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STATE FEEDBACK DESIGN CONSIDERING OVEREXCITATION

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Abstract

The state equation describing the relationship between the input signal $u(t)$, the state variable $x(t)$and the output signal $y(t)$ of a linear, time invariant n^{th} order SISO process is: *dx/dt=Ax+Bu, y=Cx+Du.* The transfer function between the output signal and the input signal of the process is: *y(s)/u(s)=*$W_p(s)$ and the time constants characterizing the delays of signals due to energy storage elements result from the eigenvalues of the state matrix A. In the classical feedback control system, the controller computes the control signal according to the expression $u(s) = W_c(s)\ u_a(s) - y(s)$. The reduction of signal delay in the process is implemented by the PID algorithm described by the transfer function $W_c(s)$that accelerates the feedback system by *overexciting* the control signal to a specified extent. The reduction of signal delay in the process can also be implemented by negative feedback of the state variables x. If the process is state controllable and the control signal is computed according to the algorithm $u = k_c u_a - Fx$, the time constants of the feedback system can be freely specified by appropriate selection of F and k_c. The design of the feedback gain F can be performed using the *Ackermann* formula; the system is accelerated by means of *overexcitation* of the control signal to an appropriate extent even in this case. The paper presents the fact that the gain can be chosen according to $k_c = \ C(A - BF^{-1}B\ ^{-1}CA^{-1}B$, and the overexcitation ratio of the control signal can be calculated using the relationship $u(0)/u(\infty) = \ 1 + F(A - BF)^{-1}B\ ^{-1}$. This overexcitation ratio is in connection with the rate of pole transfers that can be expressed analytically. It occurs frequently that the state variables x of the process cannot play any part in the computation of the control signal since the state variables cannot be measured. In such cases, the state feedback can be implemented from the state variables $x*(t)$ of a state observer according to the expression $u = k_c u_a Fx*$. The paper presents the fact that the state feedback implemented based on the state observer – as opposed to the common concept – can also be interpreted as a state feedback of the process model, with the task of computing the control signal that fulfils the requirements of acceleration. This signal is applied at the input of both the process model and the real process.

Keywords: fedback control system, controller, overexcitation, observer.

Symbols

A, B, C, D	process parameter matrices,
$u(t), x(t), y(t)$	input signal, state variable, output signal of the process,
y_A	set point, i.e. the preset value of controlled variable y,
$W_p(s) = m_p(s)/n_p(s)$	transfer function of the process,

n	order of process,
$k_p = y_0/u_0 = -CA^{-1}B$	dc gain of the process,
$Co = [B\ AB \ldots A^{n-1}B]$	state controllability matrix,
$Ob = [C\ AC \ldots A^{n-1}C]^T$	state observability matrix,
$p_i = \lambda_i$	poles of the process, eigenvalues of A, roots of $n_p(s) = 0$,
$\det(\lambda I - A) = n_p(\lambda)$	characteristic polynomials of the process,
$n_i\ (i = 1, 2, \ldots n)$	coefficients of characteristic polynomial,
$u_a(t)$	set point of the system,
$W_c(s)$	transfer function of the controller,
k_c, T_I, T_D, T	parameters of the PID controller,
$F = [0\ 0\ \ldots\ 0\ 1]\,Co^{-1}n_R(A)$	feedback matrix of the state variables, row vector,
$A - BF$	state matrix of the feedback system,
$p_{Ri} = \lambda_{Ri}$	poles of the feedback system, eigenvalues of $A - BF$,
$\det(\lambda I - (A - BF)) = n_R(\lambda)$	characteristic polynomial of the feedback system,
$n_{Ri}\ (i = 1, 2, \ldots n)$	coefficients of the characteristic polynomial,
$n_R(A)$	the characteristic polynomial with substitution $\lambda = A$,
$[0\ 0\ \ldots\ 0\ 1]$	last row of $n \times n$ dimension unity matrix,
$k_c = [C(A - BF)^{-1}B]^{-1}CA^{-1}B$	gain factor,
$k_R = y_0/u_{a0} = -C(A - BF)^{-1}Bk_c$	resulting dc gain of the feedback system,
$u_t = u(0)/u(\infty) = [1 + F(A - BF)^{-1}B]^{-1}$	over-excitation ratio,
$x^*(t)$	state variable of the state observer,
$A - GC$	state matrix of the observer,
$p_{Mi} = \lambda_{Mi}$	poles of the state observer, eigenvalues of $A - GC$,
$\det[\lambda I - (A - GC)] = n_M(\lambda)$	characteristic polynomial of state observer,
$n_{Mi}\ (i = 1, 2, \ldots n)$	coefficients of the characteristic polynomial,
G	feedback matrix of the state observer,
A_R, B_R, C_R, D_R	parameter matrices of the system with state observer,
$y_R(t) = [y(t)\ u(t)\ h(t)]^T$	output signals of the system,
$x_R(t) = [x(t)\ x^*(t)]^T$	state variables of the system,
$h(t) = x(t) - x^*(t)$	difference between the state variables of the process and the observer.

1. Introduction

Let the n^{th} order linear SISO process be *state controllable* and *state observable*. Based on the knowledge of the physical function of the process or the measurements performed on it, the mathematical model of the process can be determined. If this model is described by the state equation (1), its parameter matrices are known and are as follows: $A(n \times n)$, $B(n \times 1)$, $C(1 \times n)$ and $D(1 \times 1)$.

$$\begin{aligned} \frac{dx(t)}{dt} &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{1}$$

As a result of delays in signals due to the energy storage elements, generally $D = 0$. This means that under the effect of step change in the input signal u the output signal y will not jump. In fact, in case of $D = 0$, the output signal y will change only under the effect of the state variable x; however, since the state variable is the output of an integrating element, in principle x is unable to undergo any step change. After all, this means that a time delay is present between the output signal y and the excitation u. The transfer function of a SISO process is:

$$W_p(s) = y(s)/u(s) = C(sI - A)^{-1}B + D = m_p(s)/n_p(s).$$

Since $D = 0$, the degree m of the numerator $m_p(s)$ is necessarily smaller than the degree n of the denominator $n_p(s)$ in the transfer function ($m < n$), the value of $v_p(t)$ step response at $t = 0$ is $v_p(0) = 0$. The block diagram of the process is as follows:

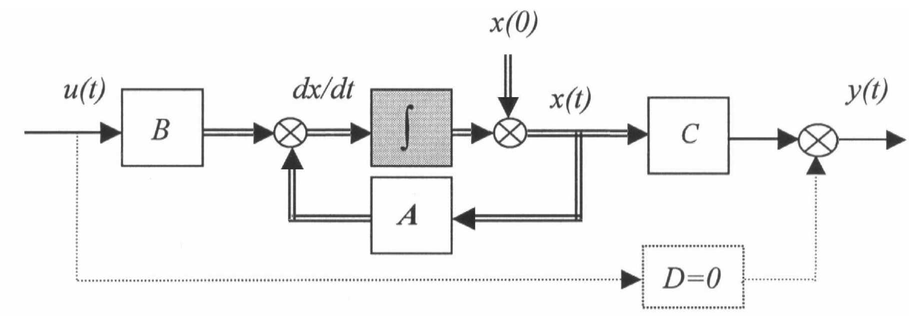

Fig. 1. Block diagram according to the state equation of the process

If the process is proportional with $u = u_0$ constant input signal in steady state, the steady state value of x state variable will be $x_0 = -A^{-1}Bu_0$ and the steady state output signal will be $y_0 = Cx_0 = -CA^{-1}Bu_0$. The dc-gain is $k_p = y_0/u_0 = -CA^{-1}B$. The steady state is reached when the transients are settled (in principle at time $t = \infty$) and the transients 'decay' according to the function $exp(p_i t)$ where

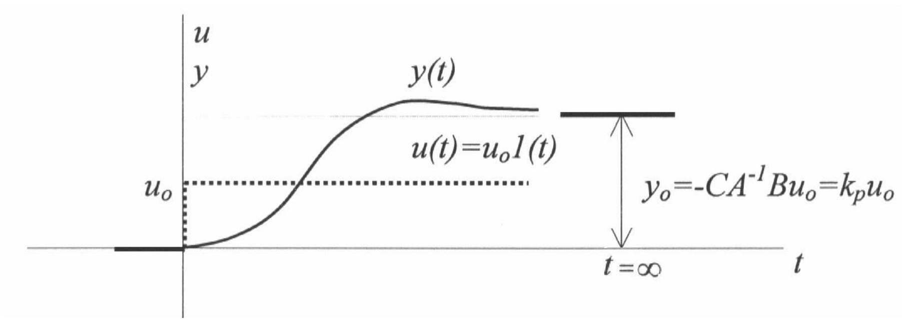

Fig. 2. Step response of the proportional SISO process

$p_i = \lambda_i$ $1, 2, \ldots n)$ are the negative eigenvalues or the eigenvalues with negative real part of the state matrix A, i.e. the poles of the transfer function of the system.

