ANALYTICAL AND ECONOMIC INVESTIGATION OF PLUTONIUM PRODUCTION IN NUCLEAR POWER REACTORS

A. LÉVAI, I. KOVÁTS and G. BÜKI

Institute for Thermal Power Stations, Polytechnic University, Budapest

(Received Sept. 10, 1958)

(Second United Nations International Conference on the Peaceful Uses of Atomic Energy. 15/P/1727)

1. Introduction

Economic problems characterizing nuclear power stations constitute the most heavily discussed themes in connections with the utilization of the new energy source. The discussion is still mainly of a theoretical character, no practical operational experience being available, which would permit the elaboration of rational economic parameters and the establishment of their interrelations for different types of nuclear reactors and nuclear power stations, as well as for conventional power plants. Uncertainties extend to both the domains of capital and to running costs. Among the problems discussed the valuation of plutonium produced in power reactors and its economic equivalent, *i. e.* plutonium credit stands at first place, at least so far as the uncertainty of valuation is concerned. In the present paper it is proposed to discuss this question, being of basic significance for the operation economies of nuclear power stations, but not yet cleared to its proper depth, only on theoretical foundations, according to the present state of affairs.

The relevant literature [1] reflects different viewpoints, so far as the valuation of the plutonium produced in nuclear power stations is being concerned and these may be essentially grouped into the following classes :

a) Most frequently the value of plutonium is referred to the fixed price of the isotope U^{235}, thus taking a unit price for the plutonium at $ 15 000 to 30 000 per kg. This unit price, however, — beyond ignoring the differences between the two fissile materials, so far as their equivalent heat content and their behaviour in reactors is concerned — should obviously be regarded as the upper limit of plutonium valuation, *e. g.* because it takes the plutonium yield — which in its pure state may become usable only after years in breeder reactors — at once at its full value, at the very moment of its production. A similar effect can be noticed as a consequence of this fact that the price of the isotope U^{235} is to-day determined by other considerations not of economic character.

b) In contrast to the preceding conception, another group tends to find a relation between the price of plutonium and that of natural uranium

(of its fissile material content), in which case the price of natural uranium at $ 40 per kg will entail the value of plutonium per kg to be about $ 5600. This method of evaluation — beyond all the backdraws of the former method is also valid for the present case — does not take into account that the plutonium thus produced is a highly concentrated fissile material. In this case plutonium will obviously be underpriced.

c) If one has a free hand to choose the appropriate energy carrier and the problem is being considered on a purely energetic basis, the price of plutonium may be referred as that of the cheapest fuel to be fired in a power station (*e. g.* fuel oil). If one wishes to take into consideration the fact that capital costs of present-day nuclear power stations usually amount to two or three times the specific investment necessary for conventional power stations, and therefore fissile material is being taken with one half of the usual value as a cost factor, then along this line — *i. e.* using only the concept of thermal equivalence — a plutonium price of $ 14 000 per kg will be obtained.

d) Essentially a modification of the preceding paragraph can be seen in the method of evaluation, where the price of plutonium is determined on the basis of the lowest unit price of electrical power of a given country or a region. This procedure leads to the evaluating method proposed by the authors, if the quantity of plutonium to be evaluated is converted into electric power, not only by taking into account its theoretical best content, but also by considering the reactor cycle utilized in the process.

Evaluating procedures hitherto described are hampered by the following principal errors :

- the evaluating process is based on more or less arbitrarily assumed energy carrier prices ;
- the procedures ignore the fact that the behaviour of plutonium in a reactor differs from that of uranium ;
- the isotope content of the plutonium produced, depends on the design and operation method of the reactor producing it ;
- plutonium production is attached to significant capital and running costs ;
- the plutonium produced — depending on the cycle to be chosen — many times will be utilized only after a substantial elapsed time etc.

If, however, one defines as a leading principle that the plutonium produced in nuclear power reactors is worth the value of the electrical power to be developed from it, then obviously the possibility of peaceful utilization will form the basis of evaluation and — as to be seen from our discussion — the grave uncertainty factors inherent with the preceding valuation procedures will greatly be eliminated. While difficulties are thus transferred to the fields of reactor engineering calculations and operational uncertainty factors, these, however, may be successfully approximated by mathematical and physical

methods and, therefore, resulting uncertainties will be less than with most of the evaluating procedures known from literature. For a similar evaluating method the value of plutonium is defined essentially by the way plutonium is utilized. Thus for determining the value of plutonium the method of utilization (*i. e.* the fuel cycle) must first be defined. With due regard to the large number of theoretically available variations, it is proposed to base our investigations on the following assumptions — not necessarily the most favourable ones, so far as plutonium valuation is being concerned :

- a heterogeneous thermal reactor is used for plutonium production ;
- plutonium shall be reutilized in a similar reactor ;
- in our reactor engineering calculations the simplifications to be enumerated in Section 3 will be introduced ;
- in our computations only the varations of fuel costs will be taken into account, while neglecting the effect of capital cost, because of lack of reliable data. (According to the authors' opinion consideration of the latter cannot decisively influence the results to be obtained by these procedures.)
- the fact that burnup levels in a reactor are frequently determined, not by reactivity requirements, but by corrosion considerations, will be ignored for the time being (the latter especially applies to fast reactors).

Having introduced the above simplifying assumptions, two procedures will be analyzed later on, in order to present the principles of plutonium valuation :

1. By means of reactivity analysis it is proposed to determine the burnup or irradiation level to be attained, in a reactor using natural uranium, and for given initial conditions. According to our assumption to be later motivated, the surplus burnup to be realized subsequent to the first burnup and the extraction of fission products can be taken as net profit due to the plutonium produced in the reactor (para. 4.1.).

2. Plutonium produced in a reactor burning natural uranium will be utilized — after extraction and mixing with natural uranium — as the basic fuel for a second fuel cycle. Thus a self-sustaining cycle may be realized (para. 4.2.).

