MIRROR MICROMETERS

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Micrometer is the general designation for instruments of special construction, employed for precisely measuring fractions of angles or of lengths. These instruments fall into two large categories. The first of these categories comprises micrometers in which readings of the main scale or of fractions thereof are taken by a vernier scale sliding along the main scale. Sliding calipers, and micrometer screws belong to this first group, while slip gauges representing values which are constant and established are also based on the same principle. The common characteristic of these instruments is that mechanical reading or measurements are made by them.

The second category of micrometers is characterized by the feature that here the vernier scale, or its image, is displaced in relation to the main scale, or vice versa. The images of the scale or of the sighted point, formed by means of optical arrangements, are optically displaced in relation to each other, when the readings are taken. This is achieved by bringing into coincidence the images of different position of the sighted point. The instruments working on this principle are the optical micrometers proper, also called *compensators*, to distinguish them from the micrometers first referred to.

A brief survey of the history of micrometers is presented before proceeding to the description of mirror micrometers.

According to literary records, the first, rather simple micrometer should be credited to Montanari, dating back to 1674. It consisted of a scale engraved into a glass plate and placed in the focal plane of the objective of a simple astronomical telescope, fractions of the scale being so related to the focal length as to produce certain angular values in the object field. While the working principle of this instrument was strictly adhered to, with the passage of time the instrument itself underwent many changes. It is still in use for measurements of less accuracy, such as *e. g.* measuring target distances and target dimensions with the aid of prism binoculars. However, the results in this manner arrived at should be considered as estimates rather than measures.

If, however, the image of the scale is projected onto the field of vision, instead of on the scale itself, the process is termed autocollimating angular measurement, a method still widely used in goniometres.

The movable-hair-micrometer, designed by Gascoigne in 1640 — independent of Montanari — marks a distinct advance. In this instrument the means of measurement is a stadia line fixed on a carriage or nut arranged in the image plane of a surveying telescope. The nut can be shifted by means of a leading screw of high precision. Fractions of one revolution of the leading screw can be read off a graduated drum, keyed to one end of the leading screw, but adjustable. The readings of fractions are also taken by estimation.

The accuracy of movable-hair-micrometers mainly depends on the accuracy of the pitch of the leading screw. Errors of the pitch run can be either

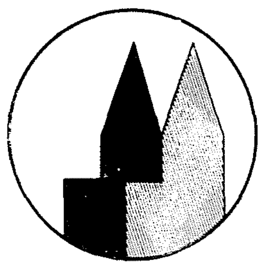

Fig. 1. Super positioning field of view

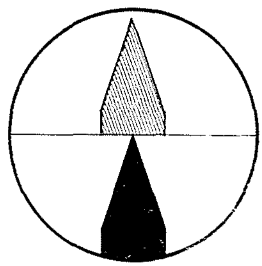

Fig. 2. Field of view with separating line

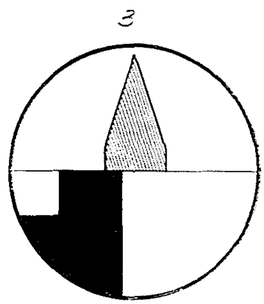

Fig. 3. Field of view with separating line, with the image cut in two

continuous or periodical. Another factor responsible for the accuracy is the appropriate bearing of the carriage, and the precise run of the leading screw.

The most up-to-date type of movable-hair-micrometers is the Bauersfeld spiral micrometer. in which no complicated mechanical means is required for displacing (controlling) the stadia line. In another known design displacement is effected by means of a cardioid cam.

There are, of course, various existing types of movable-hair-micrometers. For fuller particulars, reference should be made to pertinent literature [1].

Let us now revert to the type of instrument referred to above, in which measurement is effected by the relative displacement of the two images of the sighted point, achieved by optical means. Accuracy of measurement is obviously depending on the precise coincidence of the images. Hence, in the course of time due consideration was given to the shape, dimensions and the precincts of the object to be measured, as well as to the various aspects of the mental process of human vision, many of which are still unaccounted for.

Considering the oldest and most simple design of such an instrument, the two objectives form two images of similar construction and magnification in the field of vision of the instrument (Fig. 1). Taking an instrument adjusted to infinity, the images are in exact coincidence, while their relative displacement increases with the target distance. Coincidence is achieved by using any kind of an optical micrometer referred to above.

For obtaining more accurate results, one can horizontally divide the field of vision (Fig. 2), the two images are then formed in the thus obtained two fields. Since a constant mark is required for bringing the images into coincidence, it is advisable to divide the image by a parting line as represented in Fig. 3. If the second image is an inverted mirror image of the first, accuracy can be further improved. (Fig. 4).

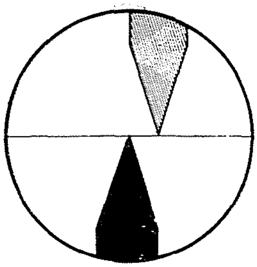

Fig. 4. Inverted coincidence field of view

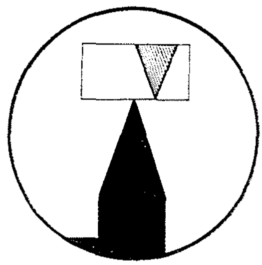

Fig. 5. Window field of view

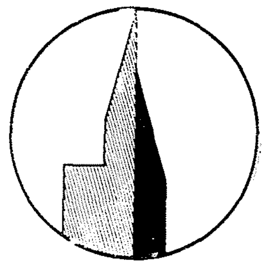

Fig. 6. Field of view with vertical separating line

The precincts of the point to be covered often disturb measuring. The so-called window fields of vision (Fig. 5) utilize only a small portion of the inverted mirror-image for coincidence.

For the purpose of measuring certain objects or portions of the field, vertical division of the image is preferred, for instance in naval range finders (Fig. 6). Some designers have extended the principle of the window field to cover the full breadth of the field of vision. This results in the so-called horizontal or vertical band field of view. The most frequent type is of the inverted coincidence kind, but there are, of course, instruments with entirely different fields of vision designed for special purposes. The two images are produced by a prism system provided with two objectives of equal relative apertures and equal magnifications.

For bringing into coincidence the image pairs displaced in the function of distance is largely achieved by inserting an optical element, such as a lens or wedge, into one of the objectives path of rays.

Lens micrometers, in most cases consisting of two components, such as the Abat micrometer designed in 1777, are applied in photographic cameras and in range finders of a short internal base line. As references to these instruments are rather scarce in literature, it was found advisable to give their brief description.

The refractive indices and the radii of curvature R of the two lenses represented in Fig. 7 are equal. In a normal position, the plano-convex and plano-concave lenses if not separated by air form a planoparallel plate ABCD. In this position, beams coming from infinity pass the plate unrefracted, hence, undeviated. If, now, the plano-convex lens is rotated about its centre of curvature O through an angle α, the axis point E_1 is shifted to A_2, hence, the wedge angle φ is changed. This type of system can, therefore, be called a prism of variable refractive angle.

Another Abat lens micrometer is illustrated in Fig. 8. It is of an even simpler construction, one of the lenses being displaced along a common contacting plane surface. The plano-convex lens alone condenses the pencils incidentally parallel to the axis in the focus F. Inserting a negative lens of

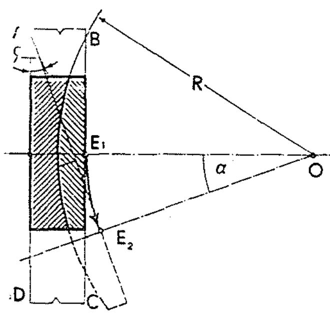

Fig. 7. Abat rotatable lens micrometer

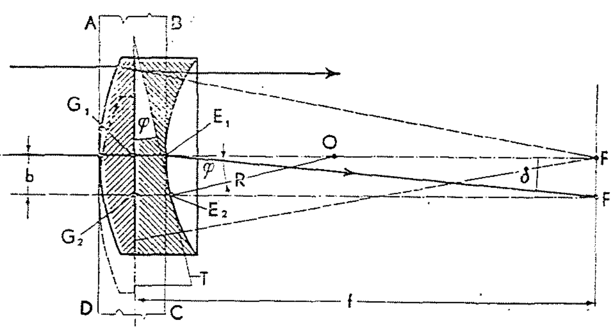

Fig. 8. Abat sliding lens micrometer

similar focal length and refractive index, the emerging pencil leaves the system parallel to the axis, an arrangement representing a simplified version of the common opera glass. Let us displace the positive lens by a length b. G_1, along with the optical axis, will then be displaced to G_2, and the emerging pencil refracted at E_2 will travel over focus F_1. When emerging, the parallel incident rays run parallel to the line E_1F_1. The tangent T laid across point E_2 of the radius of curvature R, having a centre O, makes a prismatic wedge angle with the common surface of the lenses, subject to the rate of displacement. The angle φ continually varies with the displacement of the positive lens, so that the emerging ray has a deviation δ.