For example, in the case of an asymptotically stable third order process with damped oscillations in its step response, the eigenvalues λ_i of state matrix A, i.e. the poles p_i of the transfer function are: $\lambda_1 = p_1$, $\lambda_2 = p_2$, $\lambda_3 = p_3$. In a general case,

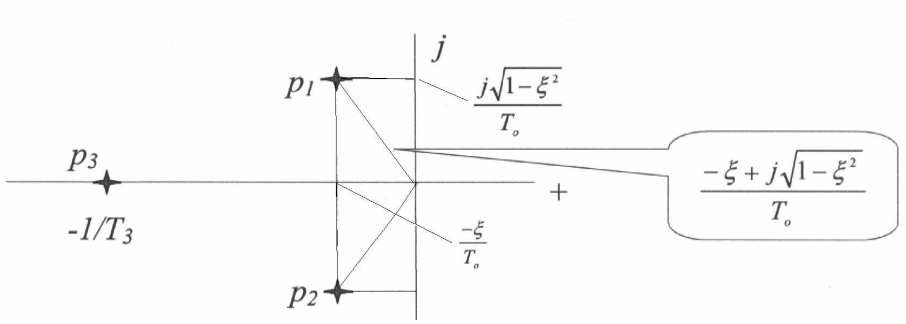

Fig. 3. Eigenvalues of the state matrix A, i.e. the poles of process

the poles are located either on the real axis of the complex plane or symmetrically to the real axis. The characteristic equation of the process is:

$$\det(\lambda\mathrm{I}\text{-}\mathrm{A}) = \mathrm{n}_p(\lambda) = \lambda^n + n_1\,\lambda^{n-1} + \ldots + n_{n-1}\,\lambda + n_n = (\lambda\text{-}p_1)(\lambda\text{-}p_2)\ldots(\lambda\text{-}p_i)\ldots(\lambda\text{-}p_n) = 0$$

The roots p_i $(i=1,2,\ldots n)$ of this equation determine the transient behaviour of the process, i.e. whether it is stable or unstable. Note that the value of the coefficient n_n is equal to the product of the poles of the process: $n_n = (-1)^n p_1 p_2 \ldots p_i \ldots p_n$.

NOTES. In the traditional control structure, the control signal u is set by the controller according to the control algorithm, from the difference $u_a - y$ between the set point u_a representing the set value y_A of the manipulated variable and the effective value y of the manipulated variable. Very often this control algorithm is characterized by $W_c(s)$ transfer function having PID characteristics, and it is the PD part of the controller that, by overexciting the $u$signal, brings about the effect that causes the system to be, so to say, accelerated. The design of the controller using serial compensation is widespread in control engineering. The classical control algorithm of the control signal u and its overexcitation ratio are as follows:

$$u(s) = W_c(s)\,[u_a(s) - y(s)] = k_c \left(1 + \frac{1}{sT_I} + \frac{sT_D}{1+sT}\right)[u_a(s) - y(s)]$$
$$u_t = \frac{u(0)}{u(\infty)} = k_c k_p \left(1 + \frac{T_D}{T}\right)$$

The controller design based on compensation resumes at determining $W_c(s)$ (its parameters k_c, T_I, T_D and T). Taking these into consideration, the block diagram of the classical feedback system is the following:

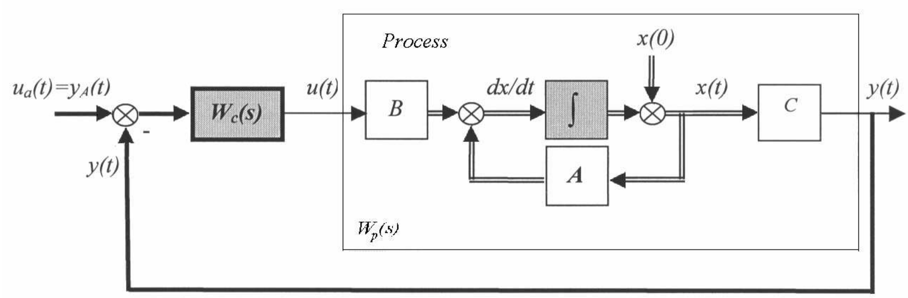

Fig. 4. Block diagram of the feedback control system with serial compensation

2. State Feedback

The system transients can also be accelerated by feeding back the state variables of the process. Using state feedback on $x(t)$ with a matrix (row vector) F and inserting a scalar gain k_c into the structure, the input signal of the closed loop system will be the reference $u_a(t)$,while the input of the process will be the control signal, according to the algorithm $u(t) = k_c u_a(t) - Fx(t)$. So, by selecting F and k_c properly, the transients can be shortened, which appears as if the poles $p_1, p_2, \ldots p_i, \ldots p_n$ of the process were replaced by the poles $p_{R1}, p_{R2}, \ldots p_{Ri}, \ldots p_{Rn}$ of the closed loop system. Due to the acceleration, the condition $real(p_{Ri}) < real(p_i) < 0$ must be fulfilled, since this results in the transients of time function $exp(p_{Ri}t)$ to settle quickly. Based on the above, the control structure established by using state feedback is as follows:

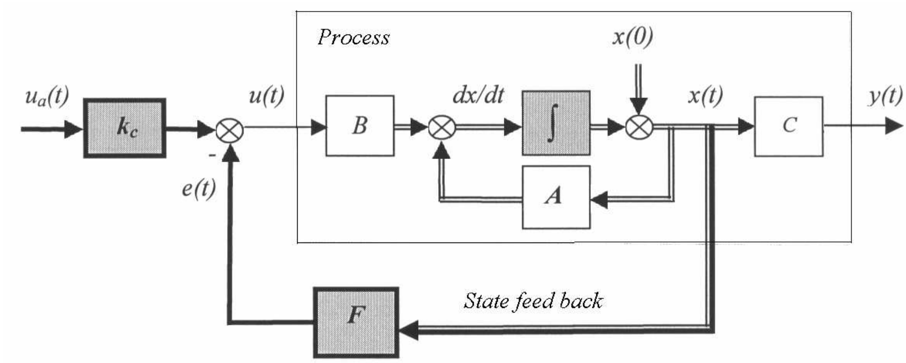

Fig. 5. Block diagram of state feedback

The state equations of the feedback system are:

$$\begin{aligned} \frac{dx(t)}{dt} &= Ax(t) + Bu(t) \\ u(t) &= -Fx(t) + k_c u_a(t) \\ y(t) &= Cx(t) \end{aligned}$$

and:

$$\begin{aligned} \frac{dx(t)}{dt} &= (A - BF)x(t) + Bk_c u_a(t) \\ y(t) &= Cx(t) \end{aligned} \tag{2}$$

Based on the state equations, the block diagram of the resulting system will be: The state matrix of the resulting system is A-BF, thus, its p_{Ri}=λ_{Ri} eigenvalues

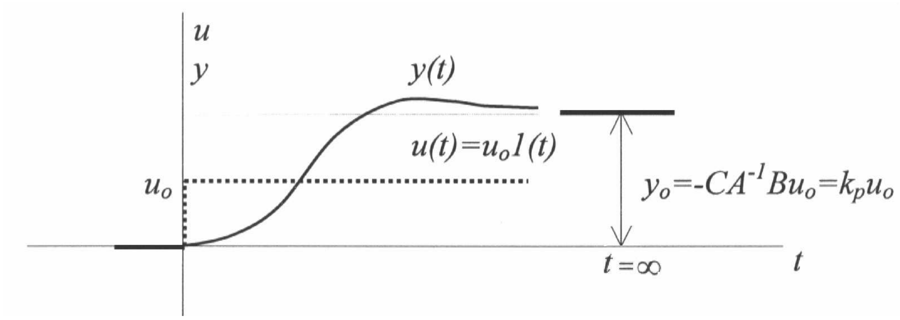

Fig. 6. Block diagram of the system with state feedback

can freely be specified with appropriate selection of F (*dimensioning for specified eigenvalues*). If the feedback system is also required to track a step reference signal, the resulting system gives the responses x_0=-$(A$-$BF)^{-1}Bk_c u_{a0}$, and y_0=Cx_0=-$C(A$-$BF)^{-1}Bk_c u_{a0}$, respectively, to a constant input signal u_{a0} in steady state. Accordingly, the dc-gain of the resulting feedback system is $k_R = -C(A - BF)^{-1}Bk_c$.

