# WIDE-ANGLE IMAGE FORMING SYSTEMS

(DISTORTION-FREE PANORAMIC PROJECTION)

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(Received March 28, 1957)

## Introduction

The problem of a large field of vision or a wide image angle has always played an important part in the development of optical instruments. Wide-angle objectives are being used in a certain special class of photography only, hence, less attention is paid to their improvement as is the case with telescopes where a large field of vision is still a critical factor, particularly for opera glasses, field glasses, etc. However, considerable difficulties have been encountered in connection with the lens corrections required for large relative apertures.

It has been attempted, in the course of time, to realize in practice all the suggestions put forward by those skilled in the art, and to apply in the design of both telescopes and photographic objectives all available technical improvements. The author's aim was to give a brief chronological summary of only the most important types of instruments.

Wide-angle projection has recently come into the foreground once more. This has induced the author to set forth some ideas with respect to wide-angle and panoramic projection in connection with the phenomenon of after-images in human vision.

## I. General

Optical systems forming real images may be grouped into two categories, when considered from the viewpoint of the path of rays. In the case of image formed by means of photographic lenses the path of rays is discontinuous, whereas it is continuous for telescopes and microscopes.

Images formed by photolenses may be received on various kinds of screens, such as ground glass plates, or photosensitive layers, etc. Although the image is viewed on the ground glass plate, it is the photographic procedure that will fix the image. The photographic picture representing the scene as viewed in the moment of exposure is, of course, permament and available at any time. Incident rays are more or less dispersed by the grainy surface of the ground glass so that the image is produced by means of dispersed radiation. Actually, one may call the procedure one of double image formation, as the crystalline

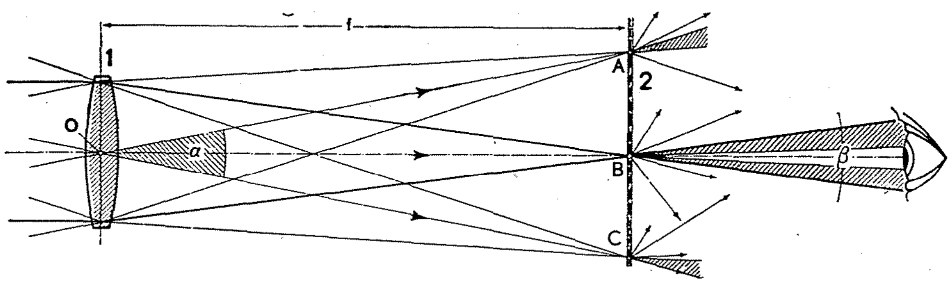



*Fig. 1.* Intermittent path of ray, illustrating the principle underlying photographic cameras. The image becomes visible to the eye, the radiation being dispersed by the opaque screen. The path of rays and the field of image are indefinite.

lens reproduces a second retinal image from the one originally formed by the photolens and received on the screen (Fig. 1). Only a portion of the dispersed radiation issuing from the image points A, B and C formed by the chief rays indicated by arrows reaches eye 3. Thus, in order to cover the entire image formed on a large-size ground glass plate not only the eye but also the head must be turned. Only the central part of the image can be seen distinctly. When observing the image on the glass plate, the *objective* image is produced by objective lens 1, and the *subjective* image by the crystalline lens. The ground glass plate plays no part in the latter procedure, hence the path of rays and, in consequence, image formation, is not continuous but intermittent. Both these images are real ones. The angle $\alpha$ enclosed by the straight lines connecting the centre O (that is, the posterior principal point) of lens 1 with points A and B at the margin of the picture but still included in the perception range of the screen, is called *image angle* of the lens for photographic cameras, and *field of vision* for telescopes and microscopes.

If the image received on the ground glass plate of the camera is viewed through magnifying glass 3 placed behind it (Fig. 2) while removing the glass plate from the path of rays, one finds that the picture, instead of disappearing,

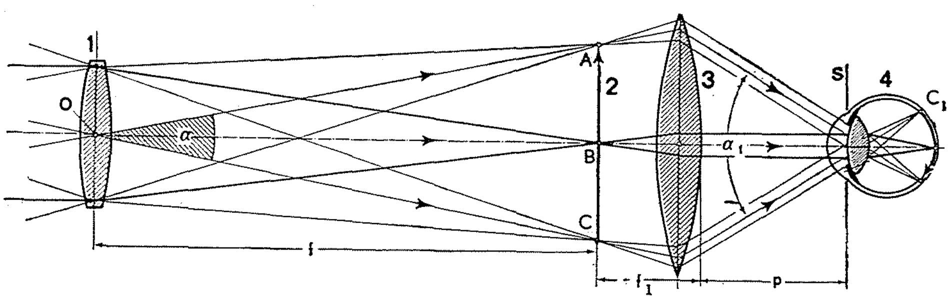



*Fig. 2.* Continuous path of rays. The principle underlying telescopes, microscopes and most optical instruments. The image is produced by the objective in the aperture of the screen limiting the field of vision. The path of rays and the field of vision behind the image are definite, that is, directed

remains visible. Thus the camera has been converted into a telescope, with photolens 1 acting as objective lens, and magnifying glass 3 acting as eye-piece. The pencils of rays are directed by the lenses, in strict conformity with the laws of optics, to eye 4 on whose retina the subjective image $A_1$, $C_1$ is formed. In this type of system the image is produced by *directed* beams, and not by dispersed ones. The telescopic image can, of course, be seen only as long as one looks into the telescope. In general, the image $ABC$ cannot be reproduced ; nevertheless, it is possible, though not usual, to photograph the said image.

The angle of vision $\alpha$ of the image formed by objective lens 1 (Fig. 2) is called *real* field of vision, expressed in terms of angular measure. This image, subject to the magnifying power of the telescope, is seen through magnifying glass 3 at an angle $\alpha_1$, representing the *virtual* field of vision of the telescope. Objective lens 1, eye-piece 3 and crystalline lens of eye 4 make a complete optical system. The design of such an instrument must, therefore, take into account the optical properties of the eye as well as psychological factors. The path of rays suffers no interruption in its passage from the objective lens to the retinal image, hence, image formation is *continuous*. Consequently, the individual elements for image formation, such as lenses, spherical mirrors, etc. cannot be positioned at will. The same applies to the visual distance $p$ (representing the distance between the eye and the eye-piece), it being imperial for obtaining a full panoramic view of the complete field of vision, as well as for exploiting the luminous intensity of the optical system that the eye be placed to the intersection of the beams emerging from the system at the smallest section point, that is to plane $S$ of the exit pupil. Optically, this means that the exit pupil of the system must coincide with the entry pupil of the eye. As a result of the aberrations in image formation of the lenses, the exit pupil is, in general, not a plane surface but a surface of higher order, with a maximum luminous intensity in the centre, gradually fading off toward the margins. This phenomenon may well be observed in connection with common opera glasses.

Obviously, telescopic observation is a direct process dependent upon time, whereas photographic representation is independent of time. Photography produces indirectly visible pictures, since the so-called *latent* image generated on the negative by radiation can only be made visible by inserting a separate step, that is, by development into a negative picture, and then by a subsequent positive procedure. The negative picture permits making as many copies as necessary. Any period of time may lapse between exposure, development and copying. A telescopic image, on the other hand, can only be seen while looking into the telescope. Perception and generation of the image take place simultanously while the instrument is placed in front of the eye. In other words, a telescope only performs its function when placed before the eye. The same applies to magnifying glasses, microscopes, and other optical instruments subject to similar principles of optics.

Let us remove glass plate 3 from the path of ray (Fig. 1) while observing the image. Image *ABC* will continue to be seen, unless accommodation of the eye to the image distance in question is changed. The ability of maintaining accommodation may easily be acquired by some practice. In this case, eye 3 accommodated to point *B* will perceive only the pencils of rays of an angular field $\beta$ whereas further pencils, indicated in the figure by dotted lines, intersecting one another at the other points and travelling towards the image plane, do not even reach the pupil of the eye.

Examination of Figs. 1 and 2 will facilitate distinction between continuous and discontinuous paths of rays. In the case of Fig. 1 only a small portion of the "directed" rays can reach the eye. Hence, it is impossible for the stationary eye to command the view of the entire image, whereas in the case illustrated in Fig. 2 all the pencils received by eye-piece 3 travel towards the eye. Thus the continuous path of rays secures simultaneous observation of the entire picture.

