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RESEARCH ARTICLE

Analytical and simulation comparison of sinusoidal and resistive modulation strategies for network-friendly three-phase grid-connected inverters

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Abstract

The growing numbers of consumers distort AC networks with harmonics. Therefore suppression of the network pollution should be considered. This problem can be solved by using "network-friendly" converters.

In our study we examined two modulation strategies of three-phase grid-connected inverters. If these methods are used, converters behave like sinusoidal or resistive current loads of the network, which enables "network-friendly" operation. The examined sinusoidal and the resistive modulation strategies are known, but the differences between the two methods have not been studied before [1,2]. This paper deals with the comparison of these two strategies. First, the analytical examination is presented. By comparing their consumed RMS currents we defined a coefficient (k[%]), which depends only on the total harmonic distortion of the network voltage (THD_u). We demonstrated that at high THD_u resistive modulation method is more favorable. Then simulation examination is discussed, by presenting our model of the three-phase four quadrant converter. Finally simulation results are represented in this article.

Keywords

converter control · harmonics · modulation strategy · optimal control · power quality · Park vector theory · pulse width modulation (PWM)

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1 Introduction

Nowadays high amount of electrical energy is converted by power electronic devices. Equipments such as variable speed motors, large uninterruptible power supplies (UPS), computers, discharge lamps and bridge rectifiers used in power electronics, are the primary cause of harmonic distortion. Most of these devices contain simple diode rectifiers (Fig. 1a). These rectifiers consume non-sinusoidal current and produce current harmonics (Fig. 1b). Therefore the RMS (root mean square) current load of the AC network substantially grows. Extra losses reduce the electric energy transmission capacity of the network. The harmonic currents generated by the non-linear load, have to flow in the circuit via the source impedance and all other parallel paths. As a result, harmonic voltages appear across the supply impedance and are present throughout the network. Eventually, qualitative parameters of the energy supply are affected: potential difficulties include various kinds of economic and technological problems, breakdowns, switching surges, overheating, and electromagnetic disturbances [2].

Because of the increasing number of consumers polluting the network with harmonics, both the suppliers and the consumers have to face dangerous phenomena. To solve this problem, electric consumers should be equipped with "network-friendly" converters. The most common way of implementation is the use of high frequency pulse width modulated (PWM) converters [2, 3].

2 Network-friendly grid-connected inverter

Several types of grid-connected converters are capable of "network-friendly" operation. Simple converters allow unidirectional power flow. In our study, we used three-phase full-bridge converter connected to the three-phase AC network and containing PWM controlled semiconductor switching elements: IGBTs (Fig. 2).

As this converter is capable of four-quadrant operation, the power flow can be bidirectional between the load (connected to P-N terminals) and the AC network (connected to AC11-AC12-AC13 terminals). If the network supplies power, the converter operates as a "network-friendly" rectifier, in the other case, it operates as a regenerative inverter. In fact, this converter is a

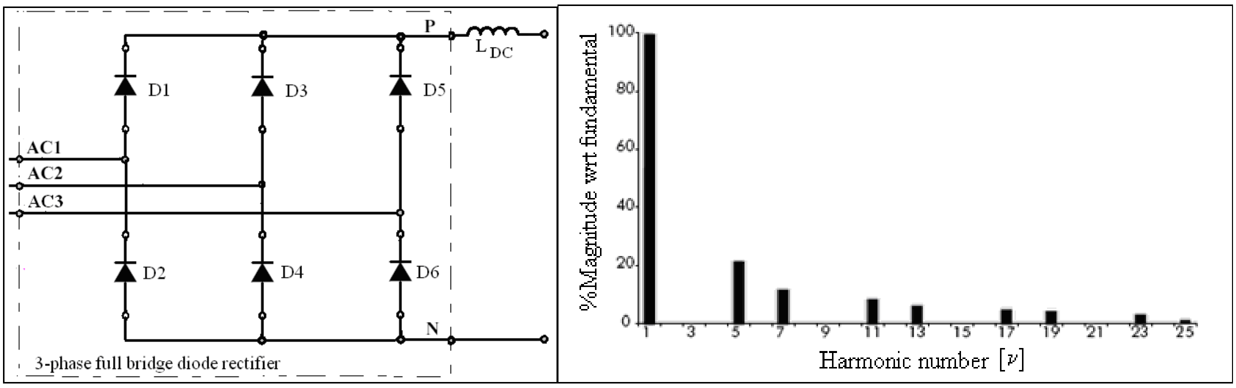

Fig. 1. a. Schematic circuit diagram of a three-phase full bridge rectifier b. 1[|ν| > 1)
Harmonic spectrum (if the overlap is zero and $L_{DC} = \infty$ then $I_\nu = I_1/|\nu| >$

