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RESEARCH ARTICLE

Independent design of decentralized controllers in the frequency domain

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Abstract

The paper presents an original frequency domain decentralized controller design technique for guaranteed performance. The novelty consists in that the designed decentralized controller guarantees the required performance for the full system. Interactions are considered in local designs by means of a chosen characteristic locus of the interaction matrix used to modify mathematical models of isolated subsystems thus defining the equivalent subsystems [7]. *The developed graphical design method is insightful and promising from the viewpoint of further application in the robust control design* [8]. *Theoretical conclusions are supported with results obtained from the solution of several examples.*

Keywords

multivariable system · decentralized controller · frequency domain · independent design AMS Subject Classification: 93 D15

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1 Introduction

Industrial plants are complex systems typical by multiple inputs and multiple outputs (MIMO systems). Usually they arise as interconnection of a finite number of self-contained units - subsystems. To control such systems multivariable or decentralized controllers are used. Compared with centralized full-controller systems decentralized controller structure constraints bring about a certain performance deterioration; however, this drawback is weighted against important benefits such as hardware simplicity, operation simplicity and reliability improvement. Due to them, decentralized control (DC) design techniques remain probably the most popular among control engineers, in particular the frequency domain ones which provide insightful solutions and link to the classical control theory.

Development of decentralized control (DC) techniques started in the 70's and has attracted much attention over the next few decades. With the come up of the robust frequency domain approaches in the 80's, robust approach to the decentralized controller design has become very popular and many practice-oriented techniques were developed along with computationally useful tools used to assess the closed-loop performance under decentralized controllers (e.g. [3, 4, 15] and references therein). The DC design includes two main steps: 1) selection of a suitable control configuration (pairing inputs with outputs); 2) design of local controllers for individual subsystems. There are two main design approaches which can be applied in Step 2): according to the independent design [3],[6],[7],[15], local controllers for individual subsystems are designed without considering interactions with other subsystems. The effect of interactions on the full system is assessed first and then transformed into bounds for individual loops that are to be considered in the local controller designs in order to guarantee stability and a desired performance of the full system. Main advantages with this approach are direct design of local controllers with no need for trial and error. The limitation consists in that information about controllers in other loops is not exploited therefore the resulting stability and performance conditions for individual loops are only sufficient and thus potentially conservative.

The sequential or dependent design [1],[4] involves designing local controllers sequentially. Usually, the controller corresponding to a fast loop is designed first and this loop is then closed before the design proceeds with the next controller. Thus, information about the "lower level" controllers is directly used as more loops are closed. If the performance of the overall system is not satisfactory, the design procedure repeats with a more corrective design. The list of main drawbacks include lack of failure tolerance when "lower level" controllers fail; strong dependence on the order of loop closing; the design proceeds by "trial and error".

Depending on the specific application there are performance objectives of two basic types [14]: a) achieving a required performance in the different subsystems or b) achieving a desired performance of the full system, in both cases either independent or dependent designs can be applied.

In this paper a novel design technique is proposed to guarantee a required performance of the full system by applying the independent design for the "equivalent subsystems" [7, 11]. The effect of interactions on the overall system is assessed using characteristic loci (CL) of the matrix of interactions; the CLs are then used to modify frequency responses of isolated subsystems thus defining the equivalent subsystems. Local controllers are designed for the equivalent subsystems by the independent design approach using any frequency-domain design method. Resulting local controllers designed for equivalent subsystems guarantee fulfilment of performance requirements imposed on the full system.

The paper is organized as follows: problem formulation and theoretical preliminaries of the proposed technique are surveyed in Section 2, main results along with the proposed design procedure are presented in Section 3 and verified on examples in Section 4. Conclusions are given at the end of the paper.

2 Preliminaries and problem formulation

Consider a MIMO system $G(s)$ and a controller $R(s)$ in the standard feedback configuration (Fig. 1)

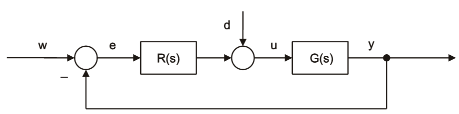

Fig. 1. Standard feedback configuration

where $G(s) \in R^{m\times l}$ and $R(s) \in R^{l\times m}$ are transfer function matrices and w, u, y, e, d are respectively vectors of reference, control, output, control error and disturbance of compatible dimensions. Hereafter, just square matrices will be considered, i.e. $m = l$.

The problem studied in this paper can be formulated as follows: Consider that the system $G(s)$ consists of m subsystems ($m = l$) and can be split into the diagonal $G_d(s)$ and off-diagonal $G_m(s)$ parts. The transfer matrix collecting diagonal entries of $G(s)$ is the model of decoupled subsystems; interactions between subsystems are represented by the off-diagonal entries of $G(s)$, i.e.

$$G(s) = G_d(s) + G_m(s) \tag{1}$$

For the system (1), a decentralized controller is to be designed

$$R(s) = \mathrm{diag}\{R_i(s)\}_{m\times m} \quad \det R(s) \neq 0 \tag{2}$$

where $R_i(s)$ is transfer function of the i-th subsystem local controller, using the independent design philosophy according to which the effect of interactions is to be appropriately quantified and included in the design of local controllers so as to guarantee a specified performance (including stability) of the full system. In this paper the proposed decentralized controller design procedure for MIMO systems reduces to independent controller design for equivalent SISO subsystems. In the decentralized controller design procedure any frequency domain performance criterion suitable for equivalent subsystems can be applied. If the performance measure applied for equivalent subsystems is identifiable for the MIMO system, then the same performance measure is guaranteed for MIMO system; for example if for all equivalent subsystems the same degree of stability has been achieved then the same degree of stability is guaranteed for the global system.

