn up, which when calculation had been completed, substituted the results obtained into the original formula, so that any error would emerge. Even on calculations performed with a maximum of accuracy errors would occur mainly resulting from the translation of fractions, and also, because during the process in unit A values of less than 2^{-27} would overflow.

There is a point of some importance, however, which may not be self-evident, and which, therefore, requires particular consideration.

Actually telephone technics are in a state of revolutionary changes. These changes are closely related to the metamorphosis of switching technics into the science of the fundamental principles embodied by computers, and to the introduction of electronics into telephone technics.

The appearance of computers has effected radical changes in the general outlook of telephone technics. Earlier telephone technicians were wont to talk of switches, their functions, dial pulses, etc., there was talk of traffic as a measurable quantity, which might be routed or transacted at certain definable losses. To-day the vernacular of telephone technics has come to be the connotation of the terms of computer technics. Technicians speak of informations, their methods of storing, circuits have come to be based on logical relations, there are "And" or "Or" circuits, gate circuits, circuits known from pulse technics, while the building elements in first order are semi-conductors or ferrites.

The Hungarian telephone industry may now look back to a past of six decades at least, and may be called on to lend a hand in tackling the momentous problems the future has in store. One of these is the development of a telephone exchange type built up entirely of electronic elements. Although this problem is being worked on within the framework of the Council of Mutual Economic Aid, and with the participation of the competent organs of friendly countries, nevertheless Hungary's share in the work is rather substantial, and the country's resources are heavily taxed for both manpower and financial means. It is the task of the telecommunication chairs of the Technical University to take a hand in this work, and train a staff of highly skilled engineers equipped with the knowledge enabling them to stand the mettle.

The essence of this educational work is to inculcate in the minds of those graduating as engineers *a novel conception of switching technics*. A useful aid in this training work is the computer just completed. By means of the computer solutions of certain *logical problems* may be visualized, and the trainees may then be led *to look upon the new problems of telephone technics from a more elevated level of switching technics*. They may then be convinced of imbuing the spirit of these new vistas opened for the art of telecommunication, and thus become equal to the duties future keeps in store for them.

With these considerations in mind we can only be glad that the scheme could be translated into physical reality. For this achievement I should like to give thanks on behalf of the Chair, and also on my behalf to the Technical Division of the Academy, for the valuable aid extended to the promoters of the scheme and also for the very effective assistance of the superior authorities of the Polytechnical University.

IV. Appendices

Appendix 1

Arithmetical unit

An adding unit should be designed so as to conform to the block schematic in Fig. 7. Of units of this type there are 2×27 in series. Between any of these units and any other there is a link each. Each unit has three inputs (A, B, C_{i-1}) and two outputs (S, C_{i+1}).

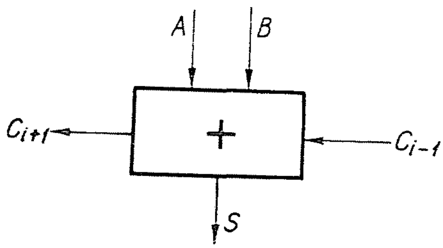

Fig. 7. Block schematic of the adding unit

Earth appears on point C_{i-1} of unit i coming from unit i—1, provided there is a carry-over. In like manner i applies earth to unit $i + 1$ over lead C_{i+1} when there is a carry-over after addition has been performed in unit i. A and B are the two numbers to be added, and S is the result of addition.