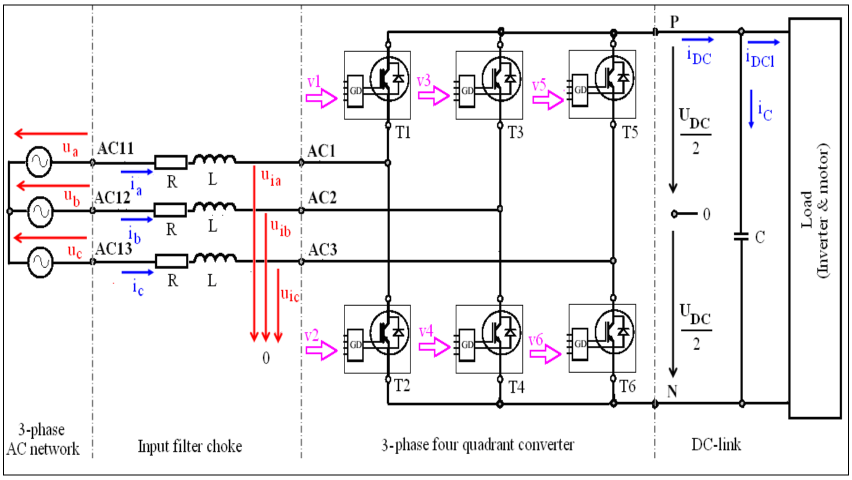

Fig. 2. Schematic circuit diagram of the three-phase grid-connected inverter

two-level inverter that allows switching between seven different voltage vectors on the AC side of the converter [$\overline{u}_I$, $\overline{u}_{II}$, ...$\overline{u}_{VII}$] (Fig. 3). DC load is connected to P-N terminals that could be a voltage source inverter-fed electric motor. This kind of converters is called DC-link frequency converter and can be applied in drives of renewable energy resources [4, 5].

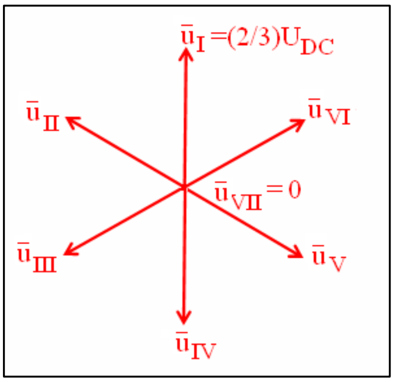

Fig. 3. Voltage vectors

Let us study the case when several other consumers distort the network voltage waveform which feeds the network-friendly converter. At every time instant, if the currents of each phase [$i_a(t), i_b(t), i_c(t)$] are controlled to be proportional to the network voltages instantaneous value [$u_a(t), u_b(t), u_c(t)$], the *u(t)/i(t)* ratios will be constant. It means that the converter behaves like a three-phase resistive load. For the adequate operation, the modulation signals of the control circuit should be proportional to $u_a(t), u_b(t)$ and $u_c(t)$. This is the resistive modulation method. The three-phase values can be transformed to Park vectors, $\overline{i}$ represents the threephase currents [$i_a(t), i_b(t), i_c(t) \Rightarrow \overline{i}(t)$] and $\overline{u}$ the three-phase voltages [$u_a(t), u_b(t), u_c(t) \Rightarrow \overline{u}(t)$]. Therefore at resistive modulation the $\overline{i}$ current vector coincides with $\overline{u}$ voltage vector and the vector amplitudes ratio is constant [*u/i=const*].

There is an other modulation strategy, when the PWM modulation enforces sinusoidal network currents, which are proportional to the fundamental of the distorted network voltage waveforms [$u_{a1}(t), u_{b1}(t), u_{c1}(t)$]. In this case, the converter operates as a three-phase sinusoidal load. The modulation signals of the control circuit should be proportional to $u_{a1}(t), u_{b1}(t)$,and $u_{c1}(t)$. It is called sinusoidal modulation method. If the three-phase values are transformed to Park vectors then at sinusoidal modulation the $=\ _1$ current vector coincides with $\overline{u}_1$ fundamental voltage vector and the vector amplitudes ratio is constant [u_1/i_1=*const*].

The question is which control method is more favorable? To answer this question we compared the sinusoidal and the resistive strategies by examining the root-mean square (RMS) value of the consumed AC network currents [I_{RMS}] in both cases, while keeping constant the DC-side power of the converter. (It is an important requirement of modern "network-friendly" converters to reduce the I_{RMS}) [6, 7].

3 Analytical comparison of sinusoidal and resistive modulation strategies

The distorted three-phase network voltage contains positive, negative and zero sequence harmonics. The AC network voltage waveform (which can be observed at AC11-AC12-AC13 terminals of the grid-connected inverter) does not contain zero sequence harmonics that caused by other non-linear loads, because the neutral point of the DC link capacitor is not connected to the neutral wire. Therefore Park vector transformation can be used for the three-phase time-dependent values.

The distorted AC network voltage waveform of the three-phases and the fundamental components are as follows:

$$u_a(t) = \sum_\nu U_\nu \cos(\nu\omega_1 t + \varphi_\nu), \qquad (1.a)$$

$$u_{a1}(t) = U_1 \cos(\omega_1 t) \qquad (1.b)$$

$$u_b(t) = \sum_\nu U_\nu \cos(\nu\omega_1 t + \varphi_\nu \pm 120°), \qquad (2.a)$$

$$u_{b1}(t) = U_1 \cos(\omega_1 t - 120°) \qquad (2.b)$$

$$u_c(t) = \sum_\nu U_\nu \cos(\nu\omega_1 t + \varphi_\nu \pm 240°), \qquad (3.a)$$

$$u_{c1}(t) = U_1 \cos(\omega_1 t - 240°) \qquad (3.b)$$

(Generally [the three-phase voltage is symmetrical but contains harmonics] ν=1+6k harmonic numbers [for positive sequence harmonics: k=0,+1,+2... and the phase angle of each phase is -120°; for negative sequence harmonics: k=-1,-2... and the phase angle of each phase is +120°], U_ν: harmonic voltage amplitudes, φ_ν: phase angles of harmonics, ω_1=2πf$_1$, f$_1$: fundamental frequency.)

