SOME PROBLEMS OF APPLICATION AND PRACTICAL DESIGN OF NEUTRON AMPLIFIERS I.

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In the last few years several methods have been developed for the calculation of subcritical neutron-multiplying systems [1—6], but the results published could not be used directly for the design of a system with a given multiplication.

In the early summer of 1955, we began to carry out calculations concerning the mass multiplication relations of subcritical systems [7], for which there were no data in the literature. The fact, that certain subcritical systems are succesfully employed indeed in university teaching [8], raised the question of practical dimensioning of such systems.

The main problems arising are the following : what is the extent of multiplication to be obtained by using different quantities of variously enriched fuel and different moderators and reflectors, further, how easily can the multiplication be adjusted through the feeding-in of fissionable material ; what is the stability of the systems, what are the economic aspects?

In the following neutron amplifier means any subcritical system, which contains fissionable material and moderator and is built for the purpose of continuously increasing the neutron flux produced by the extraneous source by means of fissionable material.

The first part of this article contains our calculations relating to the static neutron amplifier — i. e. the case, when the position and intensity of the extraneous neutron source is independent of time. Our considerations concerning sources varying with time will be described in the second part.

1. The amplification factor

According to the general reactor theory [9], the neutron density in a finite, bare, homogeneous medium containing a neutron source, independent of time, can be expressed as follows :

$$\varrho(\bar{r}) = l_0 \sum_{n=1}^{\infty} \frac{S_n Z_n(\bar{r}) \bar{P}_\infty(B_n^2)}{1 + L^2 B_n^2 - \dfrac{k_\infty}{p} \bar{P}_\infty(B_n^2)}, \tag{1}$$

where the $Z_n(\bar{r})$ autofunctions satisfy the equation :

$$\Delta Z + B^2 Z = 0 \tag{2}$$

and the requirement

$$Z(\bar{r}_{\mathrm{extr}}) = 0 \tag{3}$$

r_{extr} referring to the extrapolated boundary of the system with eigenvalues B_n^2. The extraneous source is represented by

$$S(\bar{r}) = \sum_{n=1}^{\infty} S_n Z_n(\bar{r}) .$$

$\bar{P}_\infty (B_n^2)$ is the Fourier transform of the infinite slowing-down kernel at thermal energies. Calculating it for a neutron point source located in the origin of the coordinate system, the results, according to the Fermi age theory and the m — group slowing-down theory, respectively, are the following :

$$\bar{P}_\infty = p \exp[- B_n^2 \tau], \tag{5}$$

$$P_\infty = p \left[\prod_{i=0}^{m=1} (1 + L_i^2 B_n^2) \right]^{-1} \tag{6}$$

Here p is the resonance escape probability, l_0 the average lifetime of thermal neutrons in an infinite medium, τ the Fermi age of thermal neutrons and L_i the diffusion length in the i-th energy interval.

Now, starting from [1], we can define the multiplication at any point of the system as the ratio of neutron densities with and without fissionable material :

$$A(\bar{r}, k_\infty) = \frac{\varrho(\bar{r}, k_\infty)}{\varrho(\bar{r}, 0)} = \frac{\sum\limits_{n=1}^{\infty} \dfrac{S_n Z_n(\bar{r}) \bar{P}_\infty(B_n^2)}{1 + L_0^2 B_n^2 - \dfrac{k_\infty}{p} \bar{P}_\infty(B_n^2)}}{\sum\limits_{n=1}^{\infty} \dfrac{S_n Z_n(\bar{r}) \bar{P}_\infty(B_n^2)}{1 + L_0^2 B_n^2}} . \tag{7}$$

The ratio of thermal neutrons contained in the system is in the two cases

$$\bar{A}(k_\infty) = \frac{\int\limits_V \varrho(\bar{r}, k_\infty)\, d\bar{r}}{\int\limits_V \varrho(\bar{r}, 0)\, d\bar{r}} = \frac{\sum\limits_{n=1}^{\infty} \dfrac{S_n \bar{P}_\infty(B_n^2)}{1 + L_0^2 B_n^2 - \dfrac{k_\infty}{p} \bar{P}_\infty(B_n^2)} \int\limits_V Z_n(\bar{r})\, d\bar{r}}{\sum\limits_{n=1}^{\infty} \dfrac{S_n \bar{P}_\infty(B_n^2)}{1 + L_0^2 B_n^2} \int\limits_V Z_n(\bar{r}) d\bar{r}} . \tag{8}$$

Expressions (7) and (8) may serve as definitions of neutron amplification for a bare medium with time independent flux.

These expressions are suitable for the calculation of any bare, homogeneous and with appropriate averageing, of any heterogeneous, fuel-moderator arrangement. In the latter case, naturally, the microscopic behaviour of neutron density does not alter the multiplication value. The multiplication property of the fuel here is characterized by k_∞ alone, the diffusion and slowing-down by L_0 and τ and L_i, respectively.

2. The "minimal" amplification factor

For the study of a chain reaction, it is necessary that the flux distribution in the neutron amplifier differ significantly from the case without multiplication. Depending on the size of the system, this condition is fulfilled for various k_∞, in these cases, however, the corresponding values of $\bar{A}$ are nearly identical.

To investigate the dependence of the relative neutron density $\varrho(k_\infty, z)/\varrho(k_\infty, 0)$ on $\bar{A}$, let us employ spherical geometry and suppose, we can apply the age theory. The application of spherical geometry does not mean a restriction, if we think of the "equivalent transformation" of other geometries [10], which is usually performable with satisfactory accuracy.