Each terminal may take on two conditions, *viz.* they may be either earthed, or off earth, or, with the notation of switching algebra, their state may be expressed as 1 or 0. The conditions may be tabulated as follows :

Inputs			*Outputs*	
A_i	B_i	C_{i-1}	S	C_{i+1}
0	0	0	0	0
1	0	0	1	0
0	1	0	1	0
1	1	0	0	1
0	0	1	1	0
1	0	1	0	1
0	1	1	0	1
1	1	1	1	1

This table must be realized. With the notations as used in switching algebra the following may be written :

$$f(s) = A\bar{B}\bar{C}_{i-1} + \bar{A}B\bar{C}_{i-1} + \bar{A}\bar{B}\bar{C}_{i-1} + ABC_{i-1}$$

after manipulating :

$$= (A\bar{B} + \bar{A}B)\,\bar{C}_{i-1} + (\bar{A}\bar{B} + AB)\,C_{i-1}$$

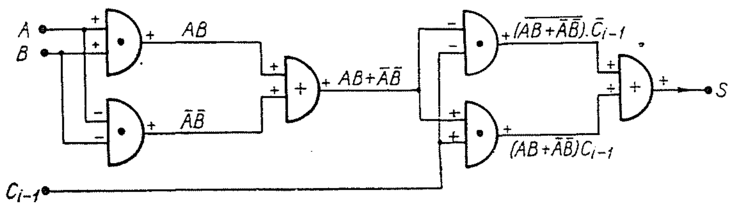

Fig. 8. Symbolic representation of the adding unit

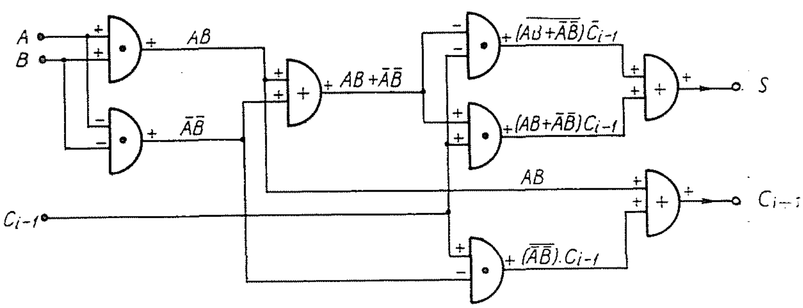

Fig. 9. Adding unit

whence after transformation :

$$(A\bar{B} + \bar{A}B) = (A + B)(\bar{A} + \bar{B}) = \overline{(AB + \bar{A}\bar{B})}$$

$$f(s) = \overline{(AB + \bar{A}\bar{B})}\,\bar{C}_{i-1} + (AB + \bar{A}\bar{B})\,C_{i-1}$$

The symbolic realization of this expression is shown in Fig. 8.

$$f(C_{i+1}) = AB + (A + B)\,C_{i-1}$$

and transformed :

$$= AB + \overline{\bar{A}\bar{B}}C_{i-1}$$

The realization of this expression completed by $f(s)$ is shown in Fig. 9.

The symbolic circuit in Fig. 9 may be realized with relays as follows : Input C_{i-1} and output C_{i+1} are each split into two terminals :

$$\begin{array}{ll} N_{i-1} & V_{i-1} \\ N_{i+1} & V_{i+1} \end{array}$$

on which earth will appear according to, if there is or not a carry-over or transfer.