The network voltage Park vector and the fundamental vector can be defined from the three-phase voltage waveforms:

$$\bar{u} = \sum_\nu \bar{U}_\nu e^{j\nu\omega_1 t} = \sum_\nu U_\nu e^{j\varphi_\nu} \cdot e^{j\nu\omega_1 t} = \sum_\nu U_\nu e^{j(\nu\omega_1 t + \varphi_\nu)}, \qquad (4.a)$$

$$\bar{u}_1 = \bar{U}_1 e^{j\omega_1 t} = U_1 \cdot e^{j\omega_1 t} \qquad (4.b)$$

If ν >0 the harmonic vector rotates in the same direction as the fundamental, otherwise the harmonic vector rotates in the other direction (Fig. 4).

By using the *sinusoidal* modulation strategy, on the AC-side the converter enforces current ($\bar{i}_{\sin}$) which is proportional to the instantaneous value of the network fundamental voltage (k_{sin} [A/V]: coefficient). By applying the Park vector of the network fundamental voltage (4b), the current Park vector is:

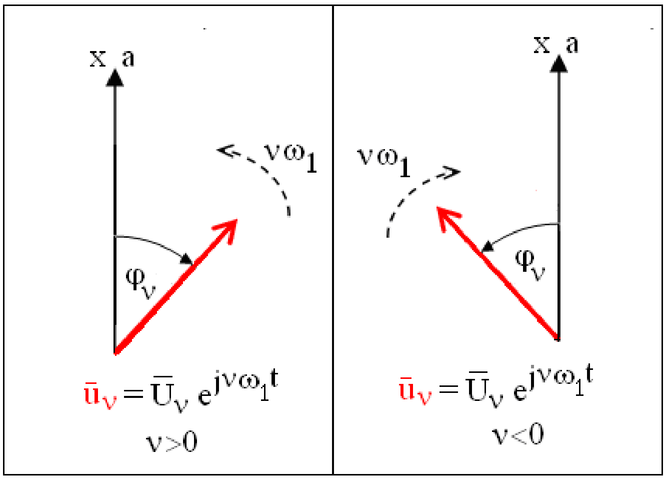

Fig. 4. Harmonic voltage vectors

$$\bar{i}_{\sin} = \bar{i}_1 = k_{\sin} \cdot \bar{u}_1 = k_{\sin} \cdot U_1 \cdot e^{j\omega_1 t}. \qquad (5)$$

If the instantaneous values are equal as in (5), then the previous equation is valid for the amplitudes of the vectors also:

$$I_{1\_\sin} = k_{\sin} \cdot U_1 \Rightarrow U_1 = I_{1-\sin}/k_{\sin} \qquad (6)$$

By using the *resistive* modulation strategy, on the AC-side the converter enforces current ($\bar{i}_{ohm}$) which is proportional to the instantaneous value of the distorted network voltage (k_{ohm} [A/V]: coefficient). By applying the network voltage Park vector (4a), the current Park vector is:

$$\bar{i}_{ohm} = k_{ohm} \cdot \bar{u} = k_{ohm} \cdot \sum_\nu U_\nu \cdot e^{j(\nu\omega_1 t + \varphi_\nu)}. \qquad (7)$$

If the instantaneous values are equal as in (7), then the previous equation is valid for the amplitudes of each harmonic vector also:

$$I_{\nu\_ohm} = k_{ohm} \cdot U_\nu \Rightarrow U_\nu = I_{\nu\_ohm}/k_{ohm} \qquad (8)$$

If the converter operates as a rectifier then k_{sin} >0,k_{ohm} >0 otherwise in inverter state: k_{sin} < 0,k_{ohm} < 0.

We assumed that the consumed powers of the two strategies are equal (at same DC-side load) [$P_{sin} = P_{ohm}$]. It can be computed with the amplitudes:

$$P_{\sin} = \frac{3}{2} U_1 I_{1\_\sin} \stackrel{!}{=} P_{ohm} = \frac{3}{2} \sum_\nu U_\nu I_{\nu\_ohm}. \qquad (9)$$

Substituting (6a) and (8a) into (9):

$$P_{\sin} = \frac{3}{2} k_{\sin} U_1^2 \stackrel{!}{=} P_{ohm} = \frac{3}{2} k_{ohm} \sum_\nu U_\nu^2. \qquad (10)$$

The definition of the Park vector RMS value is:

$$U_{RMS} = \sqrt{\frac{1}{T}\int_T \bar{u} \cdot \hat{u}\, dt} = \sqrt{\frac{1}{T}\int_T |\bar{u}|^2 dt} \qquad (11)$$

where $\hat{u}$ is the complex conjugate of $\overline{u}$.

