SENSITIVITY STUDY OF A CLASS OF FUZZY CONTROL SYSTEMS

Stefan PREITL and Radu-Emil PRECUP

Department of Automation and Applied Informatics
"Politehnica" University of Timisoara
Bd. V. Parvan 2, RO-300223 Timisoara, Romania
Phone: +40-2564032-29, -30, -26, Fax: +40-256-403214
e-mails: stefan.preitl@aut.upt.ro, radu.precup@aut.upt.ro

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Abstract

The paper performs the sensitivity study with respect to the parametric variations of the controlled plant in case of a class of fuzzy control systems dedicated to servo systems. The presentation is particularized to fuzzy control systems to solve the tracking control problem in case of wheeled mobile robots of tricycle type with two degrees of freedom. There is proposed a new development method for Takagi-Sugeno PI-fuzzy controllers based on the application of the Extended Symmetrical Optimum method to the basic linear PI controllers in a cascaded control system structure. There are derived sensitivity models, validated by considering a case study concerning the speed control of a servo system with DC motor as actuator in mobile robot control. Experimental results validate the development method for Takagi-Sugeno PI-fuzzy controllers.

Keywords: sensitivity study, servo systems, PI controllers, Takagi-Sugeno PI-fuzzy controllers, Extended Symmetrical Optimum method, tracking control.

1. Introduction

The considered class of controlled plants (abbreviated CPs) as part of servo systems is characterized by the transfer functions $H_P(s)$:

$$H_P(s) = k_P/[s(1 + sT_\Sigma)], \tag{1}$$

where k_P represents the plant gain and T_Σ is the small time constant or time constant corresponding to the sum of parasitic time constants.

The CPs with the transfer functions (t.f.s) in (1) can approximate sufficiently well the servo systems used in several applications including mobile robot control.

Fuzzy control has been widely used in servo systems control due to the flexible nonlinear input-output static map ensured by the fuzzy controllers (FCs) that can compensate – based on the designers' experience – the model uncertainties, nonlinearities and parametric variations of the CP. Applications of fuzzy control in the field of mobile robots are those reported in [11, 15, 18]. But, in spite of the fact that available simulation and experimental results are acceptable, relatively small research effort has been focused on the systematic analysis of these fuzzy control systems (FCSs) including their stability or sensitivity study due to the nonlinearity of the FCs and of the CPs [5].

The sensitivity study of the FCSs with respect to the parametric variations of the CP is necessary because the behaviour of these systems is generally reported as "robust" or "insensitive" without offering any systematic analysis tools. The sensitivity study performed in this paper is based on the approximate equivalence between the FCSs and the linear control systems, in certain conditions. This approach is fully justified because of two reasons:

- concerning the controller part of the FCS, where the approximate equivalence between linear and fuzzy controllers is generally accepted [7, 20];
- concerning the CP part of the FCS, where the support in using an FC dedicated to the controlling of a plant with linear or linearized model is in considering this CP model as a simplified model of a relatively complex one that can employ nonlinearities or variable parameters (in particular, this is the case with servo systems).

This paper is organized as follows. The next Section is dedicated to the problem setting in tracking control of a class of wheeled mobile robots of tricycle type with two degrees of freedom. Then, a development method for the Takagi-Sugeno PI-fuzzy controllers (PI-FCs) used as tracking controllers is presented in Section 3. Section 4 addresses the sensitivity study by deriving four sets of sensitivity models the FCSs involved. Section 5 contains simulation and experimental results corresponding to a case study to validate the proposed sensitivity models and the proposed Takagi-Sugeno PI-FCs, respectively. Section 6 is focused on the conclusions.

2. Problem Setting in Tracking Control of Wheeled Mobile Robots

By extending the dynamic model in [21], the dynamic model of the class of wheeled mobile robots of tricycle type with two degrees of freedom can be expressed in terms of (2):

$$\begin{aligned} &\dot{x} = v\cos\theta \\ &\dot{y} = v\sin\theta \\ &\dot{v} = a_v \\ &\dot{\theta} = \omega \\ &T_{\Sigma 1}\dot{a}_v + a_v = k_{P1}(u_1 + d_1) \\ &T_{\Sigma 2}\dot{\omega} + \omega = k_{P2}(u_2 + d_2), \end{aligned} \tag{2}$$

where (see *Fig. 1.a* and *Fig. 1.b*): (x, y) – coordinates of the mobile robot's rear axis centre; v and a_v – forward velocity and acceleration, respectively; θ – angle between the heading direction and the x-axis; ω – angular velocity; u_1 and u_2 – control signals, proportional to the generalized force variables; d_1 and d_2 – disturbance inputs due to the contact with the robot environment; k_{P1} and k_{P2} – gains; $T_{\Sigma 1}$ and $T_{\Sigma 2}$ – small time constants (equivalent to the effects of the actuator dynamics, the measuring device dynamics, the control equipment dynamics and of the parasitic time constants).