2. Method of plutonium valuation based on the fuel costs of electric power to be developed in a nuclear power station

The specific variable cost resulting from the fuel consumption needed for producing the electric power fed by the nuclear power station into the grid can be expressed by the following general relation :

$$k = \frac{4170}{Q \cdot \eta} [p_1 + i_1 \cdot p_2 + i_2 \cdot p_R - g_{Pu} \cdot p_{Pu} + p_a] \text{ mills/kWh}_{el} \qquad (1)$$

In the above formula the following symbols have been adopted :

Q — the burnup or irradiation level, *i.e.* the amount of heat energy to be produced from one ton of fuel in the reactor (MWd/ton) ;

η — the efficiency of the nuclear power station (ratio of heat energy introduced into the heat exchanger to electric power fed into the mains, per cent) ;

p_1 — fuel unit price taking reactor purity grade metal ($ per kg) ;

i_1 — the number of fuel element production stages for realizing the burnup level Q ;

p_2 — fuel element production costs ($ per kg) ;

i_2 — number of reprocessing operations necessary for realizing a burnup level Q ;

p_R — costs of a single reprocessing of one kg fuel ($ per kg) ;

g_{Pu} — plutonium yield of one kg nuclear fuel after reaching the burnup level Q (kg per kg) ;

p_{Pu} — value of 1 kg of plutonium produced in the burnt-up fuel ($ per kg) ;

p_a — costs of extracting and disposing the radioactive wastes referring to one kg of fuel ($ per kg, the latter factor being, however, neglected during our investigations).

Numerical examples as published in literature usually take a relation similar to that given above as a base for plutonium valuation, but the price of plutonium (p_{Pu}) is determined according to different assumptions as is enumerated under a) to d) in Section 1. In accordance with our aims, we are trying to find — in place of these — a different valuating process, where the value g_{Pu} of plutonium produced is defined by the electric power or excess heat energy (Q, *MWd* per ton) to be developed from it. It can be shown that plutonium value is thus defined by the relation

$$p_{pu} = \frac{1}{g_{pu}} \left[\frac{\Delta Q}{Q} (p_1 + p_2) - (\Delta i_1 \cdot p_2 + \Delta i_2 \cdot p_R) \right] \$/\text{kg} \qquad (2)$$

where the new symbols denote :

Δi_1 the number of additional fuel processing stages in order to realize the excess burnup level ΔQ to be obtained by utilizing plutonium

Δi_2 the number of reprocessing operations needed for same.

In our equation (2) giving the value of the plutonium produced we find, besides the terms Δi_1 and Δi_2 as defined above, the original burnup level of the reactor (Q), the excess burnup level to be realized from utilizing plutonium (ΔQ), the plutonium yield (g_{Pu}), as well as the costs of fuel (p_1), fuel element production (p_2) and reprocessing (p_R). In the course of economic analysis, the following numerical values have been adopted for different cost factors, in accordance with literature sources [2, 3] :

$p_1 = \$40$ per kg (natural uranium)
$p_2 = \$16$ per kg (natural uranium)
$p_2 = \$22$ per kg (plutonium-enriched fuel)
$p_R = \$13-25-37$ per kg.

In our proposed method of valuation the price of plutonium will be rationally determined as a function of reprocessing costs, the latter varying between wide limits.

The other factors ($Q, \Delta Q, g_{Pu}$) are greatly dependent on the actual type of nuclear reactor, its operating cycle and the method of utilization of the plutonium produced — taking the simplifying assumptions already enumerated into account. It seems therefore advisable to discuss — before turning to actual examples of plutonium valuation (see Section 4) — a few important reactor engineering problems necessary for the proper valuation of plutonium, namely: general analyses of plutonium production and determination of attainable burnup level (Section 3).

3. Analysis of plutonium production in a nuclear reactor using natural or slightly enriched uranium

3.1. *Variations of fuel composition during the burnup cycle*

During permanent irradiation of reactor fuel elements several important changes occur in the fuel and in the interest of taking these quantitatively into account, the following simplifying assumptions must be introduced:

a) Only thermal neutron flux (Φ) will be considered. This simplifying assumption, however, does not apply to several terms of the multiplication constant (k), thus the fast fission factor (ε), the resonance escape probability (p) and the neutron leakage factor (P).

b) Variations of thermal neutron flux (Φ) within the reactor itself will be ignored, utilizing only the mean value. It is also assumed that the reactor is operated at constant heat release rate which obviously involves a neutron flux only slightly varying with time. Time dependence of the neutron flux has been, however, considered, as far as possible, when determining the burnup level.

c) The quantity of the isotope U^{238} placed into the reactor has a constant value and is identical to the starting value (*i. e.* $s = \text{const}$).

d) Pu^{239} is produced from U^{238} only as a result of the fission of U^{235} and Pu^{239}.

e) Among the factors figuring in the multiplication constant ε, p and P during irradiation remain unchanged.

f) Fission products are produced according to the simplifying assumptions as enumerated in para. 3.2. and their behaviour during irradiation is in conformation to the description given there.

It is known that in reactors burning natural or slightly enriched uranium as a fuel, plutonium production is a result of the so-called internal conversion and may be computed from several phases. Production of the isotope Pu^{239} (its quantity being z) proceeds along the following lines :

1. Thermal neutrons are captured by the isotope U^{238} (the probability of capture being given by σ_{a8}) and the isotope Pu^{239} will be produced as a result of radioactive decay. The quantity to be produced within a differential time interval (dt) will thus be given by the term

$$s \cdot \sigma_{a8} \cdot \Phi \cdot dt$$

where $s = \text{const}$, denoting the number of U^{238} atoms.

2. During slowing down of the fission neutrons one part $(1 - p)$ of them suffers resonance capture in the U^{238} according to the definition of the probability (p) of resonance escape. As fission neutrons represent products of the actual U^{235} (u) and the Pu^{239} (z) content and of fast fission products (ε) (ignoring the fission of other materials, *e. g.* Pu^{241}), the number of Pu^{239} atoms produced by resonance capture during the time element may be given by the formula :

$$[u \cdot \sigma_{a5} \cdot \eta_5 + z \cdot \sigma_{a9} \cdot \eta_9] \cdot \Phi \cdot \varepsilon \cdot [1 - p] \cdot P \cdot dt$$

where η denotes the number of fission neutrons per captured neutron and P given the leakage factor.