By using (10) and based on (11), we got:

$$k_{\sin} U_{1RMS}^2 = k_{ohm} U_{RMS}^2. \quad (12)$$

Substituting (6b) and (8b) into (9):

$$P_{\sin} = \frac{3}{2}\frac{I_{1\_\sin}^2}{k_{\sin}} \stackrel{!}{=} P_{ohm} = \frac{3}{2}\frac{1}{k_{ohm}}\sum_{\nu} I_{\nu\_ohm}^2. \quad (13)$$

By using (13) and based on the definition of the RMS, we got:

$$k_{ohm} I_{\sin\_RMS}^2 = k_{\sin} I_{ohm\_RMS}^2. \quad (14)$$

Based on (12) and (14):

$$\frac{k_{\sin}}{k_{ohm}} = \frac{I_{\sin\_RMS}^2}{I_{ohm\_RMS}^2} = \frac{U_{RMS}^2}{U_{1\_RMS}^2} = \frac{U_{1\_RMS}^2 + U_{harm\_RMS}^2}{U_{1\_RMS}^2}. \quad (15)$$

By computing the square root of (15) and based on the definition of the total harmonic distortion ($THD_u = U_{harm\_RMS}/U_{1\_RMS}$), we defined a coefficient $k[\%]$:

$$k[\%] = \frac{I_{\sin\_RMS} - I_{ohm\_RMS}}{I_{ohm\_RMS}} \cdot 100 = \left(\sqrt{1 + THD_u^2} - 1\right) \cdot 100. \quad (16)$$

Fig. 5 represents the relative deviation of the sinusoidal and resistive loads. In the case of sinusoidal modulation the current load of the grid is $(k+1)$-times higher than at resistive modulation.

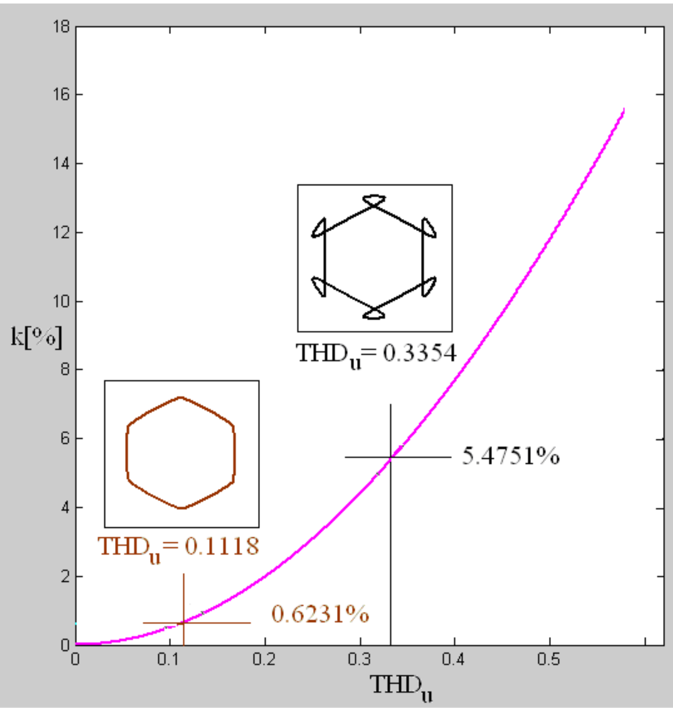

Fig. 5. The relative deviation of sinusoidal and resistive loads (with same voltage vectors)

4 Comparison of sinusoidal and resistive modulation strategies by simulations

To compare the sinusoidal and the resistive modulation strategies we built up a model of a network-friendly three-phase grid-connected inverter in the environment of Matlab Simulink. Our model consists of two main parts: the model of the power electronic circuit and the model of the control circuit. We built up our model by using per-unit quantities.

4.1 Model of the power electronic circuit

We wrote the adequate equations of the power electronic circuit (Fig. 2):

- The equation of the input filter choke:

$$\bar{u}(t) = R \cdot \bar{i}(t) + L\frac{d\bar{i}(t)}{dt} + \bar{u}_i(t)$$

- The power equations that can be written for both sides of the converter (lossless converter assumed):

$p_{AC}(t) = p_{DC}(t) \Rightarrow \bar{u}_i(t) \bullet \bar{i}_i(t) = \frac{2}{3} u_{DC}(t) \cdot i_{DC}(t)$ (where $\bullet$ represents scalar product)

- The current of the DC-link:

$$i_{DC}(t) = i_C(t) + i_{DCl}(t)$$

- The current of the buffer capacitor:

$$i_C(t) = C\frac{du_{DC}(t)}{dt}$$

4.2 Model of the control circuit

To fulfil the requirements of network-friendly equipments, the grid-connected inverter needs to have an adequate control circuit. In our study it was equipped with a cascade control (Fig. 6). The primary control loop of the cascade is a DC-voltage-control (u_{DC}) and the secondary loop is phase currents (i_a, i_b, i_c) control. This structure guarantees a high power factor and an adequate line current shape.