In accordance with the wave propagation of radiating energy, the process of representation (image formation) is similar for all optical systems. The differences are restricted to the path of rays, and are noticeable in connection with observation only.

The objective and subjective efficiency of any optical systems is determined by

1. magnification
2. field of vision
3. luminous intensity
4. resolving power
5. contrasts
6. psychological factors.

All these are closely interrelated in accordance with the laws of geometrical optics as well as those of the wave theory of radiation.

In the following observations we will confine ourselves to the investigation of the *visual field* in order to ascertain what may be expected from the latest developments in optical systems and their application in practice in the various spheres of use. This category of apparatus comprises the various types of projectors suited for displaying to a number of observers the picture obtained by means of either photographic cameras, microscopes, or telescopes. In this field the so-called cinematoscopic process meant an important step forward, whereas the problem of large-surface or wide-angle projection including the extreme case of panoramic (180°) projection field by optical and mechanical means still remains to be solved. The author will attempt to find the answer by taking into account the psychological reactions of the eye, too.

In our investigations, we shall first be concerned with photographic cameras, then with telescopes, and, finally, with apparatus comprising both kinds of systems, from optomechanical as well as historical point of view.

## II. Wide-angle photographic cameras

Leonardo da Vinci invented the *pinhole camera* or camera obscura, a simple device by which, in principle, images of up to 180° angles may be produced without the use of optical elements such as lenses and spherical mirrors. Here, the basis of image formation is the phenomenon of interference of radiating energy, the image point being the *Airy disc* possessing a certain diameter, thus being mensurable. This phenomenon may be explained by the *Huyghens* elementary spherical waves. The pinhole camera is the only optical device working by the principle of wave theory, as it produces a mosaic-like image consisting of Airy discs. The diameter of the aperture of the pinhole camera is dependent on the image distance, that is the distance between aperture and image. For a pinhole camera adjusted to infinity, image distance may be termed focal length, a designation used in connection with lenses. The smaller the diameter of the aperture, the shorter the image distance and vice-versa. However, owing to refraction apt to impair good definition, the diameter of the aperture must not be reduced beyond a certain limit. The smaller the image distance the larger the object space to be covered, in other words, the image angle. Theoretically,

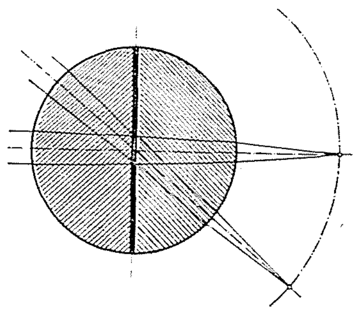


*Fig. 3.* A simple lens comprising two cemented hemispheres, with a screen of permanent aperture in the centre

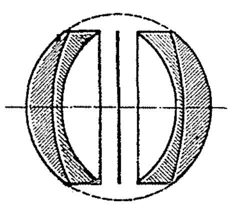


*Fig. 4.* The *Harrison* lens. A wide-angle photographic lens of permanent relative aperture (1860)

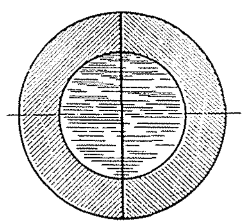


*Fig. 5.* The *Sutton* lens. A wide-angle spherical shell lens of permanent relative aperture, filled with water

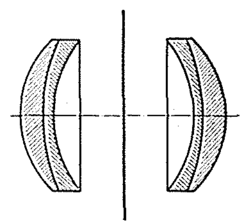


*Fig. 6.* The *Steinheil* lens. A wide-angle aplanat (1860)

it would be possible to increase the said angle up to 180°, if it were not for technical difficulties.

In view of the wave theory image distance for a given diameter of aperture and wave length is

$$k = \frac{R^2}{\lambda}$$

where $K$ represents the image distance, $R$ the radius of the aperture and $\lambda$ the wave length of radiating energy in m$\mu$. According to this formula, every wave length has a corresponding image distance, so that a picture taken by means of colourless radiation of various wave lengths cannot be sufficiently acute.

The major conclusion to be drawn from the above formula is that it is possible to compute image distances for apertures of any diameter size. Let the diameter be 60 mm ($R = 30$) and the wave length of greenish-yellow radiation, best perceived by the eye, be $\lambda = 10^{-6}$. 555 m, the necessary image distance will be

$$k = \frac{R^2}{\lambda} = \frac{30^2}{10^{-6} 555} = 1{,}621 \text{ km}.$$

Thus, the image distance obtained even for such a small aperture is very inconvenient, not to mention the extremely poor luminous intensity. This drawback may be obviated by introducing some optical elements, such as a lens, a spherical mirror, or a mirror glass in order to shorten the image distance, thereby increasing luminous intensity. Thus it is not the lens but the aperture that matters for image-forming devices, the lens being used to reduce inconvenient image distances or to increase luminous intensity to the required extent, respectively. Aware of these two factors, researchers have long been concerned with the idea of producing photographic lenses of large image angles coupled with a convenient luminous intensity.

The first embodiment of this idea was a 19th century spherical lens of the aplanatic type, consisting of two hemispheres and separated by a diaphragm. If the parallel pencils of rays incident at different angles are so directed that their optical axes intersect the centre of the diaphragm, the pencils of rays will travel through the lens without refraction. The image thus obtained is on a spherical surface, of a radius $f$. (Fig. 3)

The 1860 HARISSON-lens (Fig. 4) consists of two opposite, symmetrical achromatic doublet lenses of steep curvature. The outer surface of the two symmetrical members may be covered by a spherical surface, with the diaphragm in between. This lens arrangement has a relative aperture of 1 : 36 and gives fairly acute photos of a 90° angle. Owing to spherical aberration, caused by strong curvature, achromatization of the system is rather difficult so that the stop number must be quite large.

