PERIODICA POLYTECHNICA SER. EL. ENG. VOL. 50, NO. 1–2, PP. 141–146 (2006)

MODELLING VIA BLOCK RANDOM FEEDBACK SYSTEMS

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Received: April 28, 2005

Abstract

The paper deals with the simulation on the n variable random system developed from the well-known second order feedback system. The investigated system can be transformed into an n dimensional linear differential equation of the form $\dot{\xi}_{n+1} = A_{n+1}\xi_{n+1} + Bu$, having a system matrix A_{n+1} of random entries of dispersion σ. The main idea is that every participant is acting on the others by the rule of second order feedback. It was proved that if $c > \sigma\sqrt{n}$ the system is (a.s.) stable, where the c>0 constant measures the "headstrongness" of the participants. The simulation results show the effect of σ on stability.

Keywords: simulation, modelling, stability, Block Random System.

1. Introduction

Recent years the investigations of large complicated systems came to the front seriously. The sociological, ecological, economical etc. systems often show high dimensionality and the effects of randomness too. The block random approach seems to be one of the promising initiations for the study of such complicated systems. This means that the model consists of deterministic and even random elements, allowing to handle the regular and also the occasional events.

2. The Model

The equation of non-linear problems – after linearization - often take the form of second order linear differential equations, where T_1, T_2 are the usual parameters of the system:

$$T_2^2\frac{d^2y}{dt^2} + T_1\frac{dy}{dt} + y = b \tag{1}$$

The sufficient condition of the stability means that the above two parameters should be positive. In our study we assume the same. As it is well-known the equation is equivalent to the following two-variable system:

$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \end{pmatrix} = \begin{pmatrix} -\frac{T_1}{T_2^2} & -\frac{1}{T_2^2} \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \frac{b}{T_2^2} \tag{2}$$

The equation can be considered to describe the behaviour of one mass point (position, velocity). In a more complicated situation it is supposed that we have many participants (mass points), besides we assume that the state of a participant affects the behaviour of all other participants. Thus we have n^2 equations which - using the method in [1] – sum up to the following system:

$$\dot{\xi}_{n+1} = A_{n+1}\xi_{n+1} + Bu \tag{3}$$

Since the system is the sum of second order components, the motion of the n participants can be simulated with $2n$ state variables. In the model we use instead of the n variable corresponding to the positions the sum of these variables. Thus the system can be described by $n + 1$ so-called structural variables where the system matrix$A = A_{n+1} = (a_{i,j})$ in case of n participants is the following $(n+1) \times (n+1)$ matrix. Here the $c > 0$ constant can be called the "headstrongness", which measures the difference between the strenght of interactions of a praticipant to itself and that one to the others.

$$A_{n+1} = \begin{pmatrix} -\frac{T_1}{T_2^2} - c & -\frac{T_1}{T_2^2} & \cdots & -\frac{T_1}{T_2^2} & -\frac{n}{T_2^2} \\ -\frac{T_1}{T_2^2} & -\frac{T_1}{T_2^2} - c & & -\frac{T_1}{T_2^2} & -\frac{n}{T_2^2} \\ \vdots & & \ddots & \vdots & \vdots \\ -\frac{T_1}{T_2^2} & -\frac{T_1}{T_2^2} & \cdots & -\frac{T_1}{T_2^2} - c & -\frac{n}{T_2^2} \\ n & n & \cdots & n & 0 \end{pmatrix} \tag{4}$$

It can be seen that the matrix A has 2×2 blocks: $A_{11} = (a_{i,j}$; *i,j=1…n*), $A_{12} = (a_{i,j}$; *i=1…n, j=n+1*), A_{21}=($a_{i,j}$; *i=n+1 ,j=1…n*), $A_{22} = (a_{i,j}$; *i,j=n+1*).

Assume that the behaviour of the participants is not exactly the same, but varies around an average. Exactly, we assume that the elements of the A_{11} block of the system matrix are independent identically distributed random variables. The matrix with independent identically distributed random variables is called *Wigner* type random matrix.

The most important theorem about the stability of the above random dynamic system is the following.