In Fig. 6 notations are as follows:

- $U_{DC\_ref}, i_{a\_ref}, i_{b\_ref}, i_{c\_ref}$: reference signals of U_{DC}, i_a, i_b, i_c
- I_{ampl}: the output of the voltage controller that describes the amplitudes of $i_{a\_ref}, i_{b\_ref}, i_{c\_ref}$
- i_{sa}, i_{sb}, i_{sc} synchronized signals
- The control circuit contains a three-phase pulse width modulator. The control signals of the modulator: $u_{a\_ctrl}, u_{b\_ctrl}, u_{c\_ctrl}$. the carrier wave is a triangle signal (voltage: u_{Δ}, frequency: f_{Δ}).

The equations that describe the control circuit:

- The transfer function of the PI controller that is used for DC voltage control and for current control:

$$Y_{PI} = P + \frac{1}{sT_I}$$

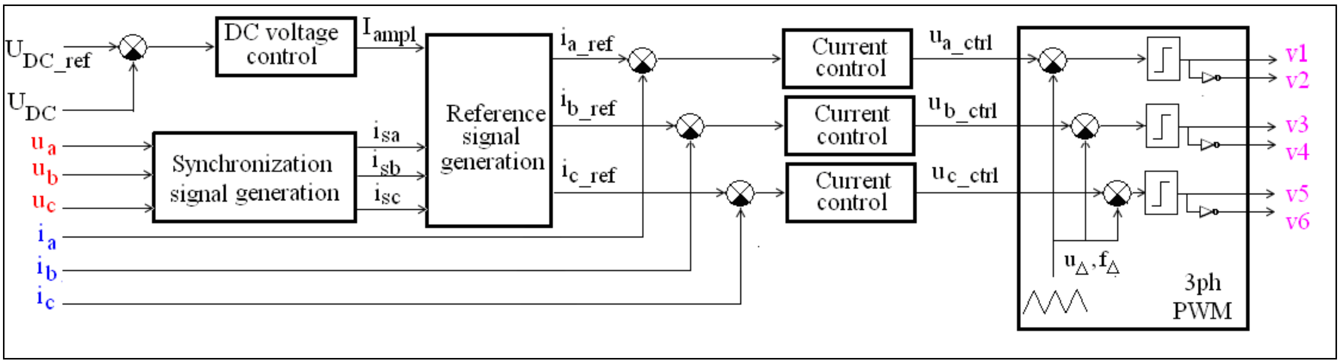

Fig. 6. Control circuit of a network-friendly grid-connected inverter

- The operation of the synchronization signal generator that synchronize the control circuit to the network voltage: (K is the gain of the synchronization signal generator)

resistive:

$$i_{sa} = K \cdot u_a(t) \qquad \text{sinusoidal}: i_{sa} = K \cdot u_{a1}(t)$$
$$i_{sb} = K \cdot u_b(t) \qquad i_{sb} = K \cdot u_{b1}(t)$$
$$i_{sc} = K \cdot u_c(t) \qquad i_{sc} = K \cdot u_{c1}(t)$$

- The reference signal generator produces the reference signals of each phase current:

$$i_{a\_ref} = I_{ampl} \cdot i_{sa}$$
$$i_{b\_ref} = I_{ampl} \cdot i_{sb}$$
$$i_{c\_ref} = I_{ampl} \cdot i_{sc}$$

- Three-phase PWM and inverter (at ideal case *Const* is the gain of the inverter; in our simulations *Const*=1 because per unit quantities are used). Ideal case means that the inverter enables switching infinite voltage vectors. The inverter voltages are defined from the virtual zero point.

ideal case: real case:

$$u_{ia} = Const \cdot u_{a\_ctrl} \qquad u_{ia} = \begin{cases} +u_{DC}/2 & if \quad u_{a\_ctrl} \geq u_\Delta \\ -u_{DC}/2 & if \quad u_{a\_ctrl} < u_\Delta \end{cases}$$

$$u_{ib} = Const \cdot u_{b\_ctrl} \qquad u_{ib} = \begin{cases} +u_{DC}/2 & if \quad u_{b\_ctrl} \geq u_\Delta \\ -u_{DC}/2 & if \quad u_{b\_ctrl} < u_\Delta \end{cases}$$

$$u_{ic} = Const \cdot u_{c\_ctrl} \qquad u_{ic} = \begin{cases} +u_{DC}/2 & if \quad u_{c\_ctrl} \geq u_\Delta \\ -u_{DC}/2 & if \quad u_{c\_ctrl} < u_\Delta \end{cases}$$

4.3 Simulations and results

In our simulations we studied the case when a network-friendly inverter was connected to the three-phase AC network which waveform is distorted by other consumers, e.g. three-phase diode rectifiers (Fig. 1). We assumed that the network voltage waveform contained harmonics of order $\nu = 1, -5, 7$.

We used the same distorted voltage waveforms during the simulations that are represented in this chapter (from Fig. 7 to Fig. 11). The waveforms of the three-phases are as follows:

$$u_a(t) = U_1 \cos(\omega_1 t) + U_{-5} \cos(-5\omega_1 t) + U_7 \cos(7\omega_1 t),$$
$$u_b(t) = U_1 \cos(\omega_1 t - 120^\circ) + U_{-5} \cos(-5\omega_1 t + 120^\circ) + U_7 \cos(7\omega_1 t - 120^\circ),$$
$$u_c(t) = U_1 \cos(\omega_1 t - 240^\circ) + U_{-5} \cos(-5\omega_1 t + 240^\circ) + U_7 \cos(7\omega_1 t - 240^\circ).$$
$$(U_1 = 1, U_{-5} = 0.1, U_7 = -0.05)$$

If these three-phase voltage waveforms are transformed to a Park vector, we get that hexagonal-shape vector which can be seen on Fig. 5. (THD$_u$=0.1118).