The specified pole distribution (e.g. in case of a third order system) is depicted below: The characteristic equation of system with state feedback is:

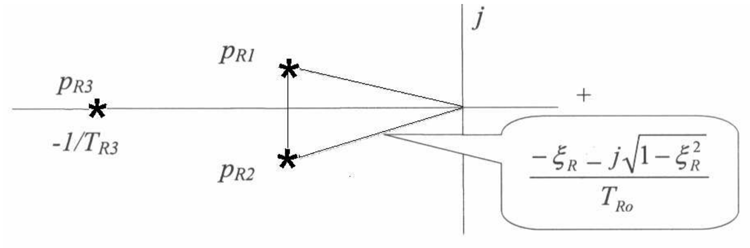

Fig. 7. The specified eigenvalues of the A-BF resulting state matrix in the system with state feedback, i.e. the p_R poles of the feedback system

$$\det \lambda I - (A - BF) = n_R(\lambda) = \lambda^n + n_{R1}\lambda^{n-1} + \ldots n_{Ri}\lambda^{n-i} + \ldots n_{R(n-1)}\lambda + n_{Rn} =$$
$$= (\lambda - p_{R1})(\lambda - p_{R2}) \ldots (\lambda - p_{Ri}) \ldots (\lambda - p_{Rn}) = 0$$

Each coefficient n_{Ri} of this equation is given, if the p_{Ri} poles are considered to be design requirements. Note that, just like in the case the characteristic equation of the process, the value of the n_{Rn} coefficient is determined by the product of the poles of the feedback system, i.e.: $n_{Rn} = (-1)^n p_{R1} p_{R2} \ldots p_{Ri} \ldots p_{Rn}$.

For design, F and k_c must be determined. The eigenvalues p_i of the state matrix A of the process are known, the eigenvalues p_{Ri} of the state matrix A-BF of the system are given as design requirements. Considering$A$, B and p_{Ri} *as known, the value of F must be selected so as to make the* λ_{Ri} *eigenvalues of A-BF state matrix of the feedback system equal to the specified values* p_{Ri}. Knowing A, B and p_{Ri} , the feedback gain F can be calculated from the Ackermann formula as follows:

$$F = [0\ 0\ \ldots\ 0\ 1]\ Co^{-1} n_R(A)$$

If the dc-gain of the original system is required to the same with the dc-gain of the feedback system, then the following condition must be fulfilled: $k_p = -CA^{-1}B = -C(A - BF)^{-1}Bk_c = k_R$. From this, after calculating the feedback matrix F, the gain k_c can be determined.

$$k_c = \ C(A - BF)^{-1}B\ ^{-1}CA^{-1}B \tag{3}$$

The calculations are also supported efficiently by the MATLAB `acker` function:

```
F = acker(A,B,pR)
kc = inv(C*inv(A-B*F)*B)*C*inv(A)*B
```

From a physical point of view, accelerating the transients of the system by means of state feedback means that, for example, under the effect of a step reference u_a

applied to the input of the system being at rest, a signal $u(0) = k_c u_a(0)$ appears at the direct input of the process at the time $t = 0$; in fact, at this point each of the x state variables is still equal to zero: $x(0) = 0$. This forced u signal accelerates the system transients. At the end of the transitory process, the feedback resets the u signal to the value $u(\infty) = k_c u_a(\infty) - Fx(\infty)$. The degree of *overexcitation ratio u(0)/u(*∞) can be expressed by the extent of pole transfer. Having the input signal $u_a(t) = u_{a0}1(t)$, the signals $u(0)$ and *u(*∞) of the process are:

$$u(0) = k_c u_a(0) = k_c u_{ao}$$
$$u(\infty) = k_c u_a(\infty) - Fx(\infty) = k_c u_{a0} - Fx(\infty)$$

Considering that the equilibrium value of state variable x is $x(\infty) = -(A - BF)^{-1} B k_c u_{a0}$, we obtain:

$$u(\infty) = k_c u_{a0} - Fx(\infty) = k_c u_{a0} + F(A - BF)^{-1} B k_c u_{a0} = \; 1 + F(A - BF)^{-1} B \; u(0)$$

From this, the u_t overexcitation ratio will be:

$$\frac{u(0)}{u(\infty)} = u_t = \left[1 + F(A - BF)^{-1} B\right]^{-1} = \frac{n_{Rn}}{n_n} = \prod_{i=1}^{n} \frac{p_{Ri}}{p_i} \qquad (4)$$

This is overexcitation of the control signal $u(t)$,and resetting the overexcitation

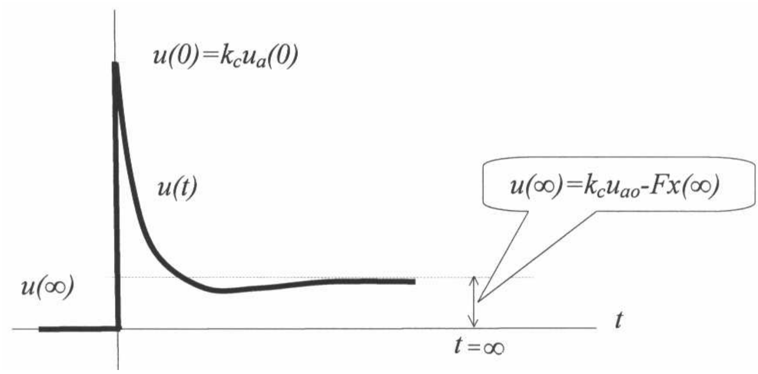

Fig. 8. Overexcitation of the input signal u under the effect of a step reference input $u_a(t)=u_{a0}1(t)$.

in an appropriate manner induces the effect that *appears* as an acceleration of the process. Physically speaking, the *overexcitation results in* acceleration, while the state feedback restrains the extent of this forced intervention. An intervention of this type can result not only from structures with state feedback; but similar acceleration can also be obtained by means of serial compensation using PD type elements [7].

If the state equation of SISO process is available in a canonical form, the design of the state feedback, i.e. the determination of the parameters F, k_c and u_t can be performed by means of very simple relationships instead of using complicated and labour intensive matrix operations [7].

EXAMPLE. *State feedback of a third order SISO process*
The transfer function of third order lag SISO process is as follows:

$$W_p(s) = \frac{y(s)}{u(s)} = \frac{m_p(s)}{n_p(s)} = \frac{m_3}{s^3 + n_1 s^2 + n_2 s + n_3} = \frac{m_3}{(s-p_1)(s-p_2)(s-p_3)} = \\ = \frac{6}{s^3 + 6s^2 + 11s + 6} = \frac{6}{(s+1)(s+2)(s+3)}$$

By using state feedback in order to shorten the transient response, let us design a system having the prescribed poles: $p_{R1} = -3$, $p_{R2} = -6$, and $p_{R3} = -9$, and characteristic polynomial as follows: $\det \lambda I - (A - BF) = n_R(\lambda) = (\lambda - p_{R1})(\lambda - p_{R2})(\lambda - p_{R3}) = \lambda^3 + n_{R1}\lambda^2 + n_{R2}\lambda + n_{R3} = \lambda^3 + 18\lambda^2 + 99\lambda + 162$. In addition, the dc-gain of the system with state feedback must be the same as the dc-gain of the process: $k_R = k_p = m_3/n_3 = 6/6 = 1$. Let us calculate the overexcitation ratio. ($n_1 = 6$, $n_2 = 11$, $n_3 = 6$, $m_3 = 6$, $n_{R1} = 18$, $n_{R2} = 99$, $n_{R3} = 162$).

The third order linear differential equation with constant coefficients will be:

$$\frac{d^3y(t)}{dt^3} + n_1\frac{d^2y(t)}{dt^2} + n_2\frac{dy(t)}{dt} + n_3 y(t) = m_3 u(t)$$
$$\frac{d^3y(t)}{dt^3} = -n_1\frac{d^2y(t)}{dt^2} - n_2\frac{dy(t)}{dt} - n_3 y(t) + m_3 u(t)$$

Based on the differential equation, or by means of direct decomposition of the transfer function $W_p(s)$, the block diagram of the process built from basic elements can be determined. This block diagram will enable us to describe the state equation in the controllability canonical form.

