A NEW INSTRUMENT FOR TESTING STEREOSCOPIC VISION

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The phenomenon of *stereoscopic* vision is known to be the result of two independent and laterally more or less displaced images, each of them being projected on the retina of one eye. The two images are blended by the optical centre of the brain, in a so far inexplored manner, bringing about the effect of space perception.

In case of *monocular* vision space perception cannot be formed on a purely geometrical basis. Nevertheless, the relative spatial positions of the objects are fairly well distinguished by the observer. The underlying reason cannot be satisfactorily explained by the laws of geometrical optics, and it is supposed that intricate psychological processes are preponderant, among which a series of empirical factors gathered in the course of time seem to play the chief part.

To begin with the simplest case, *i. e.*, with objects of identical sizes placed at various distances, the images formed on the retina by the adaptive crystalline lens will be the larger, the closer the observed objects are positioned. If their general order of magnitude is known, the object distance may be inferred upon by a comparison of the retinal images varying in shape and size, a measure far from satisfying the exigencies of tacheometry as set by geometrical optics. Consequently, except in the case of close objects, this is scarcely more than an approximate estimation.

Let us trace two straight lines to connect the extreme points of the retina with the posterior principle point of the crystalline lens (Fig. 1). The subtended angle α, i. e. the visual angle, will vary with the object distance for identical object sizes, or with the object size for a constant distance. The visual angle unambiguously determines the field of sight for the actually viewed object, not to be identified with the far more comprehensive term of the eye's general field of vision. Experience, acquired through the continual variation of the visual angle, has taught us to perceive the relative spatial position of the objects, even when viewing with one eye only, although this faculty of estimation rather than of discernment rapidly decreases with growing distances and, finally, when distance becomes very great, falls off entirely. In locating the relative positions of objects at longer distances, that is, in perceiving the respective impression, the decisive part is played by optical, physiological and psychological processes, although the faculty of estimation may be substantially led astray incorrect judgement of the object size.

On the other hand, monocular estimation of distance may considerably be supported with the effect varying in the individual case, by such factors as one object being screened to some extent by a nearer one, recognition of the details of familiar objects, intensity and direction of artificial or natural illumination, spectral composition of the luminous source, coloured and overlapping or colourless arrangement in plane of the observed objects, absolute and relative speed of the object in relation to the observer, vapour and dust

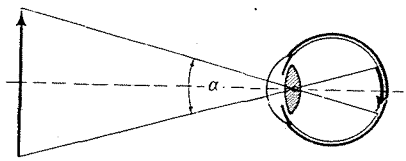

Fig. 1

percentage of the atmosphere and, finally, coloured and colourless contrast effect.

An investigation into the nature of influences exerted jointly and severally by the factors outlined above discloses a remarkably wide range of how experiences are collected. Not to be neglected is, among others, the problem, which of these factors will gain the upper hand in the course of time. May it be remembered in this context, what blunders, or even grave errors people belonging to different categories — rural and urban people, lowlanders and highlanders, etc. — are liable to commit. Town inhabitants generally measure distances in terms of time, whereas the measuring unit applied by country folk is highly varied. Furthermore, estimation of distance may be improved upon or distorted, as the case may be, due to the influence of familiar or, on the contrary, unfamiliar surroundings.

The apparent increase or decrease of the object size as caused by the changing visual angle brings us to the notion of *perspective*. The theory of perspective, together with its geometrical correlations, was first established and developed, from the artistic point of view, by the great masters of Italian renaissance. Incidentally, the constant changes in impressions of perspective also contribute to distance estimation in monocular viewing.

Let us sum the foregoings by saying that monocular space perception and distance estimation were formed out, in addition to the laws of geometrical optics, by physiological and psychological processes. However, taking into account and adding hereto the effect of surroundings and the temporal succession of phenomena, space perception is possible within rather narrow limits only, subject to further variation for the individual viewer. The collection of experiences, as a function of time, can also be but one-sided and dissimilar. It is the simultaneous cooperation of all other senses coupled with monocular viewing, giving rise to impressions based on numerous and changing phenomena, that guide us in estimating distance. Such estimation is of a relative value only. A number of physiological procedures still unknown as to origin and issue, such as after-images, optical hallucinations, inducations, etc. play an important part therein. As soon as any one factor is eliminated from the series of experiences obtained by routine, one finds that space observation issuing from estimation undergoes a considerable change.

In the case of binocular (stereoscopic) vision an independent and dissimilar image of the viewed object is formed on the retina of each eye. According to classical theories, however, space perception only arises when image pairs fall upon identical nerve endings. If that is not so, the result should be, instead of space perception, a confusion of double images unsuitable for being blended into one stereoscopic image by the visual centre. This approach to the problem, however, must be reconsidered in our days. The rods and cones of the nerve endings are supposed to be of a more or less hexagonal section, and unite into a thin layer of honeycomb pattern. Recent researches, however, suggest that their cross-section is not hexagonal ; thus, the role played by the disc of least confusion representing the image point is differently interpreted in the theory of resolving power. Investigations into the theoretical deduction of resolving power start from the supposition relying upon anatomic tests that human eye is able to separate (resolve) images received from two indentically bright point objects as long as the distance between the two point images on the retina is equal to the radius of the disc of least confusion. In terms of angular measurement, the angle subtended by the radius amounts to an average 20 seconds of arc. Nevertheless, even the most careful and precise tests were unable to produce a value approximating this figure, as the average results obtained fluctuated around 90 seconds of arc. By reading a vernier and bringing into coincidence straight lines of varying length, much better results have been arrived at, but this problem is not identical with the task of resolving two luminous points.

Rayleigh's version for the criterion of the resolving power was that the retinal images of two luminous points may be separated as long as their relative distance is equal to at least the radius of the disc of least confusion. Another explanation held that the eye is able to resolve two point objects, if the two cones stimulated by the images are separated by an unstimulated one. Researches carried out by Davson, Hartridge, Ogle, Schober, Tonner et al. suggest that, similarly to monocular vision, resolving power largely relies upon optical, physiological and psychological factors, in addition to geometrical-optical correlations. As the enumeration of said factors is beyond the scope of this paper, the reader is referred to literature for more details, particularly as regards the anatomy of the retina.

Recent researches have lead to the conclusion that, owing to psychological reasons, resolving power is a function of the size and shape of the point object-to be resolved. A maximum of resolution may be attained by viewing bidimens sional diagrams, but the critical factor for resolving two luminous points is definitely the contrast effect in relation to the surroundings. Added hereto must be the variation of illumination prevailing in the area of the retinal image and last not least, the image-forming power of the human eye.

Resolving power plays a significant part in stereoscopic vision, owing to the already mentioned importance of the parallactic difference of image pairs. It is supposed that there exists a maximum and a minimum value already or still permitting stereoscopic vision.

