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RESEARCH ARTICLE

Noninteracting control of a steering system

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Abstract

This paper describes the modeling and control of a novel steering system which makes it possible to achieve a steer-by-wire like operation with the maintenance of the mechanical contact between the steering wheel and the steered wheels. First the derivation of the dynamical model is given where linear and nonlinear holonomic constraints are introduced by two elements of the steering system, namely by a harmonic drive and a universal joint. The flatness property of the nonlinear model is shown, but, since the controlled variables are not the linearizing outputs, we give another noninteracting control algorithm for the linearized model and show the tolerance of the closed-loop performance to nonlinearities.

Keywords

electric power assisted steering system · underactuated mechanical system · noninteracting control

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1 Introduction

Up till now, power steering systems have spread in automotive industry in order to reduce the steering effort by using an external power source. The traditional power steering systems are hydraulic systems equipped with an engine-driven pump and are called *hydraulic power assisted steering systems* (HPAS). Newer solutions provide the hydraulic pressure with the help of a pump that is driven by an electric motor. These systems are called *electro-hydraulic power assisted steering systems* (EHPAS). The major advantage of EHPAS systems over the conventional hydraulic systems is that the direct influence on the hydraulic pressure enables a more sophisticated assist strategy. Obviously, the direct use of an electrical motor coupled to the rack or to the steering column allows an even more flexible assistance. Another distinct advantage of these so-called *electric power assisted steering systems* (EPAS) lies in their fuel efficiency. HPAS and EHPAS systems both have to maintain a pressure in the hydraulics constantly but EPAS systems consume energy only during power assistance. On the other hand, EPAS systems increase the energy demand from the electrical network of the car and therefore the size of vehicles where they can be used is limited. The possibilities provided by EPAS systems enable the implementation of numerous unprecedented features (such as variable assistance depending on the driving conditions or choice between different boost curves) that offer even more comfort, security, and convenience for the driver.

The intense competition in the automobile industry enforces a continuous expansion of functionalities. Such a demand motivated the setup presented in this paper. The basic idea is not new: we introduce an extra degree of freedom to allow the independent rotation of the steered wheels and the steering wheel. The result is similar to the *steer-by-wire* systems but it preserves the mechanical contact to enable a safer implementation in road vehicles. The schematic view of the steering system is presented in Fig. 1. The five rotating shafts are connected by a harmonic drive, a universal joint, and a spring. We denote in the sequel the angular position of the flexspline, wave generator, circular spline, Cardan joint and the rack axes by φ_{fs}, φ_{wg}, φ_{cs}, φ_{cj} and φ_{rk}, respectively, and the same subscripts will be used for other

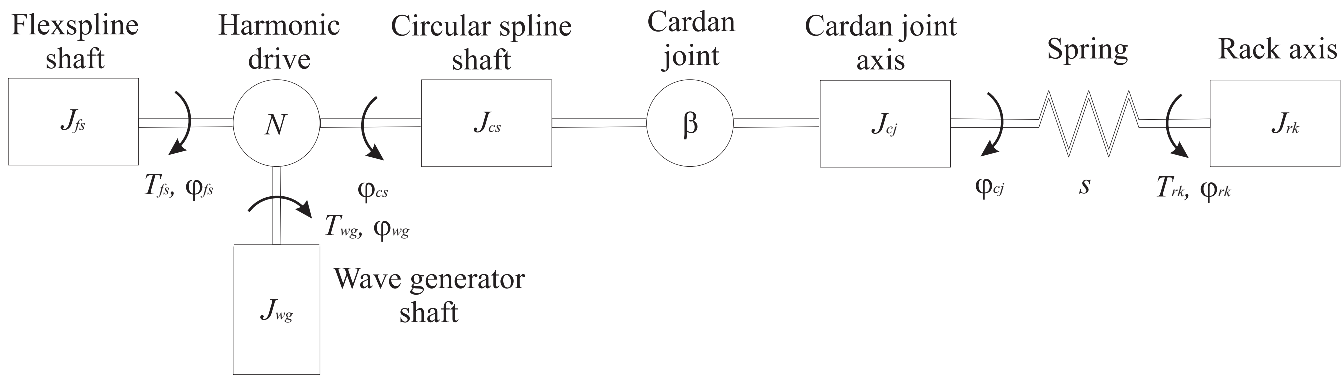


Fig. 1. Schematic view of the steering system

parameters and variables, as well. The harmonic drive and the universal joint both introduce a holonomic constraint. The constraint introduced by the harmonic drive unit reads

$$\varphi_{wg} - (i+1)\varphi_{cs} + i\varphi_{fs} + \varphi_0 = 0, \qquad (1)$$

where the constant i is the transmission ratio of the harmonic drive and the constant φ_0 can introduce offset to the position variables e.g. in the case of relative angle measurement devices such as optical encoders. We will suppose $\varphi_0 = 0$ that can be ensured at the initialization of the sensor devices. The relationship between the shafts of the universal joint can be described with the following equation:

$$\tan\varphi_{cj} = C\tan\left(\varphi_{cs} + \varphi_{cs,0}\right), \qquad (2)$$

where $C = \cos\beta$, is the cosine of the angle between the shafts of the universal joint, which will be supposed to be constant during the operation. The role of $\varphi_{cs,0}$ is similar to it of φ_0 and again without the loss of generality it will be set to zero. For values where the tangent function is not defined, the reciprocal expression is to be considered:

$$\cot\varphi_{cj} = \frac{1}{C}\cot\left(\varphi_{cs} + \varphi_{cs,0}\right). \qquad (3)$$

We will derive the dynamical model of the steering system with the Lagrangian method. In Lagrangian mechanics, holonomic constraints that do no virtual work may be used to reduce the number of variables. In our case this means that the number of variables decreases from 5 to 3 thanks to (1) and (2). The most natural way to eliminate the redundant variables is to express them as a function of the other variables. The elimination procedure will not be used here since the reduced Lagrangian is algebraically complicated because of (2) or (3). We will introduce the constraints with constraint forces and use the null space of the constraint matrix in the Pfaffian form for reduction, in the next section.

In Section 3, we present a noninteracting control that decouples two input-output channels and destroys the cross effects. In this decoupled system, we implement a position control-loop for the rack axis and the torque on the flexspline axis will be regulated in open loop. With this structure it is possible to achieve a steer-by-wire like operation since the flexspline axis is the axis of the steering wheel. We do not discuss the issue of reference signals but it is clear that, for a conventional operation, the rack position (the steering angle) will depend on the angle of the flexspline shaft where the steering wheel is mounted on. The torque on the flexspline shaft will realize a 'force feedback' for the driver, an additive torque term. The closed-loop performance is analyzed with simulations in Section 4 and the results of the research are summarized in Section 5.