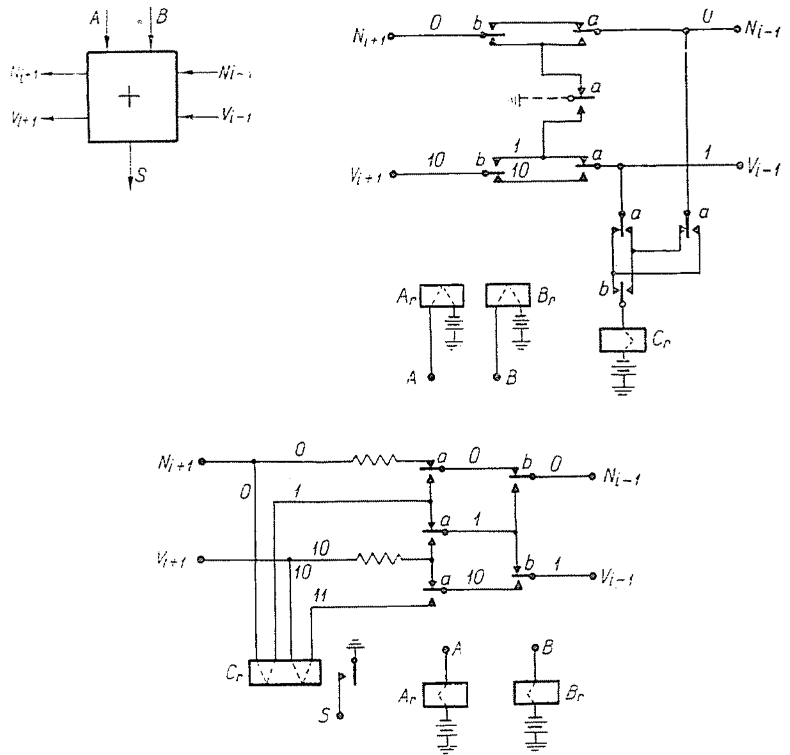

Fig. 10. Adding units with relays

The functions may be written as follows :

$$f(s) = \overline{(A\,B + \bar{A}\,\bar{B})}\;N_{i-1} + (A\,B + \bar{A}\,\bar{B})\;V_{i-1}$$

$$f(N_{i+1}) = \bar{A}\,\bar{B} + \overline{\bar{A}\,\bar{B}}\;V_{i-1}$$

$$f(V_{i+1}) = A\,B + \overline{\bar{A}\,\bar{B}}\;V_{i-1}$$

This solution is shown in Fig. 10, in two alternatives.

Appendix 2

Solution of a mixed cubic equation

A mixed cubic equation may be written in a reduced form as follows :

$$ax^3 + bx^2 + cx + d = 0$$

This equation has three roots, at least one of which is a real number. This real root is determined by way of iteration, then with the knowledge of the real root the cubic equation may be reduced to a quadratic form, so that the computer will be able to calculate the other two roots simultaneously, irrespective of whether they are real or conjugate complex numbers.

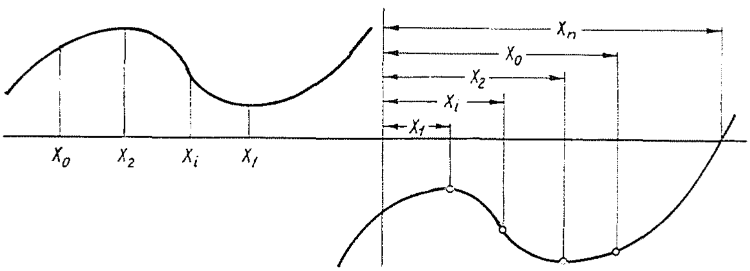

Fig. 11. Form of the function of a mixed cubic equation

An approach to the real root is by way of Newtonian iteration. The relation of two consecutive approximate values may be written as

$$X_{K+1} = X_K - \frac{f(X_K)}{f'(X_K)}$$

This cyclic operation then continues until

$$f(x_n) \leqq \delta$$

where δ is a pre-determined error value, and X_n the root wanted.

Let X_0 be the value where iteration starts. The first programme chart will then be used for determining the value of this X_0. When

$$a > 0$$

then the curve for the function may be plotted as shown in Fig. 11. (When $a < 0$ then the expression should be multiplied by —1.) Between $-\infty$ and X_1, or X_2 and $+\infty$ the function is of the increasing monotonic type. The real root may then be found in either of these intervals.

Extreme values X_1 and X_2 are obtained by differentiating the basic equation and making it equal to zero. Then

$$3\,ax^2 + 2\,bx + c = 0$$

The roots of this expression may be written as

$$x_{1,2} = -\frac{b}{3\,a} \pm \sqrt{\left(\frac{b}{3\,a}\right)^2 - \frac{c}{3\,a}}$$

further

$$x_i = -\frac{b}{3\,a}$$

which is the abscissa of the point of inflexion.