Deviation can be produced either by tilting an optical plate, but is largely made by inserting a prism of small refractive angle, a so-called wedge. In the simplest case, represented in Fig. 9, the wedge *1*, having a refractive angle φ, is rotated about the optical axis, vertically to its principal section. In a basic position, the wedge will cause the incident rays to be refracted at F_1, and the emerging ones at G. The resulting difference δ in the direction of the ray causes point B in the receiving plane to be shifted to point A. Swinging the wedge about the optical axis, A rotates in the same direction, and with the same angular velocity as the wedge, thus tracing a circle $AECDA$. Hence, the path of the image is not rectilinear but circular, with the result that measuring becomes impossible.

Let us now place behind prism *1* a wedge *2* of similar refractive angle and refractive index, and let the two rotate in opposite directions, but at similar angular velocity. This system has been widely used, also as a prism of variable refractive angle. The basic concept goes as far back as the 1777

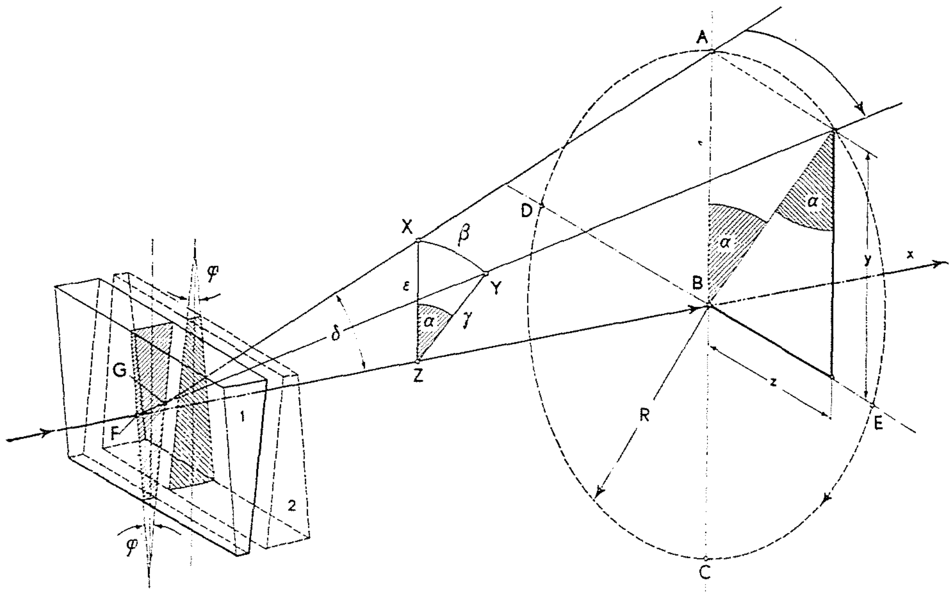

Fig. 9. Effect of wedge and wedge pair on deviating the path of rays (Boscovich pair of wedges) ·

Boscovich principle. It is sometimes called a Herschel—Rochon prism, yet we believe the first inventor$_2$ to be Boscovich.

As can be seen in Fig. 9, incident rays refracted by the wedge *1* swung about its axis, trace a conical surface having a base of radius R and a peak G, in the plane perpendicular to the axis. Assuming the maximum deviation taking place in the planes X and Y, the deviation of the wedge turned through an angle α, may be resolved to two components. From the triangle XYZ

$$\sin \alpha = \sin c \cdot \sin \alpha$$

in the case of small angles ε and γ

$$\varepsilon = \gamma \cdot \sin \tag{1}$$

also

$$\operatorname{tg} \beta = \operatorname{tg} \gamma \cdot \cos \alpha$$

and in the case of small angles β and γ

$$\beta = \gamma \cdot \cos \alpha \tag{2}$$

Let us place behind wedge *1* a wedge *2* of equal refractive angle and refractive index. Turning the two edges in opposite directions, but at equal angular velocity, the deviation is

$$\delta \sin \alpha + \delta \sin [-\alpha] = 0 \text{ in the plane } xy,$$

and

$$\delta \cos \alpha + \delta \cdot \cos [-\alpha] = 2\delta \cdot \cos \alpha \text{ in the plane } yx \tag{3}$$

The maximum deviation in the plane xy is (2δ), when $\alpha = 0$, and $(2\delta \cdot \cos \alpha = 0)$ when $\alpha = 90°$. The horizontal component z independents on the sign of the angle of rotation. Turning the wedges through an angle α from the position associated with the maximum deviation, the deviation will be

$$\delta = 2\alpha \cdot \cos \varphi$$

and will take place in the plane of maximum deviation. If the refractive angles of the two wedges are not equal, the deviated sighting line describes an elliptic cone. Such an error also occurs if the angles of incidence and emergence of the two wedges are not quite equal. As the distance between the pair of wedges located behind the objective is in proportion to the distance of said pair from the focal plane of the objective, the image displacement occurring on the rotation of the wedges, is not uniform. Hence, the rotating pair of wedges as a compensator only, be placed in front of the objective, in other words, it can only be used in parallel radiation.

The principle that governs this type of instrument is unaffected by the method applied for driving the wedge pair. Fig. 10 shows the driving mechanism for a rotating pair of wedges, designed for a Zeiss range finder of 2 m internal base line. Image displacement or coincidence depending on the distance to be measured, the driving mechanism is provided with graduations empirically established and representing the distance. A cardan shaft connects the driving shaft *11* with a graduated cylinder situated at some distance. Driving takes place in the following manner: By means of the gear *9* and pinion *14* visible behind it the gear *13* keyed to the shaft *11* drives the drum, this latter bearing the scale divisions standing for distance. The pointer *7* engaging the spiral groove of the drum slides into the dove-tailed guide *8*. Readings are taken by the mark engraved at the end of the pointer. External readings are taken by the graduated drum. At the same time, gear *15* turns gear *12*, whereupon gear *17* fixed on the shaft *16* drives gear *6* fitted on the mounting of one of the wedges. Gear *17* drives the gear *18* in an opposite directon, which in turn rotates the other wedge mounting provided with gear *5*, in opposite direction to gear *6*, but at equal angular velocity. The rotation of gear *5*