The feedback system in Fig. 1 is internally stable if and only if the transfer matrix from $[d \quad w]^T$ to $[u \quad e]^T$ given by

$$\begin{bmatrix} (I+RG)^{-1} & (I+RG)^{-1}R \\ -(I+GR)^{-1}G & (I+GR)^{-1} \end{bmatrix} \tag{3}$$

is stable. Another test for internal stability is the Nyquist encirclement criterion. Both the internal stability condition and the Nyquist stability criterion provide necessary and sufficient conditions for the closed-loop stability. Note that if the system is internally stable then it is stable with respect to all state and output variables and in the sequel it will simply be called "stable".

The multivariable stability theory relies on the concept of the system return difference [3],[6]

$$F(s) = [I + Q(s)] \tag{4}$$

where $F(s) \in R^{m\times m}$ is the system return-difference matrix, $Q(s) = G(s)R(s) \in R^{m\times m}$ is the (open) loop transfer function matrix for the system in Fig. 1 and $H(s) = Q(s)[I + Q(s)]^{-1} \in R^{m\times m}$ is the corresponding closed-loop transfer function matrix. The Nyquist D-contour is a contour in the complex plane consisting of the imaginary axis $s = j\omega$ and an infinite semi-circle into the right-half plane. It has to avoid locations where $Q(s)$ has $j\omega$ -axis poles (e.g. if $R(s)$ includes integrators) by making small indentations around these points to include them to the left-half plane. Thus, unstable poles of $Q(s)$ will be considered those in the open right-half plane. Nyquist plot of a complex function $g(s)$ is the image of the Nyquist D-contour under $g(s)$;

$N[k, g(s)]$ denotes the number of anticlockwise encirclements of the point $(k, j0)$ by the Nyquist plot of $g(s)$.

Consider the closed-loop characteristic polynomial (CLCP) of the system in Fig. 1

$$\det F(s) = \det[I + Q(s)] = \det[I + G(s)R(s)] \tag{5}$$

The closed-loop stability of the system in Fig. 1 can be determined using the Generalized Nyquist Stability Theorem [2, 12, 16].

Theorem 1 *(Generalized Nyquist Stability Theorem)*
The feedback system in Fig. 1 is stable if and only if $\forall s \in D$

$$\det F(s) \neq 0$$
$$N[0, \det F(s)] = n_q \tag{6}$$

where n_q *is the number of its open-loop unstable poles and D is the Nyquist D-contour.*

For any specific value of a complex frequency, e.g. $\widetilde{s} = j\widetilde{\omega}$, the corresponding matrix $Q(\widetilde{s})$ is a matrix of complex numbers and has its associated set of complex eigenvalues $\{q_i(\widetilde{s})\}_{i=1,\dots,m}$.

Eigenvalues of $Q(s)$ are the set of m analytic functions $q_i(s), i = 1, 2, ..., m$ satisfying

$$\det[q_i(s)I - Q(s)] = 0 \quad i = 1, 2, ..., m \quad \forall s \in D \tag{7}$$

and called characteristic function of $Q(s)$ [12]. Since we are only concerned with their frequency response evaluation, other aspects of their behaviour are not further considered here. Using the characteristic functions the closed-loop characteristic polynomial can be re-written

$$\det F(s) = \det[I + Q(s)] = \prod_{i=1}^{m}[1 + q_i(s)] \quad \forall s \in D \tag{8}$$

Characteristic loci (CL) denoted $q_i(j\omega)$, $i = 1, 2, ..., m$ are the set of loci in the complex plane traced out by the characteristic functions of $Q(s)$ as s traverses the Nyquist D-contour; this set is called the *spectral Nyquist plot* [2]. The *degree* of the spectral Nyquist plot is the sum of anticlockwise encirclements with respect to the point $(0, 0j)$ in the complex plane, contributed by the characteristic loci of $[I + Q(s)]$.

A stability test analogous to *Theorem 1* has been derived in terms of the CL's [2],[12].

Theorem 2 *The closed-loop system with the open-loop transfer function* $Q(s)$ *is stable if and only if* $\forall s \in D$

$$\det[I + Q(s)] \neq 0$$
$$\sum_{i=1}^{m} N[-1, q_i(s)] = n_q$$

where n_q *is the number of unstable poles of* $Q(s)$.

Remark 1 *In the sequel, matrices and their characteristic functions/loci be denoted by corresponding upper case and lower case letters, respectively.*

Theorems 1 and 2 are equivalent, therefore

$$N[0, \det[I + Q(s)]] = \sum_{i=1}^{m} N[0, [1 + q_i(s)]] = n_q \quad \forall s \in D \tag{9}$$

3 Main results

3.1 Theoretical development

The proposed decentralized control design technique evolves from the factorization of the closed-loop characteristic polynomial of the full system (5) under decentralized controller (2) in terms of the correspondingly partitioned system

$$\det F(s) = \det\{I + R(s)[G_d(s) + G_m(s)]\} = \tag{10}$$
$$= \det R(s) \det[R^{-1}(s) + G_d(s) + G_m(s)]$$

Existence of $R^{-1}(s)$ is implied by the assumption $\det R(s) \neq 0$.

By denoting

$$F_1(s) = R^{-1}(s) + G_d(s) + G_m(s) \tag{11}$$

and employing (9)-(12), the necessary and sufficient stability conditions of *Theorem 1* can be modified according to the following corollary.

