ELECTRIC BREAKDOWN AS A PROBABILITY PROCESS

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1. Introduction

Recent theoretical and experimental results seem to support the view of those who consider the electric breakdown to be a probability process [1]. The author of this paper made use of this assumption in 1932 in an attempt to establish a theory of the breakdown of liquids [2]. Another theory, much more general, but not very different in basic concepts, was given by Zener [3] in 1934. Experimental evaluation of this theory was carried out lately by McAfee and his co-workers in 1951 [4]. The theories already mentioned use probability concepts in such a way, as is known from the "tunnel effect" of wave-mechanics which serve as a base for them [5].

It is well-known that in this way we generally get theoretical results of statistical character, but in the course of macroscopical measurements we do not find appreciable fluctuations in the measured values. It is, however, also well-known that at the breakdown tests we find a considerable scattering of the measured values, even if a very careful technique is used and the material under test is very homogeneous. This scattering is especially great in the case of impulse voltages, that is when we are obviously faced with the so-called "electric breakdown" which is much less affected by impurities than the breakdown due to thermal instability, the so-called "thermal breakdown".

At first sight, this scattering seems to have nothing to do with the above-mentioned fluctuations, because it is far greater than the latter. Nevertheless, we will see that the existence of some, though indirect, connection between the two phenomena may be assumed.

Obviously, it is worth while to deal with the nature of the scattering, as the dimensioning of insulation is based on the data obtained by breakdown tests carried out on test samples. Since these data show a considerable scattering, it must be decided how to impart the result of the tests to the user, the design engineer. This again requires a thorough discussion of the possible factors responsible for scattering.

We will try to carry out this investigation for the case of the standard breakdown tests. This seems to be justified by two reasons. First, the scattering, being a general phenomenon, must have general causes which are also valid for the standard tests. Secondly, the investigation of the standard breakdown is of high importance for practical purposes.

Consequently, we will investigate the scattering in the case of measurements carried out in accordance with the Recommendations of the IEC. There, the size of the electrodes is specified as follows : the upper electrode is a cylinder of 50 mm diameter and of 50 mm height, its edges are rounded off with a radius of 3 mm. The other electrode is plane, its area is greater than that of the cylinder, the distance between the boundary lines of the two electrodes must be not less than 50 mm. In our tests, described later, the area of the plane electrode was much greater than required by this minimum.

The thickness of the material to be tested should not exceed 1 mm.

2. Experimental results

It is well-known that the electrode arrangement described above yields almost in every case surface discharges before the breakdown of the solid material.

The inception voltage of the glow discharges for plane electrodes is, in air,

$$U_{ig} = \frac{10^{-5}}{(\varepsilon\, \varepsilon_0)^{0,45}} \cdot a^{04,5} \quad [6],$$

where ε is the relative permittivity of the solid material under test, a is its thickness in cm, $\varepsilon_0 = 0{,}0885 \cdot 10^{-12}$ Farad/cm.

According to our tests, the formula gives the inception voltage with an accuracy of 2—3% for technically clean surfaces.

If the voltage is raised the brush discharges appear, the inception voltage of which should be

$$U_{ib} = \frac{1{,}344 \cdot 10^{-4}}{(\varepsilon\, \varepsilon_0)^{0,44}}\, a^{0,44} \quad [7],$$

where ε, a, ε_2 are the same as in the previous formula. In our tests, the constant in the numerator varied between $0{,}8 \cdot 10^{-4}$ and $1{,}1 \cdot 10^{-4}$, but did not reach the value of $1{,}344 \cdot 10^{-4}$ given in the formula for U_{ib}. When the surfaces were strongly polluted with dust, then U_{ib} was somewhat diminished and U_{ig} increased. In accordance with the Recommendations, which give no special specification for the condition of the surface the breakdown voltage itself was but slightly influenced by the condition of the surface. Naturally if, *e. g.*, the surfaces are abnormally wet, this might exert a considerable influence on the breakdown voltage, mainly in case of hygroscopic materials. Measurements were carried out on capacitor paper, oiled paper, two kinds of pressboard and cresol-formaldehyde laminate (bakelite plates) of better and poorer quality. The greatest deviations from the arithmetical mean values were, respectively, 9,5, 7,2, 5,6, 12, 4,3 and 9,7%, calculated from the first 6 measurements. It is worth mentioning that when continuing the tests, in several cases we obtained far greater deviations. This shows that the standardized 6 measurements do not always give a clear picture of the amount of the scattering.

The scattering observed cannot be put down exclusively to the measuring technique, since by measuring the breakdown voltage of air we obtain a scattering in the order of 2% only, although the methods and instruments employed are the same.

Furthermore, variations in the values of the inception voltages U_{ig} and U_{ib} seem to have no immediate influence on the scattering of the breakdown voltages. In case of the capacitor paper and of the oiled paper, the breakdown occurred before the brush discharges appeared.

Tests on glass, under widely varying surface conditions yielded very great differences in the inception voltages, but only a comparatively small change in the mean value of the breakdown voltage and in the scattering of the measured values.