Then (4), (7) and (8) can be written as:

$$S = S_0 \frac{n}{2R^2} \tag{9}$$

$$A(r, k_\infty) = \frac{\sum\limits_{n=1}^{\infty} \dfrac{n \exp[-n^2 \pi^2 \tau R^{-2}]}{1 + n^2 \pi^2 L_0^2 R^{-2} - k_\infty \exp[-n^2 \pi^2 \tau R^{-2}]} \sin n\pi \dfrac{r}{R}}{\sum\limits_{n=1}^{\infty} \dfrac{n \exp[-n^2 \pi^2 \tau R^{-2}]}{1 + n^2 \pi^2 L_0^2 R^{-2}} \sin n\pi \dfrac{r}{R}}. \tag{10}$$

$$\bar{A}(k_\infty) = \frac{\sum\limits_{n=1}^{\infty} (-1)^{n+1} \dfrac{\exp[-n^2 \pi^2 \tau R^{-2}]}{1 + L_0^2 n^2 \pi^2 R^{-2} - k_\infty \exp[-n^2 \pi^2 \tau R^{-2}]}}{\sum\limits_{n=1}^{\infty} (-1)^{n+1} \dfrac{\exp[-n^2 \pi^2 \tau R^{-2}]}{1 + L_0^2 n^2 \pi^2 R^{-2}}} \tag{11}$$

In place of the natural parameters of these expressions:

$$k_\infty, \frac{r}{R}, \frac{\sqrt{\tau}}{R}, \frac{L}{R}$$

it is often advantageous to introduce the parameter group :

$$k_\infty, \; z = \frac{r}{R}, \quad \frac{\sqrt{\tau}}{L}, \quad \frac{R}{\sqrt{\tau}} \tag{12}$$

In Fig. 1 the values of $\varrho\,(k_\infty, z)/\varrho\,(k_\infty, 0)$ are represented, which were calculated from (10) and (11), in the case of a large, natural uranium—water sytem, using the constants of Table I, further the values of L_i mentioned in Chap. 4.

The parameters are k_∞ and $\bar{A}$. It may be clearly seen the growing-up of the neutron density, especially at medium z values.

A (k_∞, z) increase very fastly with z and after a maximum — as it can be easily shown — its value is at the boundary :

$$\lim_{z \to 1} A\,(k_\infty, z) = A\,(0, k_\infty)\,.$$

In the origin it can be written : $A_c = A\,(0,\, k_\infty) \leq A\,(z,\, k_\infty)$.

Naturally, the shape of the curves in Fig. 1 depends on the size of the system. The $\varrho/\varrho_{\max}$ values of a smaller-sized amplifier are higher. But from such a representation it is always possible to find for each purpose a suitable $A_{\min}$, [a minimal amplification factor], below which, the multiplication can be considered to be only a large perturbation on the original flux.

3. The correlation of the parameters

The practical description of a neutron amplifier may consist in giving the enrichment of the fuel, the mixing ratio of fuel and moderator (in the case of a heterogeneous system the suitable lattice parameters), further the dimensions and the moderator.

Tha parameter group (12) cannot be considered suitable for such a description, these parameters, however, may be preferred to the parameters of [1] and (11), because they can be transformed in the simplest way into a set of "practical" parameters. Namely, τ (respectively the analogous L_i) in case of a homogeneous system and $\gamma \gg 1$, [9] depends only on the moderator, while k_∞ and L_0 are the functions of enrichment and dilution. We cannot, however, choose these parameters (at least three) independently, because of the properties of the known moderators. We do not deal here with the general case and in the following we shall consider systems containing U^{235} and U^{238} fuel only.

For a homogeneous system it is advantageous to introduce the following notations :

$$\gamma = \frac{N_{\text{mod}}}{N_{U^{235}}}\,, \tag{13}$$

where we shall call γ the "dilution". N is the number of nuclei in the volume element. Further

$$d = \left(1 + \frac{N_{U^{238}}}{N_{U^{235}}}\right)^{-1} \tag{14}$$

may characterize the degree of enrichment. Then it is possible to perform for all moderators the following transformation :

$$\begin{aligned} k_\infty &= k_\infty(\gamma, d) \\ L_0 &= L_0(\gamma, d) \end{aligned} \tag{15}$$

Here d and γ are now "practical" parameters.

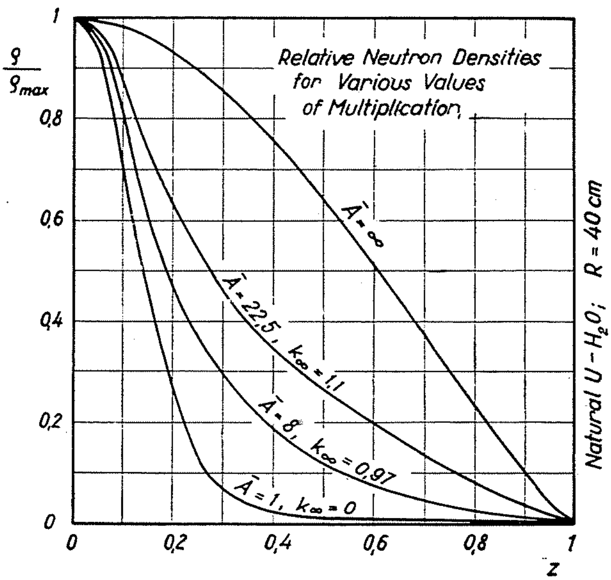

Fig. 1

Fig. 2 represents such a transformation in case of water, calculated [7] from the original definitions, on the basis of the published cross sections. It is noticeable, that L_0 varies weakly with enrichment and that above 10% k_∞ does not grow appreciably.