The Park vector of the three-phase voltage waveforms (Fig. 7a):

$$\bar{u} = \sum_\nu \bar{U}_\nu \cdot e^{j\nu\omega_1 t} = \sum_\nu U_\nu \cdot e^{j\nu\omega_1 t + \varphi_\nu} =$$
$$= U_1 \cdot e^{j\omega_1 t} + U_{-5} \cdot e^{-j5\omega_1 t} + U_7 \cdot e^{j7\omega_1 t}$$

The voltage vector can be discussed in synchronous rotating reference frame (Fig. 7b)

$$\bar{u}^* = \bar{u} \cdot e^{-j\omega_1 t} = U_1 + \sum_{\nu \neq 1} U_\nu \cdot e^{j(\nu-1)\omega_1 t + \varphi_\nu} =$$
$$= U_1 + U_{-5} \cdot e^{-j6\omega_1 t} + U_7 \cdot e^{j6\omega_1 t}$$

We compared the sinusoidal and the resistive modulation strategies in ideal and real case.

The simulation settings were as follows (per unit quantities were used):

U_{DC_ref}=2, R=0, L=0.05, C=9, Voltage controller: P=5 T_I=3, Current controller: P=1 T_I=0.001, the load was simulated with a DC current source i_{DCl}=0.4.

Ideal case:

Sinusoidal modulation (see Fig. 8)

4.4 Comparison of analytical and simulation results

We compared the analytical and the simulation results by computing the $k[\%]$ coefficient in both cases. After running

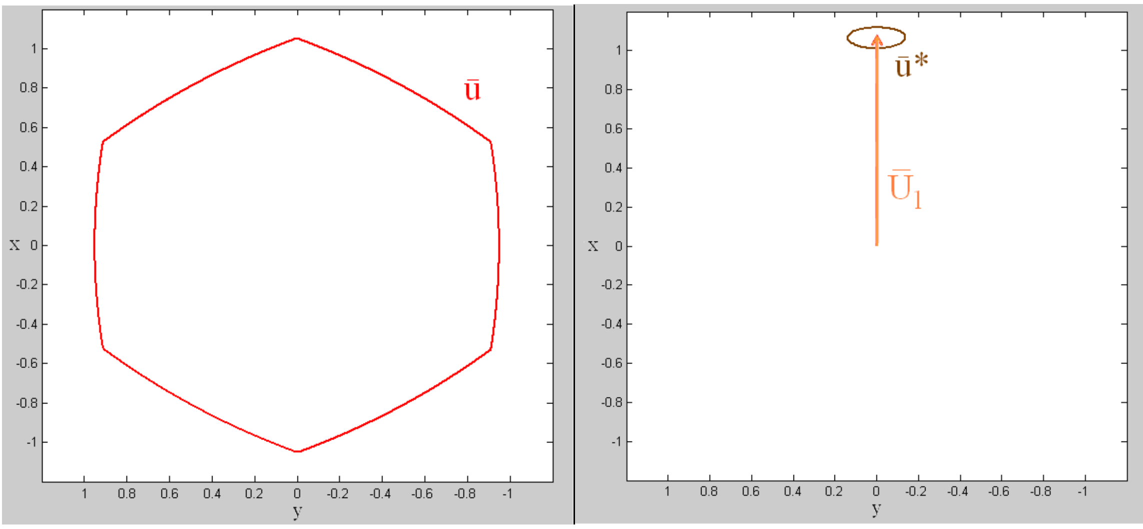

Fig. 7. a. Network voltage Park vector in stationary reference frame b. Network voltage Park vector in synchronous rotating reference frame