3. The actual number of Pu^{239} atoms will be burnt up by the capture of thermal neutrons (σ_{a9}), partly through fission, partly through conversion into isotope Pu^{240}. This term is given by

$$- z \cdot \sigma_{a9} \cdot \Phi \cdot dt$$

The approximate differential equation of Pu^{239} production will be therefore given by

$$dz = s \cdot \sigma_{a8} \cdot \Phi \cdot dt + (u \cdot \sigma_{a5} \cdot \eta_5 + z \cdot \sigma_{a9} \cdot \eta_9) \cdot \varepsilon \cdot (1 - p) \cdot P \cdot \Phi \cdot dt - z \cdot \sigma_{a9} \cdot \Phi \cdot dt \quad (3)$$

In view of the fact that the actual quantity of the U^{235} isotope is defined by the relation

$$u = u_0 \cdot e^{-\sigma_{a5} \cdot \Phi \cdot dt} \quad (4)$$

the solution of the general differential equation may be given by the following relation :

$$z = z_0 \cdot e^{\varepsilon(1-p) P \eta_5 \sigma_{a5} \Phi t - \sigma_{a9} \Phi t} + \\ + \frac{\varepsilon \cdot (1-p) \cdot P \cdot \eta_5 \cdot u_0 \cdot \sigma_{a5} \left[e^{-\sigma_{a5} \Phi t} - e^{\varepsilon (1-p) P \eta_9 \sigma_{a9} \Phi t - \sigma_{a5} \Phi t} \right]}{\sigma_{a9} - \varepsilon \cdot (1-p) \cdot P \cdot \eta_9 \cdot \sigma_{a9} - \sigma_{a5}} + \quad (5)$$

$$+ \frac{s \cdot \sigma_{a8}}{\sigma_{a9} - \varepsilon \cdot (1-p) \cdot P \cdot \eta_9 \cdot \sigma_{a9}} \left[1 - e^{\varepsilon (1-p) P \eta_9 \sigma_{a9} \Phi t - \sigma_{a5} \Phi t} \right]$$

where the following notations have been introduced :

u_0 — the number of U^{235} atoms present in the reactor at the start-up (equals unity, if not otherwise specified) ;

z_0 — the number of Pu^{239} atoms present in the reactor at the start-up (equals zero, if not otherwise specified).

In economic and operational research concerning reactor economy the so-called heavier isotopes of plutonium may play an important role, consequently their quantitative variations must also be taken into account. Differential equations established on the basis of nuclear physical relations as well as analytical or numerical solutions for these will be omitted here, only the final results are presented on Fig. 1. showing variations of Pu^{239}, Pu^{240}, Pu^{241} and

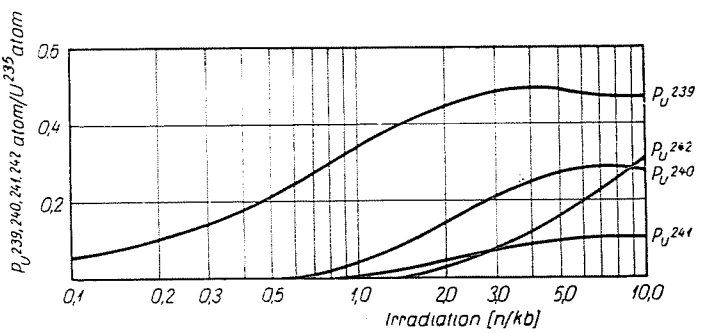

Fig. 1. Variation of specific plutonium isotope yields versus irradiation level (for natural uranium ; reactor characteristics according to Appendix)

Pu^{242} isotope yields — referred to one initial U^{235} atom — as a function of burnup (Φ. t n per kilobarn). (Nuclear constants used for numerical solutions of the equations, as well as characteristics of the hypothetical reactor may be found in para. a, and b, of the Appendix.)

3. 2. *Determinations of attainable burnup level*

The burnup or irradiation level to be realized in a reactor (in MWd per ton) is of decisive importance, so far as the unit cost of electric power produced by a nuclear power station and the utilization of fuel are concerned [see formula (1)]. Its actual value is defined — apart from variations in fissile materials as defined in the preceding Section — by time variations of the neutron — absorbing cross sections, and obviously by running requirements concerning reactor operation. As already pointed out, burnup level limitations due to corrosion are to be ignored.

Laws governing the production — during a fission reaction — of fission products and of non-fissile isotopes of fissile elements will not be dealt with here. The problem is amply discussed by literature[4]. It should be, however,

noted here that reactor poisons — essentially defining burnup level — are usually classified into three group [5];

a) The first group comprises all strong poisons, with an effective neutron capture cross section exceeding 1000 barns which, however, will become saturated within a short interval as a consequence of their intense neutron capture activity. Within this group, as usual, let us stress the importance of Xe^{135} ($\sigma_{aXe} \cong 3{,}5 \cdot 10^6$ barn) and Sm^{149} ($\sigma_{aSm} \cong 5{,}3 \cdot 10^4$ barn).

b) The second group contains poisons of medium intensity ($1000 > \sigma_a > 10$ barn), characterized by their way of formation and the compositions of the initial material. They cause, as a rule, a maximum poisoning effect a short

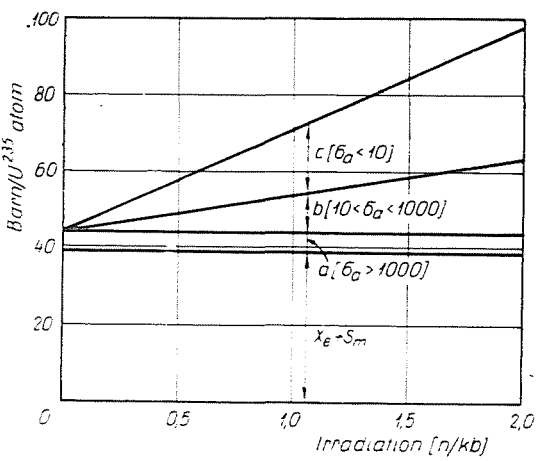

Fig. 2. Neutron capture cross sections of reactor poisons as a function of irradiation (natural uranium)

while after starting reactor operation and this may decrease, somewhat later. Certain poisons, however, show a saturation effect.

c) The third group comprises weak poisons ($\sigma_a < 10$ barn), which do not show any saturation effects and which have a poisoning effect increasing with time.