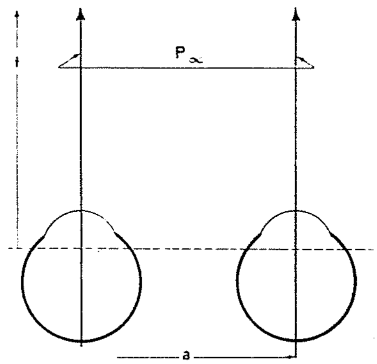

Fig. 2

The retina elements of the two eyes are correlated. When viewing objects at infinity, the visual axes are parallel while upon viewing close objects, they are convergent (Fig. 2). The magnitude of convergence is determined by the object distance t and by the viewer's visual distance a as a base. Convergence, or its variation may suggest the idea of space estimation. This, however, holds true only for near-by objects, while our sensory organs are not suited for determining the magnitude of convergence for medium and long distances. Thus, convergence as well as accomodation offer but limited possibilities for the development of space perception. A point of primary importance is the lateral displacement value of the two point image pairs, called *parallactic difference* (Fig. 3). Distance estimation is termed *real* (absolute) when it concerns the distance T between the viewer and point P, and is called *relative* when it means the distance t between points P and (P). In both cases the relative displacement of the point image pairs is characteristic of the parallactic difference.

When viewing a point object P at infinity, the visual axes are parallel, and their distance from each other is equal to the visual distance a of the observer as a base. There is no lateral displacement between retinal images P_1 and P_3, consequently, no vision of depth can arise. Images from point objects P positioned at a distance T are formed at P_2 and at P_4. The left-hand image is displaced by $P_1P_2 = x_1$, and the right-hand one by $P_3P_4 = x_2$. The difference, $(x_1 - x_2)$ is called parallactic difference, irrespective of whether the

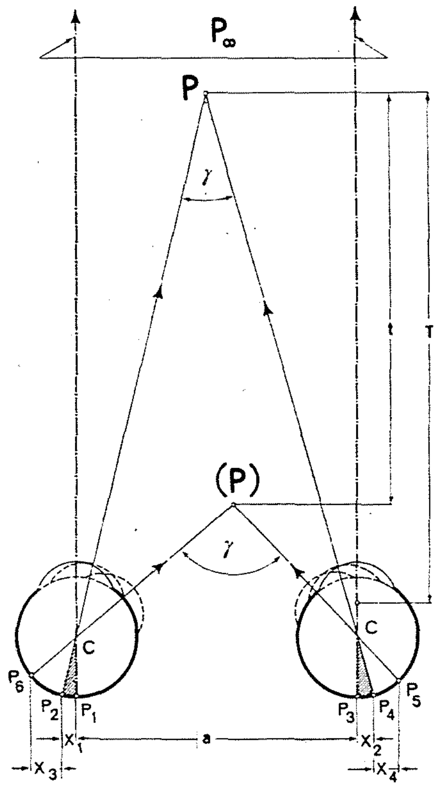

Fig. 3

sign is negative or positive. On this basis an estimate may be formed of the *real* distance T from point P for a visual distance a. Images P_5 and P_6 of the still closer point (P) arise at distances x_3 and x_4 from the above point images P and (P), respectively. Parallactic difference $(x_3 - x_4)$ permits estimating the relative distance t between point images P and (P). Distance T is called *depth perception* whereas distance t represents *space depth.*

Whether the observer's estimation is concerned with real distance from a far-away or from a near-by object, or with the relative distance between these two, the points will spring into relief against infinity — that is, they will be viewed stereoscopically — only if the value of the parallactic difference is not less than the resolving power of the eye. According to earlier anatomical data and other investigations into the nature of the retina, the generally accepted value of the human eye's resolving power is 1 minute of arc. However, values of 30, 20, 10 or even 2 seconds of arc have been encountered, as Pulfrich found in his tests accomplished by means of stereoscopic range-finders with travelling signals. Upon first thought, one might attribute this to a finer structure of the retina. The wave theory derivation with respect to the disc of least confusion, however, disclosed that a finer structure of the retina would not

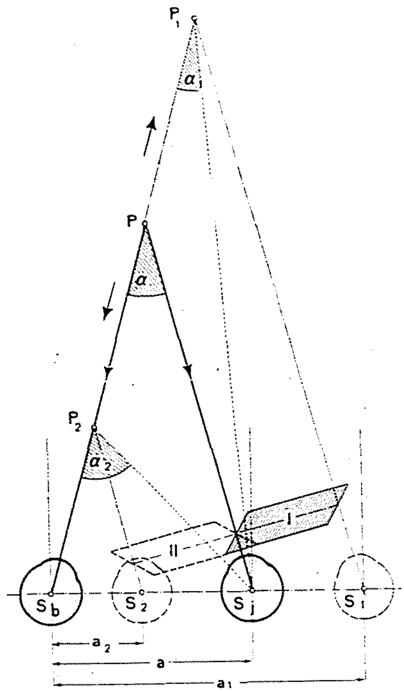

Fig. 4

render vision more acute. Just like in the event of results obtained by reading a vernier, or bringing into coincidence straight lines, such superior resolving power can only be ascribed to the shape of the retina elements, and physiological as well as psychological factors.

Fig. 3 leads to the conclusion that depth perception for binocular vision increases in direct proportion to the visual distance and to the depth difference of the two point objects to be compared, while decreasing in square proportion to the real distance of the points. Consequently, the value of parallactic difference may not fall below a certain minimum required for recording a sensation of space. As one but seldom encounters single points in space, in the majority of cases sensation of depth is a relative estimation rather than depth perception.

The varying sensation of depth concomitant to an intentional extension or cutting down, respectively, of the visual distance may be illustrated by the following experiment (Fig. 4) :

Let us place before the right eye a rhombic prism in position No. I. In this manner the unaided left eye will view directly, while viewing by the right eye will be intercepted by the prism. Thus, the original visual distance a of the right eye will be increased to a_1, at the same time reducing the angle of con-

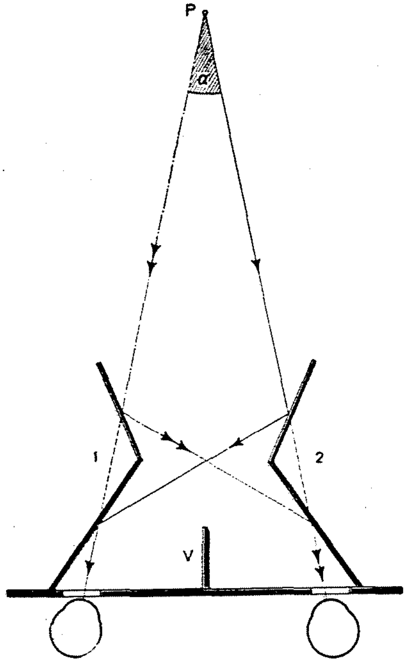

Fig. 5

vergence to —1. The mind then records a sensation of the objects being widely spaced in depth, hence they will appear considerably smaller. By turning the prism into position No. II, the base is reduced to a_2, whereas the convergence angle is increased to α_2. The objects thus contracted in depth appear much larger.