If $f(x_i) > 0$, then

$$\begin{aligned} x_0 &= 2\,x_1 - x_i \\ &= -\frac{b}{3\,a} - 2\sqrt{\left(\frac{b}{3a}\right)^2 - \frac{c}{3\,a}} \end{aligned}$$

If $|f(x_0)| \leq \delta$, then x_0 is the real root wanted. Iteration begins in fact at x_0, and the result of this iteration will then be x_n. If x_n is known, the cubic equation may be reduced to $(ax^3 + bx^2 + cx + d) : (x - x_n) = Ax^2 + Bx + C$ where

$$C = \frac{-d}{x_n}$$

$$B = \frac{C - c}{x_n}$$

$$A = \frac{B - b}{x_n}$$

The roots of the quadratic equation will then yield the 2nd and 3rd roots of the mixed cubic equation.

Appendix 3

Calculation of logarithm

Any decimal number may be expressed as

$$N = M\,2^k$$

where

M denotes the mantissa of the number, and
k the binary characteristic (order of magnitude)

$$\log N = \log M + k . \log 2$$

where necessarily

$$1 < M < 2$$

The second term on the right side is easy to determine, since order of magnitude k is the result of conversion, and this has to be multiplied by the binary form of the value of

$$\log 2 = 0.30103 \ldots .$$

The logarithm of M may be determined by way of the following infinite series :

$$\ln M = \ln (1 + x) = \frac{x}{1} - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$$

where, when $x \ll 1$ then the series is a rapidly convergent one.
When

$$x = \frac{1}{8} \qquad \text{(0.001 in the binary system)}$$

then the 8th term will read

$$\frac{x^8}{8} = \left(\frac{1}{8}\right)^8 \frac{1}{8} = 2^{-27}$$

As the sum of the series of alternating signs may be written as

$$\sum_{k=9}^{k=\infty} (-1)^{k+1} \frac{x^k}{k} < \frac{x^8}{8}$$

and as the capacity of the computer is 27 binary digits, it will suffice to calculate the series up to the 8th term.

Let M in its binary form be divided by numbers easy to handle, e.g.

$$\alpha = \frac{3}{2} \left(= 1 + \frac{1}{2} = 1.1 \quad \text{in the binary form}\right)$$

$$\beta = \frac{5}{4} \left(= 1 + \frac{1}{4} = 1.01 \quad \text{in the binary form}\right)$$

$$\gamma = \frac{9}{8} \left(= 1 + \frac{1}{8} = 1.001 \text{ in the binary form}\right)$$

When $M > a$, then by dividing

$$M_1 = \frac{M}{a}$$

Let M_1 be compared with β. When

$$M_1 > \beta$$

then by dividing

$$M_2 = \frac{M_1}{\beta}$$

When

$$M < a, \text{ then } M_1 = M \ (a = 1) \text{ and}$$

$$\text{when } M_1 < \beta, \text{ then } M_2 = M_1 \ (\beta = 1)$$

Similarly when

$$M_2 > \gamma$$

then by dividing

$$M_3 = \frac{M_2}{\gamma}$$

After one, two, or three divisions :

$$x = \left(\frac{M}{a\beta\gamma} - 1\right) < \frac{1}{8}$$

Then, when three divisions are to be performed

$$M = a \, \beta \, \gamma \, M_3$$

The numbers which have not taken part in the operation are discarded. (= 1 and ln 1 = 0.)

Accordingly,

$$\ln M = \ln a + \ln \beta + \ln \gamma + \ln M_3$$

or recast into a form suitable for programming :

$$x\left(\frac{1}{1}\right) - x\left(\frac{x}{1+1}\right) + x\left(\frac{x^2}{2+1}\right) - x\left(\frac{x^3}{3+1}\right) + x\left(\frac{x^4}{4+1}\right) - \ldots$$

The logarithm wanted will then be

$$\log N = k \cdot \log 2 + \log e \, [\ln a + \ln \beta + \ln \gamma + \ln M_3]$$

The approximate time required for determining the logarithm may be expressed as

$$k \cdot \log 2$$

a single multiplication

$$M_3 = \frac{M}{a \beta \gamma}$$

(two divisions, at most, suffice).