In our calculations the poisoning of the reactor has been determined on the basis of literature data [6]. Fig. 2 shows the variation of neutron capture cross sections characteristic of reactor poison groups and referring to an initial U^{235} atom.

It is known that a nuclear reactor remains serviceable until its effective multiplication constant ($k_{eff} = \varepsilon \cdot p \cdot f \cdot \eta \cdot P$) exceeds unity, that is until it has an effective excess reactivity, *i. e.* until

$$\delta_k = k_{eff} - 1 > 0 \qquad (6)$$

The overall number of fission neutrons for the reactor as a whole is given by the product

$$\Sigma_{\text{prod}} = \Sigma (\Sigma_{f,f} \cdot \nu) \tag{7}$$

where $\Sigma_{f,f}$ denotes the macroscopic fission cross section of all fissile fuel elements and ν denotes the number of neutrons produced by a thermal fission. The sum of these products is called the thermal productive cross section of the reactor [7].

Let us denote the total absorbing cross section of fuel by $\Sigma\, \Sigma_{a,f}$ so that final factor η (*i. e.* the number of fission neutrons for a neutron captured by the fuel) for the reactor as a whole may be given by

$$\eta = \frac{\Sigma_{\text{prod}}}{\Sigma\, \Sigma_{a,f}} \tag{8}$$

According to its definitions, the thermal utilization factor (f) is given by the ratio of thermal neutrons absorbed by the fuel to the total number of neutrons absorbed by the reactor. Apart from the fuel (denoted by index f) neutrons are also being absorbed by reactor poisons (index i), by the moderator (index m), as well as by other structural elements, such as cooling agent, fuel fans, control rods, etc. (index r). Without considering the flux differences always present in the fuel elements and the moderator as well as other materials, let us write down :

$$f = \frac{\Sigma\, \Sigma_{a,f}}{\Sigma\, (\Sigma_{a,f} + \Sigma_{a,i} + \Sigma_{a,m} + \Sigma_{a,r})} \tag{9}$$

The expression in the denominator of the preceding relation represents the overall absorbing cross section of the reactor as (Σ_{abs}) a whole, so that we have

$$f = \frac{\Sigma\, \Sigma_{a,f}}{\Sigma_{abs}} \tag{10}$$

Using the symbols as deduced above, the formula of the effective multiplication constant may be written in another form :

$$k_{eff} = \varepsilon \cdot p \cdot f \cdot \eta \cdot P = \varepsilon \cdot p \cdot P \frac{\Sigma_{\text{prod}}}{\Sigma_{abs}} \tag{11}$$

and so for the effective excess reactivity we have :

$$\delta_k = \frac{\varepsilon \cdot p \cdot P \cdot \Sigma_{\text{prod}} - \Sigma_{abs}}{\Sigma_{abs}} \tag{12}$$

The expression in the numerator represents the actual excess cross section of the reactor variable with time (Σ_{exc}) and therefore excess reactivity will be given by :

$$\delta_k = \frac{\Sigma_{exc}}{\Sigma_{abs}} \tag{13}$$

The reactor will remain workable, until it still possesses an excess cross section. *i. e.* until its productive cross section exceeds the absorbing cross section. Particulars of the productive cross sections can be calculated by using the analysis given in Section 3.1. — at least with a fair degree of approximation — while calculations of the absorbing cross sections of an actual nuclear reactor

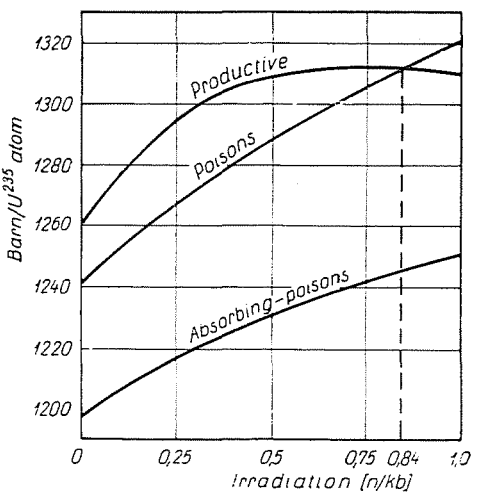

Fig. 3. Variation of productive and absorbing cross sections as a function of irradiation (for natural uranium, reactor characteristics according to para. *b*, Appendix)

may also be undertaken if one takes into account the rate of poisoning — variable with time —, the variations of temperatures, steam generation within the reactor, etc., *i. e.* all operational factors, so that for a given case variations of the productive and absorbing cross sections, as well as of excess reactivity can easily be plotted as functions of time, resp. irradiation, and thus the burnup level to be realized within the actual reactor can be determined.

A practical application of the preceding theoretical discussions is shown by Fig. 3, for a reactor complying with particulars of para. *b*, of the Appendix, where absorbing cross sections for poisons and non-poisons have been separately defined. It can be read from this diagram that an irradiation level of

0,84 n per kilobarn may be realized with the specified reactor. Fig. 4 shows the variations of excess cross-section (σ_{exc}) and excess reactivity (δ_k) with reference to one initial U^{235} atom and using the same data as in Fig. 3.