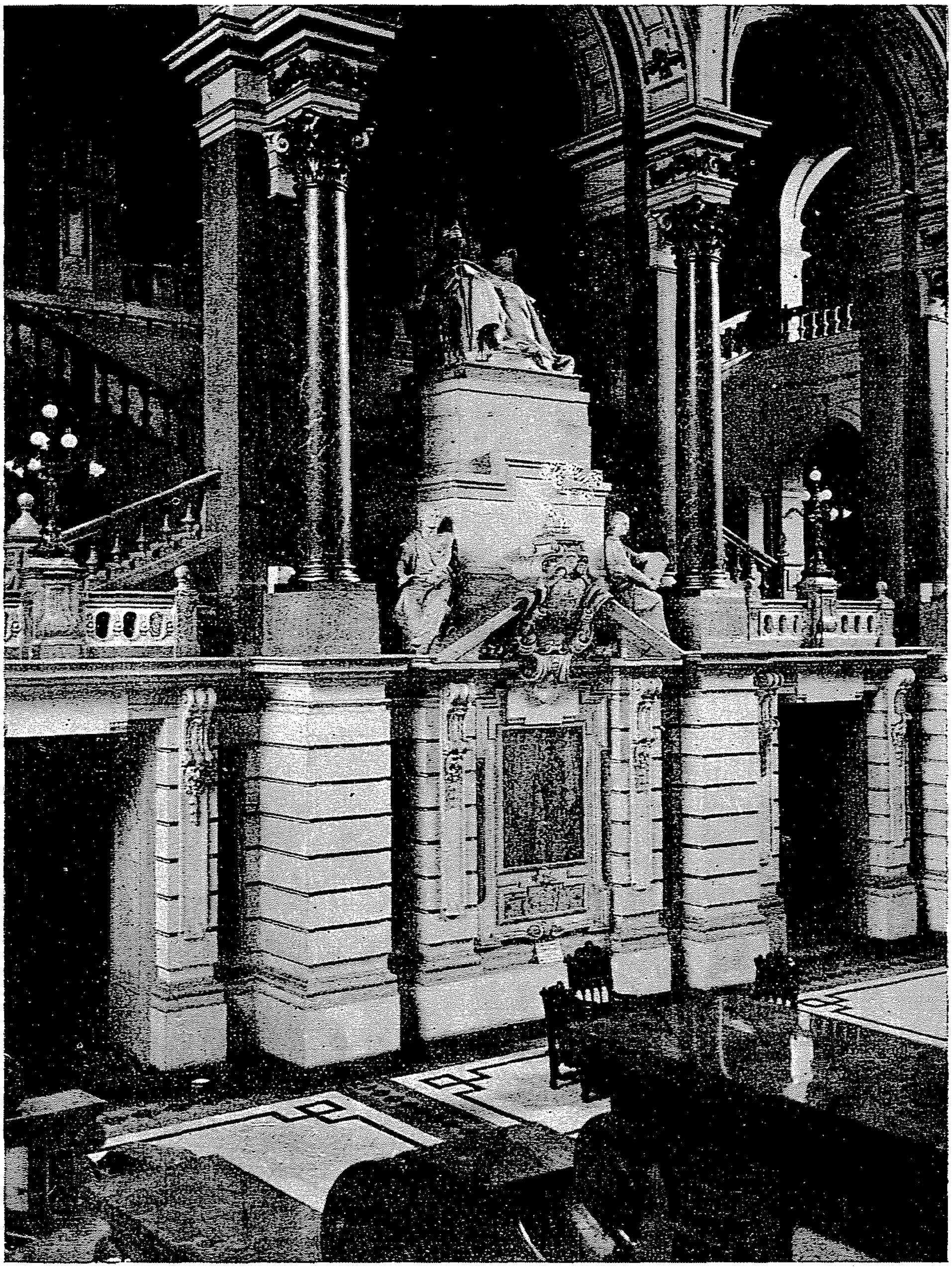


*Fig. 7.* Photography taken of the interior of a large hall. Exposure : 4 minutes (at noon in June). Relative aperture 1 : 22

Let us mention here *Sutton*'s spherical lens (Fig. 5), a system of a certain historical interest, composed of two cemented spherical shells and filled with water.

The STEINHEIL wide-angle aplanatic lens (Fig. 6) dates back to 1866. It is a four-component lens of two meniscus-shaped symmetrical doublets separated by a diaphragm. The Steinheil lens eliminates coma, distortion and chromatic differences in magnification. It has an almost 100° field angle for a relative aperture of 1 : 7 and *f*/9 cm. The picture shown in Fig. 7 was taken with a Steinheil lens.

Wide-angle lenses produce pictures of extremely marked perspective, that is, close objects appear to be very large in proportion to those in the background,

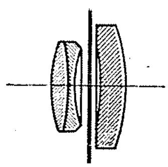


*Fig. 8.* The *Busch* Pentagonal, a wide-angle lens (1903)

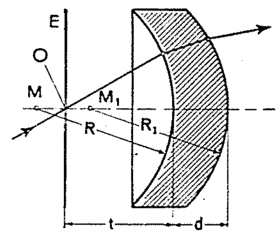



*Fig. 9.* The principle of an aplanat meniscus by *E. v. Hoegh* (1900)

and straight lines are highly convergent. Such lenses are well suited for taking pictures of monumental effect, in addition to interiors.

The BUSCH "Pentagonal" lens seen in Fig. 8 was designed in 1903. It has a 120° angle for a maximum relative aperture of 1 : 18, if the slight loss of acuity at the margins is neglected.

The 1900 HOEGH *aplanatic meniscus* is of scientific interest. This lens formed the basis for the GOERTZ "*Hypergon*", a lens of the widest angle so far known. This system was born under circumstances worthy of note. In the 1900's there was a heated scientific argument going on between Dr. PAUL RUDOLPH of the *Jena Zeiss Works* and EMIL VON HOEGH of *Goertz*, Friedenau, in connection with the elimination of image curvature and astigmatism. Contrary to RUDOLPH' views, HOEGH based his assumption on the PETZVAL-principle.[1]

The method of correction of curvature for photographic systems is set by the PETZVAL formula, the practical application of which was proved by von HOEGH using a simple lens of zero curvature (Fig. 9), bounded by two equal surfaces of radius $R$. If the lens has a thickness $d$ and a refractive index $n$, then

$$f = \frac{n}{d} \frac{R^2}{[n - 1]^2} .$$

[1] Archiv f. wissensch. Photographie II., 1900.

Owing to the magnitude of $R^2$, $f$ is always positive, and inversely proportional to thickness $d$. In practice, this type of lens may always be constructed with a rather short focal length, without having to increase considerably its thickness. Taking a lens of $R = 12$ mm, $f = 100$ and $n = 1{,}6$, then

$$d = \frac{nR^2}{f[n-1]^2} = 6{,}4$$

Von Hoegh has proved that this type of lens, if used in combination with a front-stop, produces anastigmatic and plane images, similarly to landscape-lenses. Fig. 9 shows the two surfaces of a radius of curvature $R = R_1$, the centres of curvature being $M$ and $M_1$. The lens has a thickness $d$, and the stop $E$ is at a distance $t$ from the lens. The original data stated by von HOEGH are

$$\begin{aligned} R &= 11{,}993 \\ R_1 &= 11{,}900 \\ t &= 7{,}1212 \\ d &= 6{,}0903 \end{aligned}$$

Thus, the radii of the meniscus lens are almost equal, and $f = 100$.

In accordance with the PETZVAL formula, this lens has a radius of infinite length, thus it produces a plane image, as the reciprocal value of the radius of curvature for plane images is for any lens represented by the formula

$$\frac{n-1}{n}\left[\frac{1}{R} - \frac{1}{R_1}\right]$$

The pencil of infinitesimal diameter passing through the axial centre $O$ of the screen $E$ is refracted to the right by the front surface. Subsequently, the pencil becomes divergent, so that the astigmatic image points produced by refraction are virtual. The back surface further refracts the pencil to the right, the angle of incidence being positioned opposite to the perpendicular of incidence. Following the last refraction, the emerging beams become convergent, forming almost coincident real astigmatic image points. Such beams may be called homocentric. Applying the above consideration, freedom from astigmatism as well as from curvature may be obtained, since the image point formed by the homocentric beam falls just into the focal plane. Taking an angle $\alpha = 30°$, astigmatic image surfaces will not diverge from the focal plane by more than 0,25 mm, and the distance of astigmatic image points, also called astigmatic difference, will fall off to zero. Sagittal and tangential surfaces retreat from the image plane in opposed directions.

VON HOEGH maintains that for every anastigmatic system the image may be made plane by conducting oblique incident main pencils through the system with at least two refractions of similar direction, one of the two refracting surfaces causing the pencil to diverge, and the other to converge.

For the application of the HOEGH formula, however, the "freedom" implied by the *Petzval* formula must be taken into account, that is, its application must have quite distinct limits, particularly as regards the determination of sagittal and tangential surfaces.

A two-meniscus combination, based on the HOEGH computations, was designed and produced by Messrs. GOERTZ, under the designation *Hypergon*

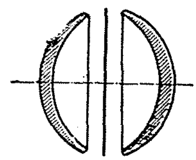


*Fig. 10.* The Hypergon of *E. v. Hoegh.* A wide-angle photographic lens comprising two aplanat menisci, based on the *Goertz* principle

(Fig. 10). This lens suffered a certain set-back in the course of time but, owing to its excellent properties, its production was lately resumed. The Hypergon consists of two similar menisci, arranged symmetrically in relation to the stop, and covers a 140° field.

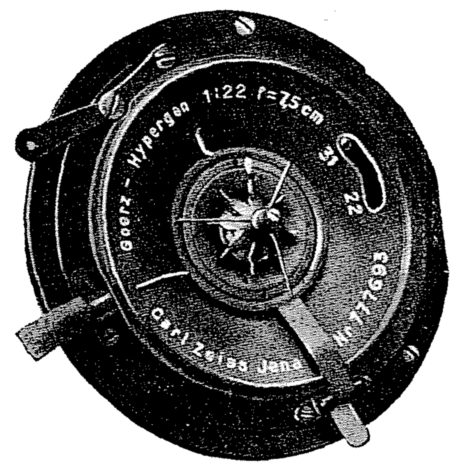



*Fig. 11.* The Hypergon, with the Stoke air-driven starwheel placed before the lens arrangement

The *Voigtländer Collinear* is a wide-angle photolens, a symmetrical lens arrangement with a relative aperture of 1 : 12,5.

It should be noted that in order to cover the entire space of a 12 by 18 cm picture a photolens of 21 cm focal length is generally required. The focal length of commercial camera lenses equals the diagonal of the rectangular negatives.