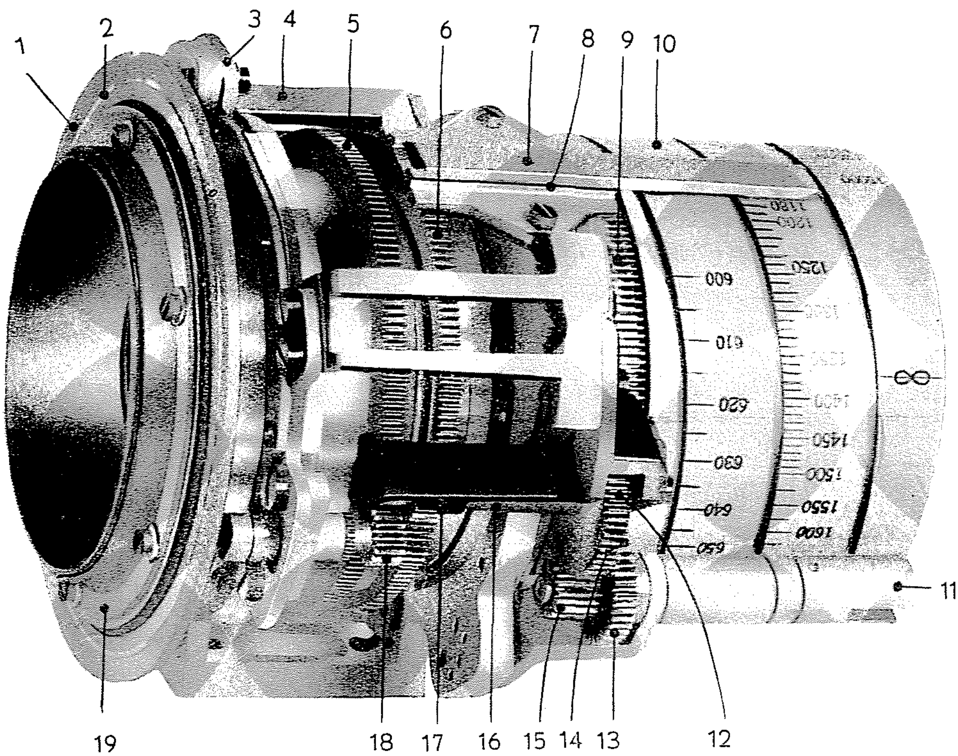

Fig. 10. Rotating pair of wedges with internal and external reading for a Zeiss range finder with internal base line

is also transmitted to the glass ring *1* bearing divisions *2* microphotographed on it. The glass ring is fixed on the mounting by means of ring *19*. The lens arrangement in mounting *3* behind the ring forms an image of the scale divisions, in an appropriate place, for the intrument's field of view. Hence, the observer is able to take the reading immediately on actual measurement. The principle of internal reading is represented by the glass ring. — The whole equipment is enclosed in the casing *4*.

The deviation of rays produced by the rotating pair of wedges has two peaks in relation to the optical axis. Assuming a continuous and uniform rotation of the pair of wedges, the object point is displaced on the screen within the limits *A* and *C* (Fig. 11). This suggests harmonic oscillation, as the path of the point may be considered as horizontal projection of a point moving at uniform speed along a circular path, the speed in point A and C being $= 0$, and reaching its maximum at point B. The curve of deviation, as a function

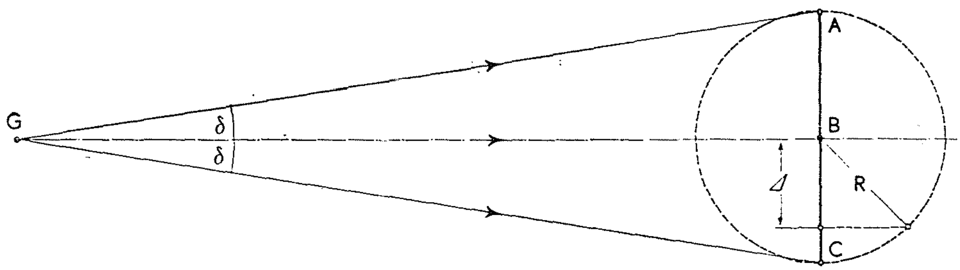

Fig. 11. The two peaks of deviation caused by a rotating pair of wedges

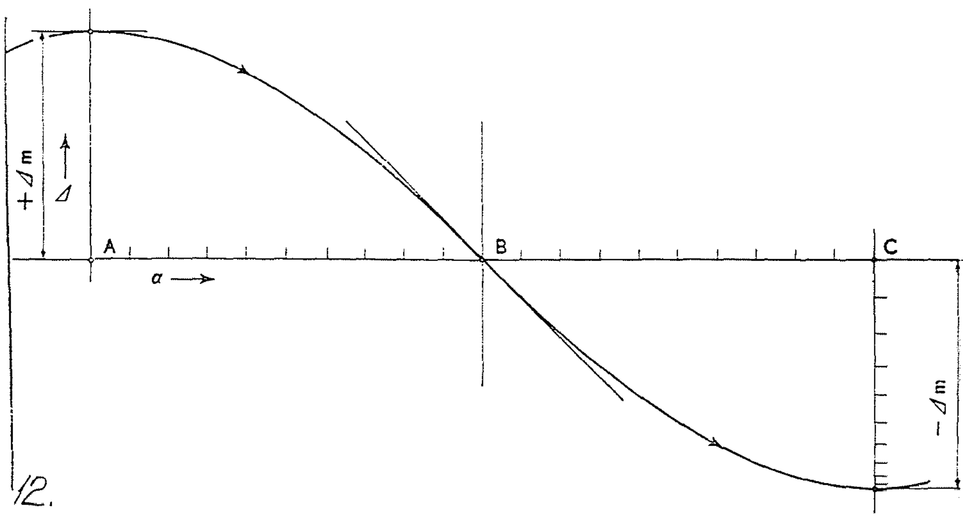

Fig. 12. Curve of deviation, as a function of angles of rotation

of the uniform angles of rotation α, is shown in Fig. 12. The extent of utilization of the curved section between the peak and the minimum is subject to the field of application of the instrument, which may be used for short range (surveying) measurement purposes, or for measuring long distances, like in military range finders. In neither case can the entire range of the curve be utilized. Along a small portion, in the vicinity of points A and C, the deviation Δm of the deviated point is practically in proportion to the uniform rotation, while deviation rapidly decreases in the precinct of the extreme values. The growing density of divisions thus arising makes reading extremely difficult. In other words, deviation is very small near the peaks, assuming uniform angles of rotation α. Hence the range of the scale cannot exceed 90°, if rotating together with the pair of wedges. Nevertheless, inspection of any range finder with an internal base line reveals that the scale covers as much as 130°. This phenomenon can be explanation in the optical design of the instrument.

As has been pointed out above, the rotating pair of wedges is apt to be used only in parallel light, that is, if located in front of the objective.

In the arrangement diagrammed in Fig. 13, the wedge is situated behind the objective. This version is given by Maskelyne, and has become known under the denomination of sliding wedge. Here the wedge can be shifted from the lens to the image plane E in the sense indicated by the arrow. If it were

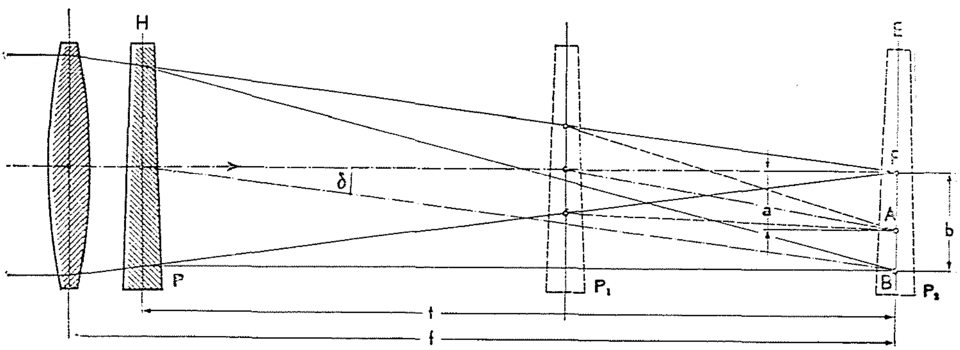

Fig. 13. Maskelyne sliding wedge

not for the wedge, the parallel incident rays would be condensed by the objective in focus F. If the wedge *2* behind the objective is replaced by a plane H normal to the principal section of the prism, in the case of a small refractive angle, the beam will be deviated by the infinitely thin wedge through an angle in the position P, point F being thus shifted to point B. The course b of the point being subject to the distance t of the wedge from the image plane E,

$$b = BF = \delta t \tag{4}$$

for small angles.

The nearer the wedge is situated to the lens, the greater the deviation and, therefore, displacement of the image, whereas these values equal 0 when the plane H of the wedge coincides with the image plane (position P_3). In an optional position P_1 image displacement is $b = AF$.