With the symbols used in the block diagram, the state equation and parameter matrices of the process are as follows:

$$\frac{dx_1(t)}{dt} = -n_1 x_1(t) - n_2 x_2(t) - n_3 x_3(t) + u(t)$$
$$\frac{dx_2(t)}{dt} = x_1(t)$$
$$\frac{dx_3(t)}{dt} = x_2(t)$$
$$y(t) = m_3 x_3(t)$$

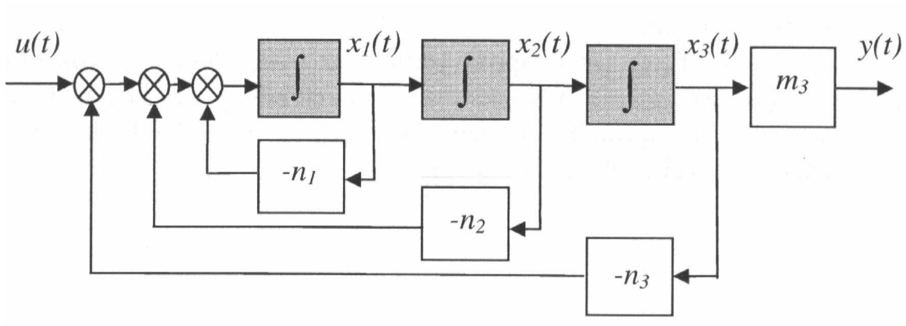

Fig. 9. Third order system – Block diagram according to the controllability canonical form

$$A = \begin{bmatrix} -n_1 & -n_2 & -n_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & m_3 \end{bmatrix} \quad D = 0$$

The dc-gain of the plant transfer function:

$$k_p = -CA^{-1}B = \frac{m_3}{n_3}$$

If the state variables $x_1(t)$, $x_2(t)$ and $x_3(t)$ are measurable with sensors, state feedback structure can be implemented. Its block diagram built with basic elements will be:

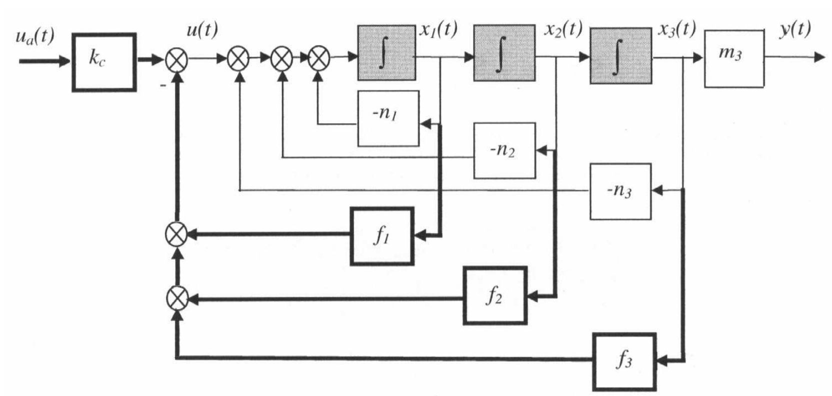

Fig. 10. Block diagram of a system with state feedback

Here the state variables are fed back through the f_1, f_2 and f_3 gains to the input. The purpose of feedback is to place the poles of the system p_{R1}, p_{R2} and p_{R3} according to the design specification. The feedback structure – since the summation elements can be freely interchanged – can be simplified. In order to do this, it can be seen that the elements with gains $-n_1$ and f_1 , $-n_2$ and f_2 as well as $-n_3$ and f_3 are connected in parallel. Therefore, the simplified block diagram is shown below:

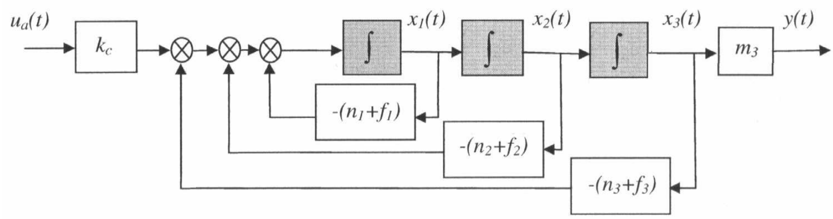

Fig. 11. Simplified block diagram of the system

The modified block diagram of the resulting system also corresponds to a canonical form of controllability. Using the symbols used in the block diagram, the state equation and parameter matrices of the feedback system are:

$$\frac{dx_1(t)}{dt} = -(n_1+f_1)x_1(t) - (n_2+f_2)x_2(t) - (n_3+f_3)x_3(t) + k_c u(t)$$
$$\frac{dx_2(t)}{dt} = x_1(t)$$
$$\frac{dx_3(t)}{dt} = x_2(t)$$
$$y(t) = m_3 x_3(t)$$

$$A_R = A - BF = \begin{bmatrix} -(n_1+f_1) & -(n_2+f_2) & -(n_3+f_3) \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad B_R = Bk_c = \begin{bmatrix} k_c \\ 0 \\ 0 \end{bmatrix}$$
$$C_R = C = \begin{bmatrix} 0 & 0 & m_3 \end{bmatrix} \qquad D_R = D = 0$$

$$k_R = -C(A-BF)^{-1}Bk_c = \frac{k_c m_3}{n_3+f_3}$$

The prescribed poles of the feedback system are: $p_{R1} = -3$, $p_{R2} = -6$, and $p_{R3} = -9$. The corresponding characteristic polynomial is: $\det[\lambda I - (A-BF) = n_R = (\lambda - p_{R1})(\lambda - p_{R2})(\lambda - p_{R3}) = (\lambda+3)(\lambda+6)(\lambda+9) = \lambda^3 + n_{R1}\lambda^2 + n_{R2}\lambda + n_{R3} = \lambda^3 + 18\lambda^2 + 99\lambda + 162$.
($n_{R1} = 18, n_{R2} = 99, n_{R3}=162$). The feedback gain F=[$f_1 f_2 f_3$] and k_c as well as the u_t overexcitation ratio is calculated as follows:

$$F = \ 0\ 0\ 1\ \ Co^{-1} n_R(A) = \ 0\ 0\ 1\ \left[B\ AB\ A^2B\right]^{-1} \left(A^3 + n_{R1}A^2 + n_{R2}A + n_{R3}I\right)$$
$$k_c = \left[C(A-BF)^{-1}B\right]^{-1}\ CA^{-1}B$$
$$u_t = \left[1 + F(A-BF)^{-1}B\right]^{-1} \tag{5}$$

The calculus of k_c and u_t must be preceded by the determination of the feedback gain F that can be calculated by means of the *Ackermann* formula. The state matrix A and controllability test matrix Co of an n^{th} order SISO process are of $n \times n$ size. The size of the input matrix B is $n \times 1$, while the matrices F and C are of $1 \times n$ size. It also follows that, even in case of the given $n = 3^{rd}$ order system, complicated matrix operations must be performed (raising to a power, inverse calculations, multiplication). It appears to be nearly hopeless without using the services of MATLAB. In order to determine the values of F, k_c and u_t, let us use the MATLAB tools for handling symbolic variables. Thus:

```
syms n1 n2 n3 m3 nR1 nR2 nR3 real A=[-n1 --n2 --n3;1 0 0;0 1 0];B=[1 0 0]';
C=[0 0 m3];D=0;                      % Parameter matrices of the process
Co=[B A*B A^2*B];                    % Test matrix of controllability
nR=[1 nR1 nR2 nR3];                  % Characteristic polynomial of the
nRA=polyvalm(nR,A);                  %                  system
F=[0 0 1]*inv(Co)*nRA;               % The feedback matrix
kc=inv(C*inv(A-B*F)*B)*C*inv(A)*B;   % The dc-gain
ut=inv(1+F*inv(A-B*F)*B);            % The overexcitation ratio
disp(F);
disp(kc);
disp(ut);
n1=6;n2=11;n3=6;m3=6;
nR1=18;nR2=99;nR3=162;
disp(subs(F));                       % F=[12 88 152]
disp(subs(kc));                      % kc=27
disp(subs(ut));                      % ut=27
```

Results obtained by using the MATLAB features are:

$$\begin{aligned} F &= \begin{bmatrix} -n_1 + n_{R1} & -n_2 + n_{R2} & -n_3 + n_{R3} \end{bmatrix} = \begin{bmatrix} 12 & 88 & 156 \end{bmatrix} \\ k_c &= \frac{n_{R3}}{n_3} = 27 \\ u_t &= \frac{1}{1 - \dfrac{-n_3 + n_{R3}}{n_{R3}}} = \frac{n_{R3}}{n_3} = 27 \end{aligned} \tag{6}$$

It must be noted again that the n_3 coefficient is the product of the poles p_i of the process, while the coefficient n_{R3} is the product of p_{Ri} poles of the system with state feedback. As shown, the u_t *overexcitation ratio is determined by the ratio of pole transfer*.

NOTES. In this example, the parameter matrices *in the canonical form of* controllability were allocated to the transfer function of the process. Due to all these properties, the characteristic polynomial of the system with state feedback can also be determined directly from the block diagram. Taking these into consideration, one gets:

$$\lambda^3 + (n_1+f_1)\lambda^2 + (n_2+f_2)\lambda + n_3+f_3 = \lambda^3 + (6+f_1)\lambda^2 + (11+f_2)\lambda + 6+f_3$$

$$\lambda^3 + n_{R1}\lambda^2 + n_{R2}\lambda + n_{R3} = \prod_{i=1}^{3}(\lambda - p_{Ri}) = \lambda^3 + 18\lambda^2 + 99\lambda + 162$$

By comparing the coefficients of polynomials, we obtain:

$$6 + f_1 = 18 \qquad 11 + f_2 = 99 \qquad 6 + f_3 = 162$$
$$f_1 = 12 \qquad f_2 = 88 \qquad f_3 = 156$$

These values are the same with the results calculated by using the Ackermann formula.

3. State Feedback Using State Observer

State feedback from the state variables $x(t)$ of the process is possible *when these state variables are measurable with sensors.* This sometimes is not possible, and even the mathematical model of the process is often unavailable. In such cases, the mathematical model of the process must be developed based on the results of measurements using certain identification procedures [8]. Typically, the identification is based on determining the step response experimentally or measuring the frequency function of the process. The final result of the identification is the transfer function of the process from which the state-space representation can also be determined.