As the procedure given in Section 3.1. permits the determination of changes in fuel composition for any arbitrary irradiation level, the appropriate integral curves will deliver the isotope quantities having suffered fission. By

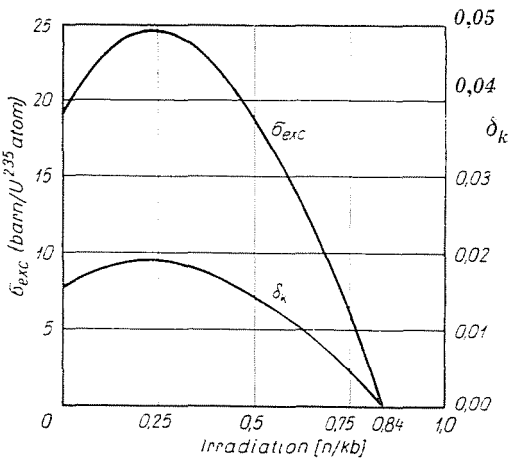

Fig. 4. Variations of excess cross section and excess reactivity versus irradiation level (for natural uranium, reactor characteristics according to para. *b*, Appendix)

knowing this, as well as the energy released during the fission of one gram of the given isotope (see para. *c*, Appendix), the heat energy produced can be determined. For our actual example an irradiation of 0,84 n/kb will yield some $Q = 3350$ MWd heat energy for one ton of natural uranium, according to the following distribution :

fission of U^{235}	2612 MWd/ton	78 %
fission of Pu^{239}	708 MWd/ton	21,1%
fission of Pu^{241}	30 MWd/ton	0,9%

4. Plutonium valuation for different fuel cycles

The governing principle of plutonium valuation as based on the concept of equivalent electric power has been described in Section 1, while calculation procedures were dealt with in Section 2. In the following paragraphs the valuating procedure will be applied to two actual plutonium utilizing processes mentioned in literature, using the result of reactor computations given in Section 3.

4.1. *Plutonium valuation of a fuel cycle with prolonged burnup*

After having been refined from accumulated fission products, reactor fuel becomes again serviceable. Using the particulars as well as the results of our preceding investigation of actual reactor service life it is found that subsequent to a fuel burnup of $Q = 3350$ MWd/ton and after reprocessing the fuel from reactor poisons ($\sigma_{ai} \approx 25$ barn) and refilling it to the reactor it will show an excess reactivity of

$$\delta_k = \frac{\sigma_{exc}}{\sigma_{abs}} = 0{,}019$$

in the interest of a prolonged burnup period.

Assuming that the fuel purified from poisons is replaced to the same reactor for another burnup period, as well as that the plutonium produced

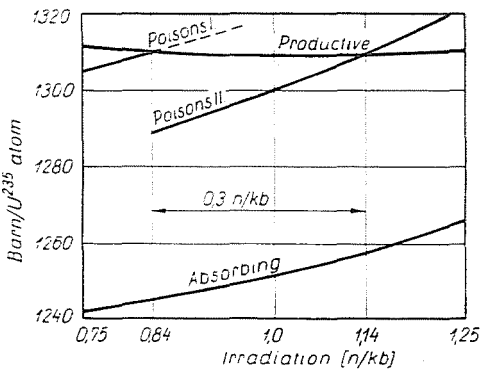

Fig. 5. Variations of productive and absorbing cross sections versus irradiation level for a prolonged fuel cycle (for natural uranium, reactor characteristics according to para. *b*, Appendix)

will not be extracted from the fuel, variations of the productive and absorbing cross sections for the additional burnup period can be determined in a way analogue to the analysis given in Section 3. It can be seen from Fig. 5 that in our example this used, but poison-free fuel, permits an additional irradiation of some 0,3 n/kb. Thus, one ton of natural uranium will yield an additional heat energy of $\Delta Q = 1100$ MWd.

Until the extraction of poisons, *i. e.* up to an irradiation level of 0,84 n/kb or a burnup level of $Q = 3350$ MWd/ton, $g_{Pu} = 2{,}4 \cdot 10^{-3}$ kg/kg plutonium will have been produced in the reactor. The additional heat energy of $\Delta Q = 1100$ *MWd*/ton as computed before, is essentially due to this quantity of plutonium. We may namely — with about the same reprocessing costs as necessary for extracting the poisons from reactor fuel, extract plutonium itself and sell or utilize it in another cycle (*e. g.* according to para. 4.2.). If, however, plutonium is retained in the reactor fuel while extracting other fission products,

thus enabling the reactor to be operated so as to achieve an additional burnup, the former considerations will retain their validity.

Through applying relation (2), plutonium value in the burnt-up fuel will be given according to the following table, for different reprocessing costs — taking into account the higher production costs of fuel elements mixed with plutonium.

Preprocessing cost (p_R) \$/kg	13	25	37
Plutonium value (p_{Pu}) \$/kg	—7530	—12340	—17350

Results are shown by curve *a* on Fig. 6. As seen from it, plutonium utilization by means of a prolonged fuel cycle — for the present reactor — proves

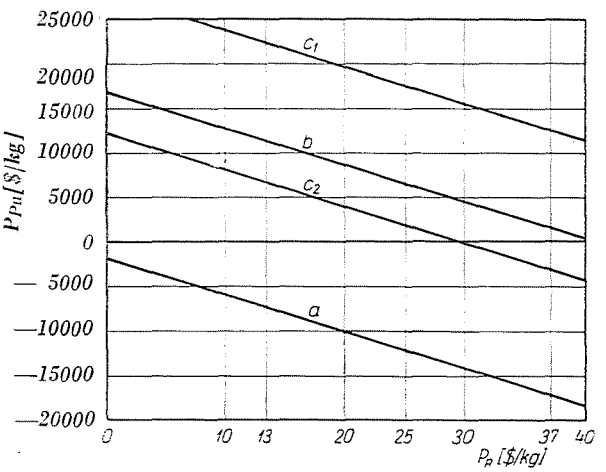

Fig. 6. Plutonium value for different valuating processes as a function of reprocessing costs

to be uneconomical even in case of low reprocessing costs, mainly because of increased fuel element production expenses. It appears, therefore, to be logical why this method of plutonium utilization was received unfavourably by the literature — at least for the time being [8].

4.2. *Plutonium valuation for an equilibrium state fuel cycle*

A possible way of utilizing plutonium for energy production is the realization of an equilibrium fuel system [9]. The governing principle in this system may be explained as follows : the reactor is started up with a fuel having a composition which ensures that — after reaching the burnup level — the plutonium produced in the reactor and mixed with natural uranium provides a fuel equivalent to the original fuel, so far as reactor operations are concerned. Thus the burnup of natural uranium in the reactor may be increased and also several subsequent, nearly equal running periods may be achieved.