The beams emerging at wide angles from the lens arrangements described above cause illumination to fall off considerably toward the margin of the picture. Thus, the ratio of illumination between centre and margins is 1 : 8 for the Hypergon. Various methods of correction are known. A front-lens similar to a filter glass, gradually deepening to darkness from centre to the margins (enixsantos-glass) is provided in the *Rodenstock* 120° field *Pentagonal.*

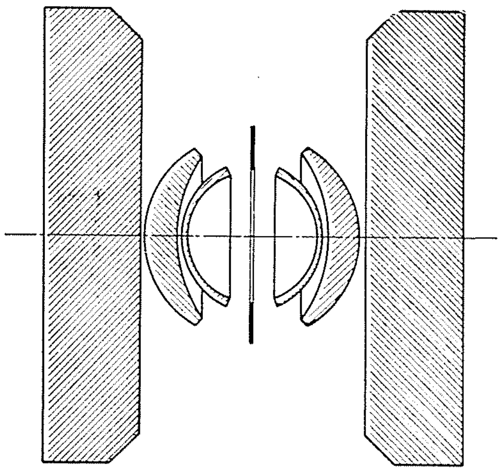


*Fig. 12.* The Topogon of Zeiss, a wide-angle lens for topographic surveying

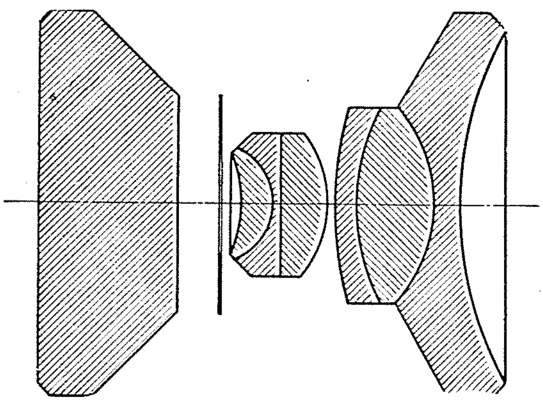


*Fig. 13.* The British Topogon lens

The Hypergon comprises a *Stocke* rotary screen (star wheel) inserted in front of the lens for good illumination (Fig. 11). This screen is rotated by a current of air, generated formerly by a rubber ball, whereas in up-to-date lens arrangements the current of air is generated by an adjustable clock-work automatism. The Hypergon works in two steps. Exposure must be eight times the unscreened exposure time corresponding to the selected stop number, when illuminating the margins of the negative with the rotary screen. The screen is then removed from the lens by means of a spring-operated arm fixed on the mount, and the normal exposure time as required for the centre of the negative is applied. This two-step operation requires a certain skill but in modern cameras the adjustment of the two exposure times as well as rotation and removal of the screen is effected automatically. The relative aperture is 1 : 3,1.

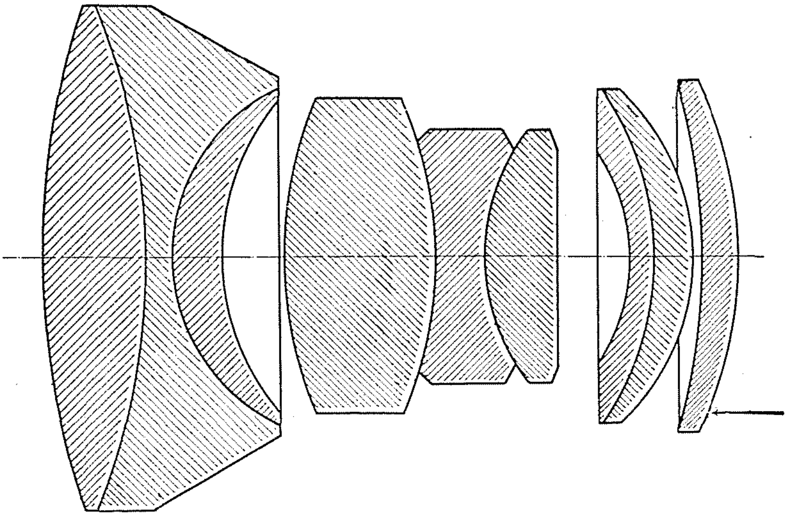


*Fig. 14. L. Berthele*'s Aviotar for topographic surveying (System Wild)

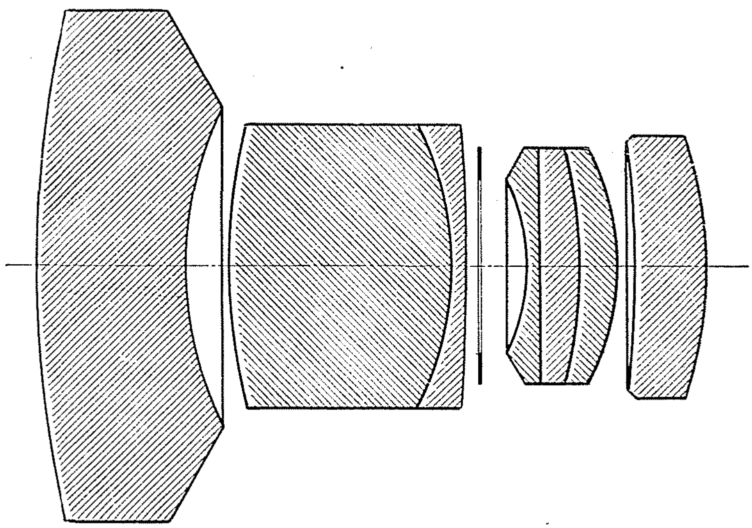


*Fig. 15.* Biogon of *W. Merté.* A wide-angle lens of large relative aperture. (System Zeiss)

The width of the angle, of course, gives rise to a certain amount of distortion at the margins which is often erroneously ascribed to the lens. In fact, this phenomenon is the result of the law of central projection, and has nothing to do with the optical aberrations of the lens arrangement. In any event, it is advisable to avoid taking photographs of arc-shaped, cylindrical or spherical objects, in situated close to the margin.

The relative aperture of wide-angle lenses is generally rather small, yet with proper illumination they allow of snapshots, too. It has been attempted to produce systems of this type with larger relative apertures.

The *Zeiss Topogon* (Fig. 12) used for topographical takings was constructed in 1935. It has a relative aperture of 1 : 6,3, and covers a field of 100°. The meniscus-like opposed lenses are bounded, in front and in the rear, by plane-parallel glass plates.

The British photolens of the Topogon type represented in Fig. 13 has a relative aperture of 1 : 6, covering again a field of 100°. The 150 Aviotar (*f*/17 and *f*/21 cm) for aerial reconnaissance (Fig. 14) was designed by *Berthele* for Messrs. *Wild.* It covers a field of 60°.

The *Zeiss Biogon f*/35 mm for commercial cameras shown in Fig. 15 belongs to the same category of lenses. It has a relative aperture of 1 : 2,8 and covers a 60° field.

It must be pointed out that, in connection with lens arrangements of large relative aperture but of short focal length, designers encounter serious difficulties when attempting to eliminate aberrations, not to mention heavy weight and high costs of production. It has been suggested to substitute them — if not for the purpose of image formation, but for the projection of light — by combinations of prisms or mirrors, such as the *Fresnel* ring lens arrangement[2] made of pressed glass. It was constructed as long ago as 1820. The central directing lens is surrounded by ring lenses of different curvatures and ring prisms of different refractive angles. Radiation is parallelized partly by refraction partly by total reflection. The Fresnel lens was originally built for lighthouse purposes. In recent ring lens systems the radii of curvature change from zone to zone. This system, too, made of pressed glass : the central luminous source is a small incandescent lamp. The system produces practically parallel, soft but intense radiation.

The zonal mirror designed by WEBER for Messrs. I. D. MOLLER secures even more accurate paths of rays. It consists of spherical mirrors of gradually increasing focal lengths fitted into one another. The system works as a mirror, the pencils being reflected by the surfaces. Combining the principles of the *Mangin* and *Schmidt* mirrors, it is used for purposes of projection. It has a 100 mm focal length and a 86 mm diameter, with a nearly 1 : 1 relative aperture.

Speaking of mirrors, mention must be made of the lowest-priced and simples wide-angle photographic device, the common glass ball. The picture is extremely distorted yet the objects can be distinguished, and with some skill, it is possible to correct the distortion. In emergencies, it can be used for the topographical taking of interiors, etc. It has the advantage of freedom from chromatic aberration, the virtual image obtained by reflection.