After determining experimentally the transfer function $W_p(s)$of the process and its parameters or the state-space representation and its parameter matrices A, B, C, a physical system can be established, represented for example in the form of an electrical network. The state variables of this are the $x*$ state variables and its parameter matrices are the A, B,C parameter matrices already identified. In addition, this physical model (the *state observer)* must be designed in such a manner that the variables $x*$ are also accessible to measurements. Having this done, if *the u input signal is applied to the input of both the process and the observer at the same time and the state feedback is implemented from the x* state variables of this physical model instead of the x state variables of the process, a similar effect is obtained as if the feedback were made from the x state variables of the process.* The state feedback from the state observer can be equivalent to the feedback from the state variables of the process if the $x*(t)$ and $x(t)$ have the same variation. (For the technical implementation of state observer, a digital computer can also be used; in such cases, it is the program running on the computer that plays the part of the process model). The structure of the process and the observer is shown in the block diagram in *Fig. 12*.

It is shown that, if the $u(t)$ is applied to the input of the process and the model of process (the state observer), and the initial conditions $x(0)$ and $x*(0)$ are the same, then $x(t) = x^*(t)$ and therefore $y(t) = y^*(t)$.This also means that the input signal of the feedback gain G of the state observer is zero, that is Gplays no part in this case. The gain G has an active role if the signal difference $y^* - y$ has a value other than zero. This may occur if $x(0) \neq x^*(0)$, that is, the initial conditions of the process and the observer (*with the same input signal u(t) applied)* are different. Thus, in case of state feedback implemented with the state observer, the design task consists in determining the feedback gain G, after having designed the gain F. In this case, $x(t)$ and $x^*(t)$ are different and, therefore, in determining G, *the design*

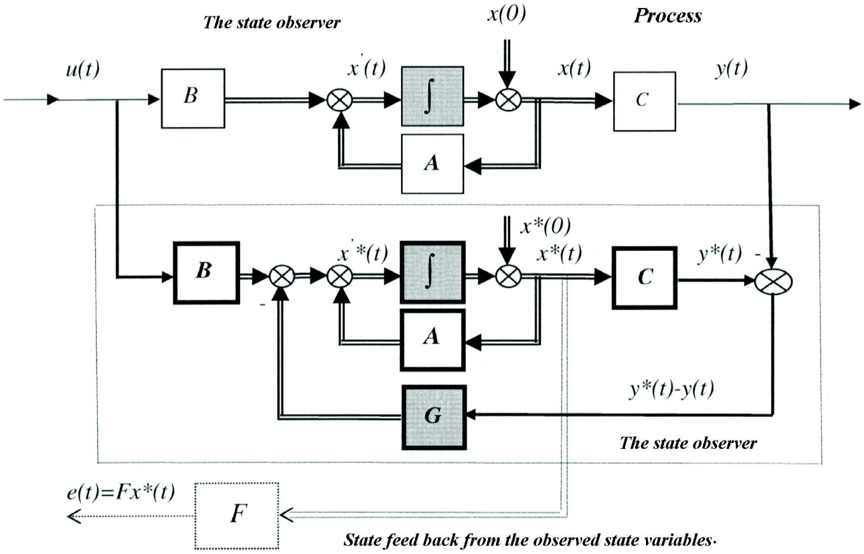

Fig. 12. Structure of process and state observer

requirement may be that $x(t)$ *and* $x^*(t)$ *approach to each other quickly* even if $x(0) \neq x^*(0)$. The design solution can be traced back to the topics related to a state feedback in which the state variable is the difference $x(t) - x^*(t)$ between the state variables of process and those of the observer. After all, the design of the state observer means the determination of Gfeedback matrix (column vector).

Based on *Fig. 12*, for the process and the state observer we obtain:

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$
$$\frac{dx^*(t)}{dt} = Ax^*(t) + Bu(t) - G\left(y^*(t) - y(t)\right) Ax^*(t) + Bu(t) - G\left(Cx^*(t) - Cx(t)\right)$$
$$y^*(t) = Cx^*(t)$$

From this the difference between the differential equations of process and observer:

$$\frac{d}{dt}\left(x(t) - x^*(t)\right) = (A - GC) \cdot \left(x(t) - x^*(t)\right) \tag{7}$$

The solution of this homogeneous state equation with the state matrix *A-GC* will be:

$$x(t) - x^*(t) = e^{(A-GC)t}\left(x(0) - x^*(0)\right) \tag{8}$$

In case of nonzero initial conditions $x(0) \neq x^*(0)$, this solution approaches to zero quickly – i.e. the $x^*(t)$ state variable of observer becomes nearly the same as the

$x(t)$state variable of process as quickly as possible – if the eigenvalues $p_{Mi} = \lambda_{Mi}$ of the matrix $A - GC$ are very small negative numbers ($p_{Mi} \ll 0$).

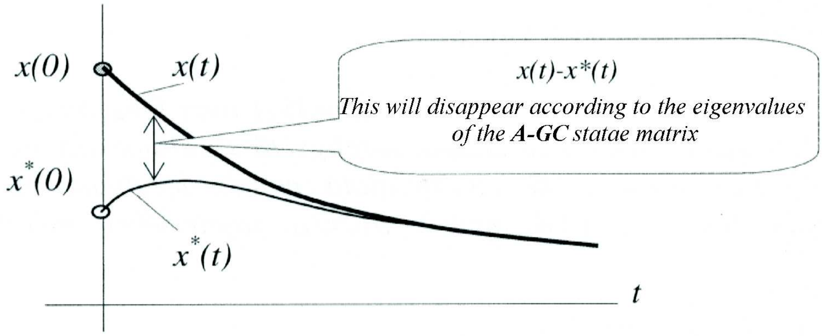

Fig. 13. Free response of the process and the observer

The characteristic equation containing the eigenvalues of A-GC matrix will be:

$$\det \lambda I - (A - GC) \;= n_M(\lambda) = \lambda^n + n_{M1}\lambda^{n-1} + \ldots + n_{M(n-1)}\lambda + n_{Mn} = (\lambda - p_{M1})(\lambda - p_{M2})\ldots(\lambda - p_{Mi})\ldots(\lambda - p_{Mn}) = 0$$

It is recommended to select the eigenvalues p_{Mi}=λ_{Mi} so as to be even less than the eigenvalues of the system λ_{Ri}accelerated by means of the state feedback $F\lambda_{Mi} < \lambda_{Ri}$). Considering the eigenvalue λ_{Mi} to be a design specification, the value of G can be determined. As the matrices A, B are controllable and the matrices A, C are observable, therefore G can also be determined by using the MATLAB function**acker**. It is important to note that the observability of matrices A, C ensures not only the theoretical possibility that the observed state variables $x^*(t)$ can be computed from the control signal u and the output signal y; instead, it also warrants for that, by appropriate selection of G, the eigenvalues λ_{Mi} of the state observer can be freely specified [3].

NOTES. In order to do this, it must be taken into consideration that, when designing the state feedback, the **acker** function can be used in respect of the *det*[λ*I(ABF)*] characteristic polynomial, while the eigenvalues p_{Ri}=λ_{Ri} of *AB**F*** are the design specifications. In this case, the feedback matrix F is *multiplied from the left* by the known input matrix B and F=**acker**(A,B,pR).

When designing the observer, the matrix Gshall be determined based on the specified root p_{Mi}=λ_{Mi} of the characteristic polynomial *det*[λ*I(A**GC**)*] and it must be taken into account that, in this case, the G to be dimensioned *is multiplied from the right* by the known C output matrix.

According to a mathematical theorem, the roots p_{Mi} of the characteristic polynomial $det\ \lambda I - (A - GC)$ are identical to the roots of the $det\ \lambda I - (A^T -$

$C^T G^T$) polynomial and, for the latter, the *Ackermann* formula of the MATLAB function **acker** can already be used:

$$G^T=[0\ 0\ \ldots 0\ 1][C^T\ A^T C^T\ \ldots (A^T)^{n-1} C^T]^{-1}[(A^T)^n + n_{M1}(A^T)^{n-1} + \ldots n_{Mn} I]$$

```
GT=acker(A',C',pM);G=GT';
```

Although setting the poles p_{Mi} according to $p_{Mi} < p_{Ri} < 0$ ensures that the difference $x(t)-x^*(t)$ disappears quickly, however, this condition gives no warranty for the initial errors. The reason is that the time functions are also influenced by the numerators of the transfer functions; these, however, cannot be kept under control by the design of this type [3].