2 Dynamical model

Holonomic constraints are used for the introduction of constraint forces. In special cases, with help of these equations with constraint forces, a minimal set of equations can be obtained for a reduced set of generalized coordinates. In this section, we will describe this method and apply it to our system.

2.1 The elimination of constraint forces

The equations of motion of a constrained system read

$$\frac{d}{dt}\frac{\partial L(q,\dot q)}{\partial \dot q} - \frac{\partial L(q,\dot q)}{\partial q} = \left[\frac{\partial c(q)}{\partial q}\right]^T \lambda + F(q)\tau, \qquad (4)$$

subject to

$$c(q) = 0, \qquad (5)$$

where q, λ and τ stand for the n-dimensional vector of the generalized coordinates, the m-dimensional ($n > m$) vector of the constraint forces and the p-dimensional ($n \geq p$) vector of the generalized forces, respectively. The $n \times p$ dimensional matrix $F(q)$ is referred to as the input mapping matrix. The coefficient matrix of the vector of constraint forces is the transpose of the Jacobian matrix of the m-dimensional vector of (independent) holonomic constraints (5) with respect to the generalized coordinates (henceforth referred to as $J(q)$). The Lagrangian L is defined as the difference between the kinetic and the potential energy:

$$L(q,\dot q) = K(q,\dot q) - V(q). \qquad (6)$$

Let us consider the time derivative of (5):

$$\frac{d}{dt}c(q) = \frac{\partial c(q)}{\partial q}\dot q = J(q)\dot q = 0. \qquad (7)$$

Note that the generalized velocity must lie in the null space of $J(q)$ to fulfil (7). Let $N(q)$ be an n-by-$(n-m)$ matrix with columns that are a maximal set of linearly independent vector fields in this null space. Now we give two identities to be reused in the sequel:

$$J(q)N(q) = 0, \tag{8}$$

$$\dot{q} = N(q)v. \tag{9}$$

Eq. (9) expresses that the generalized velocities can be determined with a reduced set of variables i.e. the $(n-m)$ coordinates for a basis in the null space of $J(q)$. For further use, let us derive its time derivative:

$$\ddot{q} = N(q)\dot{v} + \dot{N}(q)v. \tag{10}$$

Usually, (4) is written in the following form:

$$H(q)\ddot{q} + h(q,\dot{q}) = J^T\lambda + F(q)\tau. \tag{11}$$

where $H(q)$ is the n-by-n symmetric and positive definite inertia matrix and $h(q,\dot{q})$ is the n-dimensional vector of the Coriolis and centrifugal force terms that are quadratic in velocity, the potential (gravity, spring force) and possibly frictional force terms. The transpose of (8) implies that the pre-multiplication of (11) by $N^T(q)$ eliminates λ and also reduces the number of equations by the number of the constraints (i.e. by $m = \dim\lambda$). After substitution of the expressions for $\dot{q}$ and $\ddot{q}$ from (9) and (10) into (11) and pre-multiplication by N^T, we have the reduced set of equations

$$H_r(q)\dot{v} + h_r(q,v) = F_r(q)\tau, \tag{12}$$

where

$$H_r(q) = N^T(q)H(q)N(q), \tag{13a}$$

$$h_r(q,v) = N^T(q)H(q)\dot{N}^T(q)v + N^T(q)h(q,N(q)v), \tag{13b}$$

$$F_r(q) = N^T(q)F(q). \tag{13c}$$

Eqs. (12) and (9) are a set of first order ordinary differential equations (i.e. state equations because of the invertability of $H_r(q)$) appropriate to describe the dynamical behaviour of the constrained system without constraint forces. Nevertheless, the factors $H_r(q)$, $F_r(q)$ and the term $h_r(q,v)$ do not always depend on the whole set of the generalized coordinates q and if so, we do not need to take the whole set of (9).

If the time derivatives of the generalized coordinates, which are the arguments in (12) and (9) can be chosen as a subset of coordinates in the basis of the null space of $J(q)$, the following notation can be used:

$$\dot{q} = N(q_r)\dot{q}_r, \tag{14}$$

where q_r stands for the coordinates according to the basis in the null space. Then the compact set of equations of motion reads

$$H_c(q_r)\ddot{q}_r + h_c(q_r,\dot{q}_r) = F_c(q_r)\tau. \tag{15}$$

2.2 Dynamical model of the steering system

Consider the steering system depicted in Fig. 1. The kinetic energy is due to the rotation of the inertial bodies and the potential energy is stored in the spring. Accordingly, the Lagrangian of the system without constraints expressed in generalized coordinates $q = (\varphi_{fs}, \varphi_{cs}, \varphi_{wg}, \varphi_{cj}, \varphi_{rk})^T$ reads

$$L(q,\dot{q}) = \frac{1}{2}\sum_k J_k\dot{\varphi}_k^2 - \frac{1}{2}s\left(\varphi_{cj} - \varphi_{rk}\right)^2, \tag{16}$$

where $k \in \{fs, cs, wg, cj, rk\}$ and s is the stiffness of the spring. Consequently, we obtain the left-hand side of (4) in the following form:

$$\frac{d}{dt}\frac{\partial L(q,\dot{q})}{\partial\dot{q}} - \frac{\partial L(q,\dot{q})}{\partial q} = H\ddot{q} + Sq, \tag{17}$$

with

$$H = \operatorname{diag}\left(J_{fs}, J_{cs}, J_{wg}, J_{cj}, J_{rk}\right), \tag{18a}$$

$$S = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & s & -s \\ 0 & 0 & 0 & -s & s \end{bmatrix}. \tag{18b}$$

On the right-hand side of (4), the generalized force term τ will be decomposed into the terms of the friction and the external torques. The external torques are supposed to act on the flexspline and wave generator shafts and the rack axis. The friction is modeled by viscous and Coulomb friction terms. The damping of the spring (d_s) will be considered in the viscous friction term. Now the vector of the generalized torques reads

$$\tau = F_x T - D_v\dot{q} - D_c\operatorname{sgn}\dot{q}, \tag{19}$$

where $T = (T_{fs}, T_{wg}, T_{rk})^T$ and

$$F_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{20a}$$

$$D_v = \begin{bmatrix} d_{v,fs} & 0 & 0 & 0 & 0 \\ 0 & d_{v,cs} & 0 & 0 & 0 \\ 0 & 0 & d_{v,wg} & 0 & 0 \\ 0 & 0 & 0 & d_{v,cj}+d_s & -d_s \\ 0 & 0 & 0 & -d_s & d_{v,rk}+d_s \end{bmatrix}, \tag{20b}$$

$$D_c = \operatorname{diag}\left(d_{c,fs}, d_{c,cs}, d_{c,wg}, d_{c,cj}, d_{c,rk}\right). \tag{20c}$$