The servo system structure as CP (*Fig. 1 a*) is obtained by using the model (2), and it contains the kinematic subsystem (KS) and the linear dynamic subsystems characterized by the transfer functions $H_{P1}(s)$ and $H_{P2}(s)$:

$$H_{Pi}(s) = k_{Pi}/[s(1 + T_{\Sigma i}s)],\ i = \overline{1, 2}. \tag{3}$$

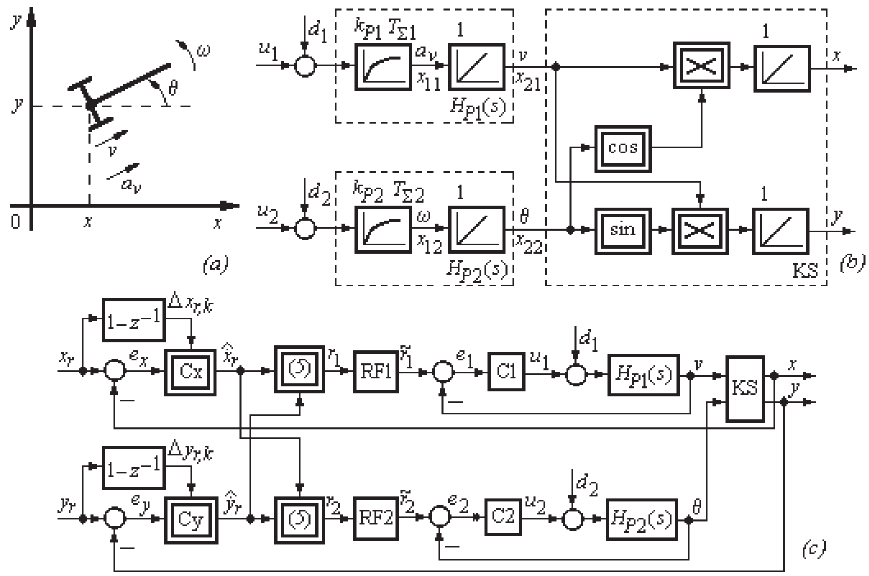

Fig. 1. Definition of mechanical variables related to (2) (*a*), servo system structure as controlled plant (*b*), control system structure (*c*)

The control aim is to solve the tracking control problem meaning to track a reference trajectory [6]. With this respect, the control system (CS) structure employed for the considered class of mobile robots is presented in *Fig. 1.c*, where: C1 and C2 – forward velocity controller and angle controller, respectively; r_1 and r_2 – reference inputs for the two control loops; RF1 and RF2 – reference (feedforward) filters; $\tilde{r}_1$ and $\tilde{r}_2$ – filtered reference inputs for the two control loops; $e_1 = \tilde{r}_1 - v$ and $e_2 = \tilde{r}_2 - \theta$ – control errors; x_r and y_r – reference positions for x and y, respectively; $e_x = x_r - x$ and $e_y = y_r - y$ – tracking errors for x and y, respectively; Cx and Cy – computational blocks to offer the estimates $\hat{\dot{x}}_r$ and $\hat{\dot{y}}_r$ of the derivatives $\dot{x}_r$ and $\dot{y}_r$, respectively; $\Delta x_{r,k}$ and $\Delta y_{r,k}$ – increments of the reference positions x_r and y_r, respectively; k – index of the current sampling interval.

To fulfill the mentioned aim the CS structure is a cascaded one, with two inner control loops to control $y_1 = v$ and $y_2 = \theta$, and the outer loops to provide the reference inputs for the inner loops by means of Cx and Cy operating as:

$$\hat{\dot{x}}_{r,k} = \begin{cases} \Delta x_{r,k}/h & \text{if } |e_{x,k}| \le \varepsilon_x \\ e_{x,k}/h & \text{otherwise} \end{cases}, \quad \hat{\dot{y}}_{r,k} = \begin{cases} \Delta y_{r,k}/h & \text{if } |e_{y,k}| \le \varepsilon_y \\ e_{y,k}/h & \text{otherwise} \end{cases}, \tag{4}$$

where: h – sampling interval; $\varepsilon_x > 0$ and $\varepsilon_y > 0$ – maximum accepted absolute values of the tracking errors e_x and e_y, respectively. The CS designer must specify the values of ε_x and ε_y, as function of the desired CS performance.

To calculate the reference inputs r_1 and r_2 fed to the two inner control loops, there can be used the first two equations in (2), transformed into (5):

$$r_1 = [(\hat{\dot{x}}_r)^2 + (\hat{\dot{y}}_r)^2]^{1/2}, \; r_2 = \tan^{-1}(\hat{\dot{y}}_r/\hat{\dot{x}}_r), \tag{5}$$

and the nonlinear blocks denoted by (5) in *Fig. 1.c* operate on the basis of these equations.