The chief advantage of the sliding wedge arrangement resides in the fact that the distance t required for displacement can be well utilized for a long focal length of the objective. The scale engraved on the pointer is directly attached to the wedge, therefore there are no dead turns in the equipment.

It is quite suitable for both internal and external readings, and its design is much simpler than that of a rotating pair of wedges.

Various other more or less different versions of image displacement by means of wedge pairs are known. Their detailed description would, however, fall outside the scope of the present paper.

Conclusion

Wedge pair arrangements entail the following inconveniences, particularly with respect to image formation:

1. Even wedges of small refractive angles tend to resolve achromatic light into a spectrum, a phenomenon which, while not too disturbing for small refractive angles, makes measuring rather inconvenient for refractive angles of some magnitude. To avoid this, wedges of high refractivity are produced by cementing wedges of different refractive indices. Since, however, actually only two wave lengths of the spectrum can be merged by using two types of glass, the disturbing effect of the secondary spectrum should be taken into account for such achromatic wedges, if measurement is to be of high accuracy.

However, the manufacture of achromatic wedges is not easy. It is essential that the refractive angle of both components should lie exactly in the same direction, — therefore, the use of a cementing collimator is required. Notwithstanding the utmost care devoted to the process of cementing, the constancy of the system is not satisfactory. This has given various researchers the idea of connecting the wedges by adhesion. When two components of plane optical surfaces are superposed, air is driven out from between them, so that atmospheric pressure combined with molecular effects cause the two surfaces to adhere to each other. Difficulties may arise with changes of temperature. Owing to the different heat expansion coefficients of the components and to gravity, the components may under certain conditions separate.

2. Oblique rays incident to an optical plate or a wedge, suffer displacement or deviation while the image is afflicted with astigmatism. The sagittal and tangential images are situated at different points of the axis. This so-called astigmatic difference is independent of the object distance for an optical plate but depends on and increases with the thickness of the plate. A similar phenomenon can obviously be observed in connection with pairs of wedges, but here the astigmatic difference is correlated to the object distance. The shorter the object distance, and the larger the refractive angle φ, the greater the astigmatic difference. The path of the rays through the wedges is one of the components responsible for astigmatic differences, this difference, of course,

increases with the thickness of the wedge. Astigmatism, therefore, varies with the length of the path of rays over the wedges.

3. Even for the simplest type of wedge pairs, four surfaces will unavoidably be in contact with the air, leading to a certain loss, due to surface reflection. It is, however, possible to reduce this loss somewhat by applying an anti-reflection surface coating.

4. Adequate image formation can only be expected from wedges bounded by precisely ground surfaces. If the wedge surfaces are not perfectly plane, the wedges will act like condensing or dispersing lenses, thus impairing uniform magnification of the images. This error, in itself, however small, contributes to aberrations of the image.

These aberrations increase with the refractive angle and with the thickness of the wedge and, in addition, their value varies in the course of rotation, a phenomenon apt to lessen the accuracy of measurement. There is no need for emphasizing the difficulties connected with adjustement to sharpness, particularly for long distances. Obviously, the slightest aberration in image function renders measurement inconvenient, not to speak of external influences affecting operation.

II.

The necessity of designing an instrument based on the principle of rotating wedge pairs, but freed from the aberrations referred to above has been felt, and an arrangement produced in which the incident light, instead of being deviated, is reflected with the aid of mirrors. A mirror micrometer of this kind is illustrated in Fig. 14 and is described below.

The mirrors are so arranged that their principal section make an angle of 45° to each other, the system thus obtained corresponds to the penta mirror of constant deviation, known in surveying practice. It will later be explained why this particular system as a starting point was preferred.

To obviate refraction, the facing surfaces BC and EF of the mirrors *1* and *2* subtending a 45^0 angle φ are anti-reflection coated. The ray J incident on BC is reflected at point K and strikes the surface EF whence it is once more reflected at point L, the angle of reflection being equal to the above angle. As the ray emerges from the system, it subtends a 90° angle at 0 with the incident ray. If, now, the air space between faces BC and EF is filled up with a glass block (bounded by a broken line in the figure) the known penta prism is obtained.

The penta prism is frequently applied for range finders of internal base line, taking the great advantage into account that incident and emergent rays always square with each other when the mirror system is rotated around its axis 0 perpendicular to the base plane, although their points of intersection vary.

The penta mirror or prism is adjustably fixed on the mounting, taking great care lest stresses should arise. The use of mirrors to replace prisms is gaining more and more ground, even for instruments of small size. The material and dimensions of the mirror mounting should so be selected that the change of its wedge angle with temperature should not exceed the measuring accuracy of the instrument.

Fig. 15 represents a modern Zeiss prop mirror mounting. The mirrors *2* and *3* are pressed to the horizontally grooved operating surfaces *6* and *7*,

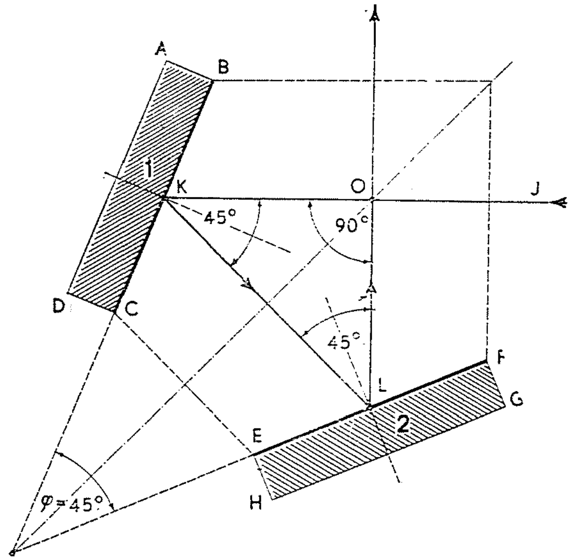

Fig. 14. Path of rays in the penta mirror

by means of bars *4* and *5* screwed onto the heat-treated steel mount *1*. Rather thin necks *10* and *11* provide connection to the mirror with the protruding sockets *8* and *9*. As the sockets are rigidly held in place by the bars, changes of temperature will cause the mirrors to suffer tensile stresses. To eliminate such stresses, the neck has been made as thin as is possible within strength limits. Corners have been rounded off or disedged, with a view to avoid excess material.

The shape of the mirror mounting suggests that its expansion, due to unilateral temperature effects, is not uniform. The operating surfaces turn in relation to each other, taking along the mirrors attached to them. This invariably leads to a change in the refracting angle φ of the mirror mounting.

The shape of the mounting is affected not only by external influences and by temperature, but by the antecedents of the system. Taking an expansion coefficient of $\alpha = 0{,}000\,012$, a distance of the near-by operating