In position No. I. of the prism the object P is seen as if viewed from eye position S, that is, as if the point object had travelled to P_1. The convergence angle being lessened to —1, the objects appear smaller. If, on the other hand, bringing the prism into position No. II., the objects viewed at an increased visual angle —2 appear to be larger, that is, as if observed from eye position S_2.

In both cases the prism placed before the eye will considerable impede not only movement in space but also orientation, as a consequence of the limited field of vision due to the apparent contraction and extension, respectively, of depth. If the prism is suitably shaped, visual distance may not only be reduced to zero but may have a negative value, in which case the rays incident from the left will be received by the right eye and vice versa. Thus, space perception will be reversed, giving rise to a pseudoscopic impression, that is, nearer objects will appear more distant and far away objects closer. This phenomenon may be created by means of a system of mirrors, schematically represented in Fig. 5. The view unfolding itself to the observer presents a fascinating world.

The two mirror arrangements 1 and 2, each consisting of two parts and subtending a certain angle, are separated by a wall *V*, in order to avoid the appearance of confusing images. Mirror 1 leads to the right eye the ray incident from the left and marked by a double-pointed arrow, while mirror 2 leads to the left eye the arrow-marked ray incident from the right. The mirror arrangement thus reverses object *P*. The angle of convergence of point object *P* projected to the eye by the two mirror arrangements equals that of the unaided eye.

The minimum value of the parallactic difference representing the acuity of depth vision is 5 to 10 seconds of arc, a value subject to a great number of factors; which, in turn, account for the surprising results obtained through experiments, resulting in much smaller, that is, more advantageous, values than expected, seemingly at variance with the laws of geometrical optics.

In describing binocular vision it is understood that the two eyes are entirely similar as to their anatomic structure, image forming power, optic data, physiological, and, in the figurative sense, mechanical action. In reality, however, such ideal eyes are very rarely encountered. In most cases the observer does not even realize major or minor aberrations. According to the current definition, the right eye forms an image of the right side of the object, and the left eye of the left side. At the slightest lateral turning, however, the first image is received by the eye first perceiving the object. Hence, in the case of continuous observation, the images are not simultaneously formed on the two retinae. Leaving aside exceptional cases, image formation on the two retinae is thus deferred in relation to each other, a phenomenon called "the race of visual fields". Simultaneous image formation is, incidentally, precluded by the fact that, in the event of a change of direction, the turning of the eyeball is not continuous but intermittent, as a result of the specific structure of the eyes' motorial muscles. The turning motion of the eyeball is not perfectly synchronized. Of the two images, the stronger prevails over the other one, attempting to check the weaker one's effect. Several investigators consider this continuous change one of the chief conditions to stereoscopic vision. *Panum* holds that stereoscopic vision may arise even if the conjugate image pairs do not fall on identical nerve endings. Let us now leave aside the section and arrangement of rods and cones, the distortion of the disc of least confusion due to aberrations in image formation, as well as the chromatic aberration caused by oblique pencils of ray, — a stereoscopic image will be formed even if image formation no longer follows the principle of conjugate point pairs. Assuming that image points do not fall on identical nerve endings, it suffices that they should be overlapping in the vicinity of the nerve endings.

In theory, even the most accomplished optical instrument is unable to form a point-like image of a pointlight lamp ; the image will be a system of mensurable surfaces comprising a nucleus and several surrounding rings. Irrespective of the image-forming device, the diameter of the disc is a function of wave-length and it is caused by the interference resulting in connection

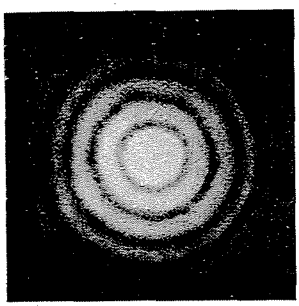

Fig. 6

with diffraction. Such a disc may be produced by means of an extremely fine bore or slit, in line with the principles of wave theory. This so-called disc of least confusion suffers a certain distortion due to the more or less considerable aberrations in image formation of the instrument in question. On the other hand, the extent said distortion offers a basis for determining the accuracy of the optical instrument. It must be remembered that the human eye is far less perfect as regards image formation than might be inferred from the conventional anatomical sections and diagrams. The shape and distortion of the disc of least confusion arising on the retina are subject to various factors, thus it differs substantially from photographic representations. The disc of least confusion shown in Fig. 6 has been produced by a very fine bore. The infinitesimal divergencies at the edge and wall of the bore were sufficient for distorting the rings of the disc.

Several investigators have drawn the conclusion from the "race of visual fields" that the law of conjugate points is far less important than the competition of fields occurring in connection with image pairs. However, no satisfactory explanation has been given so far for the real nature of physiological and psychological procedures taking place in the centre of vision. Among others, this insufficiency accounts for the awkward sensation felt when viewing a stereoscopic cinema performance. Obviously, the image projected only approximates and imitates normal space perception, but cannot replace reality until light has been thrown upon every aspect of the psychological factors participating in stereoscopic vision.

Whatever be the viewpoint from which the theories and opinions with respect to stereoscopic vision are being criticized, it is of primary importance to be able to determine the numerical value of the stereoscopic vision power of both observers and testers in certain selected and special professions, also for the sake of comparison.

Besides the Pulfrich balance, and the methods and apparatus of Mach and Dvorak, Chantraine and Luft, Hering, Garten, Monyé, etc. stereoscopic vision is mostly tested by means of the stereoscope. In such tests the observer has to determine the stereoscopic effect produced by a picture of geometric — bidimensional — lines and diagrams differently spaced in depth. In other words, he has to ascertain how the individual lines, signals and signal groups are spaced in depth in relation to each other and to the whole of the picture. Obviously, this is an estimation of depth. The relative depth of the signals can be determined numerically. Stereoscopic vision of the observer is considered best if he is able to pick out with certainty the line, signal or diagram positioned at the minimum depth. Depth determination is, of course, not continuous but intermittent, progressing from signal to signal. The tests are to be conducted under strict observation of the rules, carefully precluding any disturbing factors. The correctness of orally stated depth data should be checked by using a table. The primary conditions for successful stereoscopic tests are proper illumination as regards intensity and direction, transparent positives rich in contrast or dull positives, and, last not least, a certain ability on the part of the tested person, a condition which may cause difficulties particularly in the case of group tests. The optimum results may be attained by using trans-illuminable positives rich in details, as the confusing spots presenting themselves in connection with bright positive photos may be found very inconvenient. On the other hand, the stereoscop has the advantage of low cost, small weight, ease of handling and of transport.