Corollary 1 *The closed-loop system comprising the system (1) and the decentralized controller (2) is stable if and only if* $\forall s \in D$

$$\det F_1(s) \neq 0$$

$$N[0, \det F_1(s)] + N[0, \det R(s)] = n_q \tag{12}$$

If $R(s)$ has no poles in the open right half-plane, $N[0, \det R(s)] = 0$ the encirclement condition (12) reduces to

$$N[0, \det F_1(s)] = n_q \tag{13}$$

The diagonal term $[R^{-1}(s) + G_d(s)]$ in (12) actually comprises information on the dynamics of individual closed-loop. Using the substitution

$$P(s) = R^{-1}(s) + G_d(s) \tag{14}$$

where $P(s) = \mathrm{diag}\{p_i(s)\}_{m\times m}$ is a diagonal matrix; further manipulating of (14) yields

$$I + R(s)[G_d(s) - P(s)] = I + R(s)G^{eq}(s) = 0 \tag{15}$$

where the notation

$$G^{eq}(s) = G_d(s) - P(s)$$

introduces the diagonal matrix of equivalent subsystems. The corresponding equivalent closed-loop polynomial

$$CLCP^{eq}(s) = I + R(s)G^{eq}(s)$$

is the equivalent closed-loop characteristic polynomial.

Similarly, on the subsystem level

$$CLCP_i^{eq}(s) = 1 + R_i(s)G_i^{eq}(s) = 0 \quad i = 1, 2, ..., m \tag{16}$$

where

$$G_i^{eq}(s) = G_i(s) - p_i(s) \quad i = 1, 2, ..., m \tag{17}$$

denote the i^{th} equivalent characteristic polynomial and the i^{th} equivalent subsystem transfer function, respectively.

Combining (14) and (11) yields

$$\det F_1(s) = \det[P(s) + G_m(s)] \tag{18}$$

Consequently, the encirclement stability conditions (13) can be restated according to the following corollary.

Corollary 2 *The closed-loop system in Fig. 1 comprising the system (1) and a stable decentralized controller (2) is stable if*

- *there exists a diagonal matrix $P(s) = \mathrm{diag}\{p_i(s)\}_{m\times m}$ such that each equivalent subsystem $G_i^{eq}(s) = G_i(s) - p_i(s)$, $i = 1, 2, ..., m$ can be stabilized by its related local controller $R_i(s)$, i.e. each equivalent characteristic polynomial*

  $$CLCP_i^{eq}(s) = 1 + R_i(s)G_i^{eq}(s) \quad i = 1, 2, ..., m$$

  has roots in the open left-half plane;
- *1.*

  $$\det[P(s) + G_m(s)] \neq 0 \quad \forall s \in D \tag{19}$$

  2.

  $$N[0, \det[P(s) + G_m(s)]] = n_m \tag{20}$$

  or equivalently

  $$\sum_{i=1}^{m} N[0, m_i(s)] = n_m$$

  where $m_i(s)$ $i = 1, 2, ..., m$ are characteristic loci of $M(s) = P(s) + G_m(s)$ and n_m is the number of its unstable poles.

Besides securing the closed-loop stability, the diagonal matrix $P(s)$ can be used to implement performance requirements in the local controller design. In the next section, one method of choosing $P(s)$ is being discussed in detail.

3.2 Decentralized Controller Design for Performance

According to independent design philosophy, entries of the diagonal matrix $P(s)$ actually represent bounds for local controller designs. To be able to guarantee closed-loop stability of the full system they have to be chosen so as to appropriately consider the interaction term $G_m(s)$. The main idea of the proposed design strategy evolves from the following reasoning. According to (8), characteristic functions $g_i(s), i = 1, 2, ..., m$ of $G_m(s)$ satisfy

$$\det[g_i(s)I - G_m(s)] = 0 \quad i = 1, 2, ..., m \quad \forall s \in D \tag{21}$$

Substituting (14) into the r.h.s. of (11) and equating to zero yields

$$\det[p_i(s)I + G_m(s)] = 0 \quad i = 1, 2, ..., m \quad \forall s \in D \tag{22}$$

By comparison with (21), (22) actually defines the m characteristic functions of $[-G_m(s)]$.

Hence, if the entries in the diagonal matrix $P(s) = p(s)I$ are identical and equal to any characteristic function of $[-G_m(s)]$ then for a fixed $l \in \{1, ..., m\}$

$$\det F_1(s) = \prod_{i=1}^{m}[p(s)+g_i(s)] = \prod_{i=1}^{m}[-g_l(s)+g_i(s)] = 0 \quad \forall s \in D \tag{23}$$

For $P(s) = \mathrm{diag}\{-g_l(s)\}_{m\times m}$ the closed-loop system has some poles on the imaginary axis and no poles in the right half-plane; it is at the limit of instability as $Re \quad s \leq 0$. Similarly, shifting the imaginary axis to $-\alpha$, whereby $0 \leq \alpha \leq \alpha_m$, (23) modifies as follows

$$\det F_1(s-\alpha) = \prod_{i=1}^{m}[-g_l(s-\alpha)+g_i(s-\alpha)] = 0 \quad \forall s \in D \tag{24}$$

where α_m is the maximum feasible degree of stability of the closed-loop system. Now when $P(s) = \mathrm{diag}\{-g_l(s-\alpha)\}_{m\times m}$ closed-loop has just poles with $Re \quad s \leq -\alpha$ and its degree of stability is α. The corresponding matrix of equivalent subsystems transfer functions is

$$G^{eq}(s-\alpha) = G_d(s-\alpha) + g_l(s-\alpha)I \tag{25}$$

Thus, for the fix $l \in \{1, 2, ..., m\}$ and $\alpha > 0$ the decentralized controller $R(s)$ that stabilizes the matrix of equivalent subsystems (25) guarantees the degree of stability α for the full system. Note, that if the entries of the diagonal matrix $P(s)$ are not identical the proposed decentralized controller design procedure with guaranteed performance cannot be used. Further results on the robust decentralized controller design considering identical and non identical entries of $P(s)$ can be found in [8–10].