The maxima of k_∞ for various moderators can be read off from Fig. 3 [11], which represents the critical radii of unreflected spherical reactors with high enrichment.

In heterogenous systems, besides d and the volume ratio V_{mod}/V_{fuel} which is equivalent to γ, additional parameters are playing a role. These are the data characterizing the size of the lattice cell, the clad of uranium and the occasional air gap. References [12—15] contain their published optimal values in case of natural and slightly enriched uranium.

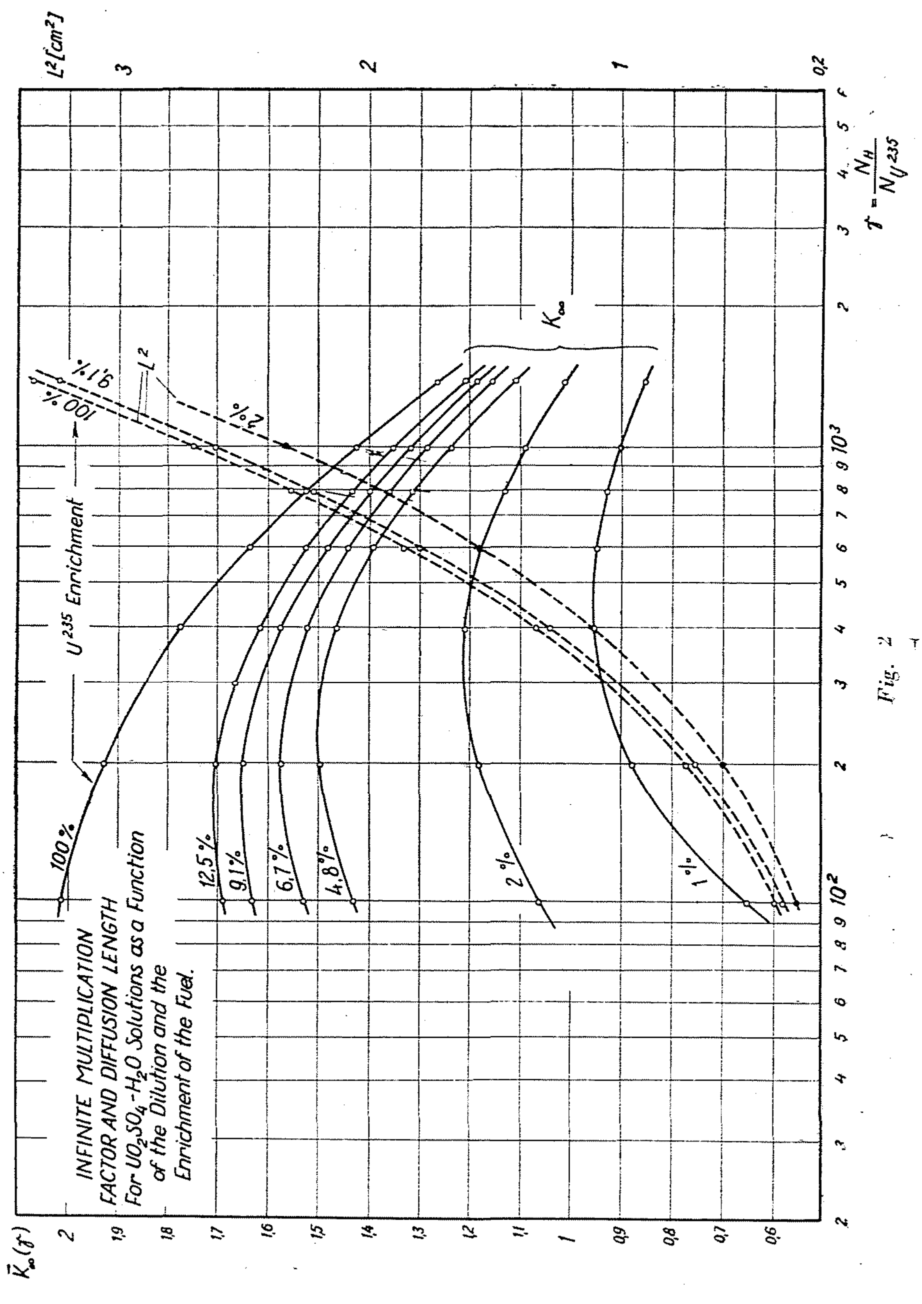
INFINITE MULTIPLICATION FACTOR AND DIFFUSION LENGTH
For UO_2SO_4-H_2O Solutions as a Function of the Dilution and the Enrichment of the Fuel.
U^{235} Enrichment
100%
12,5%
9,1%
6,7%
4,8%
2%
1%
K_∞
L^2
$L^2\,[cm^2]$
$\bar{K}_\infty(\gamma)$
$\gamma = \frac{N_H}{N_{U^{235}}}$

