EFFECT OF THE SPARK-GAP WORKING TIME UPON THE SHORT-CIRCUIT CURRENTS IN CIRCUITS CONTAINING SERIES CAPACITOR*

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In a previous article [1] we examined in detail the short-circuit currents of the system containing a series capacitor, and estimated the value of overvoltages arising on the series capacitor. It was stated that the series capacitor must be protected in almost every case. The question arises, how fast the protective device must work to comply with this requirement, that the short-circuit current should not be greater in the compensated system, than in the non-compensated one. The present article deals with this question.

The problem will be examined in detail only for the most simple case. Again a symmetrical, three-phase short-circuit arising at no-load will be supposed. First we neglect the resistance of the short-circuit loop.

Expression of the short-circuit current in this case is (see expressions (7), (8) and (22) of [1]):

$$\overline{I}(t) = \frac{U_m e^{j\left(\varphi - \frac{\pi}{2}\right)}}{X_L(1-k)} e^{j\omega t} -$$

$$- \frac{U_m e^{j\left(\varphi - \frac{\pi}{2}\right)}}{X_L(1-k)} \cdot \left(\frac{1+\sqrt{k}}{2} e^{j\sqrt{k}\,\omega t} + \frac{1-\sqrt{k}}{2} e^{-j\sqrt{k}\,\omega t}\right). \qquad (1)$$

This solution is valid for a stationary co-ordinate system. The steady-state component is a vector of constant length rotating with an angular speed ω, the two transient components are also vectors of constant lengths rotating with angular velocities $+\sqrt{k}\omega$ and $-\sqrt{k}\omega$ resp. The resultant of the three vectors can be more simply designed, if the solution is written in synchronously, with angular speed ω rotating co-ordinate system. Therefore let us multiply both sides of (1) by $e^{-j\omega t}$:

* The present paper, as well as the previous one, was realized at the Department for Theory of Operation of Electrical Machninery, Polytechnic University, Budapest.

$$\overline{I}(t)\, e^{-j\omega t} = \frac{U_m\, e^{j\left(\nu - \frac{\pi}{2}\right)}}{X_L\,(1-k)} -$$

$$- \frac{U_m\, e^{j\left(\nu - \frac{\pi}{2}\right)}}{X_L\,(1-k)} \cdot \left[\frac{1+\sqrt{k}}{2}\, e^{-j\,(\sqrt{k}-1)\,\omega t} + \frac{1-\sqrt{k}}{2}\, e^{-j\,(\sqrt{k}+1)\,\omega t}\right]. \qquad (2)$$

In this case the steady-state component is a stationary vector, while the transient components rotate with different angular speeds. The absolute value of all of three vectors is constant.

For the degree of compensation the values of

$$k = 0{,}25\,;\;\; 0{,}562\,;\;\; 1{,}5625\,;\;\; 2{,}25$$

are chosen, as in this case

$$\sqrt{k} = 0{,}5\,;\;\; 0{,}75\,;\; 1{,}25\,;\;\; 1{,}5$$

is a rational fracture further simplifying the solution of the problem.

Realization of the design itself may be seen on Figs. 1 a, b, c, d. In all cases the steady-state component is the vector $\overrightarrow{QO}$. (To simplify the design, this vector $\overrightarrow{QO}$ was taken of unit length.) Circle K_1 is the diagram of the first, K_2 that of the second transient component, the curve G is the diagram of the resultant vector.

The vector traced from point O to an arbitrary point P of curve G gives the resultant of the two transient components, while the vector drawn from point Q to point P gives the resultant $\overline{I}(t)\ e^{-j\omega t}$ of the steady-state and transient components.

Curve G was calibrated according to ωt expressed in degrees. By aid of this it may be stated for instance, that we got a resultant twice as large as the steady-state component in the first and fourth case at $\omega t = 360°$, in the second and third case at $\omega t = 720°$.

Suppose furthermore, that at a certain instant the series capacitor is short-circuited in all the three phases *at the same time*. The resistance of the by-pass circuit is then taken as zero.