To derive the Jacobian matrix for the constraints (1) and (2) or (3) we rewrite the latter two in the following forms ($\varphi_{cs,0} = 0$):

$$\varphi_{cj} - \arctan\left(C\tan\varphi_{cs}\right) = 0,$$

$$\varphi_{cj} - \operatorname{arccot}\left(\frac{1}{C}\cot\varphi_{cs}\right) = 0. \tag{21}$$

For both possible pairs of constraints (1) and (2) or (1) and (3) the Jacobian matrix has the same form of (22) and is defined on the whole domain of the variables because $0 < C \le 1$ is reasonable ($|\beta| < \pi/2$). The Jacobian depends only on φ_{cj}:

$$J(\varphi_{cj}) = \begin{bmatrix} i & -(i+1) & 1 & 0 & 0 \\ 0 & 1 & 0 & -\frac{C}{\sin^2\varphi_{cj}+C^2\cos^2\varphi_{cj}} & 0 \end{bmatrix}. \quad (22)$$

Let us express the remaining coefficient terms of (11) since H, F_x and $J(\varphi_{cj})$ are already expressed. Observe that H and F_x are constant. The term $h(q,\dot{q})$ is the sum of the friction and spring effects:

$$h(q,\dot{q}) = Sq + D_\upsilon \dot{q} + D_c \operatorname{sgn} \dot{q}. \quad (23)$$

It is easy to prove that the following vectors build a complete basis for the nullspace of $J(\varphi_{cj})$:

$$n_1 = \begin{pmatrix} 0\\0\\0\\0\\1 \end{pmatrix}, \quad n_2 = \begin{pmatrix} 1\\0\\-i\\0\\0 \end{pmatrix}, \quad n_3 = \begin{pmatrix} 0\\ \frac{C}{\sin^2\varphi_{cj}+C^2\cos^2\varphi_{cj}} \\ \frac{(i+1)C}{\sin^2\varphi_{cj}+C^2\cos^2\varphi_{cj}} \\ 1 \\ 0 \end{pmatrix}.$$

We define the matrix $N(\varphi_{cj}) = [n_1, n_2, n_3]$. Its substitution into (9) implies $\upsilon = (\dot{\varphi}_{rk}, \dot{\varphi}_{fs}, \dot{\varphi}_{cj})^T$. Note that all conditions are satisfied to build the reduced equations of motion in form of (15). We define the vector of the reduced generalized coordinates as $q_r = (\varphi_{rk}, \varphi_{fs}, \varphi_{cj})^T$ and $\dot{q}_r = \upsilon$. Consequently, the equations of motion of the steering system read

$$H_c(q_r)\ddot{q}_r + h_c(q_r,\dot{q}_r) = F_c(q_r)T, \quad (24)$$

where

$$H_c(q_r) = N^T(\varphi_{cj})HN(\varphi_{cj}), \quad (25a)$$

$$h_c(q_r,\dot{q}_r) = N^T(\varphi_{cj})H\dot{N}(\varphi_{cj})\dot{q}_r + N^T(\varphi_{cj})D_\upsilon N(\varphi_{cj})\dot{q}_r + \\ + N^T(\varphi_{cj})D_c \operatorname{sgn}(N(\varphi_{cj})\dot{q}_r) + N^T(\varphi_{cj})SPq_r, \quad (25b)$$

$$F_c(q_r) = N^T(\varphi_{cj})F_x, \quad (25c)$$

with

$$P = \begin{bmatrix} 0&1&0\\0&0&0\\0&0&0\\0&0&1\\1&0&0 \end{bmatrix}. \quad (26)$$

For brevity, we will not indicate the variable dependences of expressions from now on and so the matrix coefficients H_c, F_c in (24) read

$$H_c = \begin{bmatrix} J_{rk} & 0 & 0 \\ 0 & J_{fs}+i^2J_{wg} & -\frac{C}{R}i(i+1)J_{wg} \\ 0 & -\frac{C}{R}i(i+1)J_{wg} & J_{cj}+\frac{C^2}{R^2}\left(J_{cs}+(i+1)^2J_{wg}\right) \end{bmatrix}, \quad (27)$$

$$F_c = \begin{bmatrix} 0 & 0 & 1 \\ 1 & -i & 0 \\ 0 & \frac{C}{R}(i+1) & 0 \end{bmatrix}, \quad (28)$$

where

$$R = \sin^2(\varphi_{cj}) + C^2\cos^2(\varphi_{cj}). \quad (29)$$

The three elements of the vector h_c in (24) are as follows:

$$h_{c,1} = d_{\upsilon,rk}\dot{\varphi}_{rk} + d_{c,rk}\operatorname{sgn}\dot{\varphi}_{rk} + d_s\left(\dot{\varphi}_{rk}-\dot{\varphi}_{cj}\right) + s\left(\varphi_{rk}-\varphi_{cj}\right), \quad (30a)$$

$$h_{c,2} = -\frac{C(C^2-1)}{R^2}\sin(2\varphi_{cj})i(i+1)J_{wg}\dot{\varphi}_{cj}^2 - \\ -\frac{C}{R}i(i+1)d_{\upsilon,wg}\dot{\varphi}_{cj} + (d_{\upsilon,fs}+i^2d_{\upsilon,wg})\dot{\varphi}_{fs} + \\ + d_{c,fs}\operatorname{sgn}\dot{\varphi}_{fs} - i d_{c,wg}\operatorname{sgn}\left(-i\dot{\varphi}_{fs}+\frac{C}{R}(i+1)\dot{\varphi}_{cj}\right), \quad (30b)$$

$$h_{c,3} = \frac{C^2(C^2-1)}{R^3}\sin(2\varphi_{cj})\left(J_{cs}+(i+1)^2J_{wg}\right)\dot{\varphi}_{cj}^2 + \\ + \left(d_{\upsilon,cj}+\frac{C^2}{R^2}\left(d_{\upsilon,cs}+(i+1)^2d_{\upsilon,wg}\right)\right)\dot{\varphi}_{cj} - \\ -\frac{C}{R}i(i+1)d_{\upsilon,wg}\dot{\varphi}_{fs} + \left(d_{c,cj}+\frac{C}{R}d_{c,cs}\right)\operatorname{sgn}\dot{\varphi}_{cj} + \\ \frac{C}{R}(i+1)d_{c,wg}\operatorname{sgn}\left(-i\dot{\varphi}_{fs}+\frac{C}{R}(i+1)\dot{\varphi}_{cj}\right) + \\ + d_s\left(\dot{\varphi}_{cj}-\dot{\varphi}_{rk}\right) + s\left(\varphi_{cj}-\varphi_{rk}\right). \quad (30c)$$

In (30c) we used that for $0 < C \le 1$ (or equivalently $|\beta| < \pi/2$): $\operatorname{sgn}(\frac{C}{R}\dot{\varphi}_{cj}) = \operatorname{sgn}\dot{\varphi}_{cj}$.