It is commonly assumed that stereoscopic vision power may be improved and developed by viewing pictures widely spaced in depth. This assumption has proved to be wrong, whereas it is true that, by continual practice, stereoscopic vision may be kept on the same level. The effect of mere repetition is the clue to the phenomenon of improved stereo-vision.

Good results may be achieved by taking advantage of the stereoscopic "travelling signal", an element upon which are based military stereo-range-finders, with a base line of varying length, stereocomparators, and stereo-planigraphs or -autographs, tracing maps from aero stereophotographs. One convenient embodiment of the stereoscop is provided with a stereomicrometer (Fig. 7) mounted on the object table. The centres fastened to the ends of flat tongues vertically shiftable over the test board laid on the object table may be laterally displaced, jointly or separately, by means of a spindle bearing an indiced drum, visible on the right side of the Figure. When the centres point toward any one conjugate point of the picture, the observer will see the two centres as one united centre floating in the same depth as the observed point.

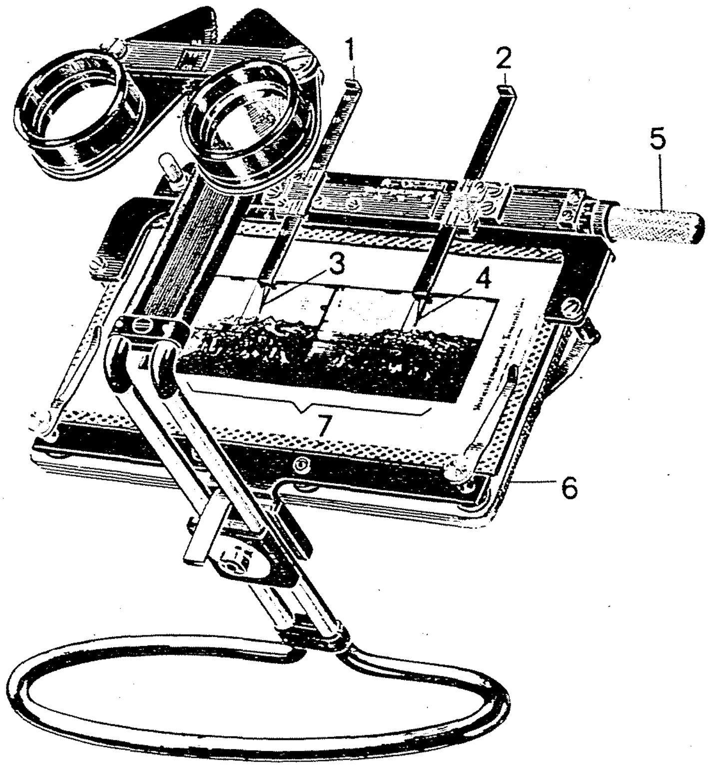

Fig. 7

Measuring is effected by shifting the right-hand centre transversely to the left-hand centre by means of the micrometer spindle. Thereupon, the centre appears to be displaced *in depth*, closer or farther away from the observer. By using conveniently selected stereo-photographs the centres can be brought into coincidence with the object points. The real or the relative distance of the point may then be computed from the exposure data of the photograph, coupled with the readings of the drum. In the absence of a micrometre only an estimate can be formed as to the distance and spatial position of the objects.

Conversations with the oculists Dr. Emil Galla and Dr. György Aczél gave the author the impetus for constructing a new instrument for the determination of stereoscopic vision power. It is to be understood that the numerical values obtained are not concerned with measuring the parallactic difference as such, but with obtaining figures and diagrams for the observer's stereoscopic vision power.

An important consideration in the design of the instrument according to the invention was to secure conditions under which — in the absence of disturbing factors acting upon the subconscious of the observer — he might concentrate all his attention upon the signal, whatever be the nature, colour and intensity of illumination.

Measuring, or rather determination is arrived at by bringing into depth coincidence an object and the corresponding image. This latter is produced

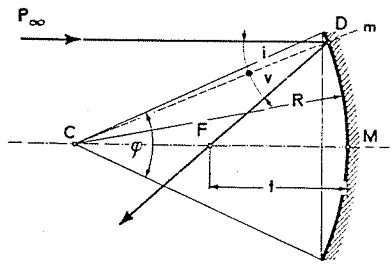

Fig. 8

by means of a concave spherical mirror. The apparatus according to the invention also causes the image to be displaced simultaneously with the object but this is done in depth and not transversely, as is the case with the stereo-micrometer. It has been found that at a certain degree of displacement of the centres confusing double images are liable to occur in connection with the stereo-micrometer. The present instrument permits a much higher degree of displacement in the direction of viewing without producing confusing double images. It is true that the familiar phenomenon of double images may occur, as a result of accomodation of object and image to varying distances. The new instrument obviates various inconveniences inherent in the stereo-micrometer. Thus, no vertical adjustment of the centres is required, and no troublesome measures are to be taken to avoid intercrossing of the centres. The new apparatus has the further advantage over the stereo-micrometer that the image perceived is more natural. Finally, the apparatus is constructed so that the observer is not disturbed by accessories, and both centre and image are viewed in depth free of all confusion.

Before describing the instrument in more details, let us recapitulate briefly the basic characteristics of spherical mirrors.

If the spherical mirror is concave, the pencils of rays parallel to the optical axis are focused after reflection in focus F (Fig. 8) situated at a focal length

f from the apex M of the mirror, the focus representing the image formed by the mirror of an object at infinitiy.

It will be noted that the mirror employed is front-coated, with a reflecting surface consisting of an aluminium deposite.

Describing now in more detail the action of the instrument, we find that it works in the following manner, represented in Fig. 9:

A ray, emitted by point P lying on the optical axis, is reflected at B, the reflected ray intersecting the axis at P_1. This latter represents the image of point P, the angle of incidence being i, and the angle of reflection being v. The above proposition holds true only for a mirror of a small angular aperture, in which case one may write $PB = PM = t$, also $P_1B = P_1M = k$, k standing

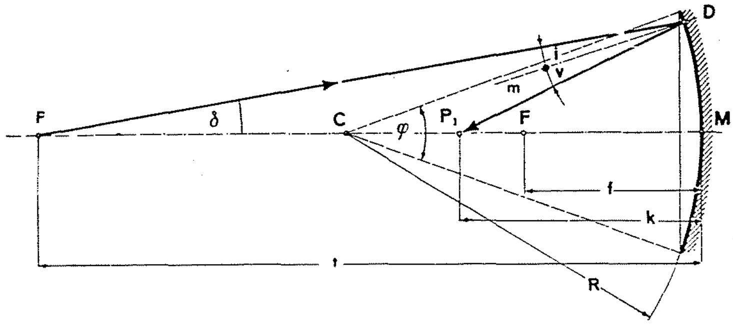

Fig. 9

for the image and t for the object distance. For a spherical mirror of infinitesimal aperture one may write, neglecting the derivation,

$$\frac{1}{k} + \frac{1}{t} = \frac{1}{f}$$

The image distance is a function of the object distance and of the radius of the mirror, but it is independent from the angle δ subtended by the radius starting from point object P, and by the optical axis. If P is at infinity, then $t = \infty$, also $l : t = 0$, that is, the parallel rays will unite in Focus F at a distance of $k = f = R : 2$. Hence, the focus of a spherical mirror of small angular aperture lies at the half of the radius of curvature. Is must be pointed out that in spherical mirrors of large angular aperture the aberrations in image formation render impossible an even approximate intersection of reflected rays in one point.