[2] A. FRESNEL, Projet d'un phare à feux tournants dans lequel réflecteurs seraient remplacés par des lentilles. Oeuvres compl. III., 73—79. Paris. Imprim. imper. 1870.

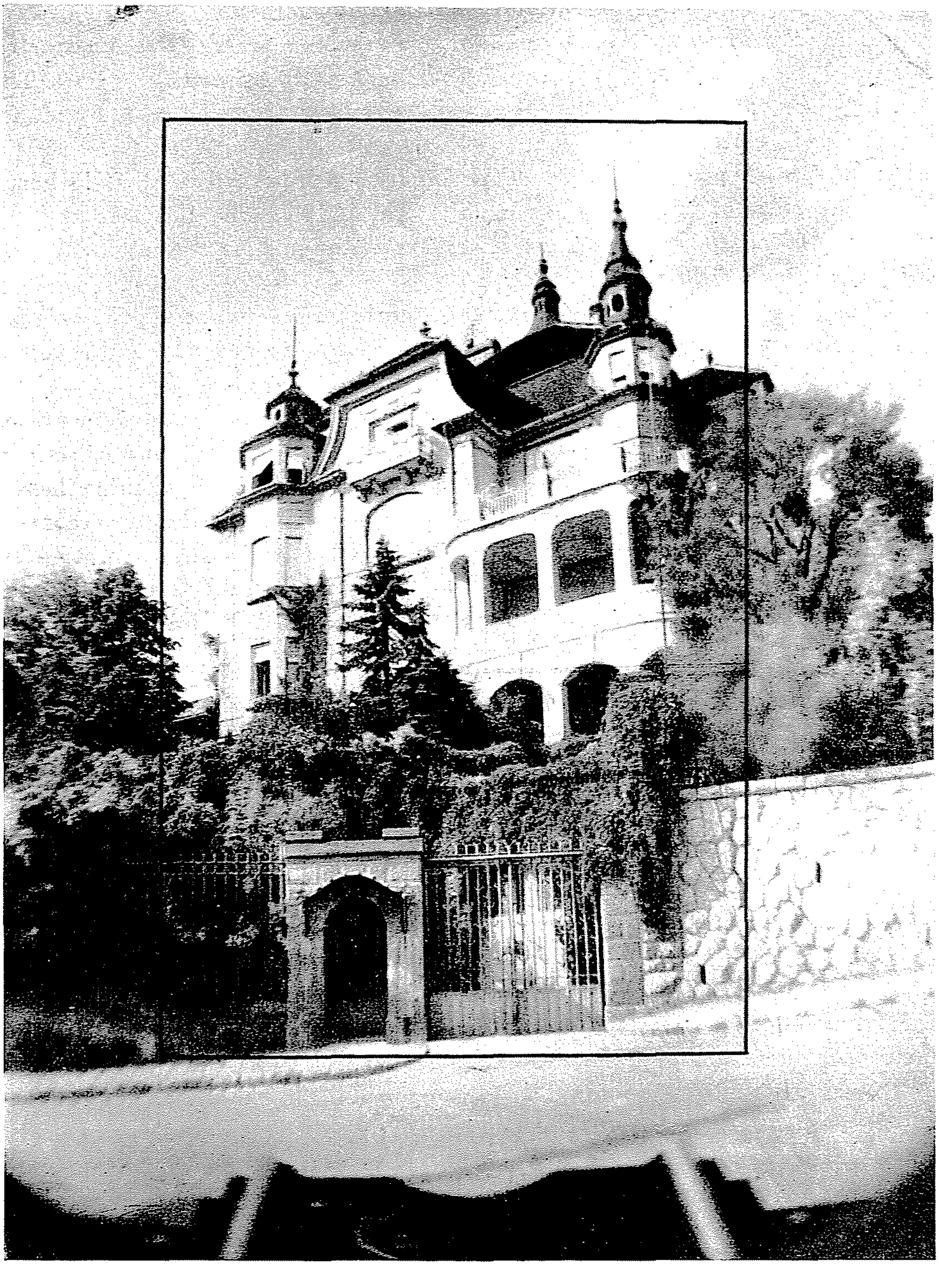


*Fig. 16.* A wide-angle picture taken using the rear element of a Tessar lens. Part of the picture, indicated in the Figure, can be well-used if the picture has been properly screened

Photography is similarly possible by using spherical mirrors of short focal lengths and of large diameters. The image formed by parabolic mirrors is real and, after magnification, the central part of the picture can be well-used. The disadvantage of both spherical and concave mirrors is, in addition to distortion, that part of the camera will unavoidably appear in the picture, covering a certain portion of the objects to be photographed.

In the following a practical method is described for transforming the conventional triplet lens arrangements of cameras into wide-angle systems.

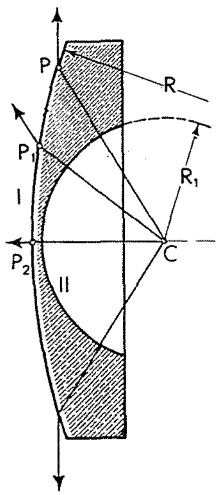



*Fig. 17.* The *Hill* panoramic front lens arrangement with inverted path of rays.

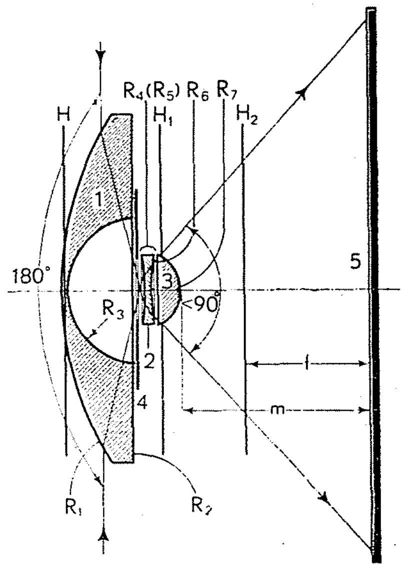



*Fig. 18.* The *Hill* panoramic lens arrangement. Field angle 180°

The method consists in dismounting the first member, usually comprising a positive and a negative element in one mount, and taking the picture with only the two-component cemented achromatic back lens. This can most conveniently be done with the *Tessar* and *Heliar* type lenses. Such an operation will, naturally, entirely upset the well-balanced correction of the system, thus the back component will be subjected to heavy curvature, chromatic aberration, astigmatism and coma. By adjusting to sharp definition either the centre or the margin, the unadjusted portion will be so blurred as to be hardly distinguished. It is therefore suggested to proceed in the following manner : the camera is adjusted to the position within the limits of dullness where the whole picture will be uniformly dull. This may be corrected by heavy screening, and a yellow filter will free the image from chromatic aberration.

The lens covers a field of 90°, thus the base board of the camera will appear in the picture (Fig. 16).

For a long time, no wide-angle lens was able to compete successfully with the *Hypergon*. Only after 25 years, in 1924, did the ingenious invention of the Englishman ROBIN HILL succeed in increasing the angle of field to 180° and above.[3]