Therefore the reasons of scattering may be supposed to originate partly in the material itself, as also in the case under discussion, that is, when there are surface discharges previous to the breakdown. It may be supposed that for these materials of restricted purity and homogeneity the scattering is partly due to macroscopic impurities.

We shall quote some facts which seem to support our statements.

a) The greatest deviation for pressboard *A*, which is of a better quality and has also a greater electric strength, is some 50% smaller than that for the pressboard *B* of poorer quality. The somewhat defective cresol-formaldehyde laminate showed also a far greater deviation from the mean value than the good one.

b) In case of capacitor paper, the relative scattering at first diminishes with increasing number of layers.

c) It seems to be clear, however, that we get scattering also by breakdown tests on very homogeneous materials, as *e. g.* glass, especially with impulse voltages. The impurities may be *one* of the causes of scattering, but certainly not their *only* cause.

d) We cannot obtain considerably greater mean values of breakdown voltages unless all edge effects are suppressed, that is, no surface discharges occur previous to the breakdown.

Another conclusion that could be drawn from our experiments is that, in accordance with the generally accepted opinion, the mean value of the breakdown voltage is much lower in the presence of surface discharges than without them ; though the statistical nature of the phenomenon seems to be the same, and the scattering of the results is identical.

We attempted to investigate some details about the mechanism of the breakdown in the presence of surface discharges, being particularly interested whether or not our previous statement, according to which the breakdown is a probability process, can be applied to this case.

In the meantime an excellent paper was published in "Electrical Energy" about the work of the ERA in the field of insulation research [8], but as it deals with another side of the subject, it will perhaps not be superfluous to describe our results.

It is well-known that in the presence of surface discharges, the breakdown very frequently occurs not between the electrodes, but near the edges or sometimes at a distance from the electrode with the smaller surface. It is also well-known that in this case the breakdown occurs along a surface discharge path, especially at the end of one, because the field is very strong there. It seems therefore that the breakdown is influenced by the shape of these discharge paths.

One might suppose, on the other hand, that the shape of these paths is influenced by macroscopic inhomogeneities on the surface or within the solid material under test. If this were true, the shape and position of these paths should be the same for repeated voltage impulses, or at least nearly the same. The known photographs from surface discharges seem to assert this assumption. It must, however, be considered that these photographs always show a fairly great number of discharges, that have occured at different times. This is due to the method frequently used to register the discharges on a photographic plate itself. We have carried out a number of experiments with separate impulses at a sequence of about 20 sec, on a bakelite surface, which were photographed separately. The five photos shown in Fig. 1 reveal the more or less different shape of the discharge path under identical circumstances.

One might conclude that the discharge paths do not always choose macroscopic inhomogeneities, because, if so, their shape should remain the same during the repetitions. Quite naturally, we find the same phenomenon with alternating voltages of 50 Hz (Fig. 2), although on these photos we have more discharges that occurred at different times. Fig 2*a* shows a series of photos taken with a time of exposure of 10^{-2} sec, on Fig. 2*b* another series is to be seen, taken with a "Zeitlupe", in immediate sequence, with a time of exposure of $\sim 10^{-3}$ sec.

Anyway it is obvious that the shape of the paths is different at different times.