Returning now to our instrument, in the case last set forth, the image distance equals the focal length, or $(k = f)$, and $f = R : 2$. Let point object P be placed in the centre C of the radius of curvature R, we will find the object distance to equal the radius that is

$$t = R = 2f$$

and

$$k = 2f.$$

If the object is being displaced, the image distance as well as the image size are changed. If the object lies in the centre of the radius of curvature

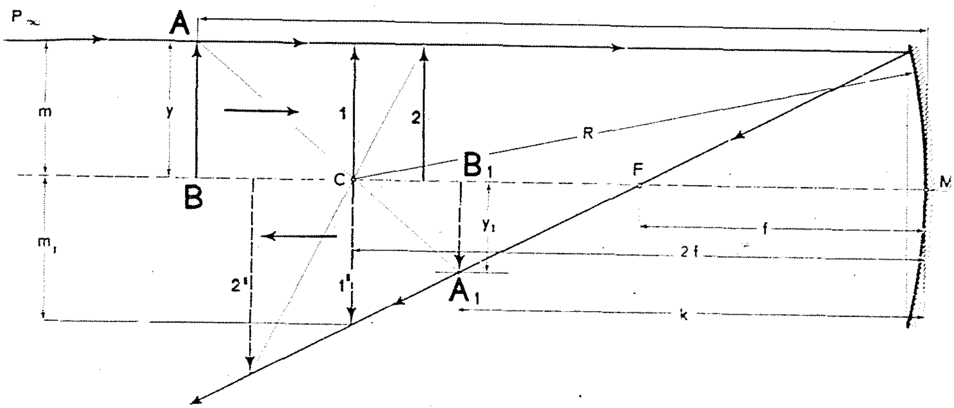

Fig. 10

it actually lies at twice the focal length from the mirror. The image distance from the mirror equals the object distance (Fig. 10). The closer the object is approached to the mirror, the speedier will the image remove therefrom, a phenomenon accompanied by a continuous increase in the image size. If the object is at the centre C of the radius, it equals the image ($m = m_1$). If the object coincides with the focus, then the rays are reflected parallelly, and the image is thus viewed at infinity.

The real image is always inverse, and is to the size of the object as the image distance is to the object distance. Let the objeet size be $AB = y$, and the image size be $AB = y_1$, then the magnification will be $N = y_1 : y = k : t$.

A further approximation of the object to the mirror within the range of focal length will cause the rays to be reflected divergently. In this case the image appears to be behind the mirror and is rectified. The analysis of this virtual image, however, falls beyond the scope of the present paper.

Figs. 11 and 12 illustrate the use of a spherical mirror in connection with the new instrument. Tests may be carried out either with the unaided eye, or with the aid of a telescope 8. In both cases, centre 4 fixed onto the upper end of the vertically shiftable metal pipe 3 mounted on carriage 6 may be displaced by means of a wheel 7. Centre 4 is illuminated by a low-tension bulb