With quite a degree of simplification, two reactor fuels (A and B) may be regarded as nearly equivalent, so far as reactor operation is being concerned, if — filled into a similar reactor — the effective multiplication constant of the reactor remains the same for both cases. (This identity from the point of view of reactor engineering does not necessarily entail that the two fuel types are equivalent so far as energy production is concerned.) Thus we have :

$$k_{effA} = \varepsilon_A \cdot p_A \cdot \eta_A \cdot P_A \cdot f_A = k_{effB} = \varepsilon_B \cdot p_B \cdot \eta_B \cdot P_B \cdot f_B \tag{14}$$

If it is wanted to determine how many kilogramms of plutonium must be added to one ton of natural uranium in order to produce a fuel equivalent to slightly enriched uranium, then it may be assumed that

$$\varepsilon_A = \varepsilon_B ; \quad P_A = P_B ; \quad P_A = P_B$$

Thus for the equivalence of reactor fuels we have the following condition :

$$f_A \cdot \eta_A = f_B \cdot \eta_B \tag{15}$$

Using our previous notations, we may write down

$$f_A \cdot \eta_A = \frac{u_A \cdot \sigma_{a5} + s_A \cdot \sigma_{a8}}{u_A \cdot \sigma_{a5} + s_A \cdot \sigma_{a8} + F} \cdot \frac{\eta_5 \cdot u_A \cdot \sigma_{a5}}{u_A \cdot \sigma_{a5} + s_A \cdot \sigma_{a8}} \tag{16}$$

or

$$f_B \cdot \eta_B \frac{u_B \cdot \sigma_{a5} + s_B \cdot \sigma_{a8} + z_B^9 \cdot \sigma_{a9} + z_B^0 \cdot \sigma_{a0} + z_B^1 \cdot \sigma_{a1} + z_B^2 \cdot \sigma_{a2}}{u_B \cdot \sigma_{a5} + s_B \cdot \sigma_{a8} + z_B^9 \cdot \sigma_{a9} + z_B^0 \cdot \sigma_{a0} + z_B^1 \cdot \sigma_{a1} + z_B^2 \cdot \sigma_{a2} + F} \cdot$$

$$\cdot \frac{\eta_5 \cdot u_B \cdot \sigma_{a5} + \eta_9 \cdot z_B^9 \cdot \sigma_{a9} + \eta_1 \cdot z_B^1 \cdot \sigma_{a1}}{u_B \cdot \sigma_{a6} + s_B \cdot \sigma_{a8} + z_B^9 \cdot \sigma_{a9} + z_B^0 \cdot \sigma_{a0} + z_B^1 \cdot \sigma_{a1} + z_B^2 \cdot \sigma_{a2}} \tag{17}$$

where

F denotes the total capture cross section of neutron-absorbing materials in the reactor — except fuel — (assumed identical for both fuel types) ;
index A: enriched uranium
index B: mixed fuel consisting of natural uranium and plutonium.

Results of the numerical solution of the above equation are shown in Fig. 7 (based on particulars given in the Appendix). It may be inferred from this that the addition of about 5 kgs of Pu^{239} isotope to one ton of natural uranium will result in a fuel which is equivalent to uranium enriched to 1,4 per cent, so far as reactor operational considerations are taken into account. (In order to simplify our calculations, the fact of plutonium consisting of a mixture of different isotopes has been ignored and only the isotope Pu^{239} has been taken into account when computing the equivalence of fuels. The error thus committed is, however, less than 1 per cent.)

The proper task of equivalence investigations is to establish the running time necessary for achieving an equilibrium fuel system, *i. e.* to determine how long a reactor must be kept in operation in order to obtain a quantity of plutonium which

a) mixed with natural uranium provides an equivalent fuel for reactors started-up with enriched uranium fuel ;

b) in case of reactors started-up with a mixed fuel of natural uranium and plutonium equals the quantity of plutonium filled-in prior at start-up.

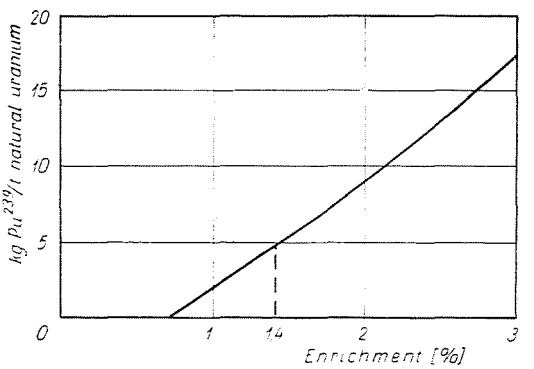

Fig. 7. Plutonium content of natural uranium equivalent to slightly enriched uranium

(For similar investigations losses occurring during plutonium extraction as well as fluctuations in plutonium composition may be neglected.)

Solutions of the differential equations determining the quantities of isotopes — provided that the reactor already contains plutonium (z_0) during start-up — have been given in para. 3.1. Let $\bar{z}$ denote the quantity of Pu^{239} in formula (5), assuming that there has been no plutonium present during start-up. In this case equation (5) may be transformed into the following general form :

$$z = z_0 \cdot e^{\varepsilon(1-p)P\cdot\eta_9\cdot\sigma_{a9}\cdot\Phi\cdot t-\sigma_{a9}\Phi t} + \bar{z} \tag{18}$$

In a reactor using equilibrium fuel, we must have necessarily that subsequent to an irradiation $\Phi \cdot t$

$$z = z_0.$$

This permits the computation of the quantity of plutonium necessary for the start-up :

$$z_0 = \frac{\bar{z}}{1 - e^{\varepsilon(1-p)P\eta_9\sigma_{a9}\Phi t-\sigma_{a9}\cdot\Phi\cdot t}} \tag{19}$$