Corollary 3 *The closed-loop system in Fig. 1 comprising the system (1) and a decentralized controller (2) is stable with the degree of stability $\alpha \in <0, \quad \alpha_m>$ if and only if for the selected* *characteristic function $g_l(s-\alpha)$ and any α_1: $0 \leq \alpha_1 < \alpha \leq \alpha_m$, $\forall s \in D$ the following conditions hold*

$$\det F_1(s) = \prod_{i=1}^{m}[-g_l(s-\alpha) + g_i(s-\alpha_1)] \neq 0$$

$$\sum_{i=1}^{m} N[0, m_{il}^{eq}(s)] = n_m \tag{26}$$

where

$$m_{il}^{eq}(s) = [-g_l(s-\alpha) + g_i(s)], \quad i = 1, ..., m$$

.

However, if $\alpha_m \to 0$ and for some $s \in D$ happens that

$$\det F_1(s) = \prod_{i=1}^{m}[-g_l(s-\alpha) + g_i(s-\alpha_1)] = 0 \tag{27}$$

i.e. if the plot of $(-g_l(s-\alpha))$ and any characteristic locus $g_i(s-\alpha_1)$ happen to cross, conditions of *Corollary 3* are not met and the closed-loop stability is not feasible under the decentralized controller $R(s)$.

Partial results of *Corollary 2* and *Corollary 3* are summarized in the following definition and lemma.

Definition 1 *For $l \in \{1, 2, ..., m\}$ and $\alpha > 0$, the characteristic function*

$$g_l(s-\alpha),$$

of the matrix $G_m(s-\alpha)$ will be called a stable characteristic function/locus if it satisfies conditions of Corollary 3.

The set of all stable characteristic functions will be denoted by P_S.

Lemma 1 *The closed-loop system in Fig. 1 comprising the system (2) and a stable decentralized controller (2) is stable with the guaranteed degree of stability $\alpha > 0$ if and only if the two following conditions are satisfied:*

1. *$p(s) = -g_l(s-\alpha) \in P_S$, $\forall s \in D$ for some fixed $l \in \{1, 2, ..., m\}$ and $\alpha > 0$;*
2. *all equivalent characteristic polynomials*

$$CLCP_i^{eq}(s) = 1 + R_i(s)G_i^{eq}(s), \quad i = 1, 2, ..., m$$

have roots with $Re \quad s \leq -\alpha$.

Proof of *Lemma 1* results from previous considerations. Note that in case of a fixed decentralized controller structure (PID) the condition "if and only if" may reduce to condition "if".

3.3 Decentralized Controller Design Procedure

Under the assumption that the selection of a suitable input-output pairing has already been accomplished, the decentralized controller design procedure has the following steps:

1. Partition the controlled system into the diagonal part $G_d(s)$ and the off-diagonal part $G_m(s)$.
2. Specify $\alpha_m > 0$ with regard to the dynamics of $G(s)$.
3. Plot the individual characteristic loci $g_i(s-\alpha)$, $i = 1, 2, ..., m$ of $G_m(s-\alpha)$ for several $\alpha \in < 0, \alpha_m >$.

Remark 2 *The set of characteristic loci $g_i(s-\alpha), i = 1, 2, ..., m$ are obtained on the frequency-by-frequency basis by calculating and plotting eigenvalues of $G_m(s-\alpha)$, $s = j\omega$, $\omega \in < 0, \infty)$ thus obtaining continuous loci.*

4. Choose

$$p(s) = -g_l(s-\alpha) \in P_S \quad l \in \{1, 2, ..., m\}$$

according to Definition 1 and Corollary 3. If no $p(s) \in P_S$ can be found, decrease the required $\alpha_m > 0$ so that the conditions of *Corollary 3* are met and repeat the procedure from Step 3. If for no $\alpha \in < 0, \alpha_m >$, $\alpha_m \to 0$, a $p(s) \in P_S$ can be found, it is not feasible to design the stabilizing local controllers using this approach; the procedure stops.

5. Design local controllers $R_i(s)$, $i = 1, 2, ..., m$ for all m equivalent subsystems (25) using any suitable frequency domain design technique (e.g. the Neymark D-partition method, Bode plots, etc.)

The proposed design technique evolves from the necessary and sufficient condition for closed loop stability under a decentralized controller, whereby the achieved performance strongly depends on the chosen controller structure. In other words, the chosen controller structure may not always provide the required performance (including stability) though if the set P_S of stable characteristic loci exists. In such a case the required closed loop performance is not feasible and the above Lemma provides only sufficient stability condition.

The proposed design procedure has been verified on examples some of which are presented in the next Section.

4 Examples

In the examples, the Neymark D-partition method [13] has been used to prove that the degree of stability achieved in equivalent subsystems uniquely determines the one of the full system.