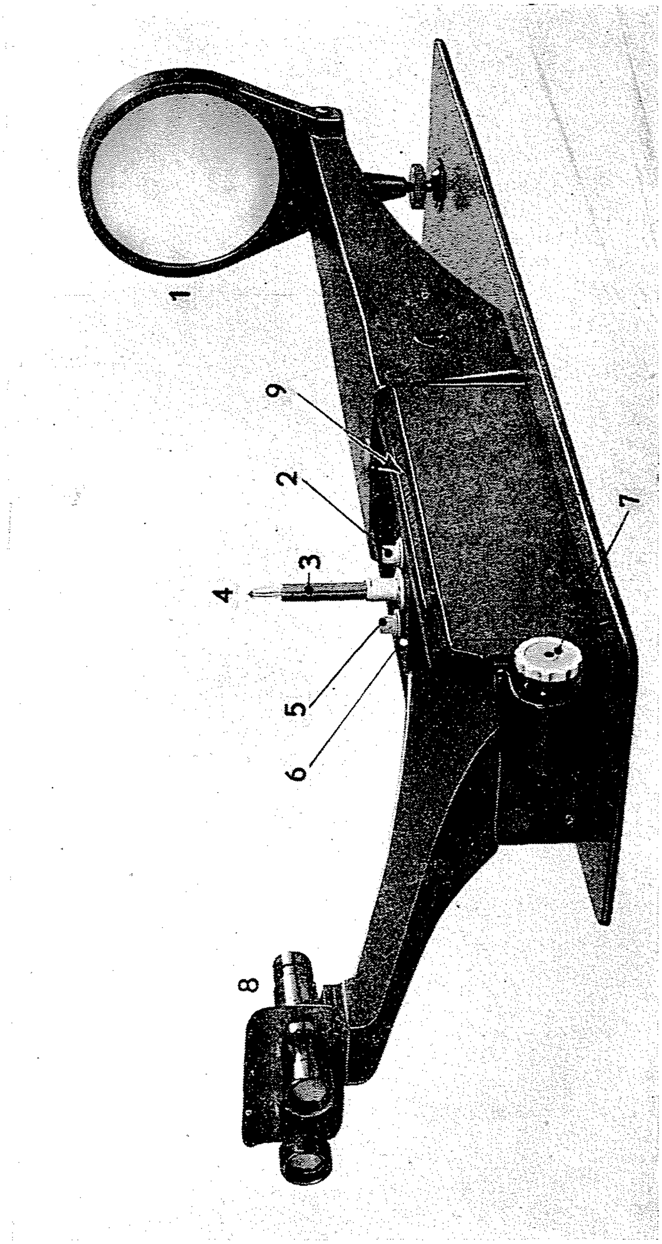

Fig. 11

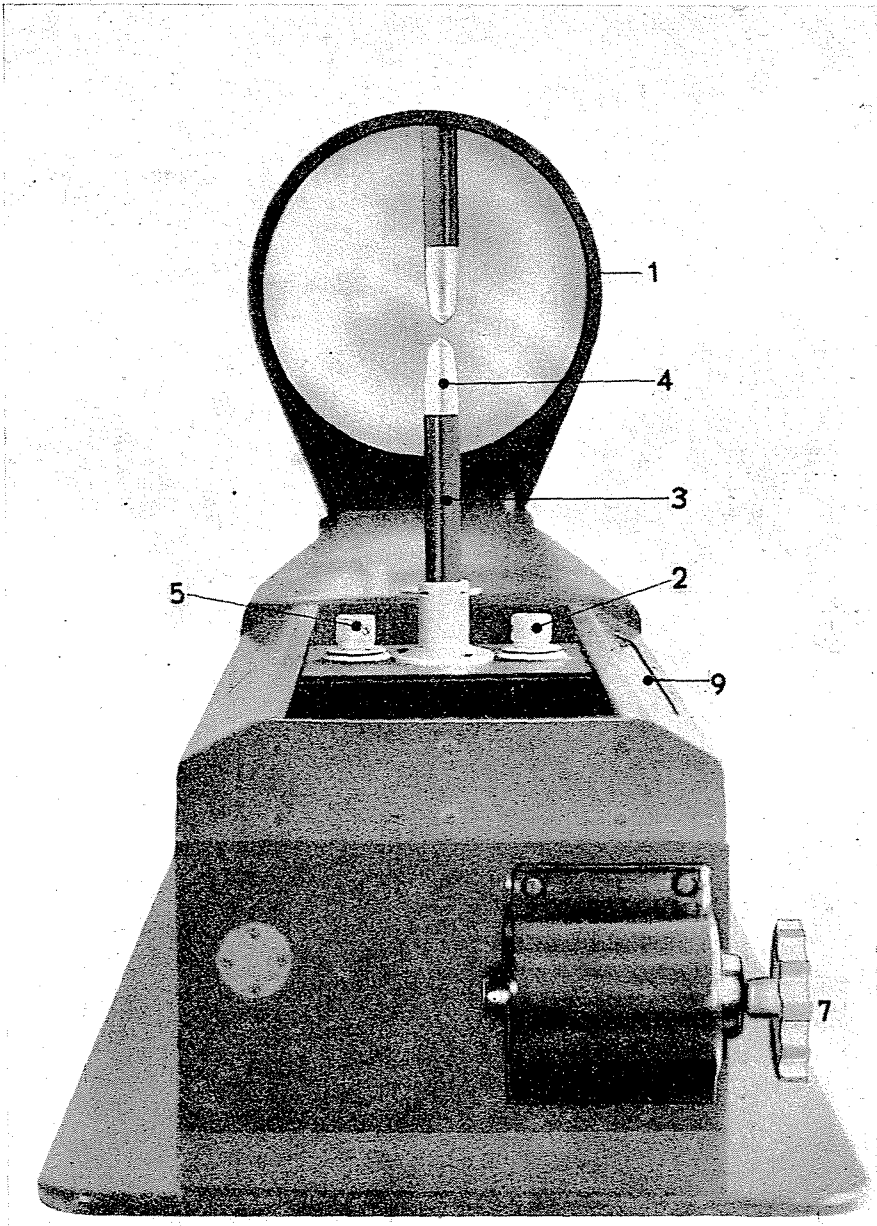

Fig. 12

fixed at the lower end of the pipe. A button 2 is provided for regulating the intensity of illumination by means of an iris diaphragm placed into the pencil of rays. The dial wedged upon the button may have linear, squared or logarithmic indices.

Coming now to the particulars of the apparatus in action, it will be noticed that under normal circumstances the centre will appear white (colourless) but may be turned red, yellow, green or blue, by means of the colour filter adjustable through button 5. If the room is dark, nothing but the illuminated centre and its image will be viewed. The spherical mirror will create an inverse floating image of the centre. If the centre is shifted nearer to or away from, respectively, the mirror, the image will travel in inverse sense. The observer's task is to bring centre and image into depth coincidence. If coincidence is established, centre and image will be of equal size, the centre being distanced at twice the focal length from the mirror. The latter position is called the basic or zero position of the carriage.

Starting now from the basic position, forward or backward adjustment may be read through window 9 from a scale engraved into the non-represented track of the carriage. There being no probability of major errors in adjustment, there are provided 150 1-mm-indices to the right and left of the zero point.

It will be noticed that the size of the travelling image will change with the shifting of the centre. Hence, the question may arise whether it will not be possible for the observer to conclude from the variation of image size upon coincidence, particularly after repeated tests. If the focal length of the mirror is very small, the size of the travelling image will considerably change on the slightest displacement of the object. To obviate this disadvantage, a mirror of great focal length would be necessary which, however, would require considerable displacement of the object in order to achieve a perceivable displacement of the image. It has been found possible to compute, from the variation of the retinal image, the maximum focal length and the corresponding change in image size where this latter does not attain the threshold of perception. Instead of such calculations, however, it has been deemed preferable to determine the focal length and the radius of curvature for the mirror by means of experiments.

Let us consider an example. It is supposed that the observer will not perceive the change in image size as long as the difference in size between the base line on the one hand and the images arising in the two extreme positions — before and behind the centre — on the other does not exceed the one minute of arc resolving power of the eye. Let us now find the focal length of the mirror which will satisfy the said requirement (Fig. 13).

Let the distance of observation between the observer and the centre P in basic position be $S = 5000$ mm, let the centre have a diameter of $d = 20$ mm, and the focal length of the mirror be $f = 800$ mm. When centre P will stop at twice the focal length ($2f$) before the mirror, object and image will coincide and their sizes will become equal ($t = k$). If this is the case, both centre and image will be in the centre C of the radius of curvature R of the mirror, that is, $t = k = 1{,}600$ mm.

The observer sights both the centre and its equal-sized image at an angle of $a = 13{,}745$ minutes of arc.

1. Shifting the centre by 150 mm, that is, from the basic position P to P_1, the object distance will be $t_1 = 1{,}750$ mm. Image K_1 is at a $k_1 = 1{,}473.68$ mm distance from the mirror, and the observer will perceive the image, magnified to $N = 0{,}842$, at a distance $S_1 = 5{,}126.32$ mm. The image will have a

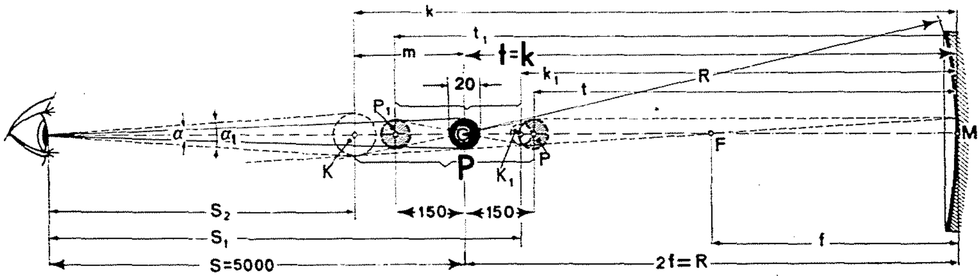

Fig. 13

diameter of 16,842 mm which appears at an angle of $a_1 = 11{,}292$ minutes of arc.