THEOREM 2 1 *Let* $\dot{\xi} = A_{n+1}\xi + Bu$ *be the above described random linear system of differential equations. Assume that* A_{11} *is a Wigner type random matrix i.e.* $a_{i,j} - Ea_{i,j}$ *(i,j=1…n) are independent identically distributed random variables of expected values:*

$Ea_{ii} = -\frac{T_1}{T_2^2} - c$ *(i=1…n) (5)*

$Ea_{ij} = -\frac{T_1}{T_2^2}$ *(i,j=1…n, i≠j) (6)*

We assume that the variance is σ. Besides, suppose that $a_{i,n+1} = -\frac{n}{T_2^2}$ (*i=1…n*), $a_{n+1,j} = n$ (*j=1…n*), $a_{n+1,n+1} = 0$. Let Q be a 2×2 matrix derived from the parameters and the dimension of the matrix A.

$$Q = \frac{1}{n+1} \begin{pmatrix} -\frac{T_1}{T_2^2} n & -\frac{n\sqrt{n}}{T_2^2} \\ n\sqrt{n} & c \end{pmatrix} \tag{5}$$

If

$$c > \iota\,(Q)\,\sigma\sqrt{n} \tag{6}$$

then the system is almost surely stable, where $\iota\,(Q)$ is the *Jordan* condition number of the matrix Q.

Remark. It was proved in [4] that

$$\iota\,(Q) \to 1 \quad (n \to \infty) \tag{7}$$

3. Simulation

During our investigations we made simulations with large number of participants ($n > 20$) and the results agreed with those in the corresponding systems with four participants. So we use a four participant random model to illustrate the behaviour of the model. A sinus was used as the excitation on the first variable, the system parameters were $T_1 = 1; \quad T_2 = 0.5; \quad c = 0, 5$.

Table 1 illustrates the statement of the theorem, showing the changes of the proportion of the stable and unstable systems in connection with different dispersions.

Table 1. Number of generated systems: 20 ($n = 4$)

σ	Number of stable systems	Number of unstable systems
0.5	8	12
0.4	10	10
0.25	18	2
0.1	20	0

If σ=*0.5* then most of the generated systems should be unstable. We selected an unstable one and followed the trajectories of it. These trajectories can be seen on *Fig. 1-3* in different time interval. The variety in the shape of trajectories is the consequence of the random part of the system. These differences can be seen in details on *Fig. 1*.

Fig. 2 already shows the first signs of instability, while in a larger time interval it becomes evident that the system is unstable (*Fig. 3*).

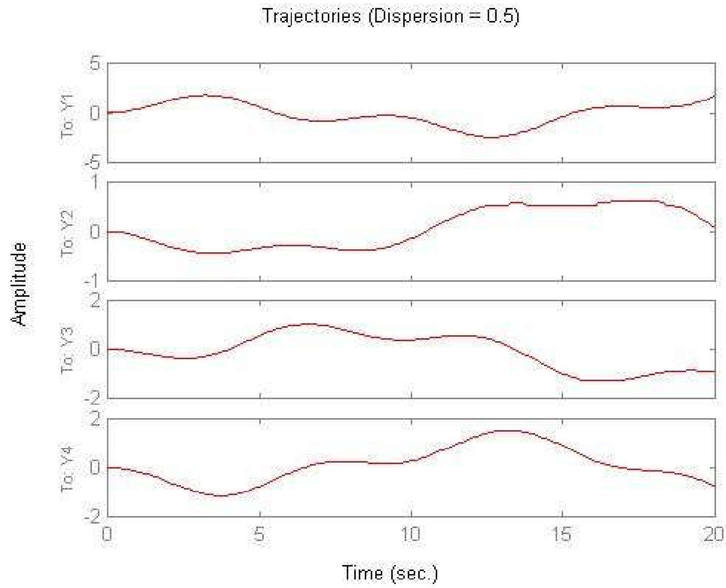

Fig. 1. The variety in the shape of trajectories.

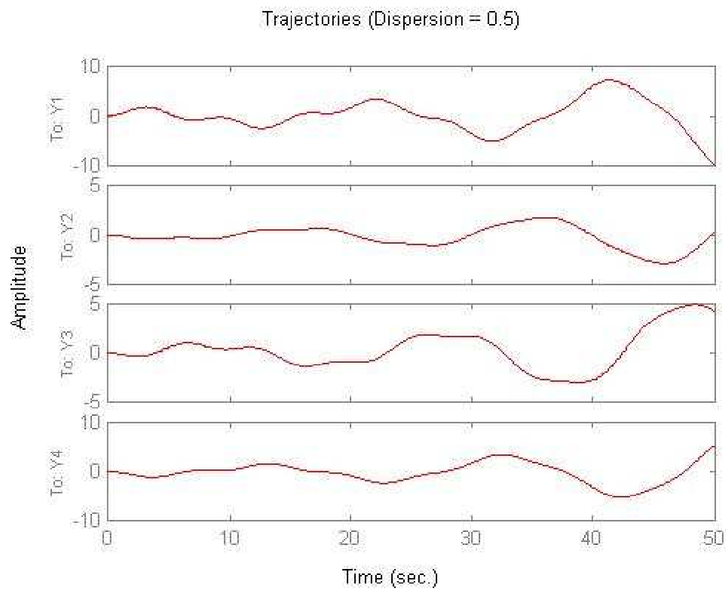

Fig. 2. The first signs of instability in the shape of trajectories.