Fig. 2

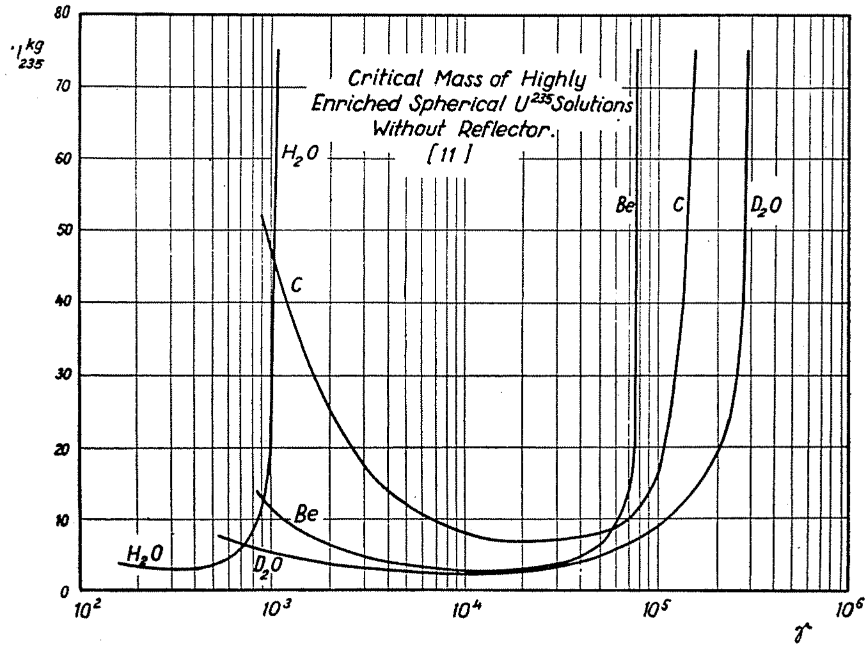

Fig. 3

4. Characteristics of unreflected, static amplifiers

With the aid of (11) and taking into account the appropriate relations between the parameters, the curves for multiplication in optimal cases may be drawn. We here discuss only a few typical examples of them.

Fig. 4 represents the characteristics of a homogeneous spherical amplifier with 9,1% enrichment, which were calculated on the basis of the four-group theory. The k_∞ and L_0 values are to be seen in Fig. 2, while the L_i were chosen according to [9] as follows : $L_1 = 4{,}49$ cm, $L_2 = 2{,}05$ cm, $L_3 = 1{,}00$ cm.

It may readily be seen that in the interval of γ which was investigated, there are critical values of mass, radius and γ^{-1}, below them the amplifier cannot become critical. The structure of the curves suggests the existence of an interesting part of the characteristics at small γ-s.

In the case of $1 < k << 2$, for heterogeneous systems, we can deduce from

$$k_{\text{eff}} = \frac{\bar{A}_1 - 1}{\bar{A}_1} = \frac{k_\infty}{1 + M^2 B_g^2} \tag{16}$$

(taking into account the first harmonic only), the following expression :

$$\frac{V}{V_{cr}} = \left[1 + \frac{1}{\bar{A}_1 (k_\infty - 1)}\right]^{-\frac{3}{2}} \tag{17}$$

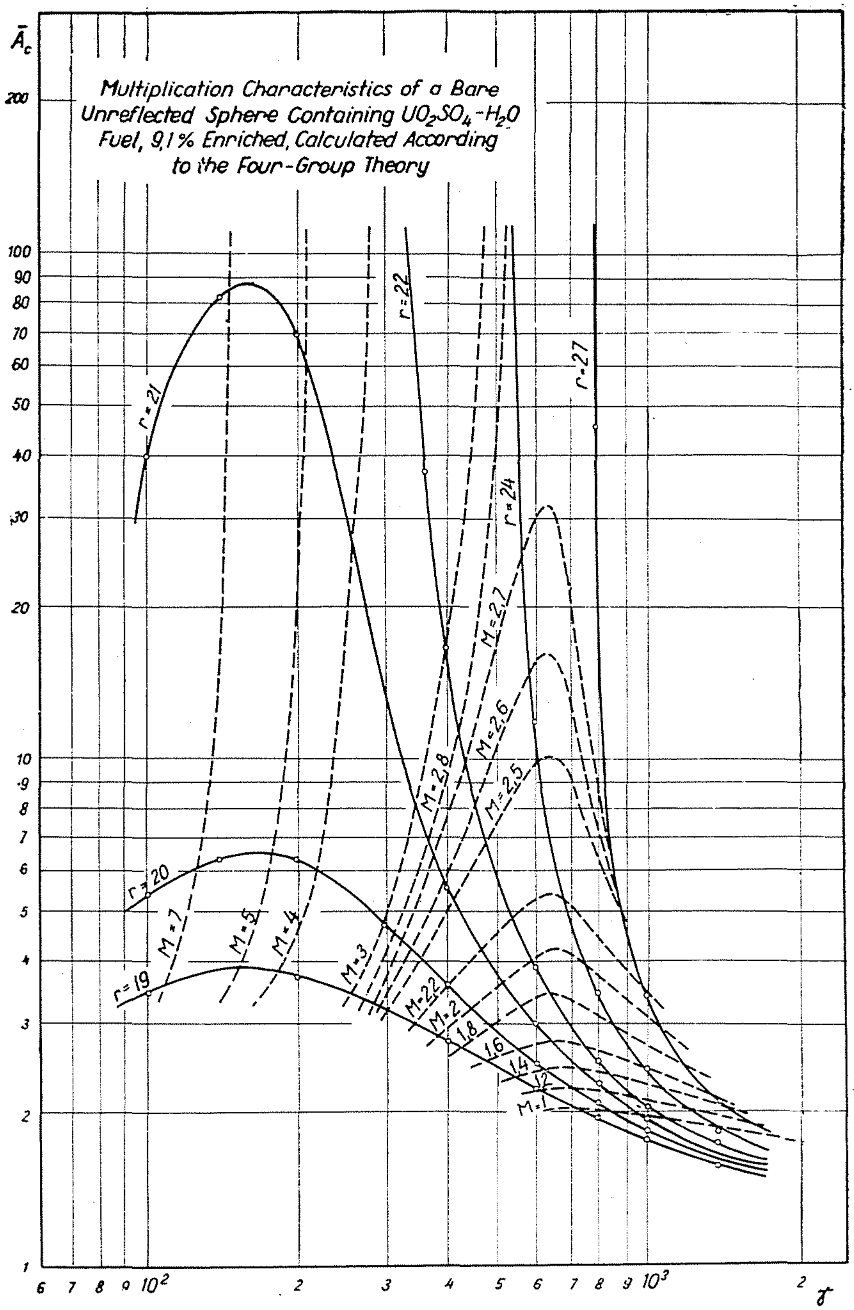
Multiplication Characteristics of a Bare Unreflected Sphere Containing $UO_2SO_4-H_2O$ Fuel, 9,1% Enriched, Calculated According to the Four-Group Theory
$\bar{A}_c$
200
100
90
80
70
60
50
40
30
20
10
9
8
7
6
5
4
3
2
1
6 7 8 9 10^2 2 3 4 5 6 7 8 9 10^3 2 γ
r=21
r=22
r=27
r=24
r=20
r=19
M=7
M=5
M=4
M=3
M=2,8
M=2,7
M=2,6
M=2,5
M=2,2
M=2
1,8
1,6
1,4
1,2
M=1