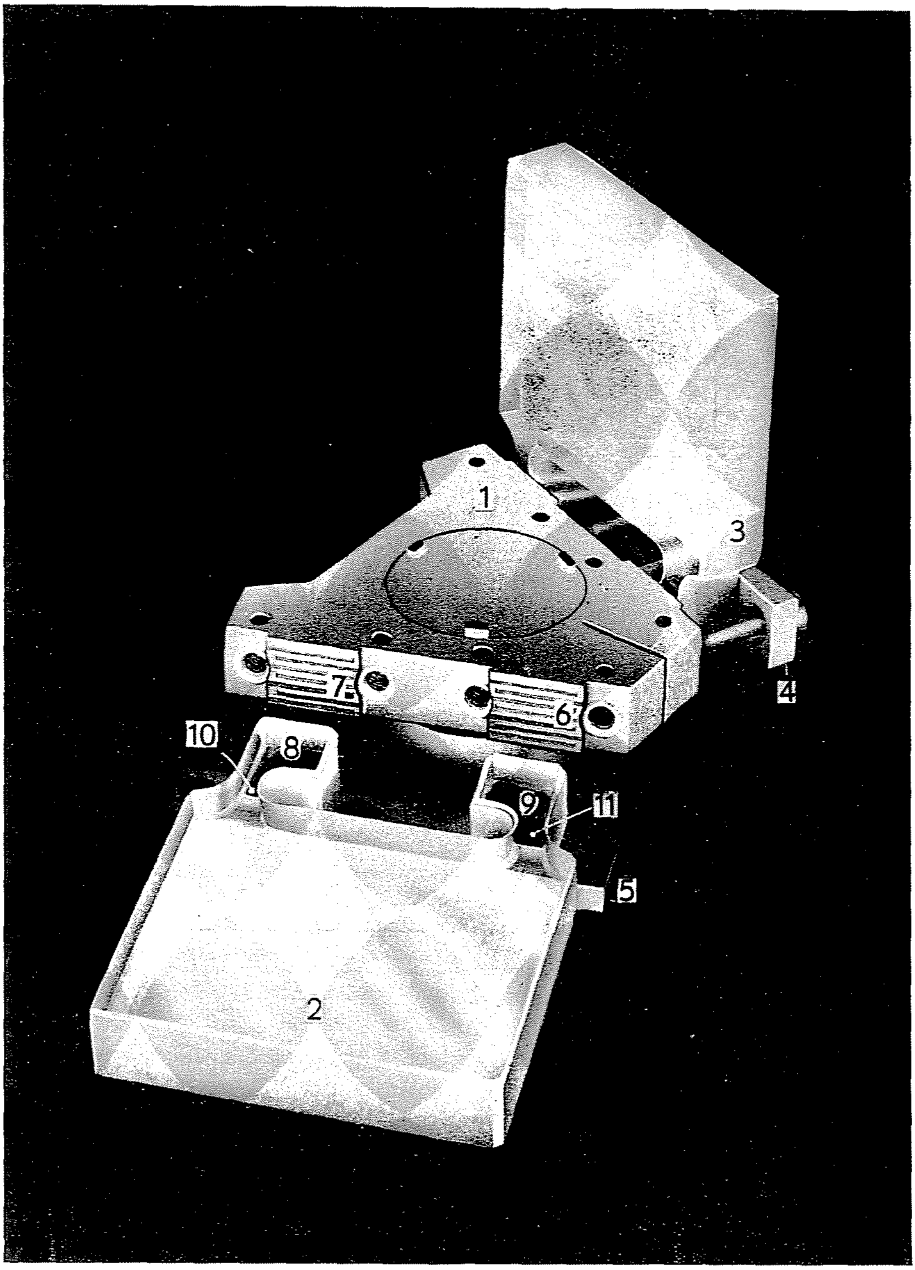

Fig. 15. Zeiss penta mirror mounting

surfaces a, a distance of the distant operating surfaces b, and a distance c between a and b, then, in the case of a uniform drop of temperature Δt from a toward b, the penta mirror suffers a change of angle that is, about 4.8 seconds from $t = 1$ Centigrade.

$$\Delta y = \frac{a. \Delta t \ (a+b)}{2c} \tag{5}$$

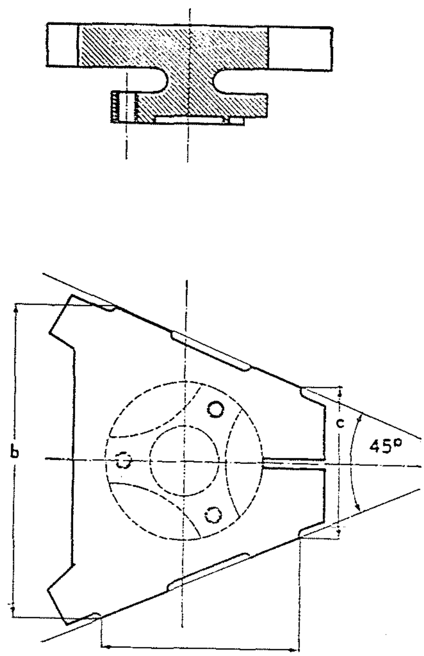

Fig. 16. Diagram of Zeiss penta mirror mounting

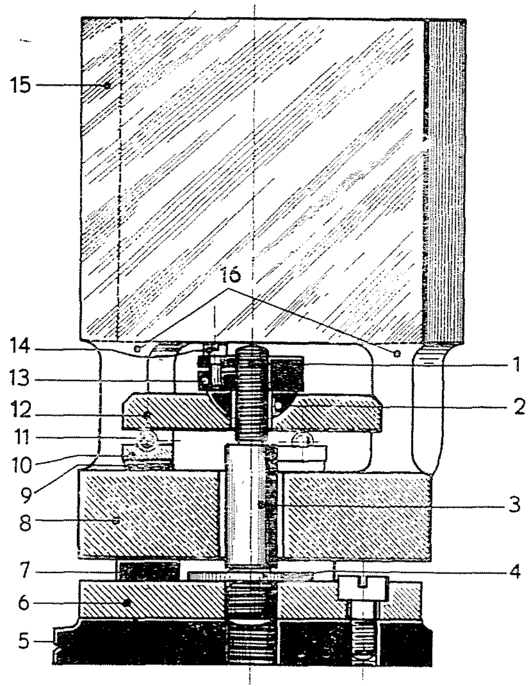

Fig. 17. Zeiss one-piece penta mirror

The mirror mounting above referred to was further developed by the Zeiss Company. Fig. 17 illustrates a section of it which from the viewpoint of precision mechanics is very instructive. The mirror *15* is connected to the glass mounting *8* by thin legs *16*. The nut *13* screwed on the threaded portion *1* of the leading screw *3* presses on disc *12* with the aid of a spherical washer *2*. Pressure of the disc lying on the balls *11* is translated through washers *10* to the rubber washers *9*, pressing the mounting *8* against the rubber washers *7* placed on the base plate *6*. The leading screw *3* screwed into the base plate *6* rests on the disc *4* of a larger diameter. The whole system can be fitted on to a part of the instrument with three screws. Nut *13* is slit, and its two halves resiliently use the screw *14* driven through it.

The Goertz mirror mounting represented in Fig. *18* has a different, but equally ingenious design. Mirror *2* and the components required for fixing it were in the figure removed. The mirrors *1* and *2* engage the optically ground surfaces *3*, *4* and *5* of the mounting. The fixing jaws *11*, *12*, *13* and *14* tightly held by the screws *9* and *11* serve to eliminate lateral displacement of the mirrors. The plate *19* with its short, rounded off legs *15*, *16* and *17* is attached to the back surface of the mirrors, and is held in place by the screw *6* threaded into fork *8* arranged on the mounting, and protruding into bore *18* of the plate.

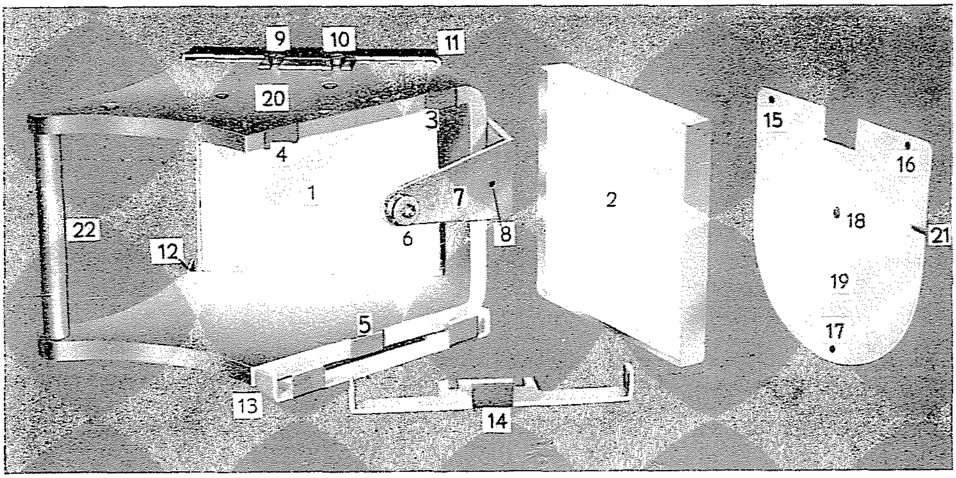

Fig. 18. Goertz resilient penta mirror mounting

A pin *8* extending into hole *20* and fixed to fork *7* secures the plate *19* against turning. The two ends of the mirror mounting *21* are rigidly connected by rod *22*.