As a summary, it can be stated that, if the mathematical model (the transfer function and the state equation, respectively) of a process is known, the poles p_i *of the process can also be considered as known. If, in order to achieve the quick settling of the transients, the poles* p_i *are "transferred" by means of state feedback to the predetermined places* p_{Ri}*, the task is solved by designing the feedback gain* $\boldsymbol{F}$ *and the gain* $\boldsymbol{k}_c$

```
F=acker(A,B,pR);
kc=inv C*inv(A-B*F)*B)*C*inv(A)*B
```

If the sensors do not have access to the state variables x of the process, the feedback F is implemented from the state variables x^* *of the state observer that models the process. The design of the state observer means the determination of the feedback matrix G based on considering the eigenvalues* $p_{Mi}\lambda_{Mi}$*of the matrix* $A - GC$ *as design specification:*

```
GT=acker(A',C',pM);G=GT';
```

Of course, in case of state feedback using a state observer, the state observer itself must also be realized. In addition to that, access to x^* shall be ensured, it is also necessary that the process and the signals u and y of the model can also be adapted to each other.

The block diagram of the feedback system with a state observer is shown below [3]:

When writing the state equation of the system, let us take the control signal $u(t)$ and the signals $h(t) = x(t) - x^*(t)$ in addition to the output signal y as further output signals, based on the block diagram. By means of tracing these signals, a comprehensive overview of the transients of the process, the observer and the complete control system can be obtained. The state equation of the system will be:

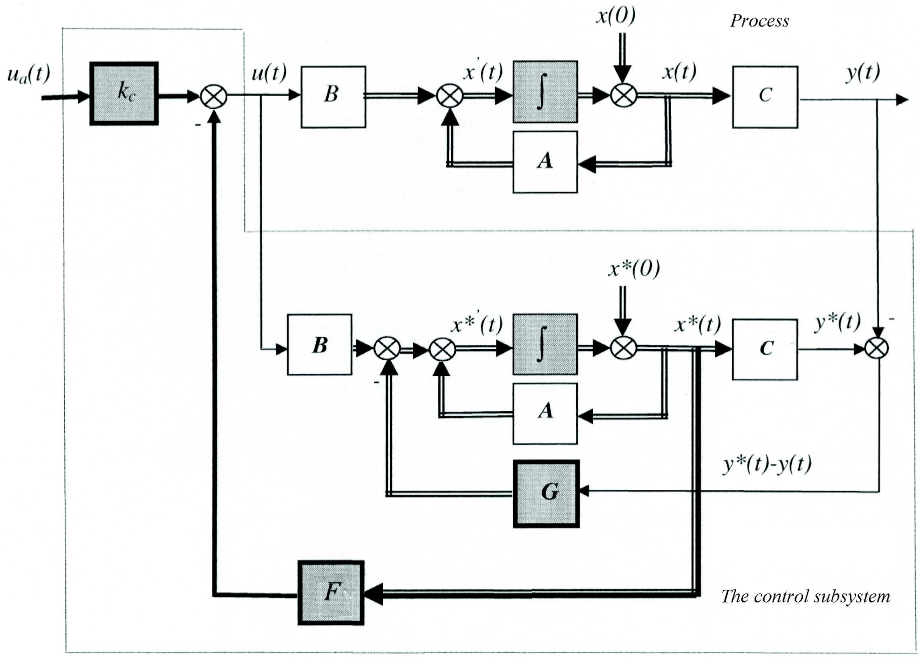

Fig. 14. Block diagram of the complete system

$$
\begin{aligned}
\frac{dx(t)}{dt} &= Ax(t) + B\left(k_c u_a(t) - Fx^*(t)\right)\\
\frac{dx^*(t)}{dt} &= Ax^*(t) - G\left(Cx^*(t) - Cx(t)\right) + B\left(k_c u_a(t) - Fx^*(t)\right)\\
y(t) &= Cx(t)\\
u(t) &= -Fx^*(t) + k_c u_a(t)\\
h(t) &= x(t) - x^*(t)
\end{aligned}
$$

Arranged to a normal form:

$$
\begin{bmatrix} \frac{dx(t)}{dt} \\ \frac{dx^*(t)}{dt} \end{bmatrix} = \begin{bmatrix} A & -BF \\ GC & A - BF - GC \end{bmatrix} \begin{bmatrix} x(t) \\ x^*(t) \end{bmatrix} + \begin{bmatrix} Bk_c \\ Bk_c \end{bmatrix} u_a(t)
$$

$$
\begin{bmatrix} y(t) \\ u(t) \\ h(t) \end{bmatrix} = \begin{bmatrix} C & 0_{1\times n} \\ 0_{1\times n} & -F \\ I_{n\times n} & -I_{n\times n} \end{bmatrix} \begin{bmatrix} x(t) \\ x^*(t) \end{bmatrix} + \begin{bmatrix} 0 \\ k_c \\ 0_{n\times 1} \end{bmatrix} u_a(t)
$$

Substituting: $dx_R(t)/dt = \; dx(t)/dtdx^*(t)dt \; ^T, x_R(t) = \; x(t)x^*(t) \; ^T$ and $y_R(t) =$

$y(t)u(t)h(t)\ ^T$:

$$\begin{array}{l}\frac{dx_R(t)}{dt} = A_R x_R(t) + B_R u_a(t) \\ y_R(t) = C_R x_R(t) + D_R u_a(t)\end{array} \tag{9}$$

The resulting parameter matrices and the block diagram of the system based on the state equations are:

$$\boxed{\begin{array}{ll} A_R = \begin{bmatrix} A & -BF \\ GC & A - BF - GC \end{bmatrix} & B_R = \begin{bmatrix} Bk_c \\ Bk_c \end{bmatrix} \\ C_R = \begin{bmatrix} C & 0_{1\times n} \\ 0_{1\times n} & -F \\ I_{n\times n} & -I_{n\times n} \end{bmatrix} & D_R = \begin{bmatrix} 0 \\ k_c \\ 0_{n\times 1} \end{bmatrix} \end{array}} \tag{10}$$

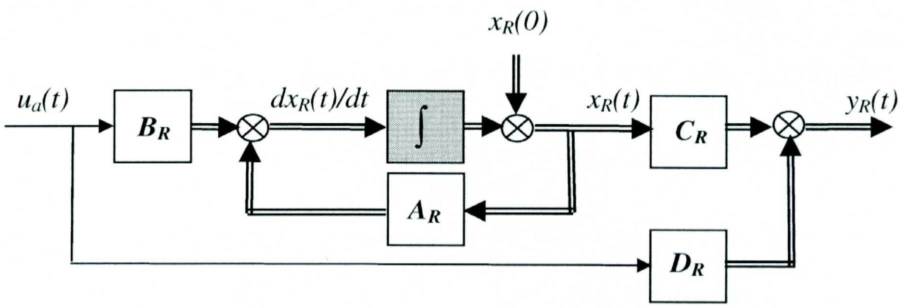

Fig. 15. Block diagram of a state control with state observer

In these system matrices, A, B, C are the parameter matrices of the state controllable and state observable SISO process. The row vector $\mathbf{F}$ and gain $\mathbf{k}_c$ are designed based on the knowledge of the poles p_{Ri} specified for the state feedback, while the column vector G is determined based on the requirements upon the observer poles p_{Mi}. If the matrices A_R, B_R, C_R and D_R are available, and using the MATLAB facilities, the analysis of the control system can easily be performed. The characteristic equation of the system will be:

$$\det(\lambda I - A_R) = \det\ \lambda I - (A - BF)\ \det\ \lambda I - (A - GC)\ = 0$$

In order to ensure the asymptotic stability, the matrices $A - BF$ and $A - GC$ each must have separate negative eigenvalues. The MATLAB program for designing and testing the system is included in the Appendix.

EXAMPLE. *State control of a third order SISO process using observer*

The third order lag process is defined by its $W_p(s)$ transfer function as follows:

$$W_p(s) = \frac{y(s)}{u(s)} = \frac{m_3}{s^3 + n_1 s^2 + n_2 s + n_3} = \frac{m_3}{(s-p_1)(s-p_2)(s-p_3)} =$$
$$= \frac{6}{s^3 + 6s^2 + 11s + 6} = \frac{6}{(s+1)(s+2)(s+3)}$$

Using a state observer and keeping the gain $k_p = m_3/n_3$ =1 of the process, let us design a system with state feedback in which the prescribed poles of the accelerated system are: $p_{R1} = -3$, $p_{R2} = -6$, $p_{R3} = -9$, and the poles of observer are: $p_{M1} = p_{M2} = p_{M3} = -10$.