Fig. 4

which is represented by Fig. 5, where the curve relating to the case $k_\infty = 1{,}45$ and $\gamma = 640$ of Fig. 4 is also drawn in.

We can read off that for identical $\bar{A}$, V/V_{cr} decreases surprisingly quickly with k_∞, while V_{cr} naturally increases.

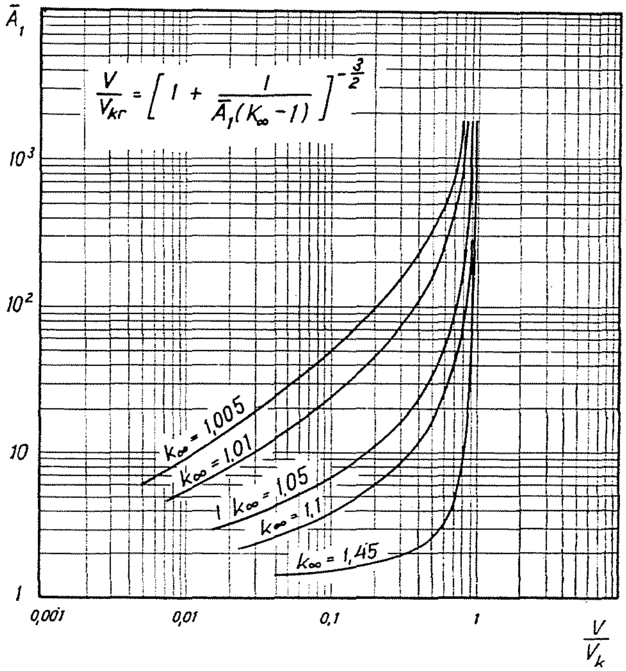

Fig. 5

Fig. 6 represents the attainable gain of natural uranium — ordinary water systems as function of volume for the lattice arrangement of Table I. The calculation was carried out according to (11), taking into account the ten first harmonics only, using the four-group model.

Table I

Moderator	V_{mod}/V_U	r_0, cm	k_∞	L_0^2, cm²	M^2, cm²	Ref.
H_2O	1,5	1,5	0,97	2,7	31	[8] [14] [15]
D_2O	29,6	2,54		125	232	[13]
C		2	1,055	402	696	[12]

Representing the multiplication calculated from the approximate expression (17) for variously moderated heterogeneous arrangements as the function of volume, with the parameters obtainable from Table I, we obtain the curves of Fig. 7. The negative values of B^2 are indicated for the sake of completeness only. It is a useful diagram for the estimation of the gain in subcritical systems, even for $k_\infty \leq 1$.

We may conclude on the basis of Figs. 5 and 7 that far from critical radii, with identical volume and moderator, the decrease of k_∞, up to the natural degree of enrichment does not mean significant decrease in the amplification.

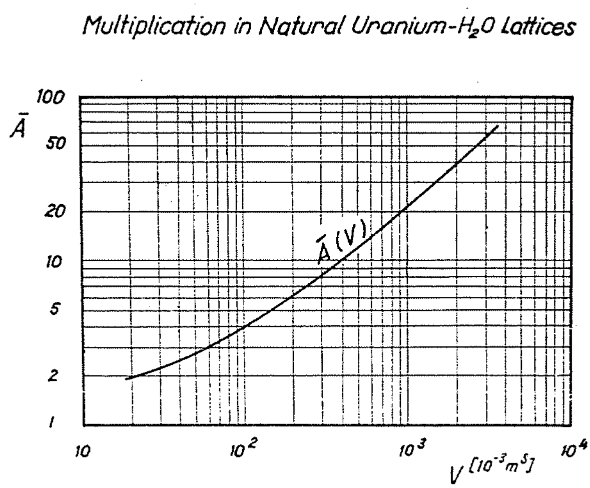

Fig. 6

5. Reflected amplifiers

The reflector savings concerning critical reactors may serve as a first approximation for estimate of the decrease in size of reflected amplifiers. The precize calculation of reflector savings becomes very circumstantial as the number of groups increases considerably. To estimate the attainable accuracy we represented in Fig. 8 two experimental curves and the critical radii and mass calculated according to the methods described in [9], for a homogeneous sphere reflected by an infinite H_2O reflector, and containing highly enriched UO_2SO_4-H_2O solution.