In the numerical example elaborated to illustrate the present method it has again been assumed that in the reactor — as specified by para. *b)*, of the Appendix — the plutonium $g_{Pu} = 2{,}4 \cdot 10^{-3}$ kg Pu per kg natural U)

produced after the irradiation ($Q = 3350$ MWd per ton) determined during the service life investigations (Section 3.2) is mixed with natural uranium and replaced to the same reactor for repeated burnup. Variations of the productive and absorbing cross-sections of the reactor are shown in Fig. 8 for the reactor started-up with a mixture of plutonium and natural uranium. Maximum

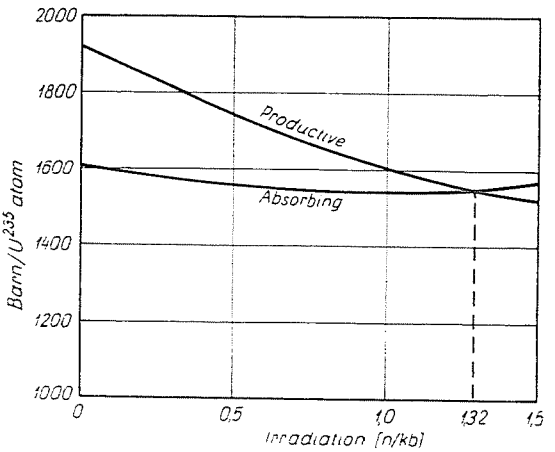

Fig. 8. Variation of the cross-sections of an equilibrium system of plutonium and natural uranium (reactor according to para. *b*, Appendix)

available burnup amounts to 1,32 n/kb and thus 1 ton of fuel will yield $Q' = 6160$ MWd heat energy. Excess heat energy — as compared to the preceding case — now amounts to $\Delta Q = 2810$ MWd per ton.

After the burnup level had been attained, the remaining Pu^{239} quantity in the reactor started-up with plutonium fuel mixture will amount to 3,32 kg per ton of fuel. This is more, than the quantity of plutonium placed into the reactor for the start-up (2,2 kg Pu^{239}) and therefore, in the given reactor equilibrium fuel system appears to be realized, even when taking all possible losses into account.

In a way similar to that given in the previous paragraph, use of relation (2) will define the value of plutonium, summarized by the following table :

Preprocessing costs (p_R) $/kg	13	25	37
Plutonium value (p_{Pu}) $/kg	11650	6660	1670

Actual data are shown as a function of reprocessing costs by curve *b* of Fig. 6, proving that for the given fuel cycle and with the reactor as specified, a very considerable plutonium value is obtained, even in case of higher reprocessing costs. This will indicate the optimistic attitude of interested experts towards the problem. (In this case the circumstance that a longer interval between plutonium production and utilization may unfavourably effect the valuation, has been ignored — see para. 5/*d*.)

5. Conclusions

In order to compare the method of valuating plutonium, as proposed by the authors and based on energy production, against the method of valuating plutonium by its fixed price as known from literature, it seems to be advisable to introduce further refinements to the latter, rather arbitrary procedure.

As already mentioned in Section 1, plutonium as a concentrated fissile material produced in a reactor is sometimes evaluated in the literature as able to substitue the fissile material content of the U^{235} — enriched uranium fuel (para. 1/*a*). For this evaluation, — specific data generally accepted by the literature will be assumed, according to which the cost of uranium enriched to 90 per cent U^{235} amounts to 15 000 or 30 000 $/kg according to enrichment costs [3]. However, for a more precise evaluation — if it is proposed to determine the price of plutonium similarly with reference to the burnt-up material in the reactor — a few other factors must be taken into account, these latter being, as a rule, ignored by literature sources. These may be summarized in the following paragraphs :

a) The thermal equivalents of U^{235} und Pu^{239} or Pu^{241} are by no means identical (see para. *c*, of the Appendix).

b) The composition of the quantity (g_{Pu}) of plutonium produced, varies with irradiation or burnup level. This composition can be characterized by the isotope ratio (m) representing the ratio of fissile Pu-isotopes to the total quantity of Pu-isotopes. This is according to the analysis given in Section 3.1. a function of reactor type and irradiation level :

$$m = \frac{g_{(239+241)}}{g_{(239+240+241+242)}} \cdot 100\% \tag{20}$$

c) Plutonium extraction costs (p_R $/kg) must also be considered when valuating plutonium.

Taking into account all the enumerated factors, the value of plutonium in the producing reactor fuel is given by

$$p_{Pu} = p'_{Pu} - \frac{p_R}{g_{Pu}} \text{ \$/kg.} \tag{21}$$

In the above relation p'_{Pu} denotes the price converted from the basic price (in $/kg) of enriched uranium in accordance with composition (m) and equivalent heat of plutonium. Fig. 9 shows the plutonium values as computed from equation (21) as a function of irradiation. In this diagram plutonium values (p_{Pu}) are shown for the different U^{235} basic prices and different reprocess-

ing cost factors (p_R). The diagram also contains a plot of the plutonium yield (g_{Pu}) and isotope ratio (m) against irradiation level.

d) The value of plutonium is influenced by the time interval between production and utilization (discount value). The later plutonium will be utilized, the less its value is at the time of production. Assuming that plutonium extraction immediately precedes utilization, *i. e.* that there is no interest charged after p_R, and if n denotes the number of years between plutonium

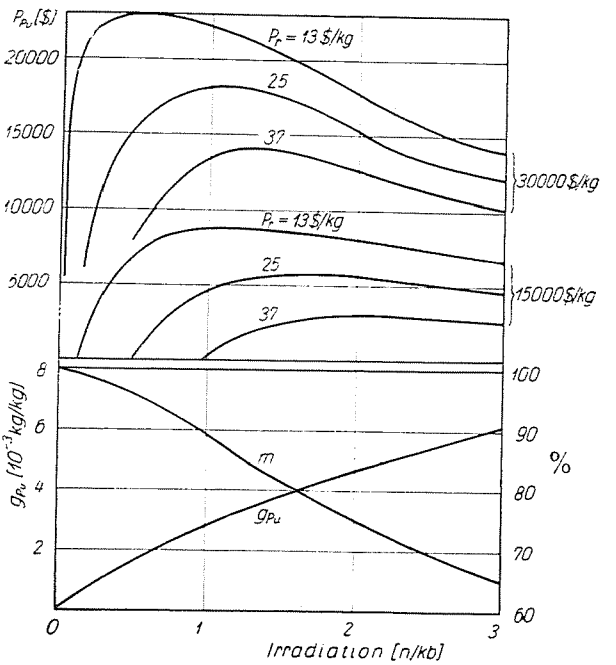

Fig. 9. Plutonium yield (g_{Pu}), isotope ratio (m) and the value of plutonium calculated using fixed basic prices (p_{Pu}) as a function of irradiation level

production and plutonium utilization and p denotes the interest rate : then the value of plutonium in the burnt-up fuel of the producing reactor will be given by

$$p_{Pu,n} = \frac{p_{Pu}}{\left(1 + \frac{p}{100}\right)} \text{ \$/kg.} \tag{22}$$