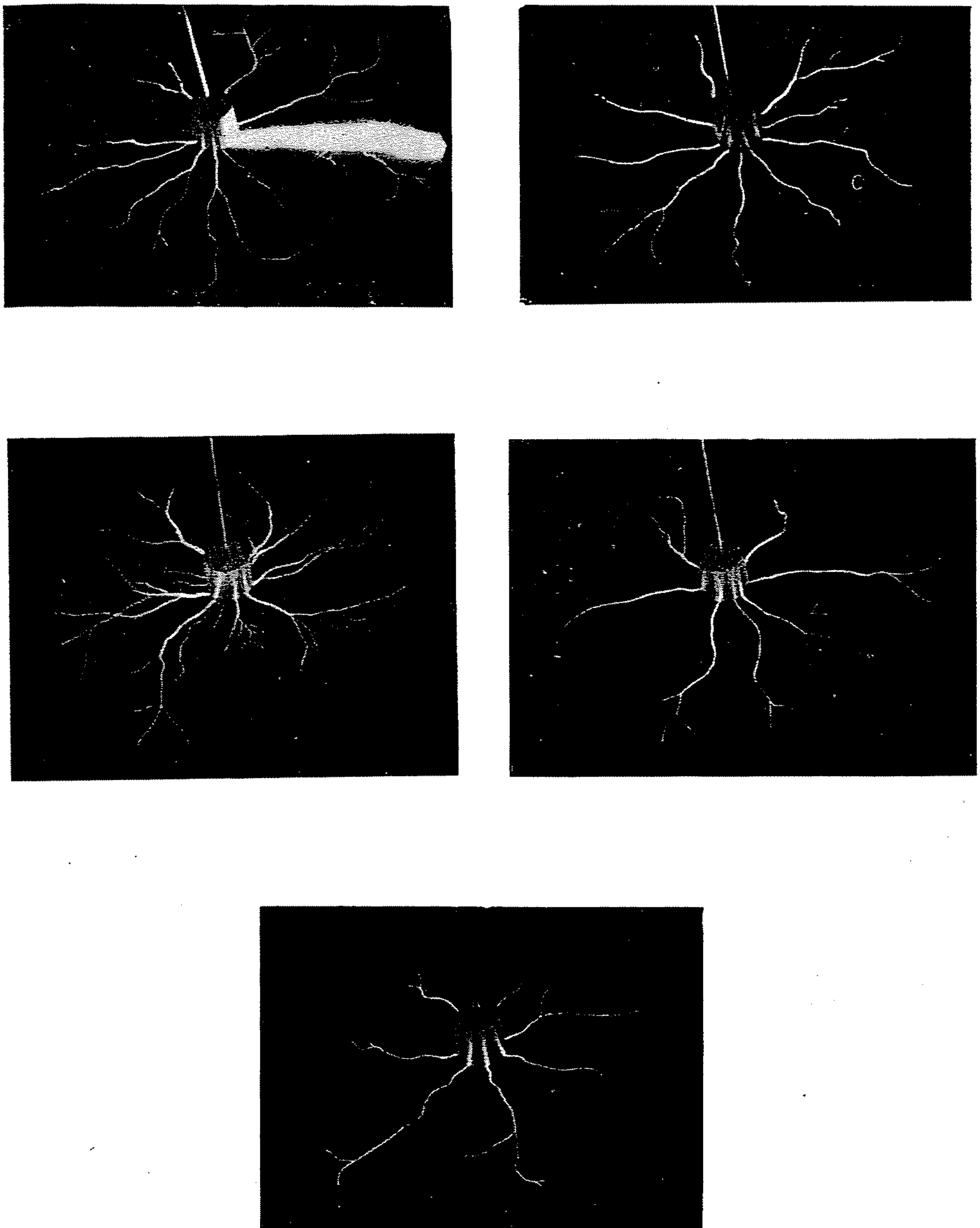

Fig. 1

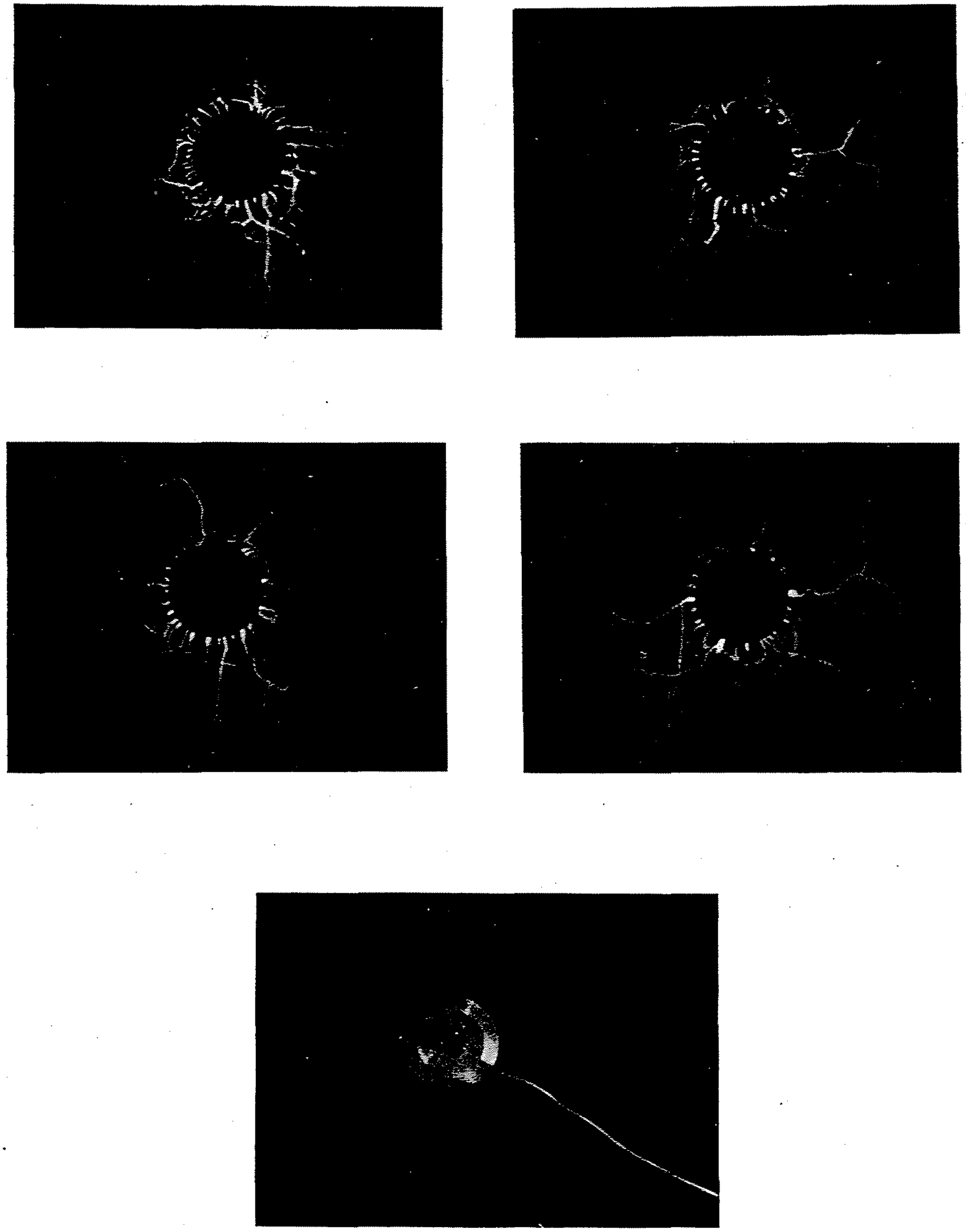

Fig. 2/a

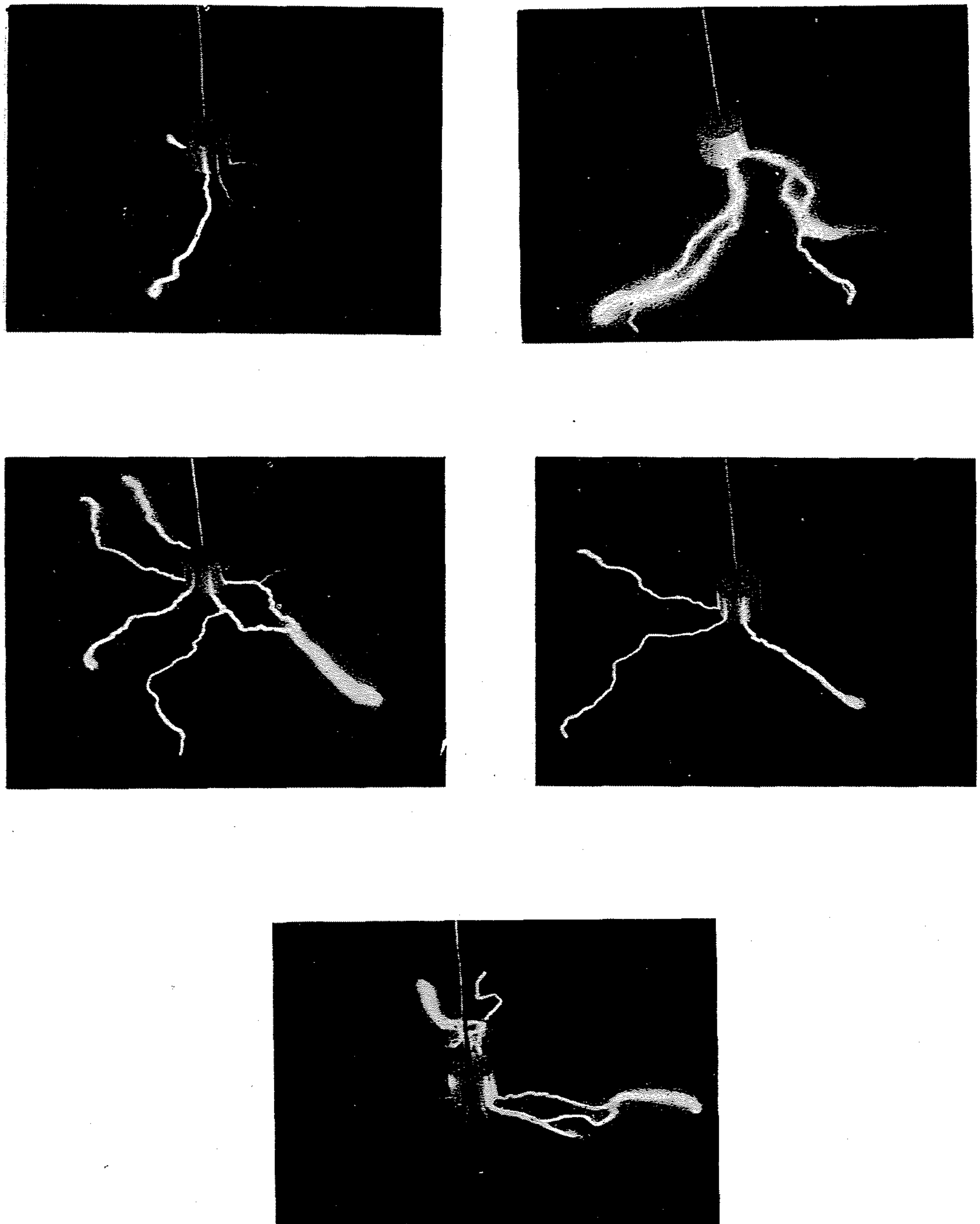

Fig. 2/b

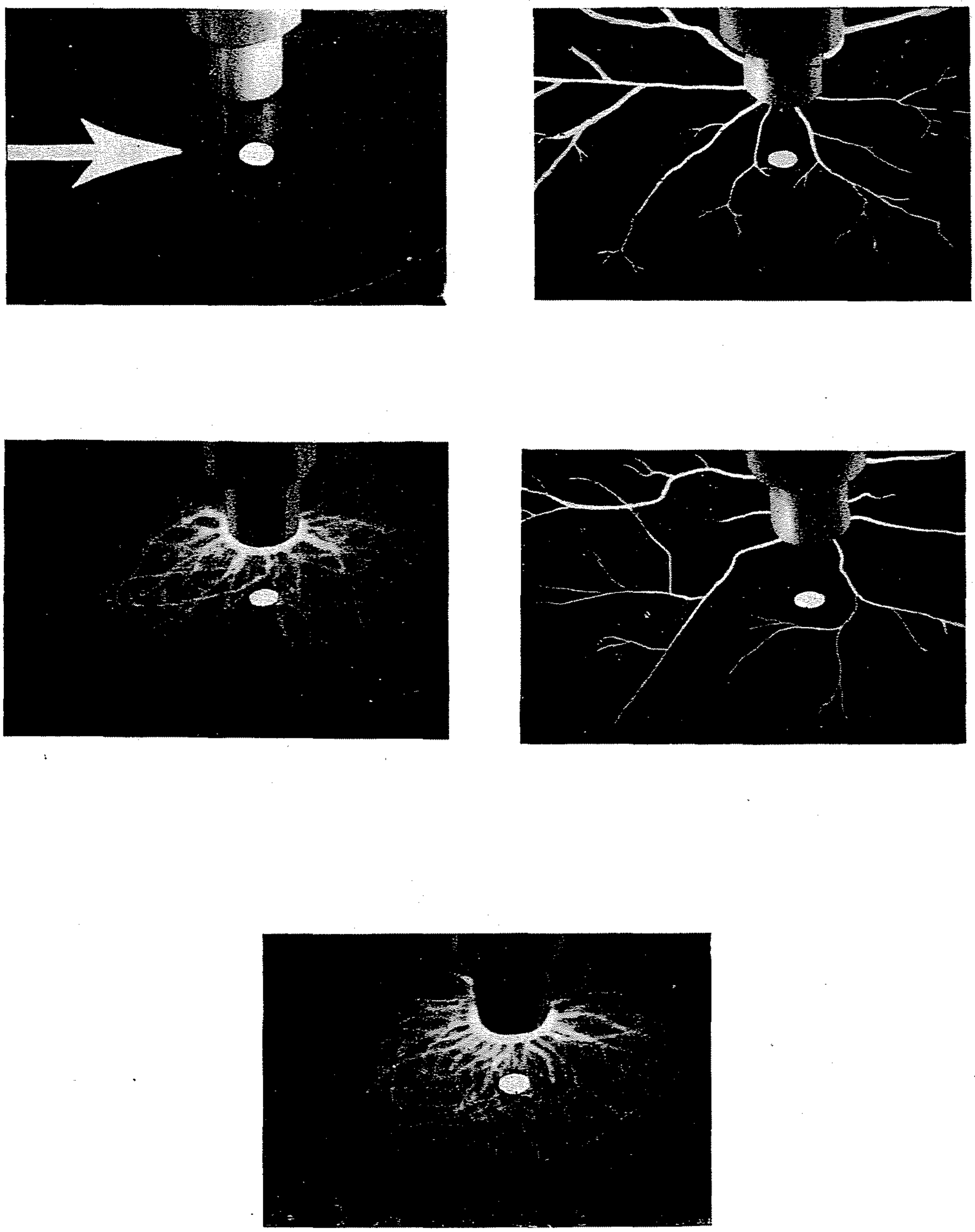

Fig. 3

The maximum distance of the end of these paths from the electrode is, as known, a function of the applied voltage, a fact which is utilized in the impulse voltage measuring instrument called klydonograph.

It can be demonstrated that the discharge paths do not always choose macroscopical inhomogeneities by drilling a hole in the material to a depth of about one third of the thickness. Impulse voltages yield sometimes a discharge path going to the hole, sometimes another going in a different direction (Fig. 3).