The block diagram, state equation and parameter matrices of third order process were already calculated. The diagram built with basic elements using state observer is the following:

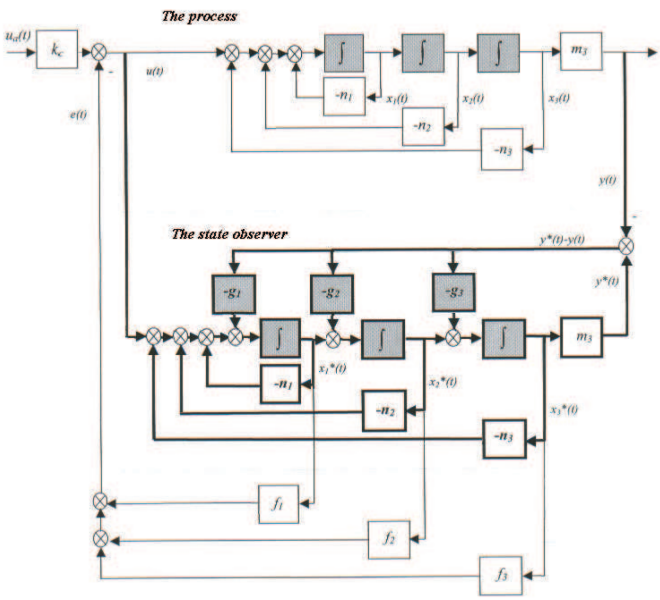

Fig. 16. Block diagram including the process, the state observer and the state feedback

For system design the program described in the Appendix was used. The obtained results are:

```
F=acker(A,B,pR)                           % f1=12 f2=88 f3=156
kc=inv(C*inv(A-B*F)*B)*C*inv(A)*B         % kc=27
ut=inv(1+F*inv(A-B*F)*B)                  % ut=27
GT=acker(A',C',pM)                        % g1=-23.33 g2=24.16 g3=4.00
```

It can be read also from the block diagram, that actually the state feedback of the estimated state variables $x^*(t)$ is implemented, and the control signal $u(t)$ resulting from the state feedback is present simultaneously at the input of both the process and the state observer. The principle mentioned is particularly expressive if the initial conditions of both the process $[x(0)]$ and the observer $[x^*(0)]$ are the same. In this case – given that $y^*(t) - y(t) = 0$ – the block diagram of the feedback system can be simplified; in fact, the G factor has no role, and therefore $G = 0$ can be assumed. See block diagram for demonstration:

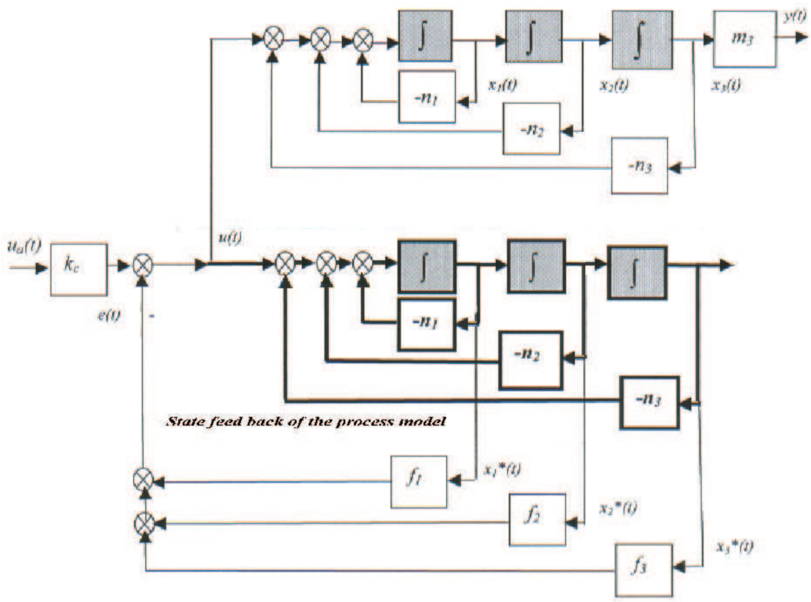

Fig. 17. State feedback using state observer with G=0.

It might be examined what are the requirements to be set relatively to the observer in order to obtain $G = 0$ as a result of the design. As indicated earlier, the *Ackermann* formula that determines the G feedback gain of the observer is the following:

$$G^T = \;00\ldots01\;\; C^T A^T C^T \ldots (A)^T)^{n-1} C^T \;{}^{-1}\; (A^T)^n + n_{M1}(A^T)^{n-1} + \ldots n_{Mn} I$$

It follows that $G^T = 0$ is possible if $(A^T)^n + n_{M1}(A^T)^{n-1} + \ldots n_{Mn} I \;= 0$. According to *Cayley-Hamilton's* theorem, A^T fulfils its own characteristic equation, therefore, $G^T = 0$ can only be ensured if $n_{Mi} = n_i$ (n_i are the coefficients of the characteristic equation of the process $\det(\lambda I - A) = 0$). Yet, this means as if the poles p_i of the process were prescribed for the poles p_M of the matrix $A - GC$. This would not be a good choice, since the error $x(t) - x^*(t)$ would disappear slowly, as a function of $x(t) - x^*(t) = \exp(At)\; x(0) - x^*(0)$ as determined by the poles p_i of the process. Even more, if any of the poles p_i were of positive value, the state feedback would be unable to stabilize the system, as a result of the lack of G feedback matrix. For all these reasons, the observer having state matrix $A - GC$ must be stable (each one of the poles p_M of the characteristic polynomial $\det(sI - (A - GC))$ shall have negative real part) what, in case of unstable A and $G = 0$ is in principle impossible.

If the prescribed roots of the characteristic equation of the matrix $A - GC$ are p_{Mi}, and the coefficients are n_{Mi} $(i = 1, 2, 3 \ldots n)$, then, in this example $n = 3$, therefore:

$$\det \lambda I - (A - GC) =$$

$$\det\left[\lambda\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} - \begin{pmatrix}-n_1 & -n_2 & -n_3\\ 1 & 0 & 0\\ 0 & 1 & 0\end{pmatrix} + \begin{pmatrix}g_1\\ g_2\\ g_3\end{pmatrix}\begin{pmatrix}0 & 0 & m_3\end{pmatrix}\right] =$$

$$= \det\begin{bmatrix}\lambda + n_1 & n_2 & n_3 + g_1 m_3\\ -1 & \lambda & g_2 m_3\\ 0 & -1 & \lambda + g_3 m_3\end{bmatrix}$$

$$\begin{aligned}&\lambda^3 + (g_3 m_3 + n_1)\lambda^2 + ((n_1 g_3 + g_2)m_3 + n_2)\lambda + (g_1 + n_1 g_2 + n_2 g_3)m_3 + n_3 =\\ &\lambda^3 + \quad n_{M1}\lambda^2 + \quad n_{M2}\lambda + n_{M3}\end{aligned}$$

Based on the identity of coefficients, we obtain:

$$\begin{aligned} g_3 m_3 + n_1 &= n_{M1}\\ (g_2 + n_1 g_3)m_3 + n_2 &= n_{M2}\\ (g_1 + n_1 g_2 + n_2 g_3)m_3 + n_3 &= n_{M3}\end{aligned}$$

Expressing the solution obtained for G in a more compact manner:

$$G = \begin{bmatrix}g_1\\ g_2\\ g_3\end{bmatrix} = \begin{bmatrix}1 & n_1 & n_2\\ 0 & 1 & n_1\\ 0 & 0 & 1\end{bmatrix}^{-1}\begin{bmatrix}n_{M3} - n_3\\ n_{M2} - n_2\\ n_{M1} - n_1\end{bmatrix}\frac{1}{m_3}$$

Based on this formula – suitable for determining the gain G – it can easily be shown that the $x(t) - x^*(t)$ error which is due to the different initial conditions of the process $[x(0)]$ and the observer $[x^*(0)]$ with coefficients $n_{Mi} = n_i$ disappears according to the poles p_i of the process and, in this case, $G = 0$.

It may be a better choice if the poles p_{Ri} of accelerated system are prescribed for the poles p_{Mi}. This choice is appropriate even if the process is unstable, but it is stabilized by using state feedback. Finally, this requirement means that the same eigenvalues $\lambda_{Ri} = p_{Ri} = p_{Mi} = \lambda_{Mi}$ are required for the matrices $A - BF$ and $A - GC$. Hence, if the coefficients are selected according to $n_{Mi} = n_{Ri}$, the above mentioned error disappears according to the poles p_{Ri} of the accelerated system.

The gain G that implements this (taking into consideration that, in this particular case, $n_{Mi} - n_i = n_{Ri} - n_i = f_i$) will be:

$$\begin{bmatrix} 1 & n_1 & n_2 \\ 0 & 1 & n_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} f_3 \\ f_2 \\ f_1 \end{bmatrix} \frac{1}{m_3} \Rightarrow \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} 1 & n_1 & n_2 \\ 0 & 1 & n_1 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} f_3 \\ f_2 \\ f_1 \end{bmatrix} \frac{1}{m_3} =$$

$$= \begin{bmatrix} 1 & 6 & 11 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 156 \\ 88 \\ 12 \end{bmatrix} \frac{1}{6} = \begin{bmatrix} -12 \\ 8/3 \\ 2 \end{bmatrix}$$

Choosing the value $p_{M1} = p_{M2} = p_{M3} = -10$ for the poles p_{Mi}, the characteristic polynomial of A-GC will be: det(λI-(A-GC))=(λ+10)(λ+10)(λ+10)=$\lambda^3 + 30\lambda^2 + 300\lambda + 1000 \cdot (n_{M1} = 30, n_{M2} = 300, n_{M3} = 1000)$. Therefore:

$$G = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} 1 & n_1 & n_2 \\ 0 & 1 & n_1 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} n_{M3} - n_3 \\ n_{M2} - n_2 \\ n_{M1} - n_1 \end{bmatrix} \frac{1}{m_3} =$$

$$\begin{bmatrix} 1 & 6 & 11 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1000 - 6 \\ 300 - 11 \\ 30 - 6 \end{bmatrix} \frac{1}{6} = \begin{bmatrix} -70/3 \\ 145/6 \\ 4 \end{bmatrix} = \begin{bmatrix} -23.3333 \\ 24.1666 \\ 4.0000 \end{bmatrix}$$

This, of course, is in conformity with the result obtained earlier.