The Laplace-transforme of the new short-circuit current is in this case

$$\overline{I}(p) = U_m\, e^{j\nu}\, \frac{p}{p - j\omega}\, \frac{1}{pL} + \overline{I}_0\, \frac{pL}{pL}\,. \qquad (3)$$

where $\overline{I}_0$ is the current flowing through inductivity L at the instant when the

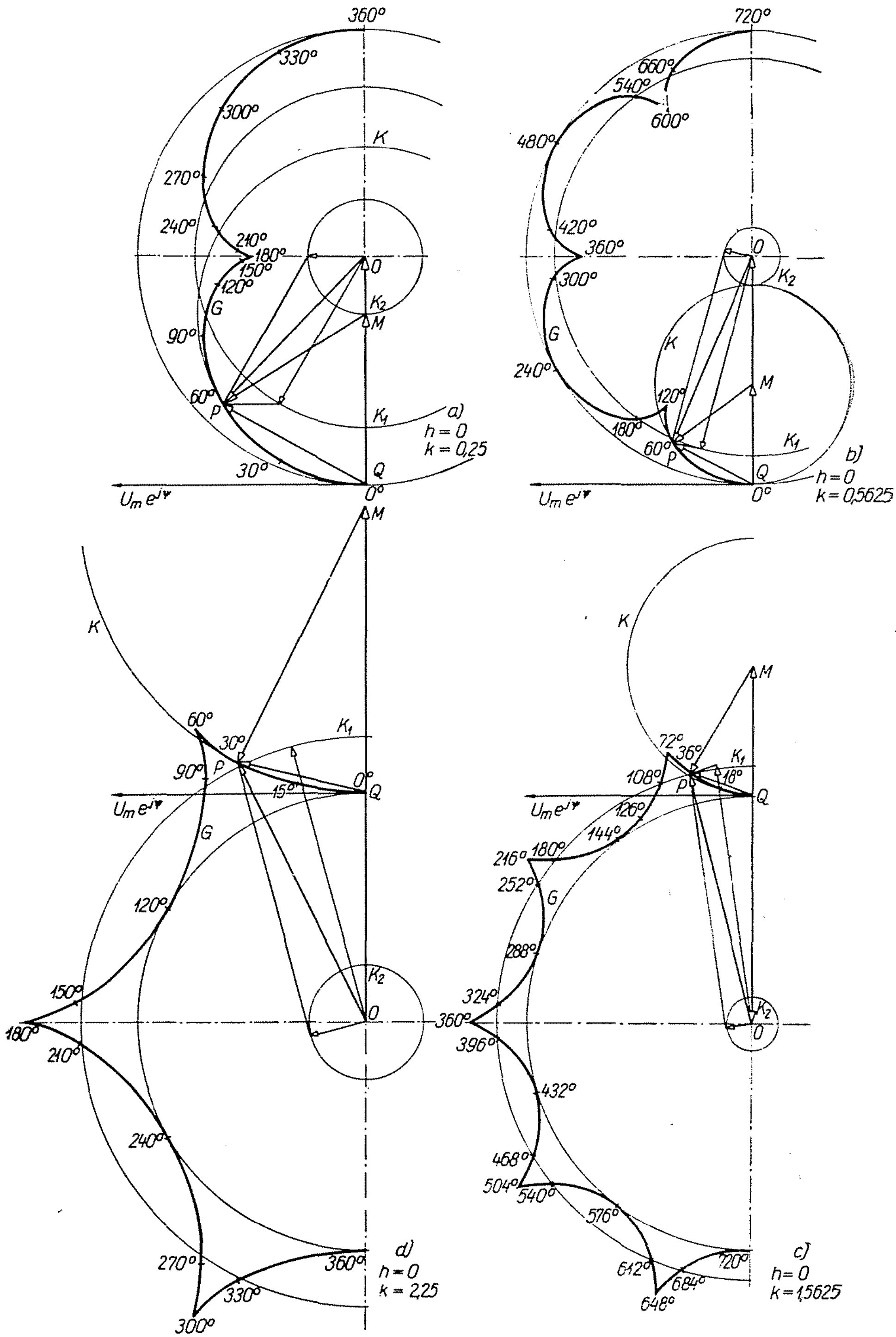

Fig. 1. Diagrams of short-circuit currents in synchronously rotating co-ordinate system. Resistance is neglected

series capacitor is by-passed. The current $\overline{I}_0$ is given by the vector $\overrightarrow{QP}$ of the previous design.

