SIMPLE DESIGN OF FERROMAGNETIC SERIES RESONANCE CIRCUITS

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1. Introduction

The ever increasing interest for circuits containing ferromagnetic inductors calls more and more new applications for, in which the nonlinear characteristics of the ferromagnetic inductor is of prime importance e. g. magnetic flip-flop circuits, stabilizators, etc. In an earlier publication [1] authors reported on a simple *ac* stabilizator as a possible application of the series resonance circuit containing ferromagnetic core. There some qualitative aspects were given about the behaviour of such a circuit and by the aid of a simple graphical construction quantitative estimations were also suggested. In the present paper by further development of the above method the ferromagnetic series resonance circuit is more generally treated and calculating methods based chiefly on graphical construction are briefly outlined for such circuits.

2. Induction coils with ferromagnetic core

The followings are restricted to series resonance circuits with inductors having ferromagnetic core. Such a circuit (Fig. 1) is composed of a condenser C and an inductivity L. It is supposed that both are ideal circuit elements but the ohmic resistance R of the inductivity is taken into account and in the Figure it is drown as a separate element.

The ferromagnetic core implies — as it is well-known — that the flux in the core will not be a linear function of the current flowing through the coil. In case of *dc* magnetization (coil with a given number of turns) the shape of that function is the same as the magnetization curve of the iron core shown in Fig. 2 (hysteresis is omitted).

With *ac* magnetizing current the situation practically will be the same because the relation between mean values of magnetic induction and magnetizing current is similar to that of the curve shown on Fig. 2. That is the permeability of the iron core and the inductivity of the inductor, too, is a function of the magnetizing current

$$\mu = \mu(i)$$

and

$$L = L(i).$$

For a given iron core this relation can be determined in good approximation

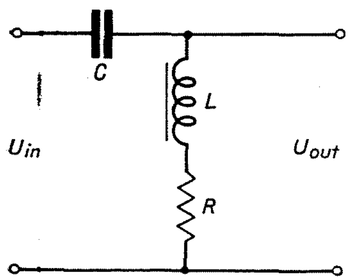

Fig. 1

Fig. 2

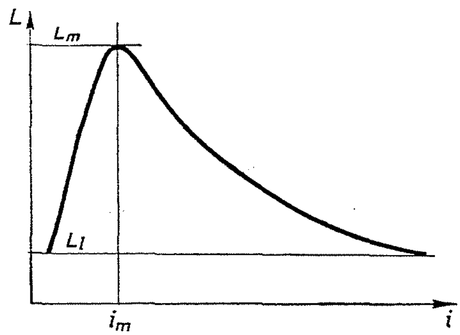

Fig. 3

by measuring the *ac* impedance of the coil as a function of current flowing through it. A typical $L(i)$ relation is shown in Fig. 3.

3. The unloaded series resonance circuit

The resonance circuit of Fig. 1 is connected to an *ac* voltage source with angular frequency ω. Let the input voltage be U_{in} and let us find the output voltage U_{out} that is the voltage on the inductivity

$$U_{\text{in}} = -j\frac{i}{C\omega} + U_{\text{out}}$$

where

$$U_{\text{out}} = i\,(R + j\omega L)$$

When the

$$L = L\,(i)$$

relation is given, then with the aid of

$$i = \frac{U_{\text{in}}}{R + j\left(\omega L - \dfrac{1}{C\omega}\right)}\,.$$

U_{out} may be calculated and

$$U_{\text{out}} = U_{\text{in}} \frac{R + j\omega L}{R + j\left(\omega L - \dfrac{1}{C\omega}\right)}\,.$$

It is very difficult, however, to use the $L(i)$ relation chiefly because it is not easy to find a suitable analytical approximation for it. To remove the difficulties the authors in their reported paper proposed a graphical method. Let be

$$Z = R + j\left(\omega L - \frac{1}{C\omega}\right),$$

so the absolute value of the impedance of a series resonance circuit will be

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{C\omega}\right)^2}\,.$$

Taking into account that the inductivity is a function of the current the impedance will be also a function of the current flowing through. This $|Z| = f(i)$ relation is derived by graphical construction. In Fig. 4 the absolute value of the impedance is plotted as function of inductivity (in arbitrary units). In this construction ω, R and C are constants.