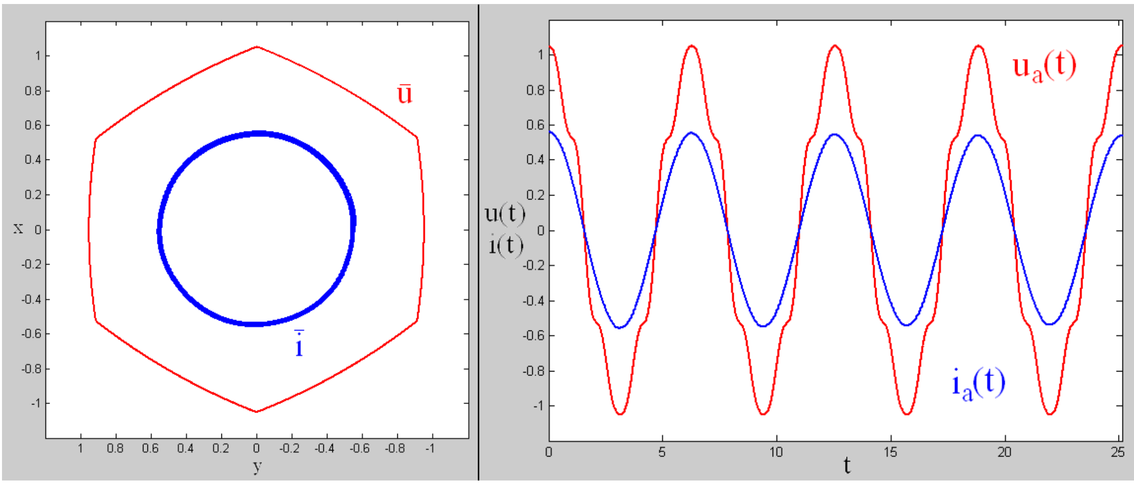

Fig. 8. Ideal case, sinusoidal modulation: network voltage and current vectors, 'a' phase time functions

Resistive modulation:

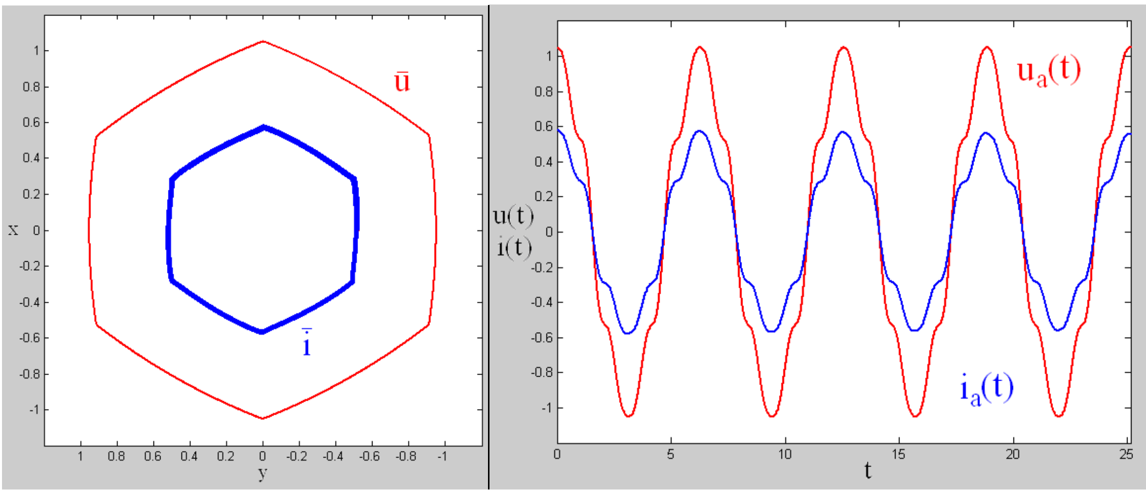

Fig. 9. Ideal case, resistive modulation: network voltage and current vectors, 'a' phase time functions

simulations of the resistive and sinusoidal modulation strategies, I_{sin_RMS} and I_{ohm_RMS} were defined at steady state. Then we calculated *k[%],* based on (16).

It turned out that our simulation gave the same results as the analytical output. Fig. 12 shows the deviation of the analytical (solid lines) and simulation (dashed lines) results at two different points.

5 Conclusions

The main requirement of "network-friendly" converters is to eliminate network current harmonics. Two appropriate modula-

Real case: (f_Δ=2.5kHz)

Sinusoidal modulation:

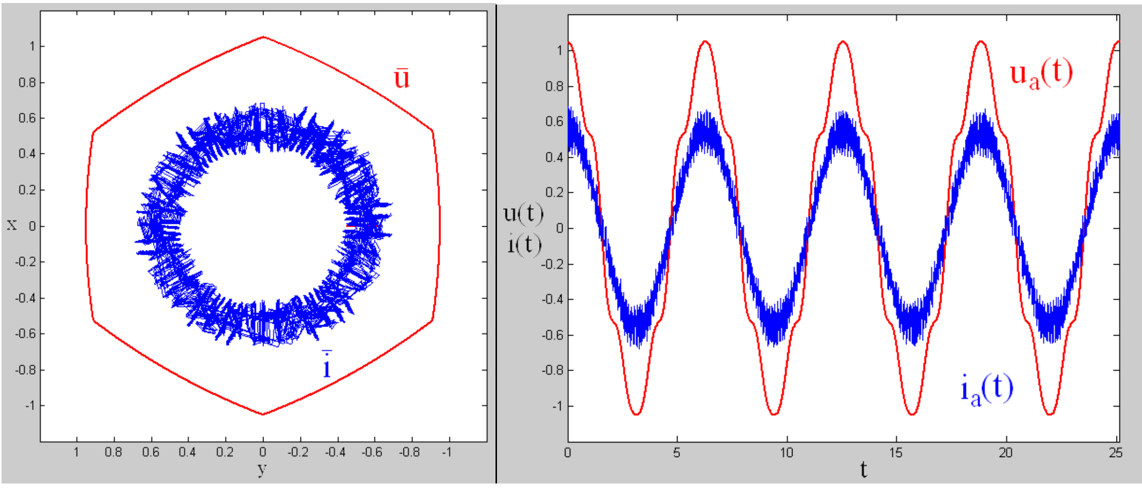

Fig. 10. Real case, sinusoidal modulation: network voltage and current vectors, 'a' phase time functions