These are facts that can be accounted for in different ways. One of the possible explanations may be the following: the instantaneous shape of the

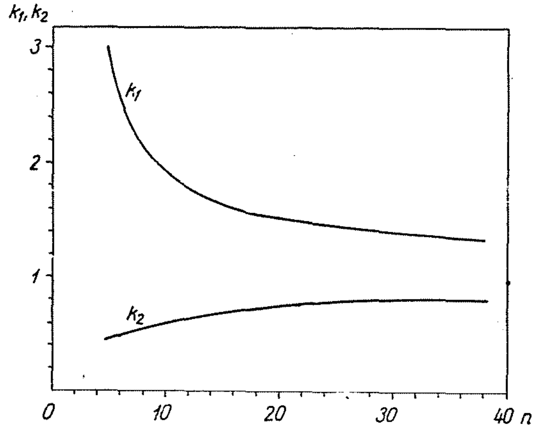

Fig. 4

discharge paths is partly influenced by the fluctuations in the values of the different parameters of the surrounding medium, partly by those of the material under test. If this is so, we can say that the discharge paths are determined by probabilities and, as the breakdown in the material itself is supposed to be a probability process, we may draw the conclusion that the breakdown in presence of surface discharges can also be considered as a probability process. If this is true, we can use the known methods of mathematical statistics for the evaluation of breakdown tests carried out according to the IEC Recommendations.

For the evaluation of our experiments we made use of the method devised by KRONDL [9].

The essential of this method is the calculation of the standard deviation

$$s = \sqrt{\frac{1}{n-1} \sum_{1} (\overline{U}_b - U_b)^2} \ ,$$

where U_b is a measured breakdown voltage, $\overline{U}_b$ the mean value of n measured

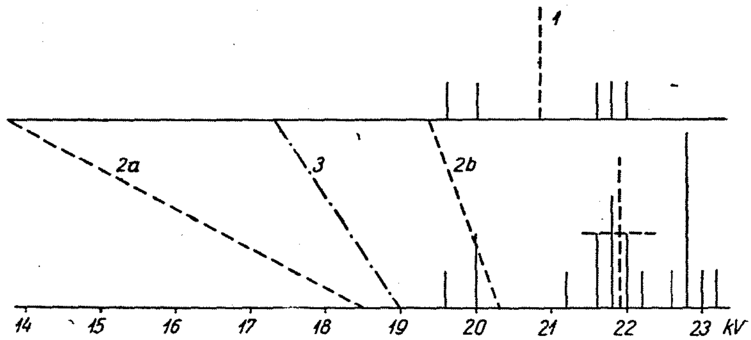

Fig. 5/a

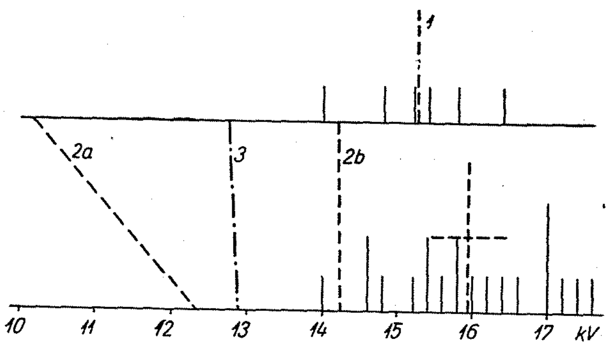

Fig. 5/b

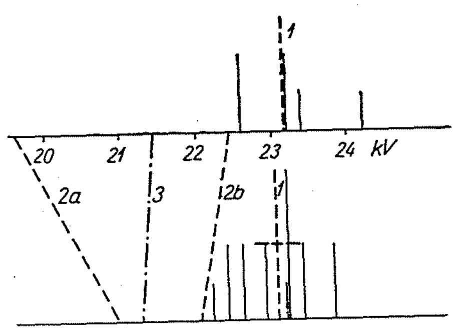

Fig. 5/c

breakdown voltages, that is,

$$\overline{U}_b = \frac{\sum U_b}{n},$$

with these values we get the two limits of the minimum breakdown voltage to be expected as

$$\overline{U}_b - k_1\, 2{,}33\, s < \overline{U}_{b\min} < U_b - k_2\, 2{,}33\, s\,,$$

where k_1 and k_2 can be taken from graphs as a function of n (Fig. 4). Examples for the evaluation are to be seen in Tables I and II. We have carried out several hundred measurements on the materials mentioned above. The results are to

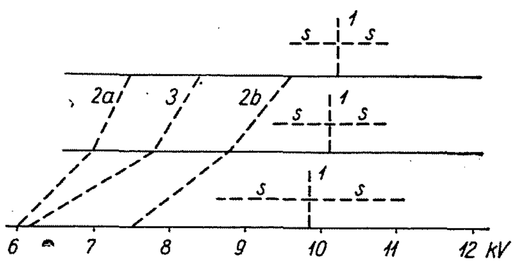

Fig. 5/d

be seen on the diagrams in Fig. 5. Lines Nr. 1 give the mean value, lines Nr. $2a$ and $2b$ give the two limits of the minimum breakdown voltage to be expected. Line Nr. 3 shows the values of U_b—$3s$, which are, as is known, the theoretical minimum values of the breakdown voltage to be expected if we can suppose a regular Gaussian distribution. It occurs sometimes, as seen on the diagram "e" of Fig. 5 that the values of the upper and lower limits of the minimum breakdown voltages are not sufficiently convergent if taken from an increasing number of tests. Thus 20—36 measurements do not seem to yield a reliable lower limit value. It may be useful to make further investigations with this, and perhaps also with other methods, because the evaluation of the probable lower limit of the breakdown voltage from a restricted number of tests is of great practical importance.