The difference in visual angle between the base line on the one hand, and of the image of the centre in the above-mentioned extreme (exterior) position is $\triangle a = 2{,}453$ minutes of arc, a sensation liable to be perceived, at least theoretically.

2. Let us now displace centre P to position P, that is, behind the basic position. The object distance will then be $t = 1{,}450$ mm and the image distance $k = 1{,}784.61$ mm. The image will be magnified by $N = 1{,}23$ with a diameter of 24,61 mm, and will be viewed by the observer at a distance $S_2 = 4{,}815.39$ mm at an angle $a_1 = 17{,}567$ minutes of arc. The difference in size between the images perceived in the basic position and in the second extreme (posterior) position will be $\triangle a = 3{,}821$ seconds of arc, a sensation falling within the range of perception. In the second case, that is, if the centre is approached to the mirror, the image size will vary at greater speed than in the first case. Hence, it will be more difficult for the observer to perceive the difference in image size than in the first case.

Temporarily accepting the above results, let us find the focal length at which the perception of the variation of image size may be excluded. According to the correlations valid for the spherical mirror and indicated in Fig. 11, that is, $t = 2f - 150$, $N = k : t$. $m = k - 2f$ one can write

$$S_2 = \frac{4850f - 750{,}000}{f - 150}$$

The mirror of unknown focal length f must come up to the requirement that the visual angle of the observer for the image of the centre be 1 minute of arc in the second — and less advantageous — case. It is therefore necessary that

$$\operatorname{tg}\frac{\alpha}{2} = \frac{10f}{4850f - 750{,}000} = 0{,}002\,291.$$

Of this, the focal length to be determined is

$$f = 1{,}546.092 \text{ mm},$$

and the radius of curvature

$$R = 3{,}092.184 \text{ mm}.$$

It is possible but has been found inconvenient to construct a mirror relying upon these results. The new apparatus was designed with a focal length of $f = 800$ mm, a distance at which the differences of size were no longer perceived by the observer. There may be various reasons for this phenomenon. First, the possibility of perception is reduced by the fact that the displacement of images, that is, the change of the image size is continuous. Surely an intermittent change of image size would not fail to strike the eye. Moreover, the mirror conveys an image of the short illuminated centre only, thus, if the test room is dark, the centre itself appears to be floating in the air. Finally, convergence variation and accomodation also exert a certain influence upon viewing, by distracting the observer's attention from perceiving the sensation caused by the varying image sizes.

The distance for bringing into coincidence object and image with the unaided eye is stipulated by international agreements to be 5 m. This, however requires much space, particularly if there is provided more than one test apparatus. In order to eliminate this inconvenience it has been suggested by the physician Dr. Sándor Lukács to add to the instrument a Galilean telescope (of negative magnification) creating the impression of viewing both centre and image from an actual distance of 5 m. At the present moment, experiments are being conducted to ascertain whether telescoping viewing may be employed without impairing the results. A series of tests will have to be accomplished before a definite answer can be given to the problem.

It was found impossible, owing to the inaccessibility of the adjusting wheel 7, to conduct the test with the unaided eye from a 5 m distance. Hence, the apparatus was designed so as to enable the observer to set the carriage in motion by means of an electric synchronizing device from any distance, including, of course, that of 5 m.

The following will describe the function of the synchronizing device.

The observer at S (Fig. 14) adjusts carriage 9 by means of a wheel 1 bedded in the upper part of column 3 at a 5 m-distance from the apparatus 5. Connection between the two parts of the apparatus is effected by an electric axis, that is, a relay servo system. The servo transmitter 2 is fixed on wheel 1, and the servo receiver is built into apparatus 5. By rotating the servo part 6 the transmission element 10 (and endless rope or chain) wedged on the axis and flung over wheel 11 pushes carriage 9 together with the centre 8 in the direction indicated by the arrows. Images 8′ and 8″ of centre 8, formed by means of the mirror 7 are viewed as the function of the centre's position. The servo transmitter 2 and receiver are connected by a five-stranded cable 4. The stationary part of the servo elements (Fig. 15), that is, of transmitter A to be rotated by the observer and of the receiver B built into the apparatus is induced by a 110 V mains a. c. Upon turning the adjusting wheel, current is induced in the rotor of transmitter A, causing the rotor of receiver B to rotate in the same direction and at identical angle. The system is out of gear if the symmetrically wound rotors are in coincidence between the poles of the induced stationary part, so that the connecting cable is free of current. Apart from a very slight slip, there is no lost motion in the function of the machines, that is, they are practically in synchronism.

The apparatus as represented in the last two figures is but a first embodiment intended for laboratory purposes. Any optical and mechanical changes that might seem advantageous will be determined and put into practice after experiments will have been terminated.

It will be noticed that the readings in connection with the centre's position, and their recording require a certain amount of time, to be felt particularly in the case of group tests. Moreover, the tables obtained after repeated tests are not sufficiently distinct. It is, therefore, intended that the next embodiment will comprise a printing device attached to the instrument for recording the repeated positions of the carriage on squared plotting paper. The curve to be obtained by connecting these points will represent the error curve and may be deposited as case history for future reference. This will facilitate the comparison of several observers' stereoscopic vision power.

The image forming power of the simple spherical mirror designed for the purpose of the instrument is far from being perfect. Yet the aberrations in image formation will not confuse the observer as both object and image are positioned in the vicinity of the optical axis. Another reason why said aberrations will not act confusingly is the small size of object and of image.

The apparatus was constructed by the Central Optical and Precision Mechanical Research Laboratory. I should like to express my gratitude therefore