Resistive modulation:

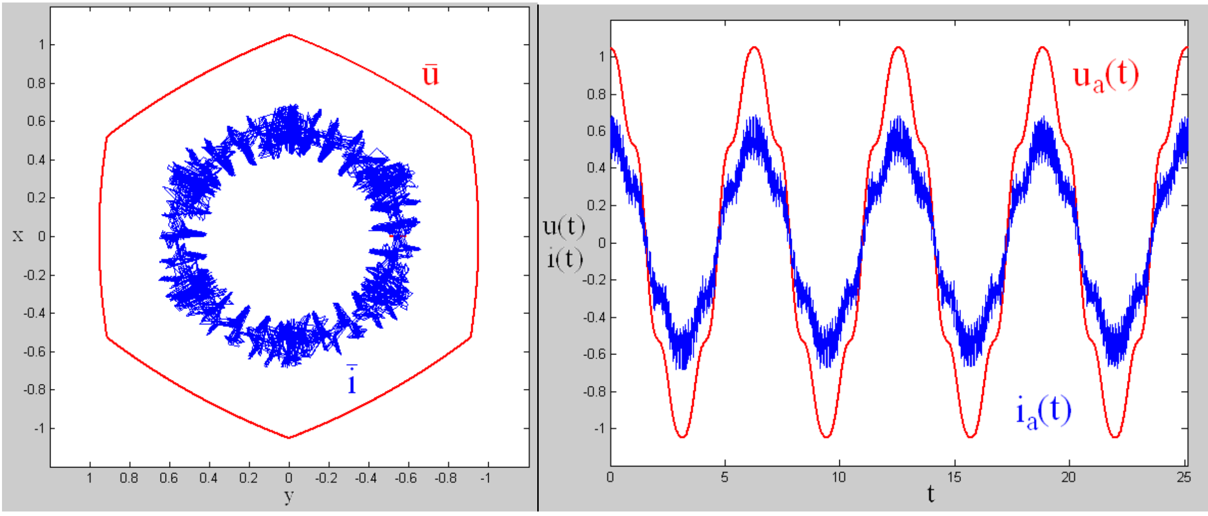

Fig. 11. Real case, resistive modulation: network voltage and current vectors, 'a' phase time functions

tion strategies were demonstrated. The sinusoidal and the resistive modulation methods were compared by analytical and simulation results. Neither the sinusoidal, nor the resistive current load produces additional harmonics to the network.

An increasing proportion of "network-friendly" converters means less harmful network pollution and an improved THD_u value of the network voltage. The waveform approaches the sine wave, the additional current load of the network decreases.

If the network voltage waveform is distorted, the resistive modulation method is more favorable than the sinusoidal. Stark differences can be observed at high THD_u values (Fig. 5).

Actually the harmonic components generate real power for the load if the resistive strategy is used. At sinusoidal strategy, only the fundamental generates real power.

It is easier to build control electronics in the case of resistive modulation, because we do not need to know the fundamental frequency of the network voltage. It is enough to map the network voltage waveform to obtain an adequate modulation signal.

On the other hand, reactive current control can be appropriately realized by sinusoidal modulation.

The grid-connected inverter can be equipped with a LC filter that tuned to the switching frequency of the IGBTs. This paper is not dealing with it but in the future we would like to observe the effect of this filter.

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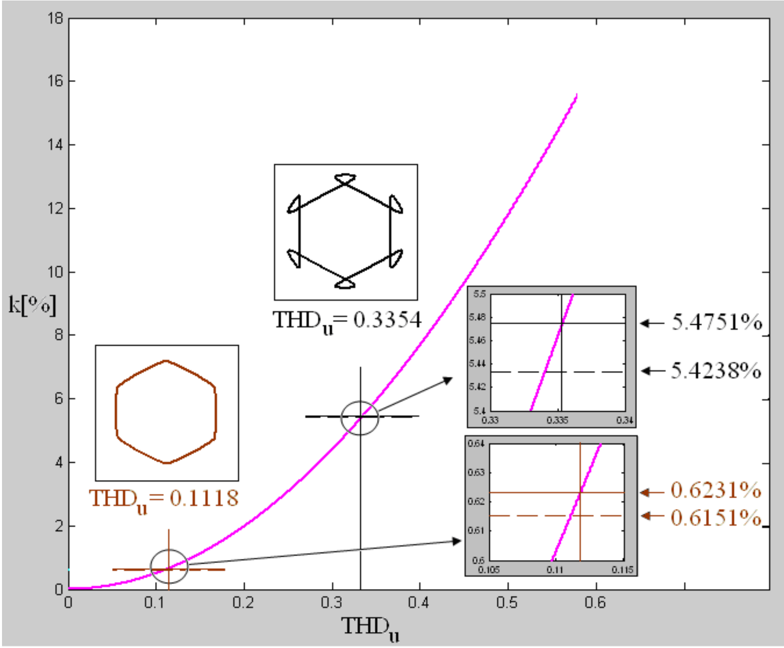

Fig. 12. Comparison of analytical results (solid lines) and simulation results at ideal case (dashed lines)

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