3 Noninteracting control

We understand by noninteracting control a feedback control where the closed loop system has the form of a set of dynamical SISO systems without cross or coupling effects.

Our physical setup is actuated by two motors on the wave generator (T_{wg}) and on the rack (T_{rk}) axes, respectively. Hence the input mapping matrix for the control input is built up of the last two columns of F_c. An outer loop namely the control loop of the driver actuates on the flexspline axis and accordingly its input mapping matrix is the first column of F_c. Our control loop with actuators on the wave generator and the rack axes has to achieve the control objectives described in Section 1 and the driver's loop is practically a position control on the flexspline axis, i.e. the steering wheel (see Fig. 2).

3.1 Nonlinear control

To show that this nonlinear system admits noninteracting control, it is sufficient to show its flatness property or, in other terms, to find outputs with trivial zero dynamics.

Theorem 1 *The system defined by* (24) *is differentially flat with flat output* $y = (\varphi_{fs}, \varphi_{cj})$.

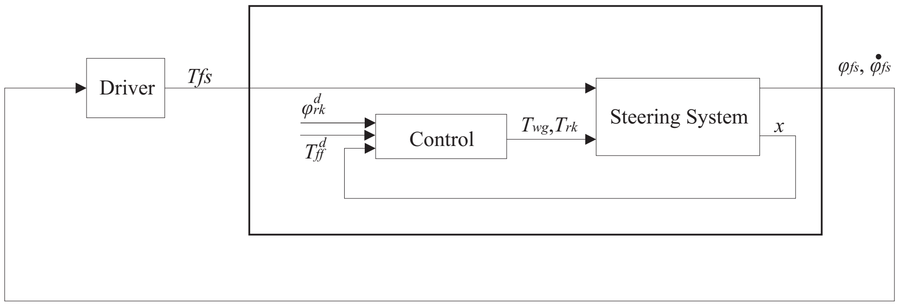


Fig. 2. Block diagram of the control structure (x stands for the state vector of the steering system)

Proof To prove the theorem, one needs to show that all variables, namely φ_{rk}, T_{wg} and T_{rk}, can be calculated as functions of φ_{fs}, φ_{cj} and finite number of their time derivatives. The second equation of (24) (with H_c, F_c and h_c as in (27), (28) and (30)) allows to calculate T_{wg} as a function of φ_{cj}, $\dot{\varphi}_{cj}$, $\ddot{\varphi}_{cj}$, $\dot{\varphi}_{fs}$ and $\ddot{\varphi}_{fs}$. Then the last equation of (24) allows to determine φ_{rk} as a function of the same arguments as before. The first equation of (24) makes it possible to express T_{rk} as a function of φ_{cj}, $\dot{\varphi}_{cj}$, $\ddot{\varphi}_{cj}$, $\varphi_{cj}^{(3)}$, $\varphi_{cj}^{(4)}$ and $\dot{\varphi}_{fs}$, $\ddot{\varphi}_{fs}$, $\varphi_{fs}^{(3)}$, $\varphi_{fs}^{(4)}$.

Nevertheless, our aim is to control the motion of the flexspline and the rack axes but they (φ_{rk}, φ_{fs}) are not the variables in the flat output. This fact suggests to simplify the model with special assumptions for the controller synthesis.

3.2 Linear control

The Cardan joint angle β is not known in practice. Usually it is a relatively small angle and its cosine C is near to 1. Accordingly, we assume $C \equiv 1$ that implies $R \equiv 1$ according to (29), as well. We also assume negligible Coulomb friction because of small Coulomb frictional coefficients and so obtain the following linear model:

$$H_l \ddot{q}_r + D_l \dot{q}_r + S_l q_r = F_l T, \tag{31}$$

where

$$H_l = \begin{bmatrix} J_{rk} & 0 & 0 \\ 0 & J_{fs} + i^2 J_{wg} & -i(i+1)J_{wg} \\ 0 & -i(i+1)J_{wg} & J_{cj} + J_{cs} + (i+1)^2 J_{wg} \end{bmatrix}, \tag{32a}$$

$$D_l = \begin{bmatrix} d_{v,rk} + d_s & 0 & -d_s \\ 0 & d_{v,fs} + i^2 d_{v,wg} & -i(i+1)d_{v,wg} \\ -d_s & -i(i+1)d_{v,wg} & d_{cj,cj} \end{bmatrix}, \tag{32b}$$

$$d_{cj,cj} = d_{v,cs} + d_{v,cj} + (i+1)^2 d_{v,wg} + d_s, \tag{32c}$$

$$S_l = \begin{bmatrix} s & 0 & -s \\ 0 & 0 & 0 \\ -s & 0 & s \end{bmatrix}, \qquad F_l = \begin{bmatrix} 0 & 0 & 1 \\ 1 & -i & 0 \\ 0 & i+1 & 0 \end{bmatrix}. \tag{32d}$$

With use of the last two inputs (T_{wg}, T_{rk}) we derive the required control algorithm. The decoupling control of similar linear systems are presented in [4,5], which are based on the theory introduced in [1,2]. They achieve the noninteracting control of the rack and the flexspline axes by state feedback but do not allow an outer control loop with the flexspline torque as an additional input. Now we present the control algorithm that can be applied to the steering system.

The first equation in (31) reads

$$J_{rk}\ddot{\varphi}_{rk} + \left(d_{v,rk} + d_s\right)\dot{\varphi}_{rk} - d_s\dot{\varphi}_{cj} + s\left(\varphi_{rk} - \varphi_{cj}\right) = T_{rk}. \tag{33}$$

The feedback

$$T_{rk} = J_{rk}u_1 + \left(d_{v,rk} + d_s\right)\dot{\varphi}_{rk} - d_s\dot{\varphi}_{cj} + s\left(\varphi_{rk} - \varphi_{cj}\right) \tag{34}$$

results in the double integrator

$$\ddot{\varphi}_{rk} = u_1. \tag{35}$$

For its (position) control a PID controller seems to be adequate.