Fig. 6 shows, for the sake of comparison, curves of plutonium valuation using the fixed basic price, but corrected according to the principles discussed (curve c_1 refers to a basic price of 15 000 \$/kg, curve c_2 to 30 000 \$/kg.) The former price, as seen from para. *c*, Section 1 is nearly consistent with the

price obtained on the fuel oil equivalence principle. [Burnup has been assumed identical with the cases given in Section 4 ($Q = 3350$ MWd/ton)]. As can be seen, the valuation curve according to the higher basic cannot be motivated by power engineering arguments, giving an unnecessarily high price to plutonium. Contrasts will become even sharper, if plutonium is produced by a different reactor, having *e. g.* a different initial conversion factor value or an initial excess reactivity, different from that assumed in our example. An energetically proper valuation will in this case show an even greater reduced plutonium value compared to the results obtained in our example. It is therefore, according to the opinion of the authors, a sound principle to avoid valuating plutonium — as a nuclear power cost factor — at fixed prices, taken arbitrarily or on the basis of any other considerations. Proper valuation must take into account design and operational characteristics of the plutonium-producing reactor, as well as the fuel cycle utilizing plutonium. It may be obvious that a similar energetic valuation possesses a lot of problems which should in the future be cleared up. Perhaps most important among these may be formulated in such a way that a solution most favourable for national economy should be sought for, by comparing different reactor types and fuel cycles. The procedure to be followed is similar to that, when during design of a conventional back-pressure type heat power plant the analysis is extended beyond the scopes of an industrial power plant and lifted to the level of national economy considerations [10]. Obviously, this will require more serious studies, but undoubtedly the solution of these problems will advance the question of peaceful utilization of nuclear energy.

Appendix

a) Nuclear constants used for numerical solutions of the equations defining nuclear fuel composition variations (normalized data).

Isotope	Thermal capture cross-section σ_c barn	Thermal fission cross-section σ_f barn	Number of fission neutrons η
U^{235}	687	580	2,08
U^{238}	2,75	—	—
Pu^{239}	1065	750	2,03
Pu^{240}	550	—	—
Pu^{241}	1480	110	2,35
Pu^{242}	30	—	—

b) Characteristics of the hypothetical reactor used for the present analysis ; in hot start-up conditions :

fuel	natural uranium
fast fission factor	$\varepsilon = 1{,}030$
resonance escape probability	$p = 0{,}8828$
thermal utilization factor	$f = 0{,}865$
	$\eta = 1{,}335$
Leakage factor	$P = 0{,}972$
initial conversion factor	$C_0 = 0{,}8$
initial excess reactivity for burnup and poisoning (without Xe and Sm)	$\delta_k = 0{,}02$

c) The heat energy released during the fission of one gramme of fissile material :

Isotope	Energy MWd/ton
U^{235}	0,94
Pu^{239}	0,878
Pu^{241}	0,79

Summary

The paper aims to consider one of the most discussed factors of the costs of electric energy produced in atomic power plants, *i. e.* to consider the value of plutonium produced. The basic consideration is that plutonium produced in an atomic power reactor is worth as much as the electric energy generated by it. This procedure is basically different from other means of evaluation — more or less arbitrary — discussed so far in the special literature, but it renders necessary to take into consideration when evaluating plutonium, the way of production in the plutonium producing reactor as well as the fuel cycle of plutonium utilizing. This procedure considers even such factors as arise from an eventually late utilization of the produced plutonium. Thus it can be supplemented at any time — when more exact data are available — with expense factors arising from investment costs. To illustrate the course of calculations there is a more detailed examination of the burnup with prolonged fuel cycle, and further of the utilization of produced plutonium mixed with natural uranium in self-surtained cycle. Whereas the first procedure in view of the costs of plutonium extraction as known today, seems by no means to be economic, the economic conditions of the second procedure — under certain circumstances — are even nowadays available. The procedures published in the paper may serve as a basis for further discussions for a further solution of the problem.

Literature

1. JUKES, J. A. : The cost of power and the value of plutonium from early nuclear power stations. Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva 1955, P/390.
2. DAVIS, W. K.: Capital investment required for nuclear energy. Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva 1955, P/477.
3. LANE, J. A.: Economics of nuclear power. Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva 1955, P/476.

4. GLASSTONE, S.: Principles of nuclear reactor engineering. New-York 1956.
5. WEINBERG, A.M.: Survey of fuel cycles and reactor types. Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva 1955, P/682.
6. JOFFE, B. L.—OKUN, L. B.: Long term reactivity changes in nuclear reactors. Atomnaja Energija 4 (1956).
7. LEWIS, W. B.: Some economic aspects of nuclear fuel cycles. Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva 1955, P/4.
8. LEWIS, W. B.: Is fuel preprocessing necessary ? Nuclear Power, February 1958.
9. DUNWORTH, J. V.: Fuel cycles and types of reactors. Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva 1955, P/403.
10. LÉVAI, A.—SÓVÁRY, E.: Planning of new power plants into a co-operative power plant system with particular regard to run-of-steam plants. World Power Conference, 1954, Rio de Janeiro.

Prof. A. LÉVAI
I. KOVÁTS
G. BÜKI
Bp. XI. Stoczek u. 2., Hungary