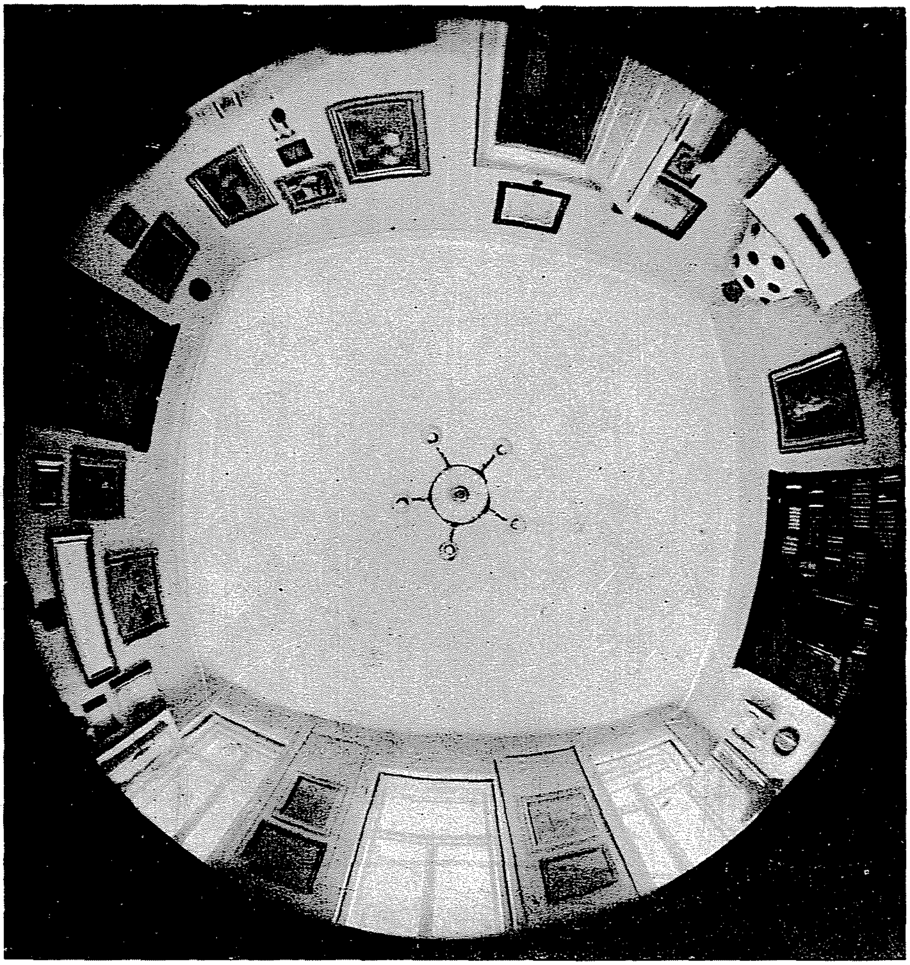


*Fig. 19.* Photography of an interior, taken with the lens arrangement represented in Fig. 18

For purposes of explanation, let us invert the path of rays (Fig. 17). The rays from the curvature centre $C$ of the surface having a radius $R_1$ of the highly dispersive lens 1 pass the back surface without refraction. On emerging at points $P$, $P_1$ of the front surface having a radius $L$, the rays are refracted. It is clear

[3] British patent granted on the 4th December, 1924. Robin Hill "Camera for photographing the whole sky". Quart. Journ. Meteor. Soc. 50 (1924), 227. — Manufactures by R. and Beck Ltd. 69 Mortimer Street, London W. 1.

that the axial ray suffers no refraction when emerging at $P_2$. The extreme emerging rays include a 180° angle. This angle of view may be widened still further, up to 220° by appropriately varying the lens sizes. Thus, a so-called "rear-view" lens is produced. The negative lens will yield only a virtual image, hence, if one

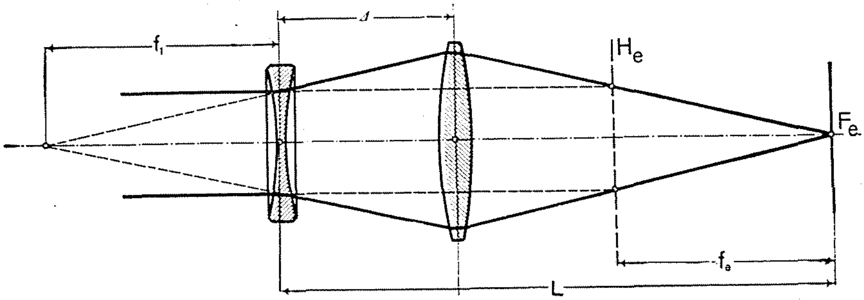



*Fig. 20.* Path of rays in the "inverted" teleobjective. Owing to the resultant principal plane situated at the rear, the mechanical length of the system is greater than the resultant focal length

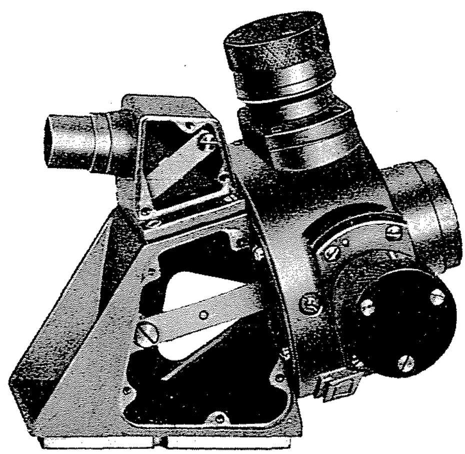


*Fig. 21.* The lens arrangement represented in Fig. 20, applied in a telescope of broken sight line. The resultant principal plane is inside the large-size lower prism

intends to produce real images, it must be combined with a positive system consisting of elements 2 and 3 (Fig. 18). The lens has a negative diameter of 58 mm, and a relative aperture of 1 : 22 will produce a panoramic image covering a 6,5 by 9 cm plate (Fig. 19). The adjustable stop 4 is inserted right behind the dispersing lens 1.

The system has a focal length of 33—46 mm. Reverting to Fig. 18, $H$ is the rear principal plane of the meniscus, $H_1$ the front principal plane of the positive components, $H_2$ the rear principal plane of the whole arrangement. It is from this latter plane that the focal length is computed. Thus, the focal length $f$ for the whole system is shorter than the intercept length $m$, *i. e.*, the distance from the rear surface of the last lens 3 to the negative 5. The arrangement reminds one of an inverted teleobjective, where the resultant focal length is, however, longer than the intercept length, so that, owing to the considerable focal length, the extension of the camera is correspondingly short. In the case of teleobjectives, the negative lens is at a certain distance behind the positive component facing the objects space. Let the negative component be placed *in front of* the positive component ; by intersecting the incident and emergent rays (Fig. 20) the resultant principal plane $H_e$ and the resultant focal length

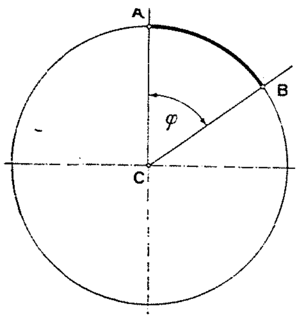



*Fig. 22.* A sphere or part of it, observed from the centre under an angle $\varphi$

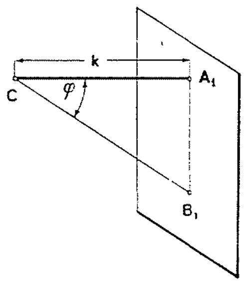



*Fig. 23.* The points represented in Fig. 22 projected on a plane surface, as observed under an angle $\varphi$

$F_e$ are obtained. In that case, as in the *Hill* lens, the resultant focal length $f$ will be shorter than the full length $L$ of the system. The length of the path of rays due to the inverting prism arrangement would prevent the application of wide-angle positive lenses for e. g. telescopes. It has been found possible to increase the mechanical length of the system by drawing the negative lens closer. In prismatic telescopes (Fig. 21) constructed according to this principle both the negative and positive lenses are fixed in a short tube provided on the left hand side on top. Behind it there is a right-angled 45° prism, and under it a penta roof prism constituting the inverting system.

Reverting to the *Hill* lens, it must be stated that the photographs taken with this lens suffer heavy distortion, particularly at the margins. As a method of analyzing the said distortion, it is advisable to investigate the different ways of projecting a sphere or parts thereof on a plane surface (Fig. 22).