The time-function of the new short-circuit current is (with the aid of the generalized expansion theorem [2, 3]):

$$\overline{I}(t) = \frac{U_m e^{j\left(\varphi - \frac{\pi}{2}\right)}}{X_L} e^{j\omega t} - \left(\frac{U_m e^{j\left(\varphi - \frac{\pi}{2}\right)}}{X_L} - \overline{I}_0 \right) \tag{4}$$

computing now the time t from the instant, when the series capacitor is by-passed.

The steady-state component is the same as it would be in a noncompensated system in the case of a short-circuit arising at no-load, but the transient component is different.

Let us again pass over on to the synchronously rotating co-ordinate system:

$$\overline{I}(t)\, e^{-j\omega t} = \frac{U_m e^{j\left(\varphi - \frac{\pi}{2}\right)}}{X_L} - \left[\frac{U_m e^{j\left(\varphi - \frac{\pi}{2}\right)}}{X_L} - \overline{I}_0 \right] e^{-j\omega t}\,. \tag{5}$$

The new steady-state component is shown by the stationary vector $\overrightarrow{QM}$ and the new transient component by vector $\overrightarrow{MP}$. Latter rotates with angular velocity $-\omega$.

Let us draw a circle from point M as a center with the absolute value of the steady-state component as a radius. As far as the final point of vector $\overrightarrow{MP}$ belonging to the transient component is within this circle K, the greatest peak-value of the short-circuit current will not be greater, than in the noncompensated case.

In point Q the circle K is exactly the circle of curvature of the curve G. Namely the velocity of the point moving on curve G equals the total velocity of the final point belonging to the two transient components. (Differentiating two times one after the other the expression (2), we get the velocity and the acceleration.) The velocity is:

$$\overline{v}(t) = -\frac{U_m e^{j\left(\varphi - \frac{\pi}{2}\right)}}{X_L (1-k)} \left[\frac{1+\sqrt{k}}{2} j(\sqrt{k}-1)\, \omega\, e^{j(\sqrt{k}-1)\omega t} - \right.$$

$$\left. - \frac{1-\sqrt{k}}{2} j(\sqrt{k}+1)\, \omega\, e^{-j(\sqrt{k}+1)\omega t} \right]. \tag{6}$$

Accordingly, the acceleration is

$$\bar{a}(t) = -\frac{U_m e^{j\left(\psi - \frac{\pi}{2}\right)}}{X_L(1-k)} \left[-\frac{1+\sqrt{k}}{2}(\sqrt{k}-1)^2 \omega^2 e^{j(\sqrt{k}-1)\omega t} - \right.$$

$$\left. - \frac{1-\sqrt{k}}{2}(\sqrt{k}+1)^2 \omega^2 e^{-j(\sqrt{k}+1)\omega t} \right] . \quad (7)$$

At the initial point of time :

$$\bar{v}(o) = \frac{U_m e^{j\left(\psi - \frac{\pi}{2}\right)}}{X_L} j\omega , \quad (8)$$

and

$$\bar{a}(o) = \frac{U_m e^{j\left(\psi - \frac{\pi}{2}\right)}}{X_L} \omega^2 . \quad (9)$$

As $\bar{v}(o)$ is perpendicular to $\bar{a}(o)$, the radius of curvature is simply

$$r = \frac{v^2(o)}{a(o)} = \frac{U_m}{X_L} , \quad (10)$$

where $v(o)$ and $a(o)$ are absolute values of $\bar{v}(o)$ and $\bar{a}(o)$. As the radius of curvature is unidirectional with the acceleration, the centre of curvature is just in point M.

Above it was shown that circle K is the circle of curvature of curve G. Circle K and curve G are near enough to each other in the initial section. If the by-pass of the capacitor takes place at the angle, or at the point of time belonging to this section, there will be no essential difference between the short-circuit currents of the compensated and noncompensated system. The values of the steady-state components are the same, these of the transient components are near to one another, only the initial angle formed by the vectors of the steady-state and transient component is different.