Example 1 Consider the mathematical model of a laboratory furnace

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$$

where

$$G_{11}(s) = \frac{0.0167s^2 - 0.1018s + 0.438}{s^3 + 2.213s^2 + 2.073s + .6106}$$

$$G_{12}(s) = \frac{0.01555s^2 - 0.0375s - 0.1106}{s^3 + 2.554s^2 + 1.783s + .5433}$$

$$G_{21}(s) = \frac{0.01325s^2 - 0.03415s + 1.018}{s^3 + 3.927s^2 + 5.815s + 3.547}$$

$$G_{22}(s) = \frac{0.01575s^2 - 0.1252s + 0.442}{s^3 + 3.514s^2 + 2.01s + .3872}$$

The characteristic loci (CL) of $G_m(s-\alpha)$ evaluated for $\alpha = \{0, 0.1\}$ are depicted in Fig. 2 and Fig. 3.

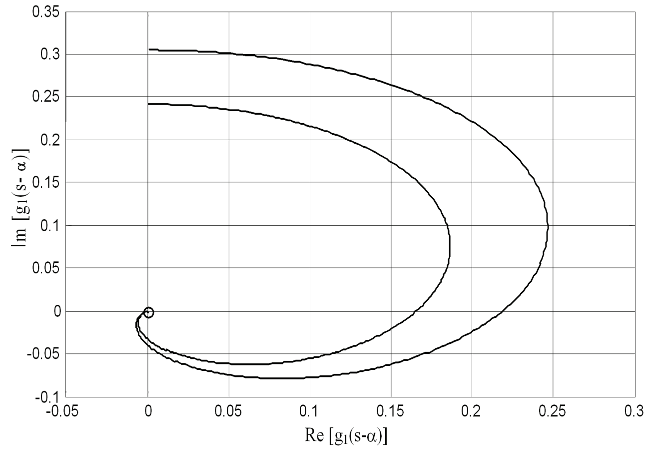

Fig. 2. Characteristic locus $g_1(s-\alpha)$, $\alpha \in \{0, .1\}$

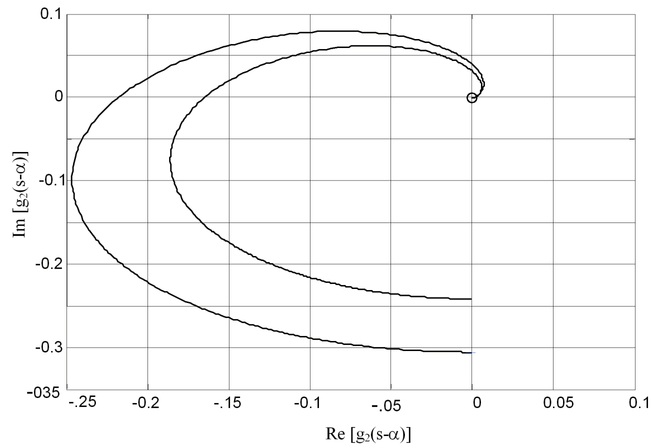

Fig. 3. Characteristic locus $g_2(s-\alpha)$, $\alpha \in \{0, .1\}$

Consider the first characteristic locus $g_1(s)$ and specify $p(s) = -g_1(s-0.1)$; the corresponding equivalent characteristic loci

$$m_{i1}^{eq}(s) = -g_1(s-0.1) + g_i(s), \quad i = 1, 2$$

are plotted in Fig. 4.

As neither of the equivalent CLs equals zero (except for $\omega \to \infty$), $p(s) = -g_1(s-\alpha) \in P_S$, in other words, it is a

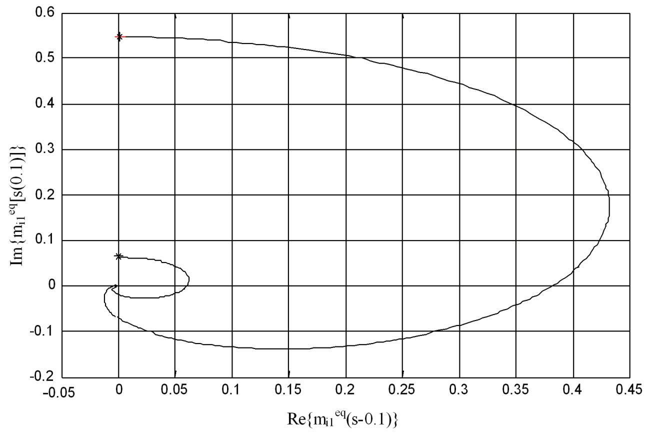

Fig. 4. Equivalent characteristic loci for $l = 1$ and $\alpha = 0.1$

stable characteristic locus. Nyquist plots of the corresponding two equivalent subsystems

$$G_i^{eq}(s-0.1) = G_i(s-0.1) + g_1(s-0.1), \quad i = 1, 2$$

obtained by modifying the Nyquist plots of decoupled subsystems are in Fig. 5 and Fig. 6. For the equivalent subsystems,

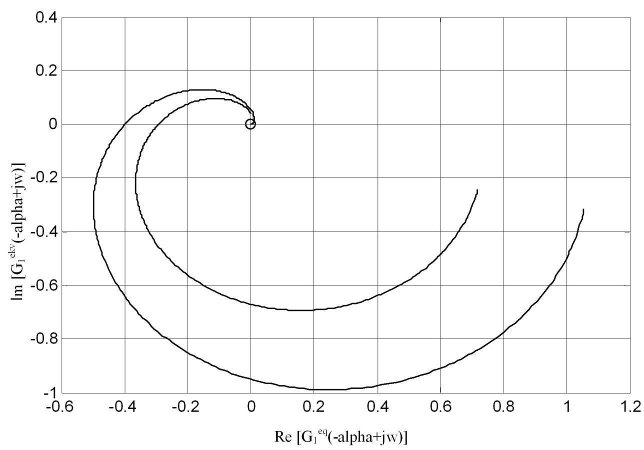

Fig. 5. Nyquist plots of the 1^{st} equivalent subsystem for $\alpha = \{0; 0.1\}$

local PI controllers with the transfer function $R(s) = r_0 + \frac{r_1}{s}$ have been designed applying the Neymark D-partition method [13] to the equivalent characteristic equations of both subsystems for $\alpha = \{0, 0.1\}$

Note that plotting the D-plots for several values of $\alpha \in < 0, \alpha_m >$ simplifies identification of pertinent stability regions in the (r_0, r_1)-plane with respect to α.