If $L_0 = \dfrac{1}{\omega^2 C}$ then resonance will occur where $L = L_0$ and the resistance of the circuit

$$|Z|_{L=L_0} = R\,.$$

Because usually $R \ll \dfrac{1}{C\omega}$ the impedance diagram is composed approximately of two straight lines. If we draw besides this diagram the $L(i)$ relation, then we can evaluate $|Z|$ as a function of i. (Fig. 5) As a result of this graphical construction we have the relation

$$|Z| = Z(i)$$

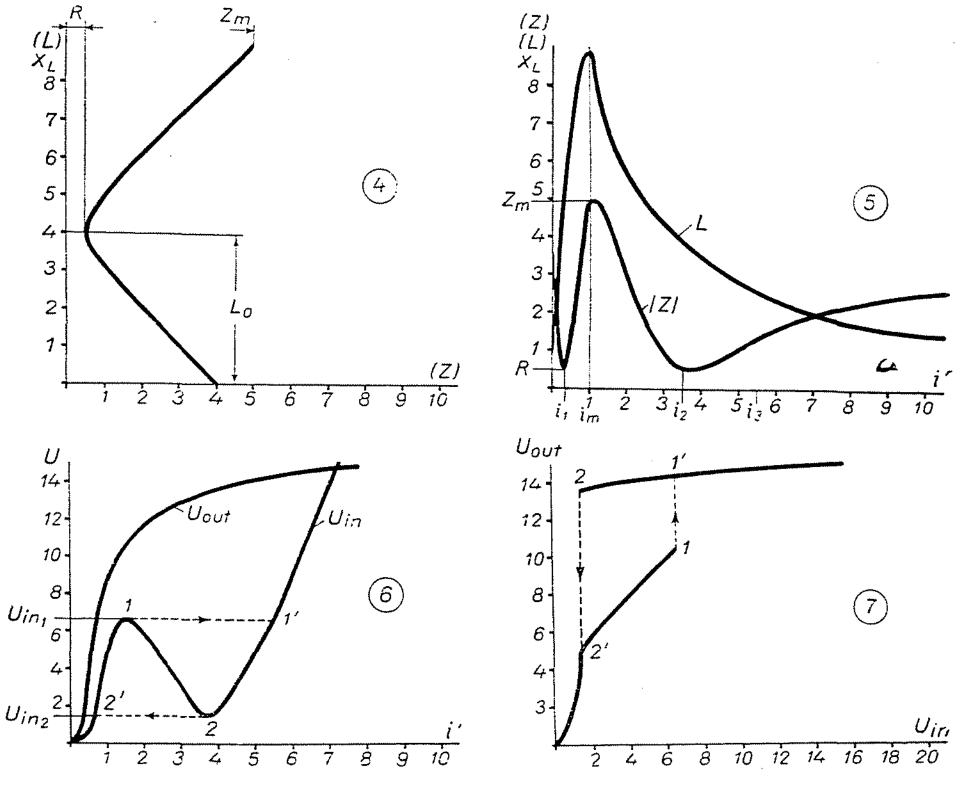

Fig. 4—7

As it can be seen there are two currents i_1 and i_2 where the inductance $L = L_0$ that is resonance occurs. Here the absolute value of impedance attains a minimum and is equal to R.

At $i = i_m$ the inductivity L has a maximum value L_m and the impedance is also a maximum Z_m.

Finally, to obtain the relation $U_{out}(U_{in})$ following products are formed

$$U_{in} = i\,|Z|$$

and

$$U_{out} = \omega L i$$

where in the second product R is neglected. These can be seen in Fig. 6 where two curves are plotted

$$U_{\text{in}} = U_{\text{in}}(i) \qquad \text{and} \qquad U_{\text{out}} = U_{\text{out}}(i).$$

The points (1) and (2) on the U_{in} curve will in good approximation belong to i_m and i_2 respectively. Furthermore

$$\begin{array}{llll} \text{if} & i = 0\,, & \text{then} & U_{\text{iu}} = 0\,, \\ & = i_1\,, & & = i_1 R_1 \\ & = i_m\,, & & = i_m Z_m \approx U_{\text{in1}}\,, \\ & = i_2\,, & & = i_2 R \approx U_{\text{in2}}\,, \\ & = i_3\,, & & = U_{\text{in1}}\,, \end{array}$$

from which even i_3 can be obtained.

While in contrast

$$\begin{array}{llll} \text{if} & i = 0\,, & \text{then} & U_{\text{out}} = 0\,, \\ & = i_1\,, & & = \omega L_0 i_1 \; = \dfrac{i_1}{C\omega}\,, \\ & = i_m\,, & & = i_m \omega L_m = U_m\,, \\ & = i_2\,, & & = \omega L_0 i_2 \; = U_{\text{out2}} \\ & = i_3\,, & & = U_{\text{out1}}\,. \end{array}$$

From this data the relation $U_{\text{out}}(U_{\text{in}})$ can be constructed (Fig. 7).