For underactuated systems, Spong [6,7] suggested the use of a set of equations of motion with number of the degree of underactuation for expressing the second derivatives of the same number of variables. This gives the clue to express $\ddot{\varphi}_{cj}$ from the last equation in (31):

$$\begin{aligned}\ddot{\varphi}_{cj} = i(i+1)\frac{J_{wg}}{\tilde{J}_{cj}}\ddot{\varphi}_{fs} + \frac{d_s}{\tilde{J}_{cj}}\dot{\varphi}_{rk} + i(i+1)\frac{d_{v,wg}}{\tilde{J}_{cj}}\dot{\varphi}_{fs} - \\ - \frac{d_{cj,cj}}{\tilde{J}_{cj}}\dot{\varphi}_{cj} + \frac{s}{\tilde{J}_{cj}}\left(\varphi_{rk} - \varphi_{cj}\right) + (i+1)\frac{1}{\tilde{J}_{cj}}T_{wg},\end{aligned} \tag{36}$$

where

$$\tilde{J}_{cj} = J_{cj} + J_{cs} + (i+1)^2 J_{wg}. \tag{37}$$

We substitute (36) into the second equation of (31) and after a long but straightforward calculation we obtain:

$$\begin{aligned}\tilde{J}_{fs}\ddot{\varphi}_{fs} + i(i+1)\frac{J_{wg}}{\tilde{J}_{cj}}d_s\dot{\varphi}_{rk} + \left(d_{v,fs} + \frac{J_{cj} + J_{cs}}{\tilde{J}_{cj}}i^2 d_{v,wg}\right)\dot{\varphi}_{fs} + \\ + i(i+1)\left(\frac{J_{wg}}{\tilde{J}_{cj}}d_{cj,cj} - d_{v,wg}\right)\dot{\varphi}_{cj} + i(i+1)\frac{J_{wg}}{\tilde{J}_{cj}}s\left(\varphi_{cj} - \varphi_{rk}\right) = \\ = T_{fs} - \frac{J_{cj} + J_{cs}}{\tilde{J}_{cj}}iT_{wg},\end{aligned} \tag{38}$$

with

$$\tilde{J}_{fs} = J_{fs} + \frac{J_{cj} + J_{cs}}{\tilde{J}_{cj}} i^2 J_{wg}. \tag{39}$$

We apply to (38) the following feedback:

$$T_{wg} = -(i+1)\frac{\tilde{J}_{cj}}{J_{cj}+J_{cs}}\left\{\frac{1}{i(i+1)}u_2 + \frac{J_{wg}}{\tilde{J}_{cj}}d_s\dot{\varphi}_{rk} + \right.$$
$$+\frac{1}{i(i+1)}\left(d_{v,fs} + \frac{J_{cj}+J_{cs}}{\tilde{J}_{cj}}i^2 d_{v,wg}\right)\dot{\varphi}_{fs} +$$
$$\left. + \left(\frac{J_{wg}}{\tilde{J}_{cj}}d_{cj,cj} - d_{v,wg}\right)\dot{\varphi}_{cj} + \frac{J_{wg}}{\tilde{J}_{cj}}s\left(\varphi_{cj} - \varphi_{rk}\right)\right\} \tag{40}$$

and obtain:

$$\tilde{J}_{fs}\ddot{\varphi}_{fs} = u_2 + T_{fs}. \tag{41}$$

For the driver's position control loop, the effective inertia of the steering wheel is $\tilde{J}_{fs}$. Notice that the 'force feedback' described in Section 1 as one objective of our control system is realized with input u_2 of the decoupled system. The driver sees this feedforward term as a disturbing torque but it always is determined by us. We can also see u_2 as a tool to modify the inertia seen by the driver when trying to steer the car.

These results can be substituted back into (36) and we obtain:

$$\tilde{J}_{cj}\ddot{\varphi}_{cj} + \tilde{d}_{cj,rk}\dot{\varphi}_{rk} + \tilde{d}_{cj,fs}\dot{\varphi}_{fs} + \tilde{d}_{cj,cj}\dot{\varphi}_{cj} + \tilde{s}\left(\varphi_{cj} - \varphi_{rk}\right) = \tilde{f}_2 u_2 + \tilde{f}_{fs}T_{fs}, \tag{42}$$

where

$$\tilde{d}_{cj,rk} = (i+1)^2\frac{J_{wg}}{J_{cj}+J_{cs}}d_s, \tag{43a}$$

$$\tilde{d}_{cj,fs} = \frac{i+1}{i}\left(\frac{\tilde{J}_{cj}}{J_{cj}+J_{cs}}d_{v,fs} + i^2 d_{v,wg}\right), \tag{43b}$$

$$\tilde{d}_{cj,cj} = (i+1)^2\frac{J_{wg}d_{cj,cj} - \tilde{J}_{cj}d_{v,wg}}{J_{cj}+J_{cs}}, \tag{43c}$$

$$\tilde{s} = \left(1 + (i+1)^2\frac{J_{wg}}{J_{cj}+J_{cs}}\right)s, \tag{43d}$$

$$\tilde{f}_2 = -\frac{i+1}{i}\frac{J_{fs}\tilde{J}_{cj}}{\tilde{J}_{fs}\left(J_{cj}+J_{cs}\right)}, \tag{43e}$$

$$\tilde{f}_{fs} = i(i+1)\frac{J_{wg}}{\tilde{J}_{fs}}. \tag{43f}$$

By identically zeroing the two outputs φ_{rk} and φ_{fs} one can calculate the appropriate zeroing inputs using Eqs. (35) and (41). If those inputs are applied to Eq. (42) we obtain the zero dynamics of the decoupled system. It does not influence the input-output behavior (due to the decoupling feedback laws) but should be stable. To check its stability we give the state equations of the decoupled system in vector-matrix form. The subscripts 'ol' refer to, that this *open-loop* system will be compensated by an outer position control loop (represented by a driver model on the flexspline axis).

$$\dot{x}_{ol} = A_{ol}x_{ol} + B_{ol}u_{ol}, \tag{44a}$$

$$y = C_{ol}x_{ol}, \tag{44b}$$

with state vector $x_{ol} = (\varphi_{rk}, \dot{\varphi}_{rk}, \varphi_{fs}, \dot{\varphi}_{fs}, \varphi_{cj}, \dot{\varphi}_{cj})^T$, input $u_{ol} = (u_1, u_2, T_{fs})^T$ and output $y = (\varphi_{rk}, \varphi_{fs})^T$. The coefficient matrices read

$$A_{cl}^T = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{\tilde{s}}{\tilde{J}_{cj}} & -\frac{\tilde{d}_{cj,rk}}{\tilde{J}_{cj}} & 0 & -\frac{\tilde{d}_{cj,fs}}{\tilde{J}_{cj}} & -\frac{\tilde{s}}{\tilde{J}_{cj}} & -\frac{\tilde{d}_{cj,cj}}{\tilde{J}_{cj}} \end{bmatrix}, \tag{45a}$$

$$B_{ol} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{\tilde{J}_{fs}} & \frac{1}{\tilde{J}_{fs}} \\ 0 & 0 & 0 \\ 0 & \frac{\tilde{f}_2}{\tilde{J}_{cj}} & \frac{\tilde{f}_{fs}}{\tilde{J}_{cj}} \end{bmatrix}, \quad C_{ol} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}. \tag{45b}$$

To conclude about the stability of (44) one has to check whether for all eigenvalues $\mathrm{Re}\{\lambda_i\left(A_{ol}\right)\} < 0$ hold true. Of course, this open-loop system is not stable because of the decoupled double integrators but we calculate the eigenvalues to see the poles of the zero dynamics:

$$\lambda_{1,2} = \lambda_{3,4} = 0, \qquad \lambda_{5,6} = -\frac{\tilde{d}_{cj,cj}}{2\tilde{J}_{cj}} \pm \sqrt{\left(\frac{\tilde{d}_{cj,cj}}{2\tilde{J}_{cj}}\right)^2 - \frac{\tilde{s}}{\tilde{J}_{cj}}}. \tag{46}$$

The invariant zeros of the system are located where the last conjugate pair of poles ($\lambda_{5,6}$) are, consequently they are the poles of the zero dynamics that correspond to damped oscillations for all physically reasonable set of parameters.