3. Conclusions

Theoretical investigations seem to justify the assumption that the so-called "electrical breakdown" is a probability process. The purpose of this paper was to show that also the breakdown in presence of surface discharges, which occurs *e. g.* by using the IEC electrodes, is regarded by author as a probability process.

Table I*

Pressboard A

U_b	$\bar{U}_b-U_b$	$(\bar{U}_b-U_b)^2$	$\bar{U}_b-U_b$	$(\bar{U}_b-U_b)^2$
20,0 kV	1,9	3,61	0,83	0,69
19,6	1,3	1,69	1,23	1,51
22,0	0,1	0,01	1,17	1,37
21,6	0,3	0,09	1,23	1,51
20,0	1,9	3,61	0,83	0,69
21,8	0,1	0,01	1,03	1,06
21,8	0,1	0,01		$\Sigma = 6,83$
22,2	0,3	0,09		
22,8	0,9	0,81		
21,8	0,1	0,01		
22,8	0,9	0,81		
23,2	1,3	1,69		
21,2	0,7	0,49		
22,8	0,9	0,81		
22,0	0,1	0,01		
22,6	0,7	0,49		
21,2	0,7	0,49		
23,0	1,1	1,21		
22,8	0,9	0,81		
22,8	0,9	0,81		
		$\Sigma = 17,56$		

From measurements 1—6

$$\sum_1^6 U_b = 125,0$$

$$\bar{U}_b = 20,83 \text{ kV}$$

$$s = \sqrt{\frac{6,83}{5}} = \sqrt{1,366} = 1,17 \quad 5,6\%$$

$$U_{b\,\min} = 20,83 - \frac{2,6}{0,54}\ 1 \cdot 17,\ 2,33 =$$

$$= 20,83 - \frac{7,10}{1,47} = \frac{13,73 \text{ kV}}{19,36 \text{ kV}}$$

$$\bar{U}_b - 3s = 17,32 \text{ kV}$$

From measurements 1—20

$$\sum_1^{20} U_b = 438,0$$

$$\bar{U}_b = 21,9 \text{ kV}$$

$$s = \sqrt{\frac{17,56}{19}} = \sqrt{0,9242} = 0,961 \quad 4,4\%$$

$$U_{b\,\min} = 21,9 - \frac{1,52}{0,72}\ 0,961\ 2,33 =$$

$$= 21,9 - \frac{3,40}{1,61} = \frac{18,50 \text{ kV}}{20,29 \text{ kV}}$$

$$\bar{U}_b - 3s = 19,02 \text{ kV}$$

Table II*

Pressboard B

15,8 kV	0,14	0,02	0,53	0,28
15,4	0,54	0,29	0,13	0,17
15,2	0,74	0,55	0,07	—
14,0	1,94	3,76	1,27	1,61
14,8	1,14	1,30	0,47	0,22
16,4	0,46	0,21	1,13	1,28
15,6	0,34	0,12		$\Sigma = 3,56$

From measurements 1—6

$$\sum_1^6 U_b = 91,6$$

$$\bar{U}_b = 15,27 \text{ kV}$$

$$s = \sqrt{\frac{3,66}{5}} = \sqrt{0,712} = 0,844 \quad 5,5\%$$

* The measurements were carried out by E. NÉMET.

14,6	1,34	1,79
14,6	1,34	1,80
15,4	0,54	0,29
17,4	1,46	2,13
17,0	1,06	1,12
17,0	1,06	1,12
16,6	0,66	0,44
17,2	1,26	1,59
16,2	0,26	0,07
16,0	0,06	—
15,8	0,14	0,02
17,0	1,06	1,12
17,6	1,66	2,75
		$\Sigma =$ 19,49

$$U_{b\min} = 15{,}27 - \frac{2{,}6}{0{,}54}\, 0{,}844 \cdot 2{,}33 =$$

$$= 15{,}27 - \frac{5{,}11}{1{,}06} = \frac{10{,}16 \text{ kV}}{14{,}21 \text{ kV}}$$

$$\bar{U}_b - 3s = 12{,}75 \text{ kV}$$

From measurements 1—20

$$\sum_1^{20} U_b = 318{,}8$$

$$\bar{U}_b = 15{,}94 \text{ kV}$$

$$s = \sqrt{\frac{19{,}49}{19}} = \sqrt{1{,}025} = 1{,}012 \quad 6{,}4\%$$

$$U_{b\min} = 15{,}94 - \frac{1{,}52}{0{,}72}\, 1{,}012 \cdot 2{,}33 =$$

$$= 15{,}94 - \frac{3{,}59}{1{,}70} = \frac{12{,}35 \text{ kV}}{14{,}24 \text{ kV}}$$

$$\bar{U}_b - 3s = 12{,}91 \text{ kV}$$

Cresol-formaldehyde laminate, good quality, 1 mm thick

23,2 kV	0,13	0,02	—	—
23,2	0,13	0,02	—	—
22,6	0,47	0,22	0,6	0,36
24,2	1,13	1,27	1,0	1,00
23,4	0,33	0,11	0,2	0,04
22,6	0,47	0,22	0,6	0,36
24,0	0,93	0,86	Σ	= 1,76
22,8	0,27	0,07		
22,8	0,27	0,07		
22,2	0,87	0,76		
22,4	0,67	0,45		
23,2	0,13	0,02		
23,2	0,13	0,02		
23,8	0,73	0,53		
23,2	0,13	0,02		
23,4	0,33	0,11		
23,0	0,07	—		
22,0	1,07	1,14		
23,8	0,73	0,53		
22,4	0,67	0,45		
		$\Sigma =$ 6,89		