But an important deviation can be observed if the curve G already left the territory of circle K. So e. g. in case of $k = 0{,}562$ (Fig. 1b) if the by-pass of the capacitor takes place two cycles after the short-circuit, i. e. at $\omega t = 720°$, the transient component would be 3,5-times greater, than the steady-state component.

The transient component will be smaller, than the steady-state component, until curve G does not intersect circle K. It may be seen from Fig. 1, that greater security is served by taking the first peak-point of curve G, instead of the point of intersection. The time t, resp. the angle ωt of the by-pass belonging to the peak-point, can be easily computed. In the peak-point namely, the vector of the two transient components is of opposite direction, so the difference of angles made by the two vectors from the initial time is exactly 180° :

$$(\sqrt{k}-1)\,\omega t_0 - [-(\sqrt{k}+1)\,\omega t_0] = 2\sqrt{k}\,\omega t_0 = \pi, \tag{11}$$

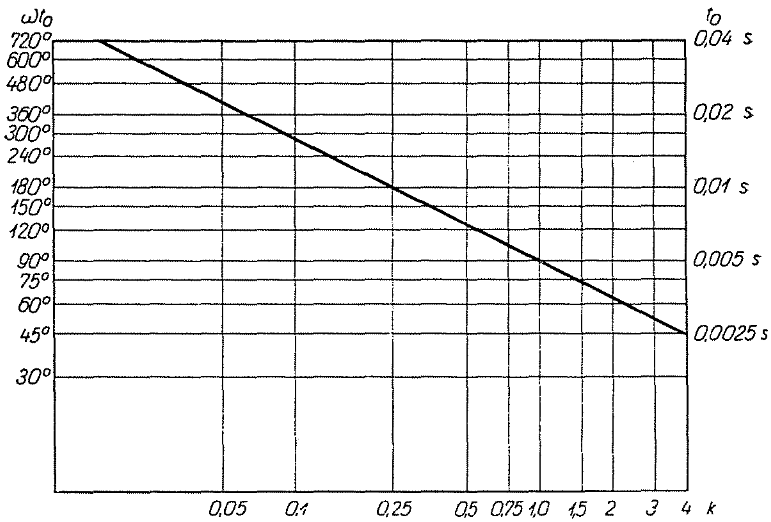

Fig. 2. Relation of the time-limit of the protective device and the degree of compensation

from which the time-limit of the protective device

$$t_0 = \frac{\pi}{2\sqrt{k}\,\omega}, \tag{12}$$

or expressed in an angle

$$\omega t_0 = \frac{\pi}{2\sqrt{k}}\ \text{radian} = \frac{180^\circ}{2\sqrt{k}}. \tag{13}$$

If the protective device works within this time-limit, the transient component of the short-circuit current will not be greater, than the steady-state component. The relation of the time-limit and the degree of compensation is shown on Fig. 2.

Until now resistance has been neglected. The question arises, whether consideration of the resistance does not alter the results obtained considerably.

To decide this, we drew up Figs. 3 a, b, c for case $h = 0{,}2$; $0{,}4$; $0{,}6$ and $k = 0{,}25$. The figures, similarly to Fig. 1. show the variation of the short-circuit current in a synchronously rotating co-ordinate system.

The diagram G of the resultant short-circuit current in the compensated system is given by the following expression :

$$\bar{I}(t) = \bar{I}_f \frac{1}{h + j(1-k)} - \bar{I}_f \frac{1}{h + j(1-k)} \cdot \frac{1}{2j\sqrt{k - \left(\frac{h}{2}\right)^2}} \times$$

$$\times \left\{ \left[-\frac{h}{2} + j\sqrt{k - \left(\frac{h}{2}\right)^2} + jk \right] e^{\left(-\frac{h}{2} + j\sqrt{k - \left(\frac{h}{2}\right)^2} - j\right)\omega t} - \right.$$

$$\left. - \left[-\frac{h}{2} - j\sqrt{k - \left(\frac{h}{2}\right)^2} + jk \right] e^{\left(-\frac{h}{2} - j\sqrt{k - \left(\frac{h}{2}\right)^1} - j\right)\omega t} \right\}, \tag{14}$$