This Figure enables us to evaluate the qualitative behaviour of the circuit. Increasing the input voltage from zero to U_{in1} (point 1) the current, which meanwhile attained the value i_m, suddenly jumps to another much higher value i_3 (point 1′). When we decrease the input voltage untill U_{in2} (point 2) the current which has here an approximate value i_2 suddenly jumps to a much lower value (point 2′).

It can be seen that in the interval $U_{\text{in2}} < U_{\text{in}} < U_{\text{in1}}$ there are two possible current values belonging to every U_{in} value. That range of the curve lying between the points (1—2) has no real meaning because in this interval the impedance is negative and the circuit is unstable. In this interval the two stable current value can be obtained with trigger like rising or decreasing input voltage.

There is also a similar jump in output voltage U_{out} in the (1—1') and (2—2') regions, respectively. When the input voltage approaches the value U_{out} the voltage across the inductivity jumps to U_{out1} and because an augmented current flows we are in the saturation region where the output voltage U_{out} will change only slowly when input voltage U_{in} is changed. If U_{in} decreases until reaches U_{in2}, U_{out} will also decrease until reaches U_{out2} where sudden jump occurs to a much lower value.

Owing to saturation the quotient

$$\delta = \frac{U_{out1} - U_{out2}}{U_{in1} - U_{in2}}$$

can have very low value and so the circuit will operate not only as a bistable nonlinear circuit but also as a simple voltage stabilizator.

The above figures give no direct information about the application of this circuit as a stabilizator in respect of power which it can deliver, therefore the loaded condition will also be considered.

4. The loaded series resonance circuit

Fig. 8 represents the loaded condition. In this figure a resistor r is connected across the inductivity as a shunting element. As compared to the real load this represents a simplification yet we will confine ourselves to pure ohmic load.

In order to use the graphical construction the load which is connected in parallel will be transformed in a series impedance (Fig. 9).

In this conversion let be

$$Z_1 = Z_1', \; Z_2 = Z_2' \text{ and } i_1 = i'$$

so the voltage across the inductivity U_{out} will be the same in both cases.

It is very easy to show, that

$$Z_3' = \frac{Z_1 Z_2}{Z_s} \; .$$

Since

$$|Z_1| = \frac{1}{C\omega}, \; Z_2 = r$$

and

$$|Z_2| = L\omega ,$$

supposing

$$R \ll L\omega ,$$

the substitution series resistance Z_3' which will be denoted by R_s

$$R_s = \frac{L}{C \cdot r} \, .$$

Naturally, since

$$L = L(i),$$

the substitution resistance

$$R_s = R_s(i)$$

will also be a function of current.

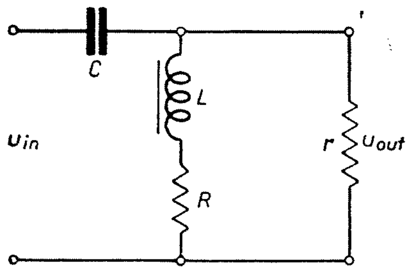

Fig. 8

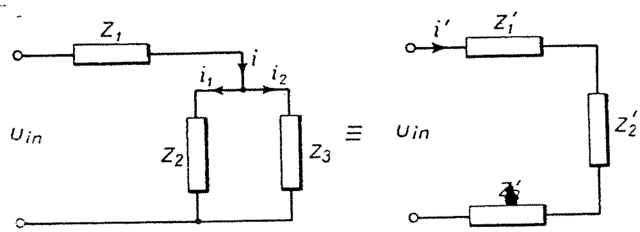

Fig. 9

As we shall see the value of the substitution resistance is important chiefly at the current i_2 where jumping occurs so as a first approximation L can be regarded to be constant and equal to L_0.

So at least approximately :

$$R_s \approx \frac{L_0}{C \cdot r} .$$

Figs. 10—13 show the influence of loading. It can be seen that with rising R_s the curve $U_{in}(i)$ straightens and finally the two stable state, and the negative impedance disappear.

Naturally, this impairs the stabilization and the load will throw out the circuit from its state of higher output voltage.

The former graphical construction can be used in this case, too, if the loss resistance R would be increased by the substitution resistance R_s of the load.

So we can say quantitatively that the load can be only so large that the range of the curve $U_{in}(i)$ between (1—2) would be just horizontal.