From measurements 1—6

$$\sum_1^{6} U_b = 139{,}2$$

$$\bar{U}_b = 23{,}2 \text{ kV}$$

$$s = \sqrt{\frac{1{,}76}{5}} = \sqrt{0{,}352} = 0{,}593 \quad 2{,}6\%$$

$$U_{b\min} = 23{,}2 - \frac{2{,}6}{0{,}54}\, 0{,}593 \; 2{,}33 =$$

$$= 23{,}2 - \frac{3{,}59}{0{,}75} = \frac{19{,}61 \text{ kV}}{22{,}45 \text{ kV}}$$

$$\bar{U}_b - 3s = 21{,}43 \text{ kV}$$

From measurements 1—20

$$\sum_1^{20} U_b = 461{,}4$$

$$\bar{U}_b = 23{,}07 \text{ kV}$$

$$s = \sqrt{\frac{6{,}89}{19}} = 0{,}602 \quad 2{,}6\%$$

$$\bar{U}_{b\min} = 23{,}07 - \frac{1{,}52}{0{,}72}\, 0{,}602 \; 2{,}33 =$$

$$= 23{,}07 - \frac{2{,}13}{1{,}01} = \frac{20{,}94 \text{ kV}}{22{,}06 \text{ kV}}$$

$$\bar{U}_b = 3s = 21{,}27 \text{ kV}$$

Cresol-formaldehyde laminate, defectuous, 1 mm thick

23,8 kV	0,77	0,59	1,17	1,37
22,8	0,23	0,05	0,17	0,03
22,8	0,23	0,05	0,17	0,03
23,4	0,37	0,14	0,77	0,59
22,6	0,43	0,18	0,03	—
20,4	2,63	6,92	2,23	4,97
21,4	1,63	2,65		$\Sigma = 6,99$
22,4	0,63	0,40		
23,2	0,17	0,03		
23,4	0,37	0,14		
22,8	0,23	0,05		
24,4	1,37	1,88		
23,4	0,37	0,14		
24,6	1,57	2,46		
22,2	0,83	0,69		
23,0	0,03	—		
23,0	0,03	—		
24,4	1,37	1,88		
23,0	0,03	—		
23,6	0,57	0,32		
		$\Sigma = 18,57$		

From measurements 1—6

$$\sum_1^6 U_b = 135,8$$

$$\bar{U}_b = 22,63 \text{ kV}$$

$$s = \sqrt{\frac{6,99}{5}} = \sqrt{1,398} = 1,18 \quad 5,2\%$$

$$U_{b\,\min} = 22,63 - \frac{2,6}{0,54}\, 1,18 \cdot 2,33 =$$

$$= 22,63 - \frac{6,61}{1,48} = \frac{16,02 \text{ kV}}{21,15 \text{ kV}}$$

$$\bar{U}_b - 3\,s = 19,09 \text{ kV}$$

From measurements 1—20

$$\sum_1^{20} U_b = 460,0$$

$$\bar{U}_b = 23,03 \text{ kV}$$

$$s = \sqrt{\frac{18,57}{199}} = \sqrt{0,977} = 0,986 \quad 4,2\,\%.$$

$$U_{b\,\min} = 23,03 = \frac{1,52}{0,72}\, 0,986 \cdot 233 =$$

$$= 23,03 - \frac{3,49}{1,65} = \frac{19,54 \text{ kV}}{21,38 \text{ kV}}$$

$$\bar{U}_b = 3s = 20,08 \text{ kV}$$

Author omitted a detailed analysis of the mechanism of the breakdown itself in this particular case, since it seems to have been made clear already [8, 10], but tried to show that the breakdown phenomenon might be attributed to fluctuations. The scattering observed seems to be a resultant of these fluctuations.

Author does not consider this to be the only cause of scattering, but it seems very likely that it is *one* of its causes. Should that be true, the use of statistical methods for the evaluation of breakdown measurements, a method for which some examples are given in this paper, is justified.

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Summary

Author considers the electrical breakdown — also in presence of surface discharges — as a probability process and supposes that at least one cause of the scattering observed is due to this circumstance. If it is true, then the known statistical methods may be used by evaluating the measurements instead of giving the mean value alone. Results of experiments, which seem to justifie this assumptions are also given in the paper.

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