For both subsystems, parameters of local PI controllers have been chosen

a) from inside of the closed areas in Fig. 7 and Fig. 8, that correspond to the values $\alpha > 0.1$;

b) from the boundaries of the closed areas in Fig. 7 and Fig. 8, that correspond to the value $\alpha = 0.1$.

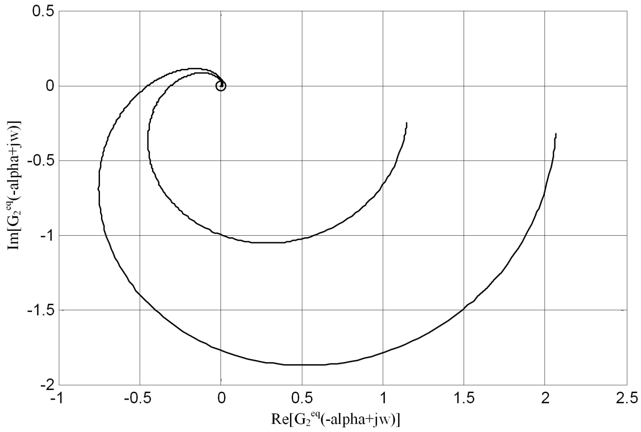

Fig. 6. Nyquist plots of the 2^{nd} equivalent subsystem for $\alpha = \{0; 0.1\}$

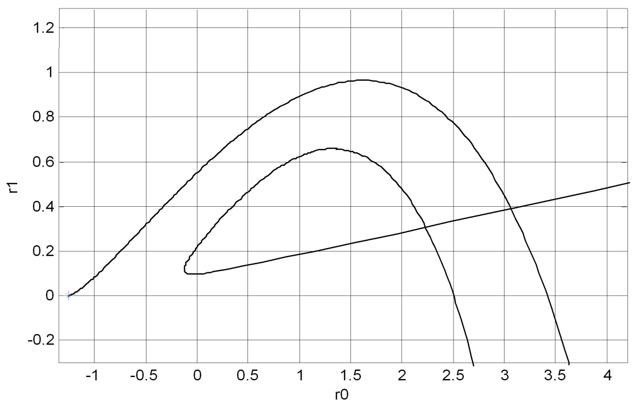

Fig. 7. Neymark D-plots for the 1^{st} equivalent subsystem, $\alpha = \{0; 0.1\}$

Design results are summarized in Table 1.

The related sets of closed-loop eigenvalues are

– for $\alpha > 0.1$

$$\Lambda_1 = \{-.1549; -.1825; -.2286 \pm .4133j; -.3464 \pm .6648j; -.4129 \pm .3888j;$$
$$-1.079 \pm .9291j; -1.2811; -1.7029; -1.8115; -2.9768\}$$

– for $\alpha = 0.1$

$$\Lambda_2 = \{-.1007 \pm .0091j; -.2499 \pm .4113j; -.36 \pm .7166j;$$
$$-.41632 \pm .3871j; -1.0801 \pm .9296j; -1.3172; -1.7043; -1.8145; -2.9918\}$$

where $j = \sqrt{(-1)}$. Closed-loop step responses in Fig. 9 and Fig. 10 verify that with a controller designed for $\alpha > 0.1$, the settling time is considerably smaller (up to 25 s) compared to the one designed for $\alpha = 0.1$ (about 40 s).

Example 2 Consider the quadruple tank process borrowed from [5]. The transfer function matrix $G(s) = \{G_{ij}(s)\}_{2\times 2}$ has

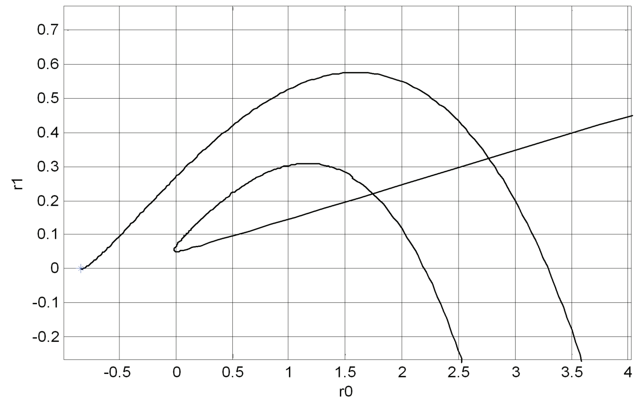

Fig. 8. Neymark D-plots for the 2^{nd} equivalent subsystem, $\alpha = \{0; 0.1\}$