Let us implement a PID control law on the rack axis for the reference φ_{rk}^d:

$$u_1 = k_{rk,P}\left(\varphi_{rk}^d - \varphi_{rk}\right) + k_{rk,I}\int_0^t\left(\varphi_{rk}^d - \varphi_{rk}\right)d\tau - k_{rk,D}\dot{\varphi}_{rk}. \tag{47}$$

We assume a similar driver model on the flexspline axis:

$$T_{fs} = k_{fs,P}\left(\varphi_{fs}^d - \varphi_{fs}\right) + k_{fs,I}\int_0^t\left(\varphi_{fs}^d - \varphi_{fs}\right)d\tau - k_{fs,D}\dot{\varphi}_{fs}. \tag{48}$$

The state-space model of the closed-loop system is given below. The state vector is extended with the integrator states: $x_{cl} = (x_{ol}^T, x_{rk,I}, x_{fs,I})^T$. The input is the vector of the control references (desired axis angles and force-feedback torque)

$u_{cl} = (\varphi_{rk}^d, T_{ff}, \varphi_{fs}^d)^T$ and the output is the same as before.

$$\dot{x}_{cl} = A_{cl} x_{cl} + B_{cl} u_{cl}, \tag{49a}$$

$$y = C_{cl} x_{cl}, \tag{49b}$$

The coeffitient matrices are given as

$$A_{cl}^T = \begin{bmatrix} 0 & -k_{rk,P} & 0 & 0 & 0 & \frac{\tilde{s}}{\tilde{J}_{cj}} & -1 & 0 \\ 1 & -k_{rk,D} & 0 & 0 & 0 & -\frac{\tilde{d}_{cj,rk}}{\tilde{J}_{cj}} & 0 & 0 \\ 0 & 0 & 0 & -\frac{k_{fs,P}}{\tilde{J}_{fs}} & 0 & -\frac{\tilde{f}_{fs} k_{fs,P}}{\tilde{J}_{cj}} & 0 & -1 \\ 0 & 0 & 1 & -\frac{k_{fs,D}}{\tilde{J}_{fs}} & 0 & -\frac{\tilde{d}_{cj,fs} + \tilde{f}_{fs} k_{fs,D}}{\tilde{J}_{cj}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\tilde{s}}{\tilde{J}_{cj}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -\frac{\tilde{d}_{cj,cj}}{\tilde{J}_{cj}} & 0 & 0 \\ 0 & k_{rk,I} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{k_{fs,I}}{\tilde{J}_{fs}} & 0 & -\frac{\tilde{f}_{fs} k_{fs,I}}{\tilde{J}_{cj}} & 0 & 0 \end{bmatrix}, \tag{50a}$$

$$B_{cl} = \begin{bmatrix} 0 & 0 & 0 \\ k_{rk,P} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{\tilde{J}_{fs}} & \frac{k_{fs,P}}{\tilde{J}_{fs}} \\ 0 & 0 & 0 \\ 0 & \frac{\tilde{f}_2}{\tilde{J}_{cj}} & \frac{\tilde{f}_s k_{fs,P}}{\tilde{J}_{cj}} \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad C_{cl} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{50b}$$

To analyze the stability of the closed-loop system we give the characteristic polynomial of A_{cl}:

$$D_{cl}(\lambda) = \left(\tilde{J}_{cj}\lambda^2 + \tilde{d}_{cj,cj}\lambda + \tilde{s}\right) \cdot \cdot \left(\lambda^3 + k_{rk,D}\lambda^2 + k_{rk,P}\lambda + k_{rk,I}\right) \left(\tilde{J}_{fs}\lambda^3 + k_{fs,D}\lambda^2 + k_{fs,P}\lambda + k_{fs,I}\right). \tag{51}$$

The invariant zeros are the roots of the following polynomial:

$$D_z(\lambda) = \left(\tilde{J}_{cj}\lambda^2 + \tilde{d}_{cj,cj}\lambda + \tilde{s}\right) \left(k_{rk,P}\lambda + k_{rk,I}\right)\left(k_{fs,P}\lambda + k_{fs,I}\right). \tag{52}$$

Note that, as expected, the poles, which determine the input-output behavior can be placed arbitrarily with help of the controller parameters and the poles of the zero dynamics are not affected by the PID feedback.

4 Simulation study

The control algorithm that was described in the last section is designed for the linearized model of the steering system. In this section, we will perform a simulation study to see the closed-loop performance where the control law is applied to the nonlinear model of Subsection 2.2. The values of the model and control parameters that will be used for the simulation study are listed in Table 1. The order of magnitude of these values are based on previous identification results [3]. The units of the parameters are according to the SI unit system. The control parameters are tuned so that one of the poles is close to the only zero of the corresponding closed-loop transfer function and together with the remaining conjugate complex pair of poles the step response of the system is reasonably fast with small overshoot (approx. 5%).