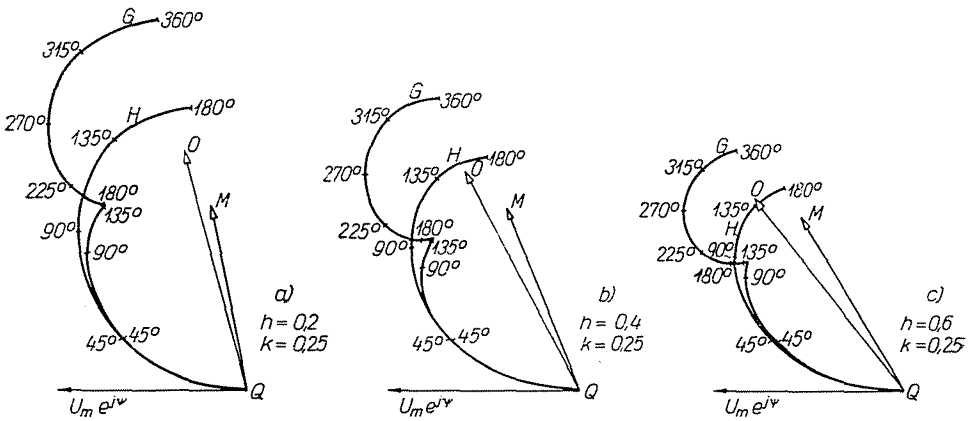

Fig. 3. Diagrams of short-circuit currents in synchronously rotating co-ordinate system. Resistance is taken in consideration

where

$$\bar{I}_f = \frac{U_m e^{j\psi}}{X_L}. \tag{15}$$

The diagram H of the short-circuit current in the noncompensated system is expressed as follows :

$$\bar{I}(t) = \bar{I}_f \frac{1}{h + j} - \bar{I}_f \frac{1}{h + j} e^{(-h-j)\omega t}. \tag{16}$$

Compared to the case of $h = 0$, the changes below may be observed : The vectors $\overrightarrow{QO}$ resp. $\overrightarrow{QM}$ of the steady-states short-circuit currents have not the same direction. The diagram G became deformed on account of damping. Furthermore, the circle K is substituted by a spiral H.

Nevertheless, it may be stated generally that in the initial point Q the circle of curvature of curves G and H are common.

The final point of the resultant current-vector of the noncompensated system moves with a velocity $\bar{v}(t)$ and with an acceleration $\bar{a}(t)$ along curve H. Differentiating twice one after the other the expression (16), we get

$$\bar{v}(t) = \bar{I}_f \frac{1}{h+j} (h+j)\, \omega\, e^{(-h-j)\omega t} \tag{17}$$

resp.

$$\bar{a}(t) = -\bar{I}_f \frac{1}{h+j} (h+j)^2 \omega^2 e^{(-h-j)\omega t} \; . \tag{18}$$

At the initial time

$$\bar{v}(o) = -\bar{I}_f\, \omega \tag{19}$$

and

$$\bar{a}(o) = -\bar{I}_f (h+j)\, \omega^2 \, . \tag{20}$$

Similarly, by differentiating expression (14) the velocity and acceleration of the final point of the vector moving on curve G can be computed for the initial time. Performing the computation we again obtain as a result, expressions (19) and (20) resp.

Consequently, the velocity, as well as the acceleration is of equal magnitude and direction in both cases. So curves G and H have a twofold contact with one another at the initial point of time. The velocity and so the tangent of curves G and H is unidirectional with vector $U_m e^{j\varphi}$. The result obtained may generally be put into words as follows : Connecting an alternating voltage to a series circuit R, L, C, with zero initial conditions, at the initial time the value of the short-circuit current, as well as its first and second differential quotient is of the same magnitude as in the case without capacitance ($C = \infty$). Therefore the initial sections of the current-curve of the compensated and noncompensated system are very near to one another.

Let us return to Figs. 3 a, b, c. With increasing resistances the curve H crosses the curve G always at smaller ωt values. So e. g. in case of $k = 0{,}25$, the point of intersection is at $\omega t = 230°$ if $h = 0$, and about at $175°$ if $h = 0{,}6$.