Let the observer be placed at the centre $C$ of a circle of radius $R$. He will then view points $A$ and $B$ at an angle $\varphi$. Fig. 23 shows the projections $A_1$ and $B_1$ of the afore-said points $A$ and $B$, seen by the observer from a distance $k$ at an angle $a$. Perspective, in other words representation, would be true if the angular distance of central observation were equal to the angular distance of the projected points, that is,

$$\varphi = a$$

or, accordingly

$$\operatorname{tg} \varphi = \operatorname{tg} \alpha .$$

It is, however, impossible to fulfil this condition for a plane image of finite size. One has to resort to other methods which, although introducing distortion

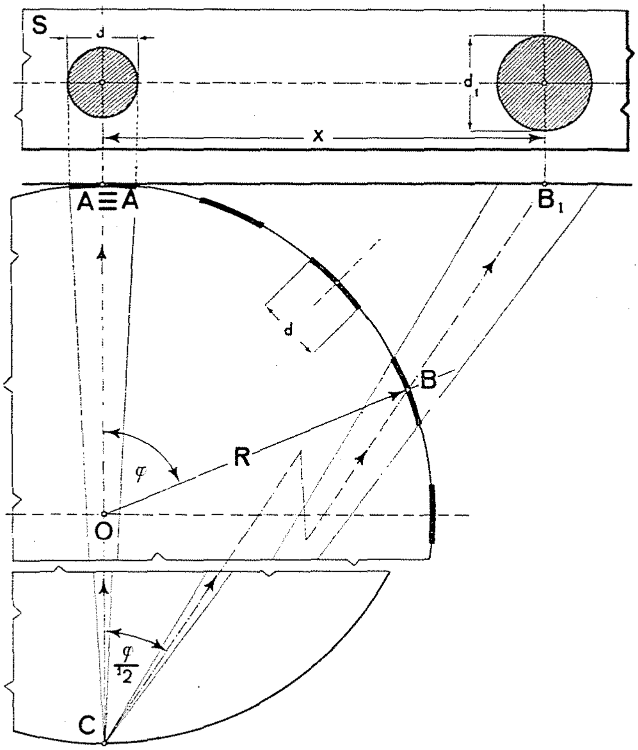



*Fig. 24.* The principle of stereographic projection

of the perspective, allow of the representation of a hemisphere in a finite plane. The following relations are characteristic of the various methods of projection.

1. For stereographic projection :

$$\operatorname{tg} \frac{\varphi}{2} = A \operatorname{tg} \alpha .$$

2. For equidistant projection :

$$\varphi = A \operatorname{tg} \alpha .$$

3. For orthographic projection :

$$\sin \varphi = A \operatorname{tg} \alpha .$$

Viewing the picture from a distance $k$,

$$A_1 B_1 = k \operatorname{tg} \alpha .$$

1. *Stereographic projection.* The image points of the hemisphere (Fig. 24) are projected from point $C$ of the sphere, upon a plane laid across point $A$ and situated opposite to point $C$. Let the radius of the sphere be $R$, then

$$\overline{A_1 B_1} = 2 R \operatorname{tg} \frac{\varphi}{2}$$

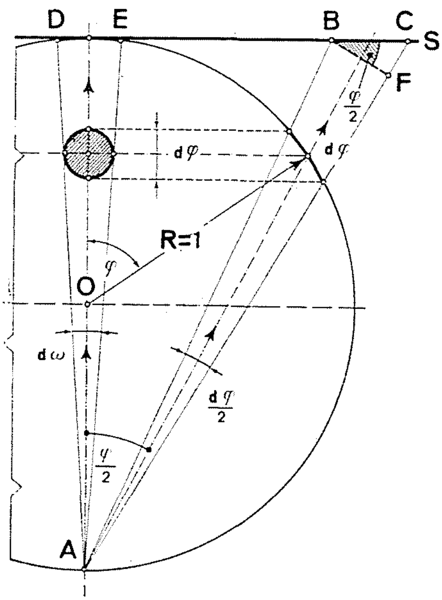



*Fig. 25.* Illustration of the principle represented in Fig. 24, according to which the ratio of distances between the individual projected points remains unchanged while projection is being magnified or lessened

If the image thus obtained is viewed from a distance $k$, then $\overline{A_1 B_2} = 2 R \operatorname{tg} \frac{\varphi}{2} =$

$= k \operatorname{tg} \alpha$ so that actually $\operatorname{tg} \frac{\varphi}{2} = \frac{k}{2R} \operatorname{tg} \alpha = A \operatorname{tg} \alpha$ where $A = \frac{k}{2R}$ .

Constant $A$ means that the ratio of the sizes of the projected points remains unchanged for magnified or diminished projections. When projecting on plane $S$ circles of diameters $d$, lying on the meridians of the sphere, circular and undistorted images will result, and only the size of the diameters $d_1$ (Fig. 25) will change.

$$\frac{d\varphi}{1 + R\cos\varphi} = d\omega \qquad \frac{2R}{AB} = \cos\frac{\varphi}{2}$$

$$DE = 2R\,d\omega \qquad AB = \frac{2R}{\cos\frac{\varphi}{2}}$$

$$DE = 2R\frac{d\varphi}{R[1 + \cos\varphi]} = \frac{2\,d\varphi}{2\cos^2\frac{\varphi}{2}} \qquad BF = \frac{2R}{\cos\frac{\varphi}{2}} \cdot \frac{d\varphi}{2}$$

$$\frac{BF}{BC} = \cos\frac{\varphi}{2}$$

$$BC = \frac{BF}{\cos\frac{\varphi}{2}} = \frac{2R\,d\varphi}{\cos^2\frac{\varphi}{2}\,2}$$

$$DE = \frac{d\varphi}{\cos^2\frac{\varphi}{2}} \qquad BC = \frac{d\varphi}{\cos^2\frac{\varphi}{2}}$$

2. *Equidistant projection* (Fig. 26). The image of each point projected with radius $R_1$ on the tangential plane laid across point $A$ lies in its own meri-

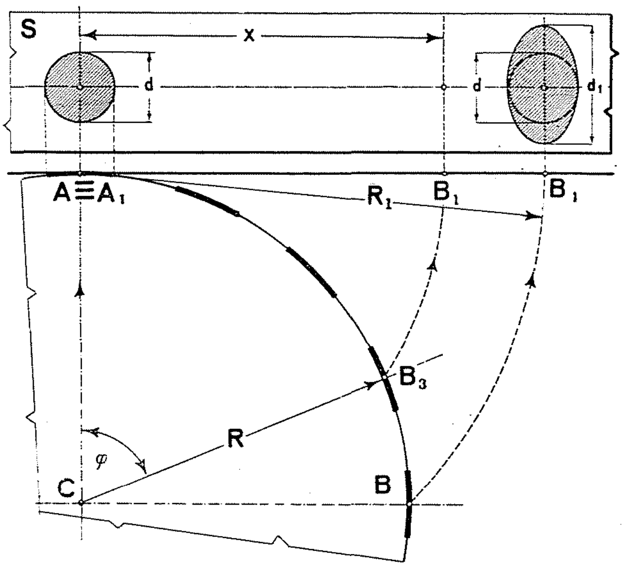



*Fig. 26.* The principle of equidistant projection

dional plane at a distance equal to the spherical distance of the projected point from $A$: $\overline{A_1 B_1} = \widehat{AB}$ In this case $\overline{A_1 B_1} = R\,\varphi$ similarly $\overline{A_1 B_1} = R\,\varphi = k \operatorname{tg} \alpha$ and finally $\varphi = \frac{k}{R} \operatorname{tg} \alpha = A \operatorname{tg} \alpha$.

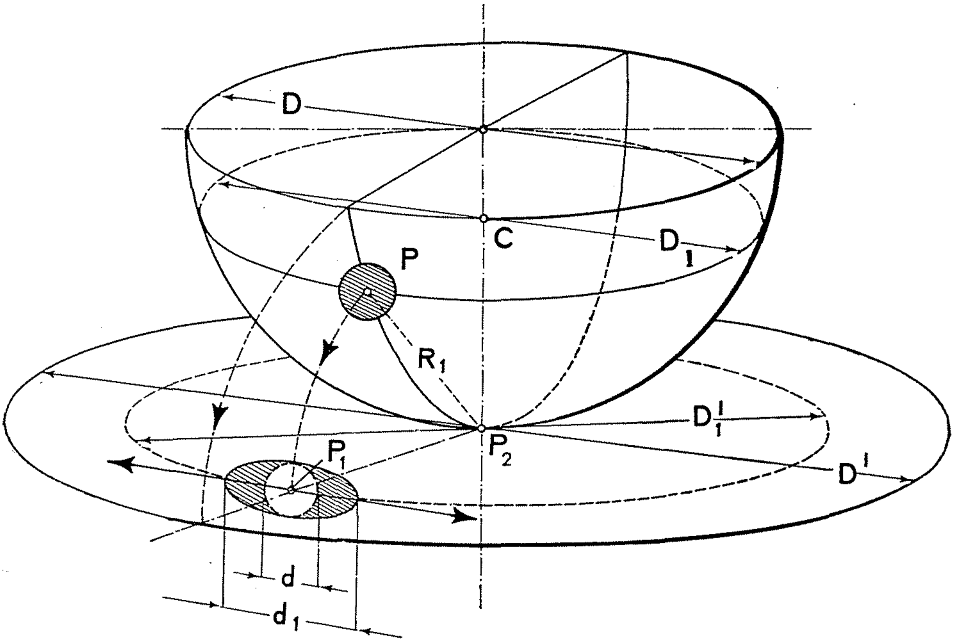



*Fig. 27.* Equidistant projection ; the diameter of circular areas lying in the meridian remains unchanged in the one direction while increasing in the other