It may be clearly seen that the curves representing the one-group theory and the two-group method go near each other and the latter approximates fairly well the experimental curves. Applying the one-group theory we used the values for B_c obtained in the unreflected case with the aid of the four-group theory. This method is very advantageous by estimating the size.

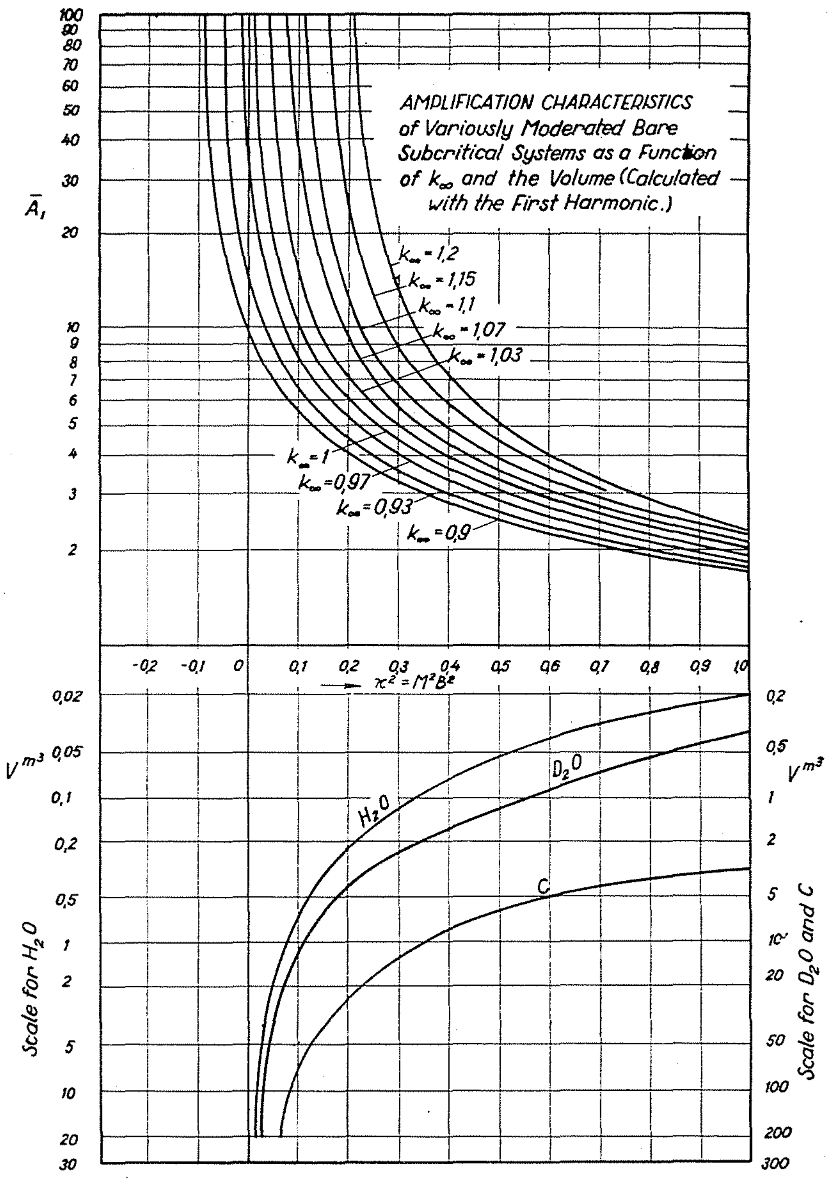

Fig. 7

We may judge the effect of usual reflectors in reducing the critical size from the data of Table II [6c], which relate to water solutions, in case of various "infinite" reflectors. We can conclude that the water reflector in spite of its large capture cross section is a relatively efficient reflector.