This is accomplished when

$$(R_s + R)\, i_2 = U_{\mathrm{in}_1} .$$

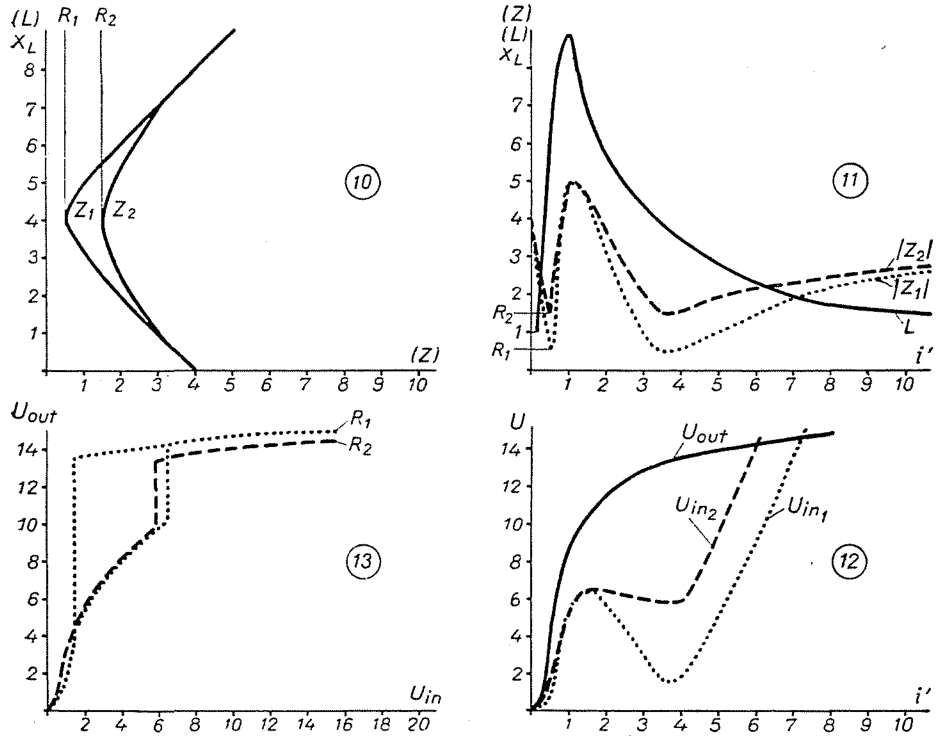

Fig. 10—13

Taking into account that

$$U_{\mathrm{in}_1} \approx i_m Z_m ,$$

so

$$R_0 = R + R_s = Z_m \frac{i_m}{i_2} .$$

If $R_s \gg R$, then $R_0 = R_s$ and the value of the loading resistance

$$r = \frac{L_0}{CR_s} \approx \frac{L_0\, i_2}{CZ_m\, i_m} \approx \frac{1}{C\omega}\, \frac{i_2}{i_m}\, \frac{L_0}{L_m} .$$

So the load carrying capacity at a given inductivity can be increased by the increase of the capacitor C. This expression will throw some light upon the desirable ferromagnetic characteristics. It is advantageous to use such iron cores which give a steeper slope and a greater drop on the curve $L(i)$ between the maximum and L_0.

5. Design of loaded series resonance circuits

At the design following data are generally given :

a) U_{in} the input voltage,

b) $\pm \Delta U_{in}$ permissible variation of input voltage,

c) W the desired power (ohmic load),

d) $L = L(i)$ function (e. g. so that the data of measurements performed on a coil with given cross section and number of turns is given at a particular angular frequency),

e) ω the angular frequency.

The data to be computed are :

a) q the final cross section,

b) N the final number of turns,

c) C the necessary capacitor,

d) R the permissible loss-resistance (this is composed of the copper losses of the coil and of iron losses),

e) δ stabilization factor.

As we have seen before :

$$r \approx \frac{1}{C\omega} \frac{i_2}{i_m} \frac{L_0}{L_m} = \frac{1}{C\omega} \frac{i_2 L_0 \omega}{i_m L_m \omega}$$

$$\approx \frac{1}{\omega C} \frac{U_{out}}{U_m},$$

where

$$U_{out} \approx U_{out_1} \approx U_{out_2}$$

and

$$U_m = i_m L_m \omega ,$$

so

$$W = \frac{U^2_{out}}{r} = C\omega U_m \cdot U_{out}$$

$$= C\omega L_0 \omega i_2 U_m ,$$

therefore

$$i_2 = \frac{W}{U_m} .$$

Since the curve $L = L(i)$ is given, U_m accross the given inductivity can be computed.