From the block diagram of the case $G = 0$, another unusual concept of the state feedback implemented by means of state observer can also be interpreted. Its essence is that, based on the mathematical model of the process, a physical model is developed with state variables that can be measured with sensors. Based on this, a state feedback is implemented in the process model thus developed which, of course, also includes the control signal as an internal signal built according to the $u(t) = k_c u_a(t) - F x^*(t)$ algorithm, that is applied at the input of the process model. This signal $u(t)$ contains forcing which accelerates the process model. As a result, if this signal $u(t)$ is also applied to the input of the real process, the actual output signal of the process is also accelerated according to the output signal of the model.

Given the technical possibilities available at present, the process model and its state feedback are implemented on a digital computer. In this case, the program running on the computer can be considered as the algorithm to calculate the discrete u_d control signal. As the control signal $u_d$can be interpreted as a series of discrete samples, therefore the discrete signal u_d is connected through Zero Order Hold, implemented by a DAC digital-analogue converter, to the input of the process. The ADC analogue-digital converter converts the continuous signal y into a series of discrete samples y_d. At the choice of the T_s sampling time the fastest transients of the system must be taken into account.

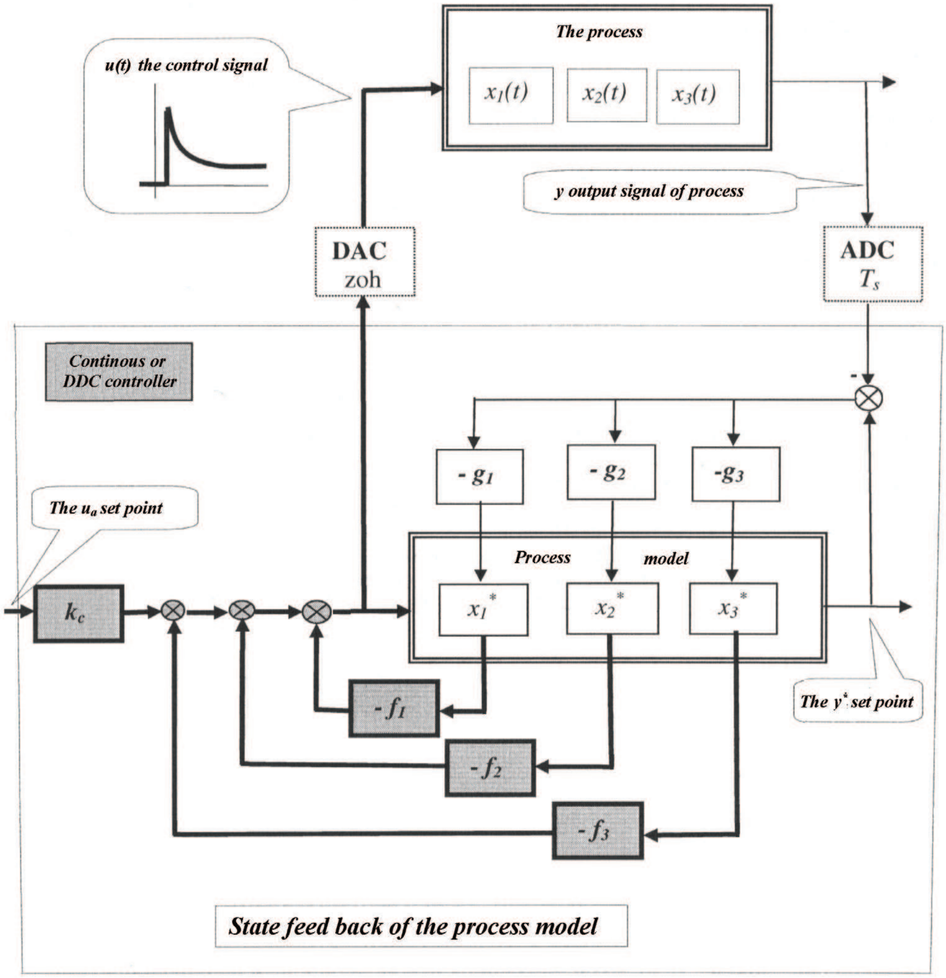

Fig. 18. Algorithm to produce the control signal

4. Conclusion

In the classical feedback control the controller generates the control signal $u(t)$ applied at the input of the process by processing the error signal — typically based on the PID control algorithm. The delay of signals caused by the lag elements of the process can be reduced if the controller applies the control signal with *overexcitation* at the process input, and then reduces this *overexcitation* gradually with an appropriate rate.

If the state variables $x(t)$ of the process can be measured with sensors, the delays due to the time constants of the process can also be reduced by means of state feedback through a gain F, as well as by inserting an appropriate gain k_c, and also the poles of the feedback system can be freely specified. The acceleration is achieved by means of *overexcitation* even in this case, *the overexcitation is determined by the ratio of pole transfer (the rate of acceleration).*

If the sensors do not have access to the state variables of the process, the state feedback can be implemented by using the state variables $x^*(t)$ of the process model. The model shall be designed so that its state variables are accessible to measurements and the state control can be implemented by their feedback. *This structure can be interpreted as a usual state feedback applied to the process model which also computes the control signal applied at the input to the model. If this control signal is also applied to the process input, the acceleration of the model, as a result of its state feedback, is also implemented upon the process.*

Appendix

The MATLAB program that supports the system design and analysis of state feedback:

```
%Data entry
mp=input('mp=');np=input('np=');         % Wp=mp/np
[A,B,C,D]=tf2ss(mp,np);                  % Parameter matrices of the process
A=input('A=');B=input('B=');C=input('C=');D=0;
step(A,B,C,D);grid;pause;                % Step response of
the process
n=lengt(A);                              % Order of the process
p=eig(A);                                % Poles of the process
%Design requirements
pR=input('pR=');                         % Prescribed poles of the system
pM=input('pM=');                         % Prescribed poles of the observer
disp([p pR' pM']);%
% Design of the state feedback
F=acker(A,B,pR);kc=inv(C*inv(A-B*F)*B)*C*inv(A)*B;%
% Design or the observer
GT=acker(A',C',pM);G=GT';
% Displaying the results of design
FGT=[F;GT];ut=inv(1+F*inv(A-B*F)*B);
disp(FGT);pause;disp([kc ut]);pause;
% Parameter matrices of the system
AR=[A --B*F;G*C A-B*F-G*C];BR=[B*kc;B*kc];
CR=[C zeros(1,n);zeros(1,n)-F;eye(n)-eye(n)];DR=[0;kc;zeros(n,1)];
printsys(AR,BR,CR,DR);pause;kR=dcgain(AR,BR,CR,DR);disp(kR);pause;
% Determination of the transfer function and step response of the system
[mR,nR=]ss2tf(AR,BR,CR,DR);               % Transfer matrix of the system
step(mR(1,:),nR);grid;pause;hold;         % The vR(t) step response
step(A,B,C,D);pause;clg;                  % The vp(t) step response
step(mR(2,:),nR);grid;pause;              % The u(t) control signal
% Simulation of the system
xpo=input('xpo='); % Initial values of the state variables of system
xmo=input('xmo='); % Initial values of the state variables of observer
xRo=[xpo xmo]';
tmax=input('tmax=');t=linspace(0,tmax,1000);
```

```
ua=ones(1,lengt(t));                        % The ua(t) set point
[yRi,xRi]=initial(AR,BR,CR,DR,xRo,t);      % The self-movement of the system
initial(AR,BR,CR,DR,xRo);grid;pause;plot(t,xRi);grid;pause;
for i=1:n
plot(t,xRi(:,i),t,xRi(:i+n));grid;pause; % Signals x(t) and x*(t) end;
[yR,xR]=lsim(AR,BR,CR,DR,ua,t,xRo);       % Forced movement of the system
lsim(AR,BR,CR,DR,ua,t,xRo);grid;plot(t,xR);grid;
for i=1:n
plot(t,xR(:,i),t,xR(:,i+n));grid;pause;  % Signals x(t) and x*(t)
end;
disp(end)
```

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