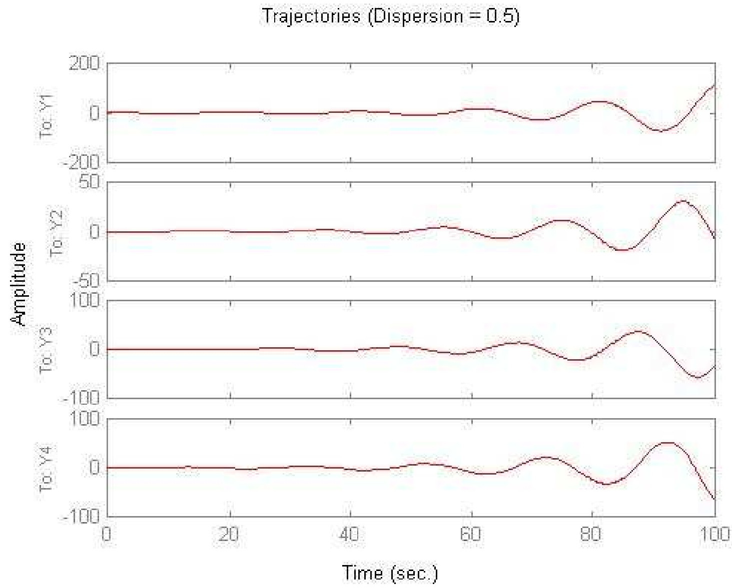

Fig. 3. Unstable system for large time intervals.

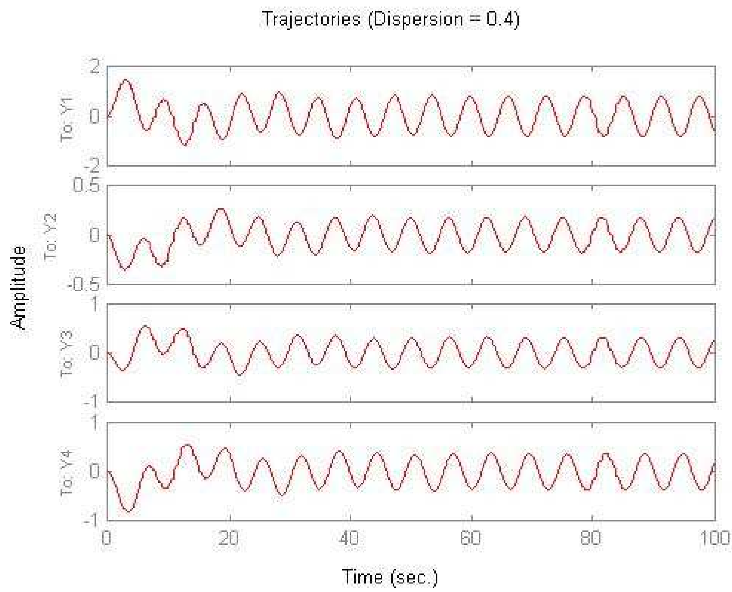

Fig. 4. Stable system in case of decrease of the dispersion of the entries.

The decrease of the dispersion of the entries resulted in turning the system into a stable one (*Fig. 4*).

4. Conclusions

The simulation results support the result of the above mentioned theorem. The increasing dispersion implies decreasing stability. On the other hand the "head-strongness" ($c > 0$) also affects the stability.

Since the model handles a number of participants together it can be considered to be a multi-body simulation technique. The close connection of this method to the turbulence offers the applications in ecology, economics, etc. [2].

References

[1] JUHÁSZ, F., On the Turbulence of Slightly Unstable Block Random Systems, *Numerical Methods in Laminar And Turbulent Flow*, **9/I** (1995), pp. 113–121.
[2] JUHÁSZ, F.– JUHÁSZ, ZS., Block Random System in Economics, Modelling and Simulation, ESM'96:432-434, 1996.
[3] JUHÁSZ, M., Másodrendű rendszerből származtatott sokváltozós véletlen rendszer stabilitása, *Elektrotechnika*, **2003/12**, 2003.
[4] SONTAG, E. D., *Mathematical Control Theory: Deterministic Finite Dimensional Systems*, SpringerVerlag, New York, 1990.