Using L_0, which belongs to the computed current i_2 we have the capacity

$$C = \frac{1}{\omega^2 L_0} .$$

and the output voltage

$$U_{out} \approx U_{out_2} = i_2 \omega L_0 .$$

If this capacity cannot be easily realized we can choose an other more realizable one. Let us denote the value of this new capacity C' then the new value of inductivity is given by

$$L_0' = \frac{1}{\omega^2 C'} .$$

If the number of turns belonging to L_0 is N then the required number of turns for L_0' is

$$N' = N \sqrt{\frac{L_0'}{L_0}}$$

since at constant cross section the inductivity varies with the square of the number of turns.

Naturally, now the value of i_m and i_2 will also change. The currents belonging to the new inductivity are

$$i_m' = i_m \frac{N}{N'} \quad \text{and} \quad i_2' = i_2 \frac{N}{N'} ,$$

respectively.

The voltages will change, too, namely

$$U_m' = U_m \frac{N'}{N}$$

and

$$U_{\text{out}}' = U_{\text{out}} \frac{N'}{N} .$$

Since

$$W = i_2 U_m = i_2' U_m' ,$$

the output power is constant.

At a given loss-resistance i_2 can not have any value because when

$$i_2 R = U_m ,$$

the circuit can not be loaded further. Therefore in practice it is advisable to limit the loss-resistance as follows

$$R \leq \frac{U_m}{10 i_2} .$$

Moreover, it is important that the minimum value of the input voltage should be greater than the turnover voltage

$$|U_{\text{in}} - \Delta U_{\text{in}}| \geq U_{\text{in}1} \approx U_m .$$

Finally, let us suppose that we have computed an inductivity L_0 and a capacity C for a given load W (the function $L = L(i)$ was measured with a given coil of cross section q and number of turns N). In this case the design procedure is as follows.

a) The cross section is given by the fact that the losses of the induction coil must not be greater than 0,1 W which fact sets a limit to the weight of iron in consequence of iron losses. With the so computed cross section we procede in a tentative calculation.

b) If we know the total and iron losses the copper losses can be calculated which in turn determins the number of turns which can be wound on the iron core and the diameter of the wire, too.

Let be the so computed cross section q',

the number of turns N',

and introduceing the following quotients

$$\nu = \frac{N'}{N},$$

$$\varkappa = \frac{q'}{q},$$

the obtainable inductance in the resonance point

$$L_0' = L_0 \varkappa \nu^2 .$$

c) The required capacity

$$C' = \frac{1}{\varkappa \nu^2} C .$$

The currents are

$$i_2' = \frac{i_2'}{\nu}, \quad i_m' = \frac{i_m}{\nu} .$$

The voltages

$$U_m' = U_m \varkappa \nu , \quad U_{\text{out}}' = U_{\text{out}} \varkappa \nu .$$

If the so computed values are not suitable we must choose another type of iron core which has smaller losses, or has different dimensions.

d) The stabilization factor δ is determined graphically.

6. Remarks

Finally, we have to examine the approximations used. The calculations were made with mean values. The most important approximations were the followings :

a) It is supposed that the points of overthrow (1) and (2) are at i_m and i_2, respectively. This approximation for i_2 is quite good, for i_m it is not so. In reality the point (1) appears at a greater current and so the overthrow voltage which can be calculated by the equation as

$$U_{in1} = i_m Z_m$$

is lesser than in reality.

b) When calculating the substitution resistance of the lood the inductivity was taken as constant, L_0. At the same time when we computed r instead of Z_m we took a greater value $L_m \omega$ which means a greater lood.

However, by the introduction of the voltage U_m the approximation in point (a) is partly compensated because

$$U_m = i_m \omega L_m > U_{in1}$$

As the current of the induction coil flows through the capacitor, the capacitor must withstand this current and also the voltage drop required accross.

If the output voltage U_{out} is not a suitable one a transformer between the load and the output terminals will serve. Practically, the inductivity may be the primary of this transformer. In this case, the iron cross-section is given merely by the winding space requirements and one has to choose an iron core in wich the losses are low enough.

A few such circuits have been accomplished and there are good agreement between the data received by graphical construction, computation and measurements respectively ($\sim$5%). With a 25 watt load in a comparatively small volume the stabilization factor attained was $\delta \sim 0{,}2$.

7. Acknowledgements

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References

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Summary

The series resonance circuit with ferromagnetic core is treated. It is shown that such a circuit can be used not only as a nonlinear network with two stable states, but also as a simple *ac* stabilizator. Design procedures are given by the aid of a graphical construction for the unloaded as well as for the loaded cases.

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