Tab. 1. Simulation Parameters

$J_{fs} = 5 \cdot 10^{-2}$	$d_{v,fs} = 1 \cdot 10^{-1}$	$d_{c,fs} = 2 \cdot 10^{-1}$
$J_{cs} = 1 \cdot 10^{-4}$	$d_{v,cs} = 5 \cdot 10^{-4}$	$d_{c,cs} = 1 \cdot 10^{-4}$
$J_{wg} = 5 \cdot 10^{-5}$	$d_{v,wg} = 5 \cdot 10^{-4}$	$d_{c,wg} = 5 \cdot 10^{-4}$
$J_{cj} = 1 \cdot 10^{-5}$	$d_{v,cj} = 1 \cdot 10^{-4}$	$d_{c,cj} = 1 \cdot 10^{-4}$
$J_{rk} = 5 \cdot 10^{-2}$	$d_{v,rk} = 1 \cdot 10^{-2}$	$d_{c,rk} = 1 \cdot 10^{-2}$
$i = 5 \cdot 10^{1}$	$k_{rk,P} = 225$	$k_{fs,P} = 5$
$s = 1 \cdot 10^{2}$	$k_{rk,I} = 50$	$k_{fs,I} = 1$
$d_s = 2 \cdot 10^{-2}$	$k_{rk,D} = 21$	$k_{fs,D} = 2$

The simulation is performed with the following values of the universal joint parameter (β): 0°, 10°, 20°, 30°. For all these values the simulation is made with the following sequence:

1 Start of simulation (0 s) with zero states and input values.

2 At 1 s stepwise change of the rack axis position reference to π (control loop).

3 At 3 s stepwise change of the flexspline axis position reference to π (driver's loop).

4 At 5 s stepwise change of the 'force feedback' torque reference to 1 (control loop).

5 At 7 s stepwise change of the disturbance torque on the rack axis to 10.

The evolution of the angle of the rack axis is shown in Fig. 3 for all values of the universal joint angles (β). It hardly changes for different Cardan joint angles. The same can be said about the evolution of the angle of the Cardan joint axis (φ_{cj}) that represents the zero dynamics (Fig. 6). It is different in the cases of the flexspline shaft angle (Fig. 4) and the driver torque (Fig. 5). The main difference can be observed after the change of the feed-forward term at 5 s. The decoupling performance is acceptable because we cannot observe any effect on the rack axis angle and the cross-term transient seen by the driver model lies in the order of magnitude of the friction, as one can see in Fig. 5 (the fast transient at 1 s). For small β, the steady-state error in the torque feed-forward is mainly caused by the Coulomb friction that is an adequate performance but for $\beta > 20°$ it is growing apace.

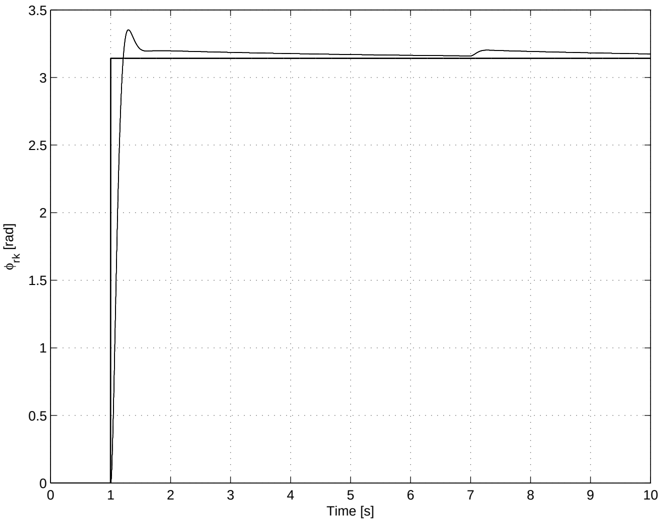


Fig. 3. Rack axis angle and reference (for the different values of β the curves φ_{rk} coincide)

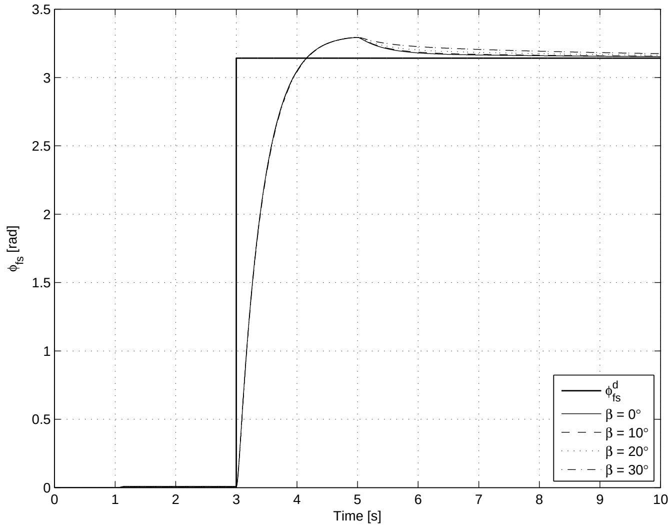


Fig. 4. Flexspline axis angle and reference

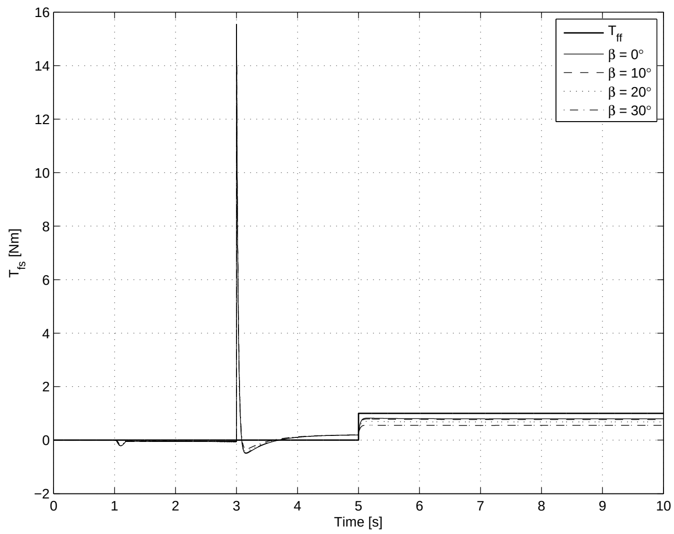


Fig. 5. Driver torque and torque feed-forward (force feedback for the driver)