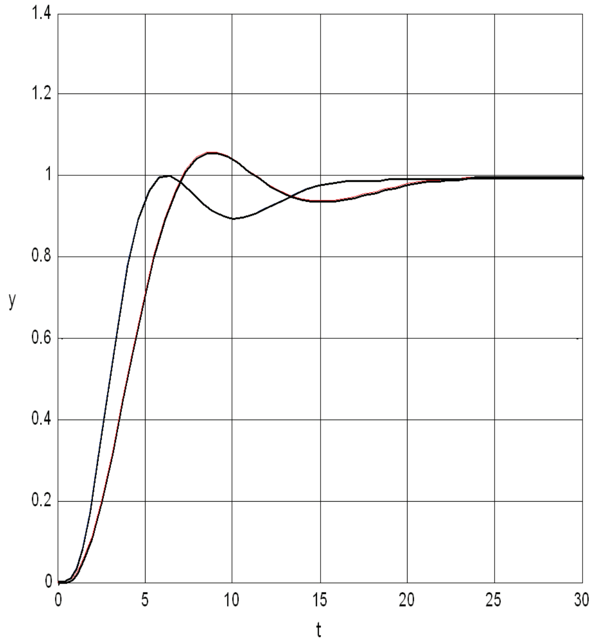

Fig. 9. Closed-loop step responses for $\alpha > 0.1$

the following entries

$$G_{11}(s) = \frac{3.11}{95.57s + 1}$$

$$G_{12}(s) = \frac{2.04}{(32.05s + 1)(95.57s + 1)}$$

$$G_{21}(s) = \frac{1.71}{(38.9s + 1)(98.67s + 1)}$$

$$G_{22}(s) = \frac{3.24}{98.67s + 1}$$

Equivalent characteristic loci for $\alpha = 0.01$ are shown in Fig. 11.

D-plots for equivalent subsystems with $p(s) = -g_1(s - .01)$ are depicted in Fig. 12 and Fig. 13 (for the 1^{st} equivalent subsystem there is just a detail of the D-plot for $\alpha = 0.01$). Parameters

Tab. 1. Results of the controller design

Subsystems	Controller	Choice of α	Achieved degree of stab. α
1	$R_1(s) = 1.2786 + \frac{.3553}{s}$	Inside of the closed reg.	0.1549
2	$R_2(s) = .9327 + \frac{.1916}{s}$	Inside of the closed reg.	0.1549
1	$R_1(s) = 1.274 + \frac{.2149}{s}$	From the D-plot	0.1007
2	$R_2(s) = .9222 + \frac{.1429}{s}$	From the D-plot	0.1007

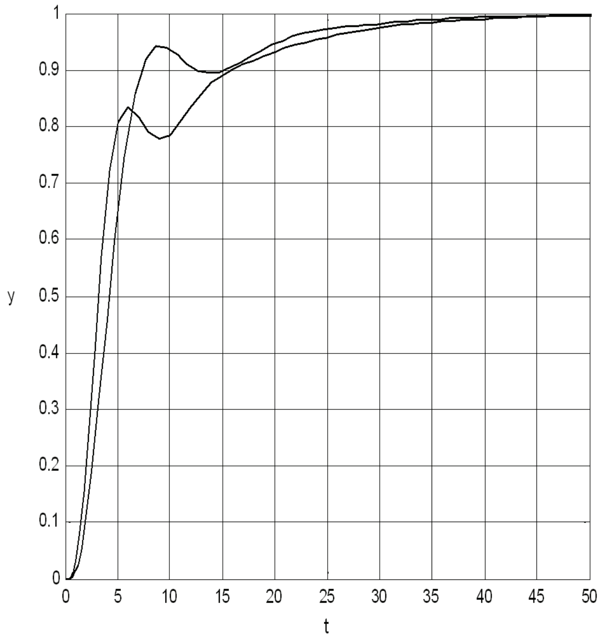

Fig. 10. Closed-loop step responses for $\alpha = 0.1$

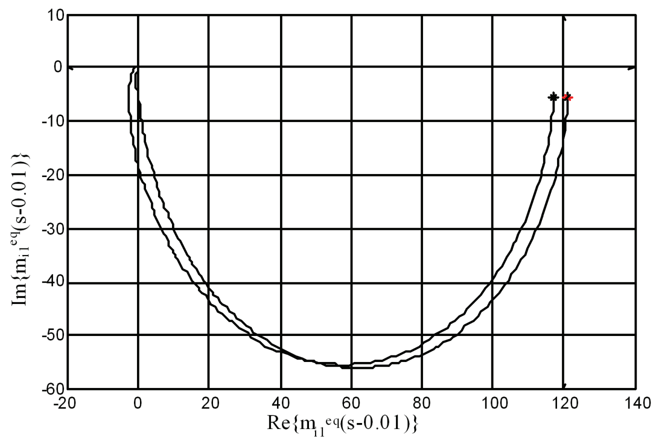

Fig. 11. Equivalent characteristic loci for $l = 1$ and $\alpha = 0.01$

of local PI controllers have been chosen from the D-plots corresponding to the degree of stability $\alpha = 0.01$