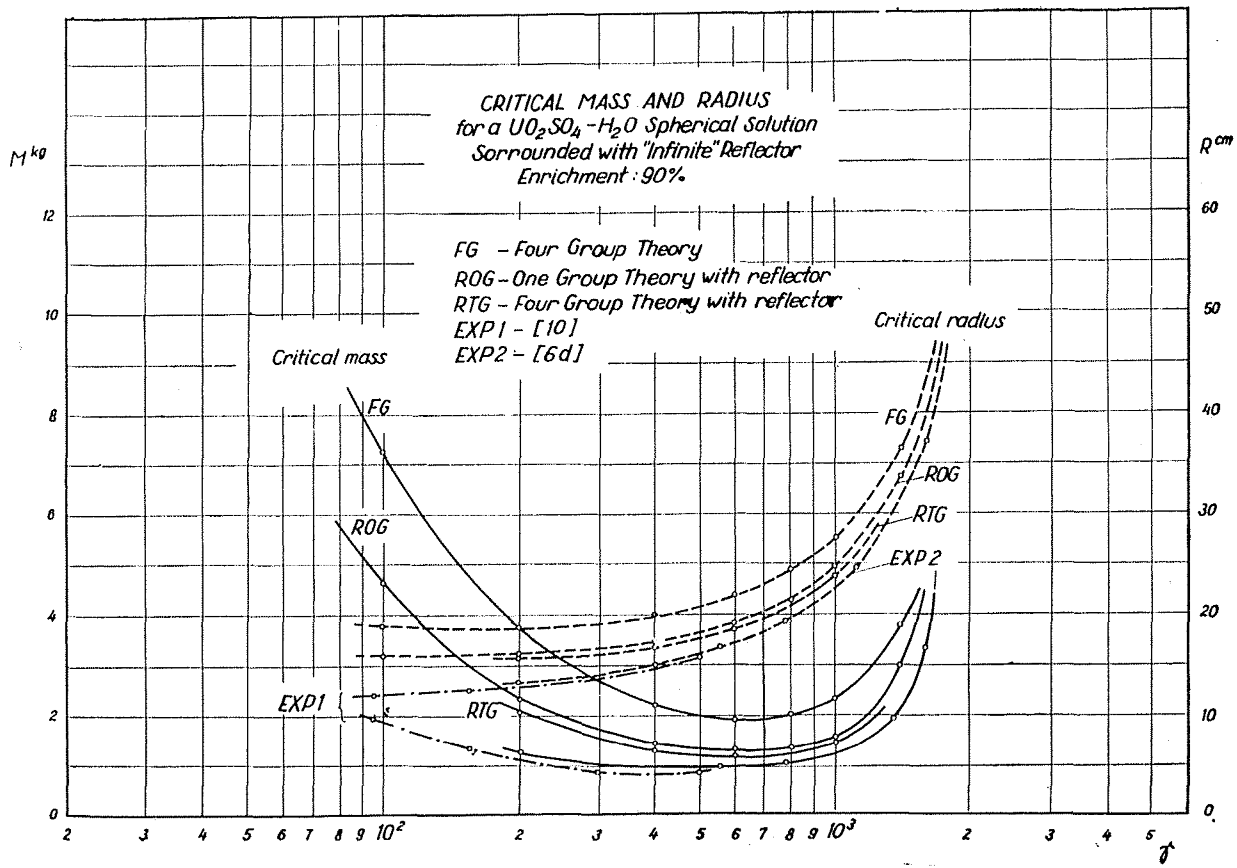
CRITICAL MASS AND RADIUS
for a UO_2SO_4-H_2O Spherical Solution
Sorrounded with "Infinite" Reflector
Enrichment : 90%
FG - Four Group Theory
ROG - One Group Theory with reflector
RTG - Four Group Theory with reflector
EXP1 - [10]
EXP2 - [6d]
Critical mass
Critical radius
M kg
R cm
FG
ROG
RTG
EXP1
EXP 2
0
2
4
6
8
10
12
10
20
30
40
50
60
2
3
4
5
6
7
8
9
10^2
10^3
γ

Fig. 8

Table II

	One-group theory		Experimental [6c]	
Reflector	r_{cr} cm	M_{cr} g	r_{cr} cm	M_{cr} g
U 235 %	9,1		12,5	
γ	600		720	
BeO 2,8 g/cm³	16,2	778	15,0	489
Graphite 1,6 g/cm³	16,85	876	16,1	600
D_2O	16,8	870	16,3	622
H_2O	22,03	1970	20,7	1273
Unreflected	24,83	2820	27,3	2920

The dimensioning of reflected neutron amplifiers may be carried out by using equation (11), equating the value of the neutron flux and current density at the core-reflector interface and satisfying the requirement that the flux should be zero at the extrapolated boundary of the whole system. Then, instead of the known one-group reflector equation :

$$\operatorname{ctg} B_c R = \frac{1}{B_c R}\left(1 - \frac{D_c}{D_r}\right) - \frac{D_r}{D_c B_c L_r} \operatorname{cth} \frac{T}{L_r} \tag{18}$$

we obtain :

$$\sum_{n=1}^{\infty} a_n \left\{ n B_c R \cos n B_c R + \sin n B_c R \left[\frac{D_r}{D_c} \left(\frac{R}{L_r} \operatorname{cth} \frac{T}{L_r} + 1 \right) - 1 \right] \right\} = 0 , \tag{19}$$

where T is the reflector thickness and

$$a_n = \frac{n \exp\left[-\tau B_n^2\right]}{1 + L^2 B_n^2 - k_\infty \exp\left[-\tau B_n^2\right]} = a_n (B_c, n) . \tag{20}$$

We can improve our results by substituting the unreflected four-group values B_c, as we mentioned above.

The values of reflector saving will become dependent on the gain because of (19). In case of systems of smaller size, when k_∞ is greater, this dependence can be neglected, for in cases in medium gains, which are interesting of practical purposes, the mass of reactor and multiplier scarcely differ. In case of small k_∞, the difference will be greater.

6. Stability

In water-uranium systems a negative temperature coefficient may be obtained [6b, 15] by choosing suitable lattice parameters in heterogeneous ceses, too. For the sake of simplicity we took the T. C. to be $3 \cdot 10^{-4}$/grad [11] and from this it is immediately apparent that in laboratory conditions a gain of about 20 cannot be described as satisfactorily stable (allowing for a variation in temperature of $\pm$ 15 C°).

In heterogeneous systems stability of temperature it is recommended, in homogeneous systems, with a greater gain, it must definitely be assured.

So as to investigate the feeding-in of fuel in homogeneous systems, we may introduce the sensitivity curve. (Under sensitivity the derivative of the gain with respect to the mass at a given radius is to be understood here). In the case of $R = 24$ cm in Fig. 4, such a relation is represented by Fig. 9.