The generation of the reference trajectory (x_r, y_r) for the CS structure in *Fig. 1.c* can be performed by drawing a virtual potential field to ensure the obstacle avoidance.

The application of the virtual potential field method [9] to the considered class of mobile robots is done by several approaches including the fuzzy logic [22] or the sliding mode [12].

3. Development Method for Takagi-Sugeno PI-Fuzzy Controllers

The two PI-FCs playing the role of the controllers C1 and C2 in *Fig. 1.c* are Takagi-Sugeno FCs or type-III fuzzy systems [17]. Both PI-FCs have the structure presented in *Fig. 2.a*, based on adding the dynamics to the basic fuzzy controllers without dynamics FC$_i$, $i = \overline{1, 2}$ by the numerical differentiation of the control error $e_{i,k}$ as the increment of control error, $\Delta e_{i,k} = e_{i,k} - e_{i,k-1}$, and by the numerical integration of the increment of control signal, $\Delta u_{i,k} = u_{i,k} - u_{i,k-1}$, $i = \overline{1, 2}$.

The fuzzification in the basic fuzzy controllers without dynamics FC$_i$, $i = \overline{1, 2}$, can be solved in the initial phase by using the input membership functions illustrated in *Fig. 2.b*. Other shapes of membership functions can contribute to CS performance enhancement. Both blocks FC$_i$, $i = \overline{1, 2}$, use the max and min operators in the inference engine and employ the weighted average method for defuzzification [2].

The development of the two PI-FCs starts with the development of the two basic original linear PI controllers. For the considered class of servo systems with the t.fs in the forms (1) or (3) the use of PI controllers with the t.f. (6):

$$H_{Ci}(s) = k_{ci}(1 + sT_{ci})/s = k_{Ci}[1 + 1/(sT_{ci})], \tag{6}$$

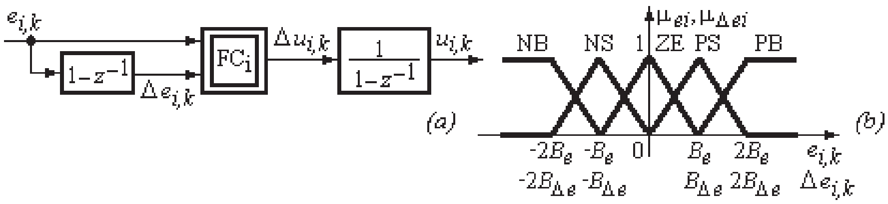

Fig. 2. PI-FC structure (*a*), initial input membership functions of FC_i, $i = \overline{1,2}$ (*b*)

with the gains k_{ci} (or k_{Ci}) and the integral time constants T_{ci}, $i = \overline{1,2}$, tuned by the Extended Symmetrical Optimum (ESO) method [13], can ensure good CS performance. The tuning equations specific to the ESO method are (7):

$$k_{ci} = 1/(\sqrt{\beta_i^3} T_{\Sigma i}^2 k_{Pi}),\ T_{ci} = \beta_i T_{\Sigma i},\ k_{Ci} = k_{ci} T_{ci}, \tag{7}$$

where β_i, $i = \overline{1,2}$, represent design parameters, only one for each controller.

The closed-loop transfer functions with respect to the reference inputs and the open-loop transfer functions will obtain their optimal expressions $H_i(s)_{opt}$ and $H_{0i}(s)_{opt}$, respectively:

$$H_i(s)_{opt} = \frac{1 + \beta_i T_{\Sigma i} s}{\sqrt{\beta_i^3} T_\Sigma^3 s^3 + \sqrt{\beta_i^3} T_{\Sigma i}^2 s^2 + \beta_i T_{\Sigma i} s + 1},$$
$$H_{0i}(s)_{opt} = \frac{1 + \beta_i T_{\Sigma i} s}{\sqrt{\beta_i^3} T_{\Sigma i}^2 s^2 (1 + T_{\Sigma i} s)},\ i = \overline{1,2}\,. \tag{8}$$

The ESO method represents a generalization of Kessler's Symmetrical Optimum method, and it does indeed an optimization by guaranteeing for the two loops maximum phase margins for constant CP parameters and minimum phase margins for variable CP gains, k_{Pi}.