III.

The use of the penta mirror as an optical micrometer is subject to the condition that the reflecting surfaces — as in the case of an instrument for setting out right angles — should be so swung in relation to each other so as to convert them into a rotating pair of wedges of variable refractive angles. To achieve this, the two wedge-shaped, externally coated mirrors *1* and *2* illustrated in Fig. 19 are rotated — in opposite direction to each other — around the axes T and T_2-normal to their exterior faces. The penta mirror is thus converted into a penta micrometer, similarly to the pair of wedges by Boscovich.

In the initial position as shown in the figure, the system represents a common penta mirror, for the faces of the mirrors *1* and *2* make a 45° angle in position I. The rays arriving from direction J are twice reflected and emerge in the direction J_1. The incident and emerging rays subtend a 90° angle at O.

Turning the mirrors into position II, the incident ray is reflected at point A of mirror *1*, then at point A_1 of mirror *2*, and finally emerges towards J_2.

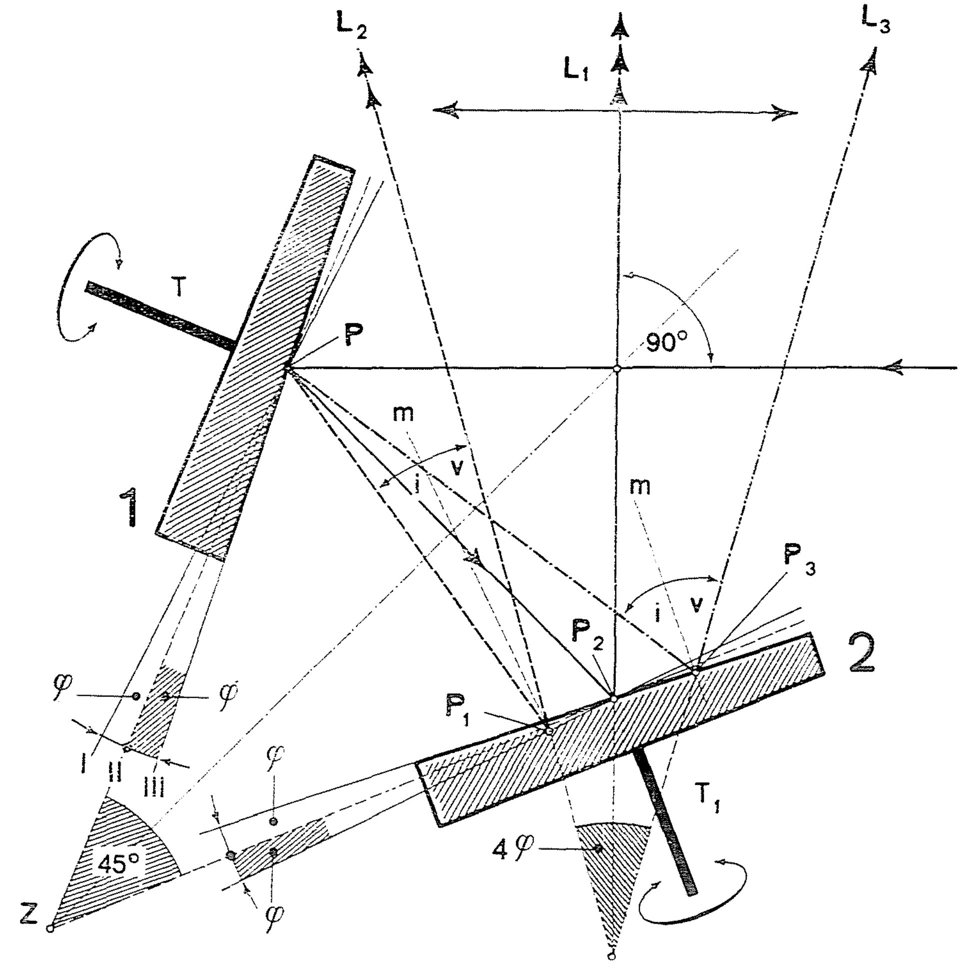

Fig. 19. A simple penta mirror, used as a mirror micrometer

If the mirrors are turned in an opposite direction, the ray will suffer a deviation from position I to position III in accordance with the arrow. In the latter position the light is reflected at point A_3 (mirror *1*) and at A (mirror *2*) and emerges toward J_3.

The deviation of the reflected rays shows that the penta mirror thus converted can replace the rotating wedge pair, while retaining its quality as an instrument for setting out constant directions and securing a constant sighting line.

The rays reaching the mirror system are reflected — without being refracted — by an air-contacting surface coated with aluminium or with some other metal. Hence, the instrument is free from the errors and aberrations in connection with rotating pairs of wedges referred to above.

Considering the path of rays, one finds that the rotation of the mirrors results in a change of the side lengths of the triangle *abc* (Fig. 20) which in

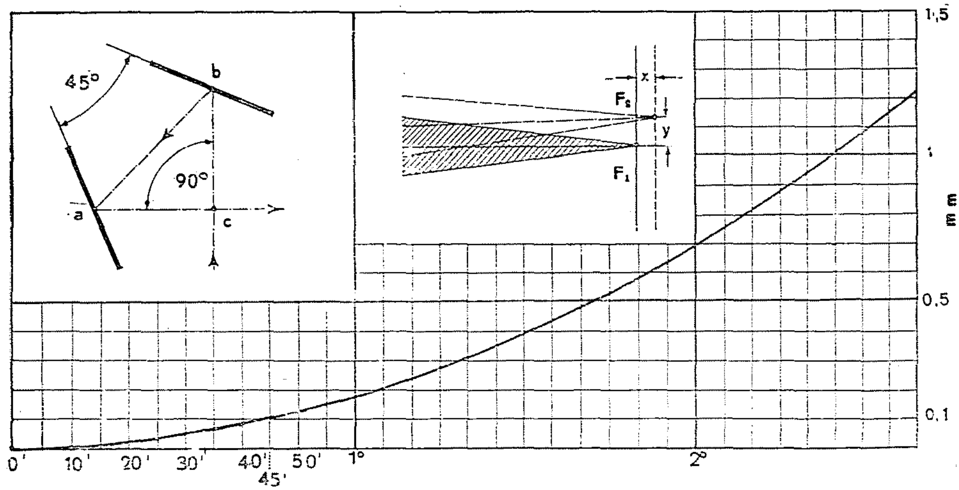

Fig. 20. Displacement of image plane resulting at the rotation of the mirror micrometer

turn means, that the image formed by the objective, that is, the focus F_1, travels to F_2, displaced by a length x along the axis. The value of x varies in accordance with the curve shown in Fig. 20 for a range finder of 0,3 m internal base line. The human eye being unable to perceive a parallax of 0,1 mm, this error does not involve inaccuracy of measurement for small mirror angles.

A laboratory model of the instrument is represented in Fig. 21. It must be emphasized that the instrument is only suitable for test purposes, as the gear mechanism drive is not able to achieve the required accuracy. In addition, the error becomes greater in accordance with formula (5).

$$\Delta y = \frac{a \cdot \Delta t \, (a + b)}{2\,c} \tag{6}$$

Apart from the mode of drive applied, correction is to be sought by selecting suitable materials and shapes, as well as appropriate heat insulation methods. Finally, it must be kept in mind that all mirrors are afflicted with a certain amount of wedge error which is apt to arise on rotation. It is, therefore, advisable to take twice this error into account.

The mirror micrometer, when used as a penta micrometer for range finders of internal base line is a combination of the penta mirror and the rotating pair of wedges. It can also be suitably used in all instruments based on the principle of micrometer measurement by means of optical deviation of the direction of the radiating energy.