Consequently, with growing resistance the time-limit of the by-pass decreases. But as the time-limit of by-pass had been stated in case of $h = 0$ with a security (e. g. in case of $k = 0{,}25$ for $180°$) the relations (12), (13) between the time-limit and the degree of compensation may be considered valid also for the cases $h \neq 0$. (Only at greater values of h must the time-limit be decreased.)

Up to now we examined the short-circuit current arising at no-load. How does the initial load current influence the results obtained? To decide this, we drew up Figs. 4a and 4b ($k = 0{,}25$, $h = 0{,}4$). In both cases the amplitude of the load current I_0 is — exaggerating — one fourth of the amplitude of the short-circuit current in the noncompensated system. In the first case the power factor is 1,0 ; in the second case 0,7. The diagram of the short-circuit current in the compensated system is G, in the noncompensated one H, resp. H_0. Curve H was drawn by taking in consideration the initial load current, curve H_0 refers to a short-circuit arising at no-load.

Already the tangent of curves G and H in point Q is not unidirectional. In the cases occuring in practice, at the initial section curve H runs farther in, than curve G, but the deviation is not important. If the by-pass of the series capacitor takes place quickly enough, the initial load will not cause a great

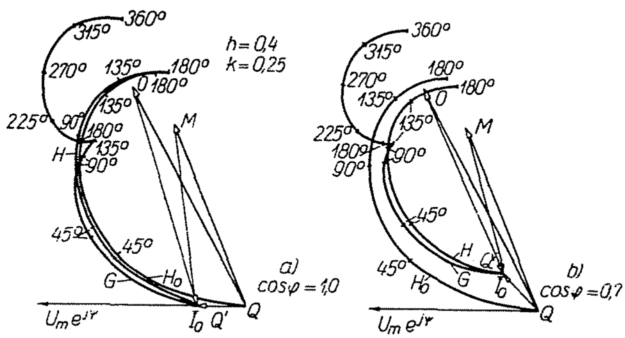

Fig. 4. Influence of initial load current on the short-circuit currents

difference between the short-circuit currents of the compensated and non-compensated systems.

Circumstances are even more favourable, when comparing curve G with curve H_0 (Fig. 4b). Curve G generally runs farther in, than curve H_0. The only exception is when the final point of the vector of the initial load current I_0 falls out of curve H_0 (Fig. 4a). But this only occurs at power factors very near to 1, which is an uncommon case in practice.

So it may be stated, that the time-limit of the by-pass is not considerably influenced, neither by the resistance of the short-circuit loop, nor by the initial load current, though strictly speaking the time-limit is not only a function of the degree of compensation, but also of these two factors.

Similarly to the reasoning in point 6 of [1], the results obtained may be generalized for the case of asymmetrical short-circuits too.

The gravest condition seems to be the obligation, that the series capacitor must be short-circuited in all phases at the same time. This cannot be realized by either of the protective devices known. But consideration of the different times of by-pass would complicate the calculations significantly.

Conclusion

In what has gone before we examined in detail, how fast the protective device of the series capacitor must work not to raise a greater short-circuit current in the compensated system, than in the noncompensated one. This question is important, not only from the point of view of short-circuit currents, but from that of the relay-protection. Neglecting the resistance of the short-circuit loop we stated a relation between the time-limit of by-pass and the degree of compensation (Fig. 2). The results obtained are not significantly influenced by consideration of the resistance and of the initial load current. For the time-limit of the by-pass we have got a relatively small value. While the spark-gap protection does not preceive the short-circuit current itself, but the overvoltage raising on the capacitor; it is of high-speed operation, and so it may prevent the development of too great short-circuit currents, though it is not certain that the spark-gap will work in every case within the time-limit determined above.

References

1. Csáki, F.: Short-circuit Currents in Circuits Containing Series Capacitor. Periodica Polytechnica **1**, 155 (1957).
2. Wagner, K. W.: Operatorenrechnung und Laplacesche Transformation. Leipzig, 1950.
3. Kovács, K. P.—Rácz, I.: Transient Performance of Alternating-current Machines. Akadémiai Kiadó, Budapest, 1954.

Summary

Joining with a previous article, in this study it will be examined, how fast the protective device of the series capacitor must work to avoid a greater short-circuit current in the compensated system, than in the noncompensated one. A relation was found between the time-limit and the degree of compensation.

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