$$R_1(s) = 0.0751 + \frac{0.0037}{s}$$

$$R_2(s) = 2.6940 + \frac{0.0273}{s}$$

yielding the following closed-loop eigenvalues

$$\Lambda_3 = \{-0.01, -0.0108, -0.0126 \pm 0.0036j,$$

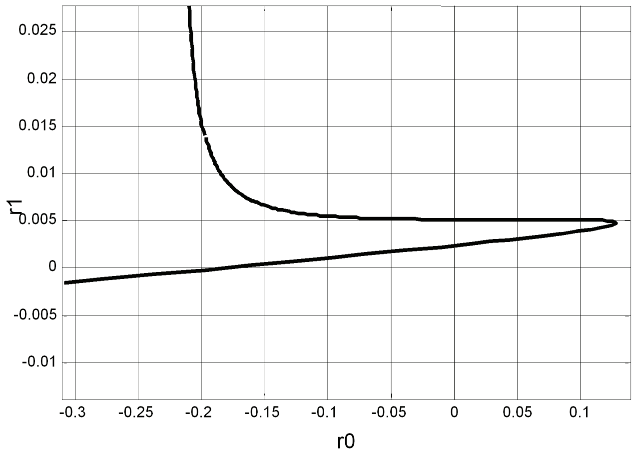

Fig. 12. D-plots for the 1^{st} equivalent subsystem

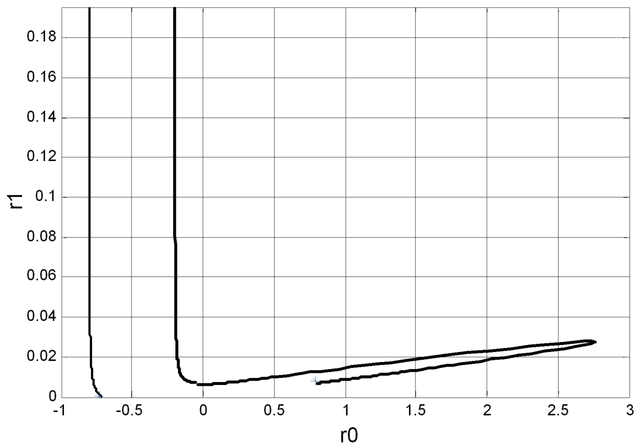

Fig. 13. D-plots for the 2^{nd} equivalent subsystem

$$-0.0337, -0.0886\}$$

Example 3 This benchmark example has been taken from [14]. The transfer function matrix $G(s) = \{G_{ij}(s)\}_{2\times 2}$ with the following entries

$$G_{11}(s) = -0.875\frac{1 - 0.2s}{(1.75s + 1)(0.2s + 1)}$$

$$G_{12}(s) = \frac{0.014}{(1.75s + 1)}$$

$$G_{21}(s) = -1.082\frac{1 - 0.2s}{(1.75s + 1)(0.2s + 1)}$$

$$G_{22}(s) = -0.014\frac{1-0.2s}{0.2s+1}$$

physically corresponds to a high purity distillation column. Parameters of local PI controllers have been chosen from the D-plots corresponding to the degree of stability $\alpha = 0.01$ in Fig. 14 and Fig. 15 (thick line)

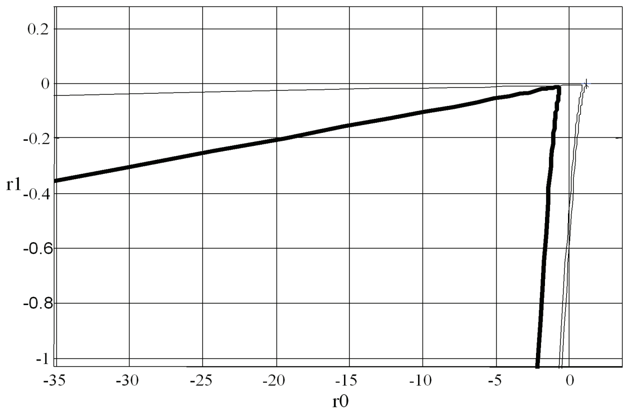

Fig. 14. D-plots for the 1^{st} equivalent subsystem and $\alpha = \{0; 0.001; 0.01\}$

$$R_1(s) = -(20.07 + \frac{.244}{s})$$

$$R_2(s) = -(53.09 + \frac{.567}{s})$$

The resulting closed loop eigenvalues are as follows:

$$\Lambda_4 = \{-.01, -.0121, -.0133, -.0281, -.246, -4.5073, -4.9801\}$$

5 Conclusion

In this paper a novel frequency-domain approach to the decentralized controller design for performance has been proposed. Its main advantage consists in that the plant interactions are included in the design of local controllers through their characteristic function, modified so as to achieve a guaranteed closed-loop performance in terms of a specified degree of stability of the full system. The independent design is carried out on the subsystem level for the equivalent subsystems which are actually mathematical models of individual decoupled subsystems modified using characteristic functions of the plant interaction matrix. Local controllers designed for equivalent subsystems guarantee a specified performance of the full system without any performance deterioration brought about by the effect of interactions. Theoretical results are supported with solutions of several examples.

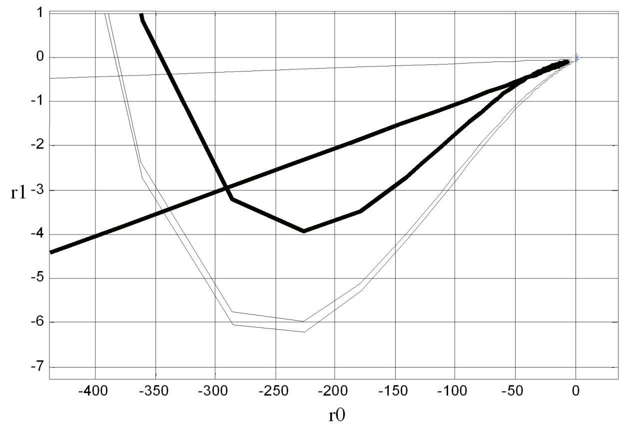

Fig. 15. D-plots for the 2^{nd} equivalent subsystem and $\alpha = \{0; 0.001; 0.01\}$

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