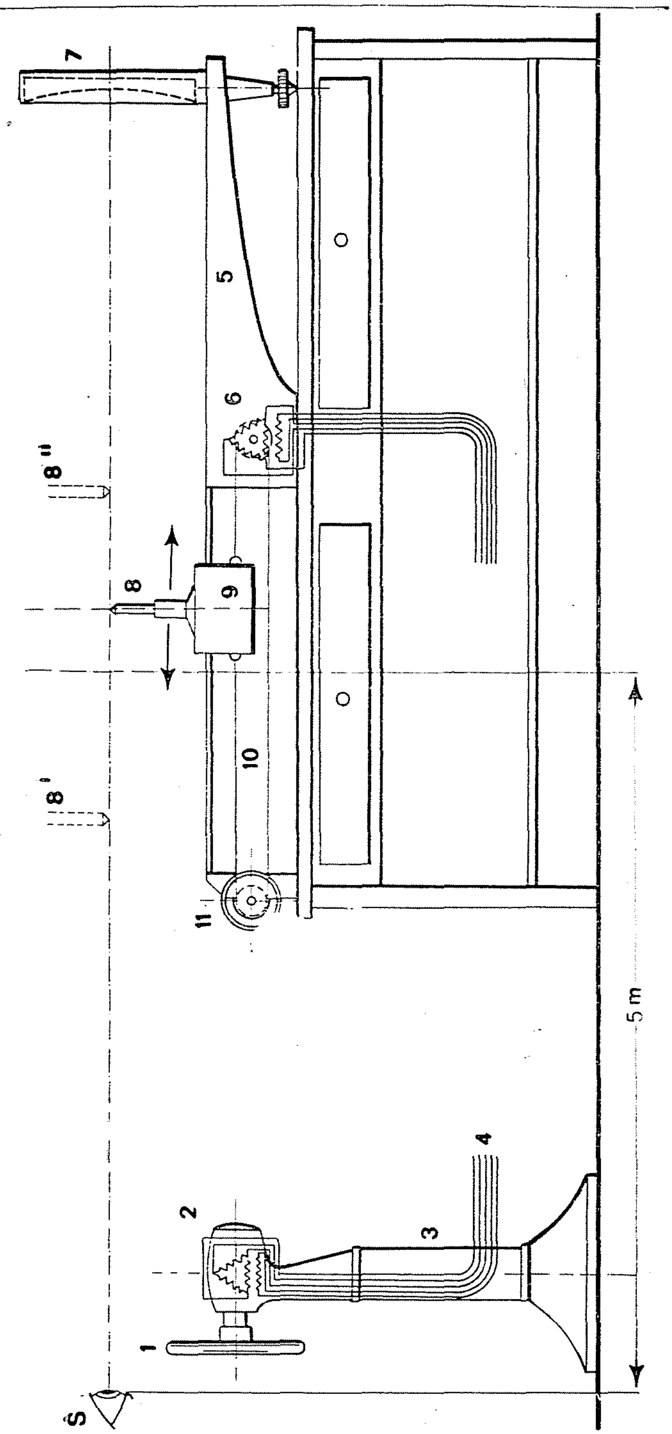

Fig. 14

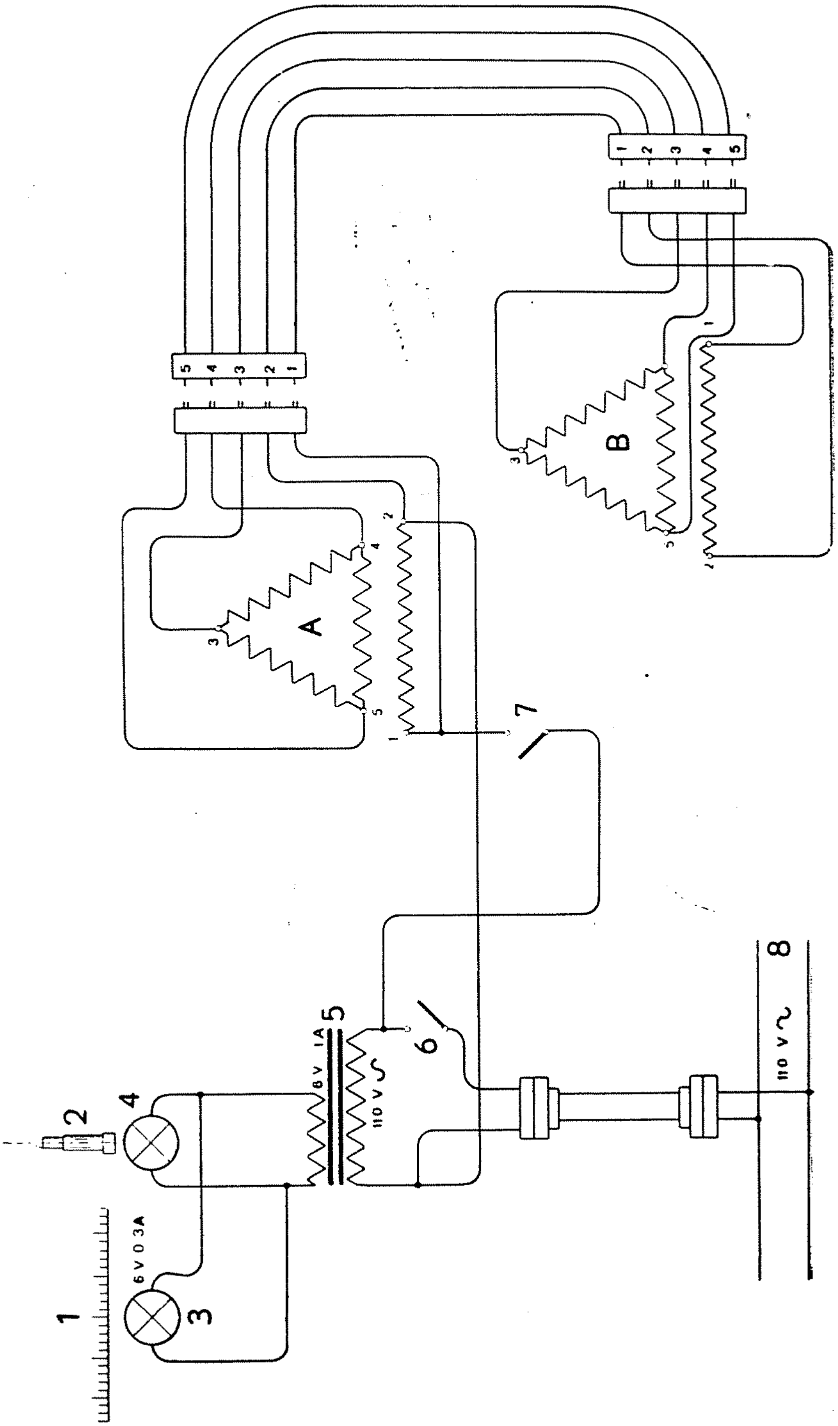

Fig. 15

to the Director of the Institute, the physicist Dr. Ferenc Szalkay whose aid rendered it possible to have the apparatus constructed. I should also like to take this opportunity to express my gratitude to the oculists Dr. Emil Galla and Dr. György Aczél for their untiring work in assisting me with information and helpful suggestions with regard to the technicalities in carrying out and putting into practice the basic idea.

Summary

There are several kinds of instruments for the assessment and determination of stereoscopic vision. In this article author describes a new instrument which can be used independently of the conditions of illumination, taking into account the optical and psychological phenomena connected with stereoscopic vision. The tests are carried out by means of radiation having different wave-lengths and whose strength of illumination can be regulated. The determination of the vision power, which hitherto took place in an intermittent manner, becomes continuous by the use of this instrument and the numerical values are capable of being laid down by means of diagrams. The stereoscopic signal is represented by a movable signal illuminated from the inside by radiation of varying wavelength and by its image produced by a spherical mirror. For the theoretical determination of the focus of spherical mirrors author deals with the general optical-geometrical relations of spherical mirrors. The signal is displaced by hand or by electric command from any distance. Using the instrument in question, it is possible to carry out — in addition to the determina tionof stereoscopic vision power — other tests, too, which latter, however, are not being dealt with in this paper and will be reserved for another publication.

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