The calculation of the 8th term of the infinite series involves eight cycles, each cycle representing a multiplication, division and addition each.

Finally, the expression in square brackets represents four additions, while the calculation of the final result each a multiplication and addition.

Appendix 4

Calculation of Erlang's formula

In its conventional form Erlang's formula may be written as

$$P_v = \frac{\frac{y^r}{r!}}{1 + y + \frac{y^2}{2\,!} + \frac{y^3}{3\,!} + \ldots + \frac{y^{r-1}}{(r-1)!} + \frac{y^r}{r!}}$$

where y denotes the traffic in terms of hours, as handled by a number of

r link circuits, and

P_v denotes the percentage of y failing to engage a link circuit immediately.

In its conventional form the formula is not suitable for programming, for it may occur that

$$y > 100, \text{ and}$$

$$r > 150$$

i. e. y^r exceeds the capacity of the computer.

The terms of the formula should therefore be divided by

$$\frac{y^r}{r!}$$

so that the formula boils down to

$$P_V = \frac{1}{1 + \frac{r}{y} + \frac{r(r-1)}{y^2} + \frac{r(r-1)(r-2)}{y^3} + \ldots + \frac{r!}{y^r}}$$

Programming of the nominator in this form is a simple matter, the calculation of each value representing a cycle each.

From term k of the nominator $(k + 1)$ will be derived:

$$N_{K+1} = N_K \frac{[r - (k - 1)]}{y}$$

E. g.

$$N_4 = \frac{r(r-1)(r-2)}{y^3}$$

$$N_5 = \frac{r(r-1)(r-2)}{y^3} \cdot \frac{(r-3)}{y} = \frac{r(r-1)(r-2)(r-3)}{y^4}$$

Finally,

$$N = \sum_{x=0}^{x=r} N_x \text{ and}$$

$$P_v = \frac{1}{N}$$

Appendix 5

Conversion by halving and doubling

Let *e.g.* 87.6875 be converted.

The integers are keyed into halving circuit F_1, the fractions into doubling circuit D_1. Fig. 12 shows how the digits vanish in course of halving and doubl-

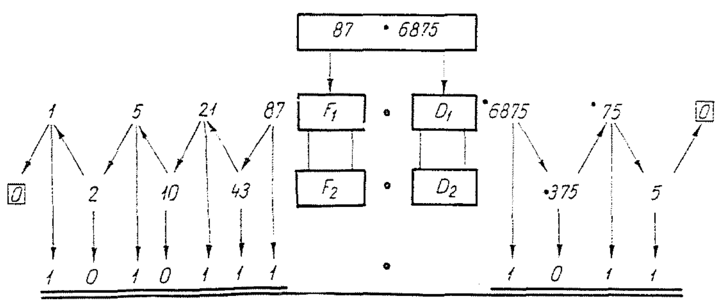

Fig. 12. Principle of conversion

ing, and how the binary form

1010111.1011

is obtained.

Summary

A concise description of the digital computer of the Chair of Wired Telecommunication at the Polytechnical University of Budapest is here offered. This computer is an automatic programme controlled equipment, operating on the binary system and built up of electromagnetic relays. Each type of operation has been allotted a programme chart made of insulating material, with definite patterns punched in it, and suitable for electrical scanning. After the constants of the problem have been keyed into the computer, this will continue to operate in an automatic way in accordance with instructions, and cause an electromagnetically operated conventional typewriter to type out the results. The numbers are carried in and out in the decimal form, translation into, and retranslation from, the binary form being effected automatically.

The computer was primarily designed for didactic purposes, however, it is capable of tackling a great variety of mathematical problems. In the appendices the programming of a few simple problems has been described.

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