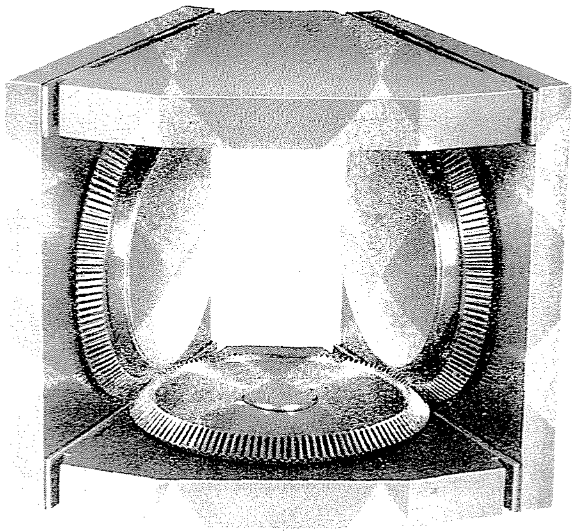

Fig. 21. The penta mirror as mirror micrometer

It has already been mentioned that the displacement of the length of the path of rays in the case of rotation, as a function of the angle a in basic position, that is, if $a = 0$,

$$x_{a=0} = a \cdot \left\{ 2 + \sqrt{2} - 1 + \left[1 - \left(\frac{\operatorname{tg} 45^\circ - 1}{\operatorname{tg} 45^\circ + \operatorname{ctg} 22{,}5^\circ} \right) \left(1 + \frac{1}{\cos 45^\circ} \right) \right] \right\}.$$

If a varies, the displacement of the image is $2a$. Thus one can write that

$$x_a = a \left\{ 2 + \sqrt{2} - \left[1 + \left(1 - \frac{\operatorname{tg} 45^\circ + 2a - 1}{\operatorname{tg} 45^\circ + 2a + \operatorname{ctg} 22^\circ 30' + a} \right) \cdot \left(1 + \frac{1}{\cos 45^\circ + 2a} \right) \right] \right\}$$

The location of the mirrors or wedges depends on the principle of the instrument's construction. Fig. 22 shows an embodiment in which the mirrors *1* and *2* are situated beside each other. In this case, however, a third auxiliary mirror *3* is required. In another version the mirrors face each other at optional distances, with their axes laterally displaced in relation to each other. The laboratory scale model shown in Fig. 22 applies plane parallel plates with front metal coating, the wedge angles are determined by the tilting of the

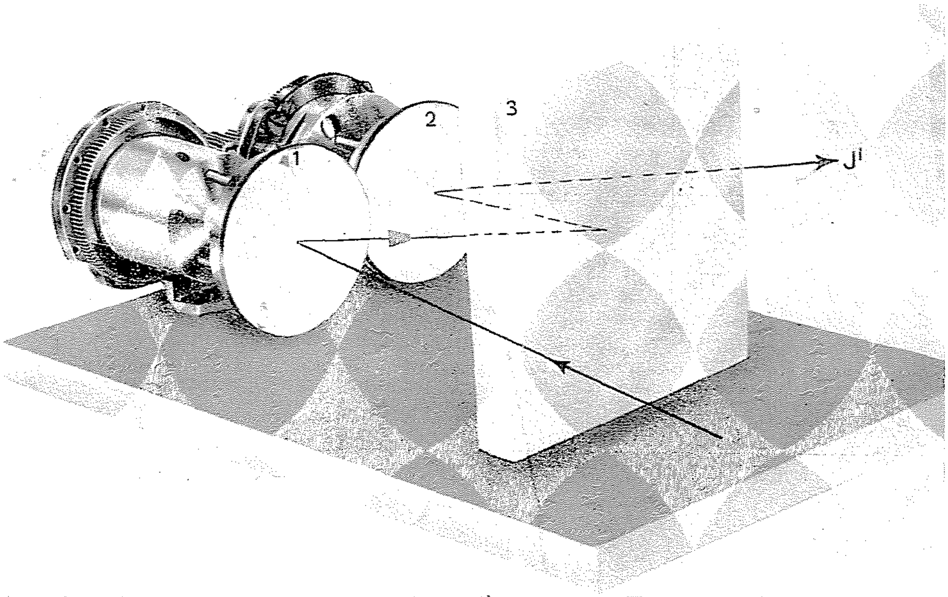

Fig. 22. Mirror micrometer with auxiliary mirror and parallel shaft drive

mirrors with a micrometer. This method is able to secure the wedge angles required for research purposes.

It is to be noted that in addition to rotating pairs of wedges and sliding wedges, other wedge combinations may also be used for measuring small angles. The pair of sliding wedges of equal refractive angles and refractive indices, but opposite in position, is an arrangement similar to the Maskelyne type sliding wedge pair. The Colzi swinging pair of wedges comprise two prisms of equal refractive angles and refractive indices, arranged around the two axes ; the wedges can be opened and shut like a pair of scissors. Systems like the Barr and Stroud arrangements have a wedge pair situated behind the objective, with an opening smaller than the free aperture of the objective. While the beams suffer no refraction at the centre of the objective, they are deviated at the margins of the ring-shaped wedges, in accordance with the position of the wedges. Another design presents pairs of wedges with no aperture, but a smaller diameter than the free aperture of the objective.

The measurement of small angles is not the sole field of application for mirror micrometers. They are adapted for various other purposes, such as *e. g.* the adjustement of revolution numbers. When keeping the revolutions of two electric motors at constant values, one may make good use of the Lissajous figures. The derivation of this phenomenon is strictly in the scope of physics, so that we shall restrict ourselves to considering the underlaying principle.

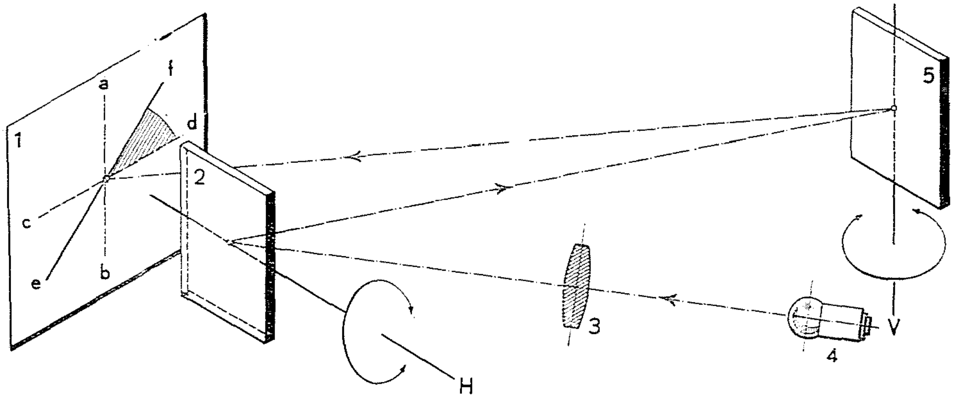

Fig. 23. Resultant path of point, due to oscillation of two mirrors with mutually perpendicular axes

It has been stated above that in the mirror micrometers emerging rays are displaced along a straight line in accordance with the special location of the mirror arrangement. Let us place a second mirror system behind the first, in such a manner, that the rays emerging from the first should travel through the second system ; and let the principal section of this latter system be normal to the principal section of the first. If, now, the mirror pairs rotate independently of each other, the point of intersection of the emerging rays will trace a curve corresponding to the resulting motion. This curve can be established either by computation or by graphical construction. This latter method has been applied in the following example, let us consider two mirrors oscillating about two rightangled axes (Fig. 23). Lens *3* forms an image of the filament of the low-voltage incandescent lamp *4* on screen *1*, the rays having been reflected by mirrors *2* and *5*. Mirror *2* oscillates around the horizontal axis *H* while mirror *5* performs rapid oscillations around the perpendicular axis *V*. The curve depends on the oscillation number, on amplitude and on the phase difference.