Diameter $d$ or the projections of the small circles on the meridian remains unchanged in the one direction, and is increased in the other. The explanation thereof is diagrammed in Fig. 27. The sphere of diameter $D$ is projected in the sense of the arrows upon the plane perpendicular to the vertical axis and laid across point $P_2$. The projection $D_1$ of the circle of diameter $D$ is larger than the original. If, now, point $P$ at a distance $R_1$ on the one meridian along with the circle with diameter $D_1$ passing through point $P$ is projected, its projection will be, in accordance with the above considerations, larger than the original. Thus, the projection of point $P$ and of circle $P_1$ remains unchanged in the direction of one $d$ whereas in the direction of $d_1$ it is extended in the sense of the arrows drawn from point $P_1$.

3. *Orthographic projection* (Fig. 28). The points of the sphere are being projected perpendicular to the tangential plane $A$. In this case $\overline{A_1 B_1} = R \sin \varphi$. Similarly to the foregoing $\overline{A_1 B_1} = R \sin \varphi = k \operatorname{tg} \alpha$ and finally $\sin \varphi = \frac{k}{R} \operatorname{tg} \alpha =$

$= A \operatorname{tg} \alpha$, where $A = \dfrac{k}{R}$. The diameter of the projected images of the meridian circles increases in the one direction and decreases in the other, until the image of point $A_2$, that is, of the circle, is reduced to a straight line.

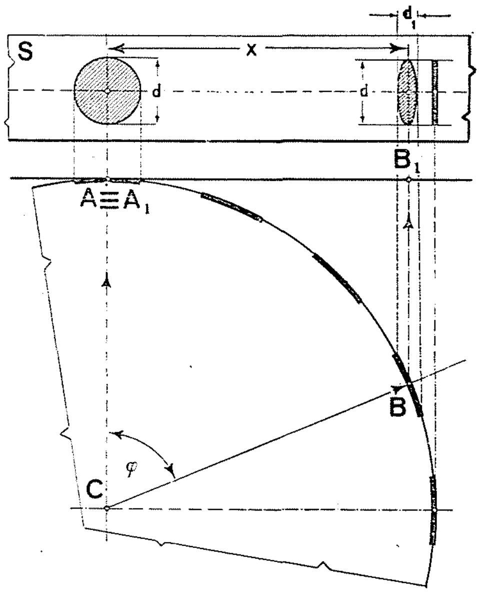



*Fig. 28.* The principle of orthographic projection

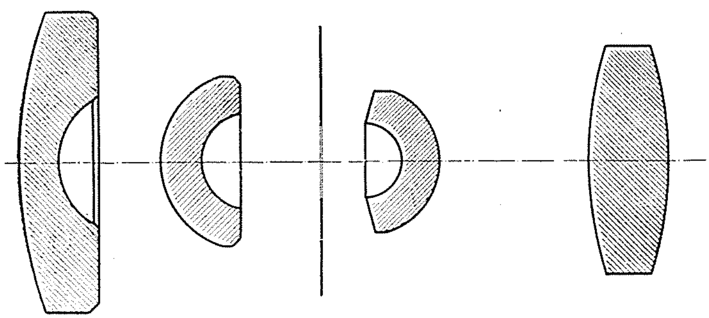


*Fig. 29.* Panoramic lens of *Schultz*

The *Hill* lens satisfies the condition set under 2, since, in case the distances on the image plane are equal, the angular distances on the hemisphere are also equal. The individual sections of the distorted image may be corrected subsequently.

Various "horizontal systems" forming panoramic images have been designed. The *Schultz* lens illustrated in Fig. 29 covers a field of 150—160°. One of the latest developments is the *Mertë* lens, represented in Fig. 30.

A variety of "optical systems" similar to those described above may be encountered in the animal world. The "eyes" or rather light reception organs of most insects work according to the principle disclosed above. They comprise a group of elementary pyramidal receptors, of largely

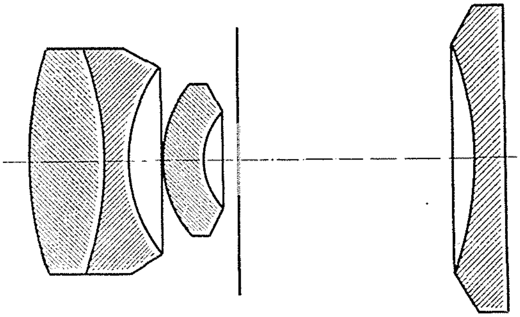
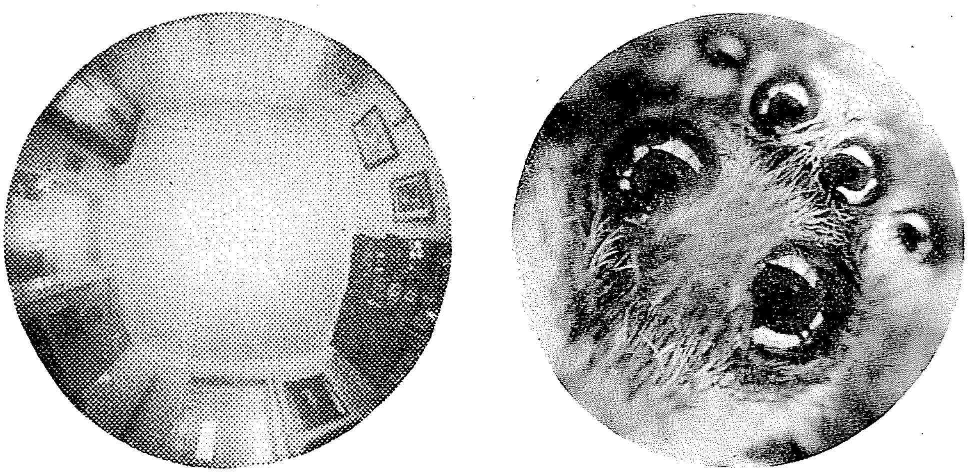


*Fig. 30.* Panoramic lens of *W. Merté*

hectagonal cross-sections, each of them pointing towards the interior, and optically separated from each other. Incident rays of light are received by the nerve endings at the peak of the pyramid, whereas laterally incident rays are absorbed by the pigment partitions, separating the receptors. Obviously, these arrangements cannot be considered as image-forming optical systems, they only promote the animals's orientation based on the sensation of light and shadow. For example, the dragon-fly's bigger-than-hemisphere eyes composed from elements as described



*Figs. 31 and 32.* The left hand figure illustrates the visual image supposed to be formed by the one eye of a May fly, the eye being composed of a group of simple eyes. This type of animal eye is, however, unsuited for image formation in the optical sense, and should be correctly called a light perception organ, as it is mainly responsive to light effects. *Fig. 32.* The right hand figure is a 70× magnification of an image formed by the simple eyes of a cross-spider which, contrary to the May fly's eyes, are able to form "visual images". The approximately 900 diopter eye is probably highly myopic. Nature has created wide-angle lenses long before man. The technicalities of printing have, unfortunately, impaired the visibility of fine structural details of the image, but the reflected images of the window panes can, nevertheless, be well seen above, are suited for forming overlapping images covering the entire surrounding space, except for that part of the space actually occupied by its own body. Human imagination is hardly able to follow the idea of a similar image. As far as one can speak of image formation at all, the dragonfly's one eye probably forms an image similar to the one represented in Fig. 31. Next to the intersection of the two hemispheres there are a few point-like eyes of steep curvature, perceivable also for an unaided eye. These assist the insect in tracing its prey.

## Summary

It has been attempted to give a short historical summary of wide-angle photographic systems operating with discontinuous paths of ray, including the entire range from pinhole cameras to the most recent optical arrangements, from the point of view of optics as well as optomechanics. A detailed and comprehensive survey of all existing types of instruments would fall outside the scope of the present paper.

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