With respect to safety, the curves $R(\gamma, M) < R_{kr}$ of Fig. 4 deserve attention, however, their realization is made impossible because of their being uneconomic, as they require great amount of fuel. In the case of homogeneous systems when greater gains are obtained it is therefore more advantageous to feed-in at optimal dilution. If, however, this operation is being carried out at above indoor temperature, with some simple safety equipment, then control of the amplifier may be reduced to temperature regulation in the appropriate k_∞ interval. As to the design of safety circuits information may be obtained e. g. from [16—17]. In case of $\delta k_\infty = 1\%$, $T - T_{\text{lab}} = 20°$ C, $dT/dt \leq 0{,}4$ grad/s : $\delta k/dt = 1{,}2 \cdot 10^{-4}$/sec and $P_{\text{min}} \gtrsim 50$ sec. Thus we can determine upper and lower limit of multiplication for the case considered.

7. Conclusions

From our results we may conclude, that when realizing amplification factors, which are not too great, the price of a neutron-amplifier equals only the price of the fissionable material and the neutron source by both the homogeneous and the heterogeneous case. The necessary quantity of fissionable material is nearly the same for an amplifier or a critical reactor is case of medium enrichment, whereas in case small k_∞, the difference is essential.

Medium enriched homogeneous or a natural-uranium heterogeneous system, with a suitable water reflector both offer certain advantages. As to the costs of

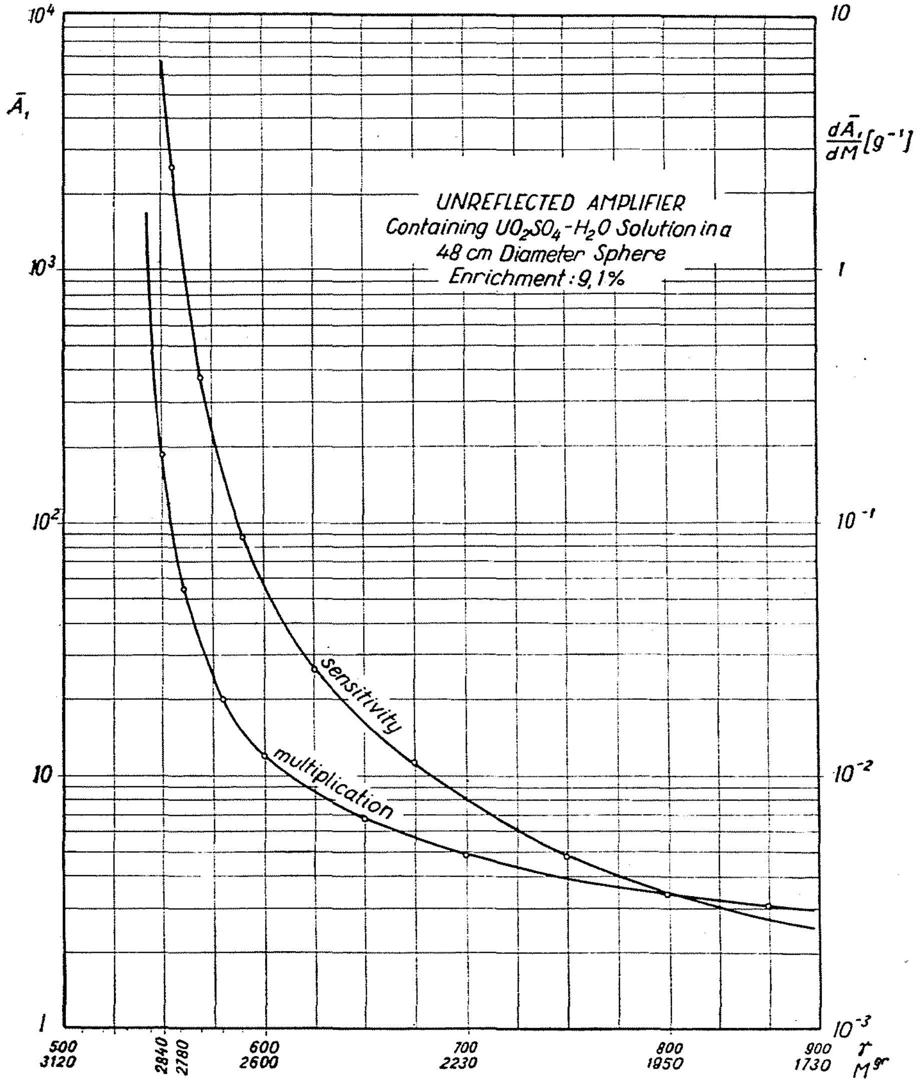

Fig. 9

fissionable material, they are roughly the same [8, 18]. However, an arrangement of greater geometrical dimensions appears in regard to its use to be more versatile.

The small-sized homogeneous system is with adequate precautions suitable for the increase of the intensity of a smaller neutron source. The bigger-sized not enriched heterogeneous system may be absolutely safe ($k_\infty < 1$), suitable for most of the usual measurements carried out in a reactor-school, and just on account of its greater size it may be operated by many persons. From these points of view it has many advantages over the water-boiler type reactor, employed in university teaching [19].

A reactor, especially for the university of a small country, is rather a valued research tool, than a versatile means of education of reactor engineers.

As the neutron amplifier does not consume the fuel, it is probable that in case of borrowing from a central pool, the use of the neutron amplifier may become an appreciated and more wide-spread means of the university education.

Finally we wish to express our thanks to all who helped us in carrying out the calculations.

Summary

Our calculations concern the value of multiplication to be obtained in static neutron-multiplying systems using different quantities of variously enriched fissionable material and several types of geometrical layouts and reflectors. The economic and safety aspects of these systems are also considered. In the second part the problems relating to the application of neutron sources varying with time will be dealt with.

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