Fig. 24 shows the graphical construction of the curve. For this purpose, the circumference of the circle was divided into 24 parts, and the division points were connected by horizontal and vertical lines. If mirror *5* is stationary (Fig. 23), but mirror *2* oscillates around axis H, the point traces the line *ab* on the screen, whereas with the mirror *2* being stationary and mirror *5* oscillating about the axis *V*, the line *cd* is traced. Both oscillations produce a resulting motion. Once more considering Fig. 24, the point passes along the line *AB* in its horizontal course, and along line *CD* in its vertical course, both courses in one oscillation requiring equal periods of time. The point starting from *O* reaches *1* during $^{1}/_{24}$th part of the oscillation period, it reaches *2* during

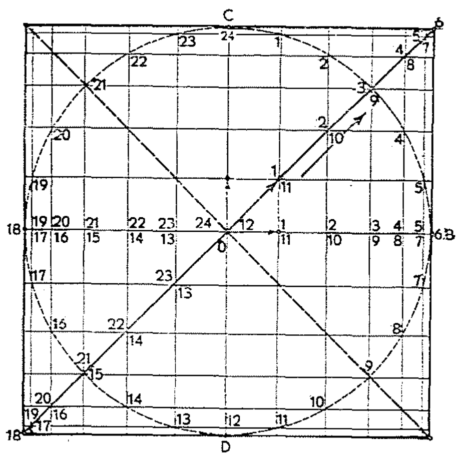

Fig. 24. Graphical construction of the path of the oscillating point

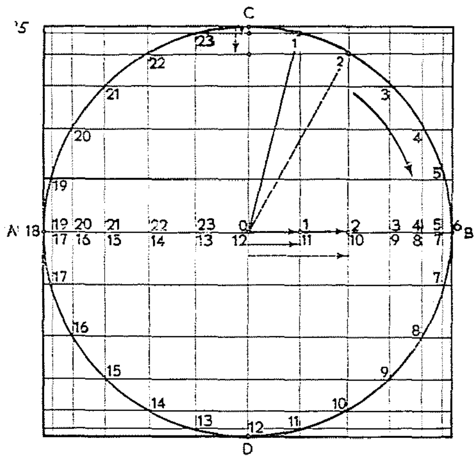

Fig. 25. Circular path: phase difference N = 1 : 4

$^{2}/_{24}$th parts, etc. The same applies to the vertical motion. If the point starts from *O* both in the horizontal and vertical direction at the same time, then its course can be readily plotted and its location determined during any given 24th part of the oscillation. The path of the point will follow the diagonal of a square.

Therefore, the oscillation number of the two mirrors is equal. For a phase difference of $N = \frac{1}{2}$ — that is, oscillation of the one mirror starting when the other mirror has already completed one half oscillation — the resulting path of the point is once more, a straight line, only perpendicular to the former one.

Considering a phase difference of $N = \frac{1}{4}$, represented in Fig. 25, the vertical motion will have brought the point to *C* when the horizontal right-handed motion begins. The motion downward from *C* is superposed on the right-hand motion, and arrives to 1 during $^{1}/_{24}$th part of the oscillation period.

If one continues the plotting, one finds that the point traces a circle *CBDAC*. Reaching point B, the right-hand motion changes to the left. For a phase difference $N = \frac{3}{4}$, the point runs along the circle in an opposite sense.

Considering a phase difference of $N = {}^1/_8 = {}^3/_{24}$ (Fig. 26), the point will have left points a, b and c on its upward course, when starting on its horizontal motion at c. Plotting the path of the point yields an ellipse, and again one of opposite direction for $N = {}^7/_8 = {}^{23}/_{24}$. Each phase difference results in an ellipse of different eccentricity.

Starting from a phase difference of $N = 0$ and taking steadily increasing phase differences, the straight line is converted into first a flat then a broader ellipse, then into a circle, and finally into an ellipse of opposite direction,

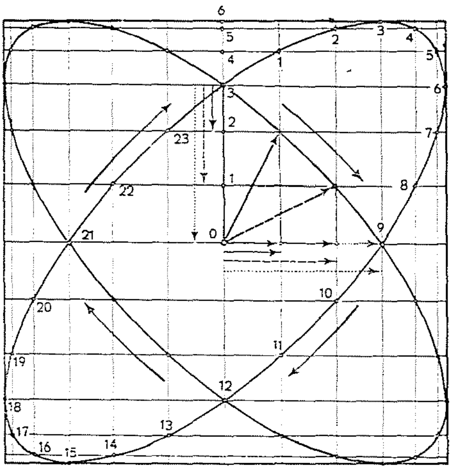

Fig. 26. Elliptic path: phase difference $N = 1 : 8$

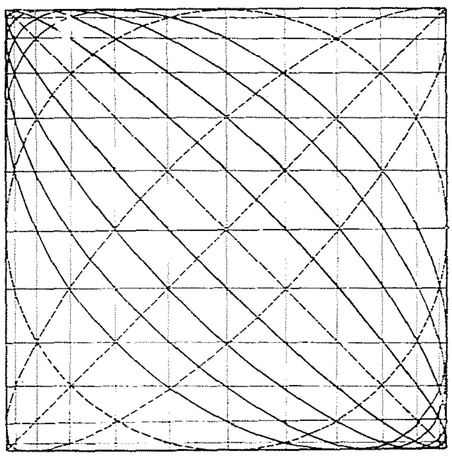

Fig. 27. Paths of rays obtained with continually increasing phase differences

giving the impression of an ellipse swinging about its own major axis. (Fig. 27). Every phase difference is associated to a certain curve, and the rate of variation depends on the duration of oscillation.

From the above brief description it follows that it is possible to produce oscillation figures by means of two penta mirrors so located that their principal sections are normal to each other. The curves traced on the screen by the ray travelling through and emerging from the two systems during rotation of the mirrors correspond to the amplitude, the oscillation or revolution number and to the phase number. If the r. p. m-s are equal, the course of the ray is a straight line subtending a 45° angle with the horizontal.

Applying this principle for securing the constancy of speed of two electric motors, two separate mirror micrometers can be operated. The emerging rays are directed to a slot-shaped photocell, the effective surface of which corresponds to the cross-section of the pencil of rays. On the event of equal r. p. m-s, the straight line traced by the luminous point will entirely cover the effective surface of the cell. As soon as the r. p. m-s are different, the straight line emerges from the slot, and the point traces the curves referred to above. Thereupon, an electric apparatus controlled by the photocell adjusts the r. p. m-s of the lagging or too speedy motor.

Let us now consider the accuracy of the instrument, that is, determine the lowest difference in r. p. m-s to equipment is still responsive. Taking two motors with revolution numbers of 400 and 400,1 per *second*, there is a difference of $^1/_{10}$ revolutions per *second*. Therefore, in 10 seconds the Lissajous-figure will have performed a full revolution, comprising all possible phase differences. Let us take another case where the Lissajous-figure is completed in one minute only. It follows that while one motor makes 440.60 = 26 400 revolutions, the other performs 26 401. The ratio between them is 26 401 to 26,400 = 1,000 038. Thus, this method is adaptable for determining very small differences of revolutions and it is possible — employing appropriate electric or electronic equipment — to keep the revolutions at constant values. The above description makes it clear, that the operation of this instrument is entirely automatic.

Summary

Some of the known micrometers for measuring small angles and short distances have been described above. An attempt was made to replace the 200 years old Boscovich rotating pair of wedges by a mirror micrometer. In the new instrument a mirror micrometer is used as a penta mirror, thus associating the penta mirror with the rotating pair of wedges. Its accuracy depends on two factors : the mode of drive on one hand, the deformation of the mirror mounting owing to changes of temperature on the other. As regards the mode of drive, a gear mechanism can only be applied on a laboratory scale, as it does not give a satisfactory accuracy in the transmission of angles. Therefore, one has to apply other modes of drive.

Temperature influences can be felt by the mirror mounting, generally used for penta mirrors too. It is therefore essential to provide for heat insulation, and to resort to the most careful methods manufacture.

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