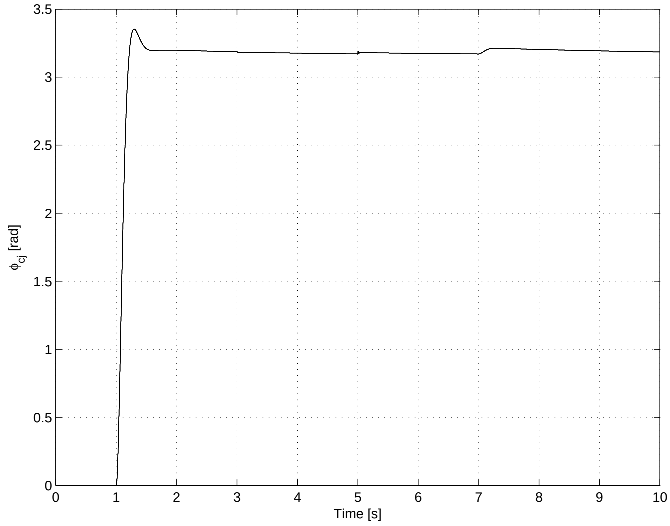
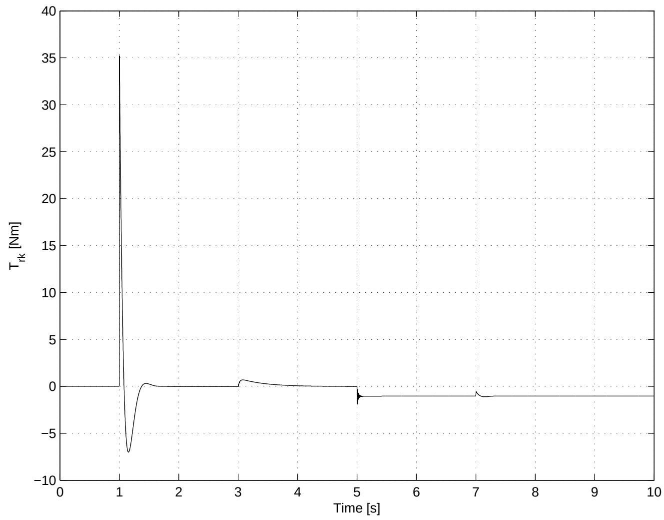
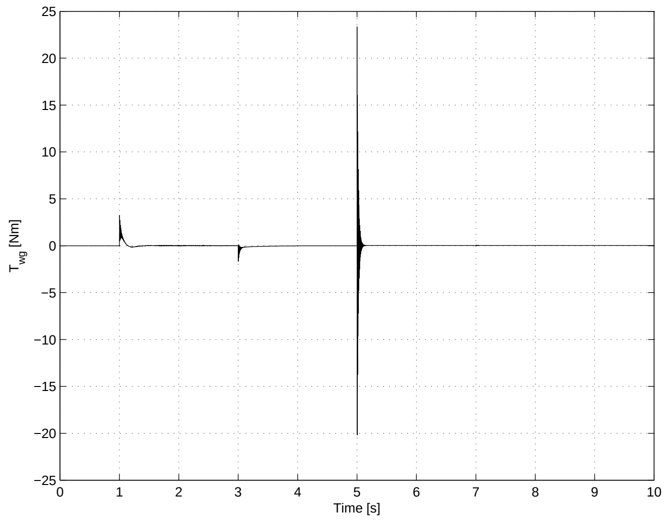


Fig. 6. Cardan joint angle (φ_{cj})

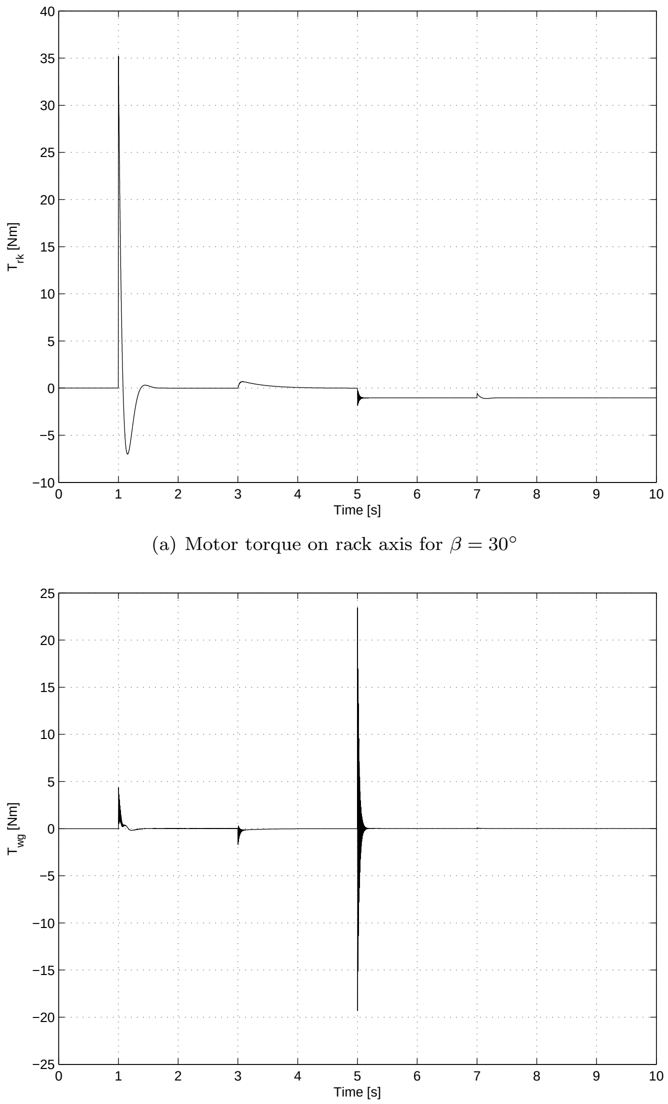


(a) Motor torque on rack axis for $\beta = 0°$



(b) Motor torque on wave generator axis for $\beta = 0°$

Fig. 7. Actuator signals for $\beta = 0°$



(a) Motor torque on rack axis for $\beta = 30°$

(b) Motor torque on wave generator axis for $\beta = 30°$

Fig. 8. Actuator signals for $\beta = 30°$

The actuator signals are shown for universal joint angles $\beta = 0°$ (Fig. 7) and $\beta = 30°$ (Fig. 8). Note that the curves for $\beta = 30°$ are barely distinguishable from the curves for $\beta = 0°$. Again, the most critical transient is after the change of the feed-forward torque at 5 s. Especially, the actuator on the wave generator shaft answers with a high but fast transient. It originates from the well-damped oscillation of the universal joint axis with a small starting amplitude of about $1°$. It is a small angle but recall the high stiffness of the spring ($s = 100$ Nm/rad). This small angular displacement on the spring corresponds to a torque of about 1.8 Nm and because of the high gear ratio of the harmonic drive we obtain a high factor in the last term in (40) that is to cancel it. Accordingly, the saturation of this torque would not destroy the performance but perhaps the driver could perceive a short vibration. Nevertheless the stepwise change of this torque is unrealistic and with realistic feed-forward references the damping of the system components excludes the possibility of such effects. The same oscillation can be observed in the torque of the rack axis motor, as well, but with much smaller gain and so it does not play such a critical role. The highest transient occurs with the stepwise change of reference, consequently during real operation this signal is also reasonably bounded.

5 Conclusion

We derived the nonlinear model of a novel steering system with a universal joint and a harmonic drive. We stated that the outputs to be controlled are not differentially flat therefore we derived the noninteracting control for the linearized model. The simulation study has shown that for small universal joint angles ($\beta < 20°$) the given linear control algorithm can be applied to the steering system in order to control the position of the rack axis and develop a feed-forward torque to the steering wheel on the flexspline axis. This scheme achieves a steer-by-wire like operation with the maintenance of the mechanical contact that is an important safety aspect. A hardware implementation may be possible on the setups of an industrial partner and accordingly the decision about the application of the presented method to the control of real EPAS systems depends on several aspects.

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