By the choice of the design parameters β_i in the recommended domain $1 < \beta_i < 20$, the CS performance indices {σ_1 – overshoot, $\hat{t}_s = t_s/T_{\Sigma i}$ – settling time, $\hat{t}_1 = t_1/T_{\Sigma i}$ – first settling time, ϕ_r – phase margin} can be accordingly modified. A compromise to these indices can be reached by using the diagrams illustrated in *Fig. 3*. These diagrams have been obtained by processing (8), and for the sake of simplicity the index i, $i = \overline{1,2}$, has been omitted in the CS performance indices. The CS performance indices can be corrected by the feedforward filters RF1 and RF2 (see *Fig. 1.c*) [13]. To develop the PI-FCs, the continuous-time PI controllers (6) are discretized resulting in the incremental versions of the PI quasi-continuous digital controllers:

$$\Delta u_{i,k} = K_{Pi} \Delta e_{i,k} + K_{Ii} e_{i,k} = K_{Pi}(\Delta e_{i,k} + \delta_i \cdot e_{i,k}),\ i = \overline{1,2}\,, \tag{9}$$

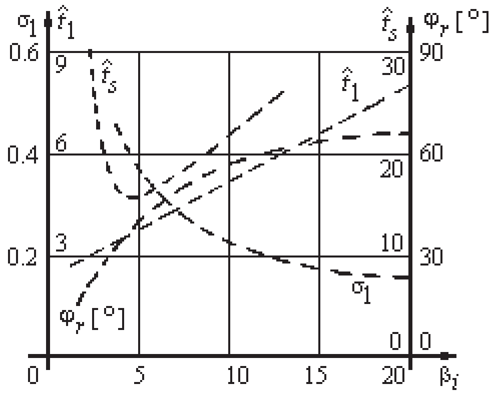

Fig. 3. CS performance indices versus $\beta_i, i = \overline{1, 2}$

where $\{K_{Pi}, K_{Ii}, \delta_i\}$ are functions of $\{k_{ci}, T_{ci}\}$:

$$K_{Pi} = k_{ci} T_{ci}[1 - (h/2T_{ci})]$$
$$K_{Ii} = k_{ci}\, h\ ,\ \delta_i = K_{Ii}/K_{Pi} = 2h/(2T_{ci} - h),\ i = \overline{1,2}\,. \tag{10}$$

The rule bases of the two blocks FC_i can be expressed in terms of the decision table shown in *Table 1*, representing the consequent part of the inference rules.

The strictly positive parameters of the PI-FCs are $\{B_{ei}, B_{\Delta ei}, B_{\Delta ui}, m_i, n_i, p_i\}$ to be tuned by the development method to be presented as follows. The parameters B_{ei}, $B_{\Delta ei}$ and $B_{\Delta ui}$ are in correlation with the shapes of the input membership functions (*Fig. 2.b*), and the parameters m_i, n_i and p_i (Table I), $m_i < n_i < p_i$, have been added to the standard version of PI-FCs to improve the CS performance by modifying the input-output static map of the blocks FC_i.

Table 1. Decision table of $FC_i, i = \overline{1, 2}$

$\Delta e_{i,k} \backslash\ e_{i,k}$	NB	NS	ZE	PS	PB
PB	$\Delta u_{i,k}$	$m_i \Delta u_{i,k}$	$n_i \Delta u_{i,k}$	$p_i \Delta u_{i,k}$	$p_i \Delta u_{i,k}$
PS	$m_i \Delta u_{i,k}$	$\Delta u_{i,k}$	$m_i \Delta u_{i,k}$	$n_i \Delta u_{i,k}$	$p_i \Delta u_{i,k}$
ZE	$n_i \Delta u_{i,k}$	$m_i \Delta u_{i,k}$	$\Delta u_{i,k}$	$m_i \Delta u_{i,k}$	$n_i \Delta u_{i,k}$
NS	$p_i \Delta u_{i,k}$	$n_i \Delta u_{i,k}$	$m_i \Delta u_{i,k}$	$\Delta u_{i,k}$	$m_i \Delta u_{i,k}$
NB	$p_i \Delta u_{i,k}$	$p_i \Delta u_{i,k}$	$n_i \Delta u_{i,k}$	$m_i \Delta u_{i,k}$	$\Delta u_{i,k}$

The development method dedicated to the two Takagi-Sugeno PI-FCs consists of the development steps 1 . . . 6:

- Step 1: Express the mathematical model of the servo system as CP in its simplified form (1) and compute the CP parameters, k_P and T_Σ; in case of the servo system used in mobile robot control, the CP is characterized by the t.fs $H_{P1}(s)$ and $H_{P2}(s)$ in (3), and the CP parameters to be computed are k_{P1}, k_{P2}, $T_{\Sigma 1}$ and $T_{\Sigma 2}$.

- Step 2: Choose the values of the design parameters β_1 and β_2 as function of the desired CS performance indices by using the diagrams in *Fig. 3* and – in case of robot control – the maximum accepted absolute values ε_x and ε_y.
- Step 3: Use (7) to tune the parameters of the basic continuous-time PI controllers $\{k_{c1}, T_{c1}\}$ (for C-1) and $\{k_{c2}, T_{c2}\}$ (for C-2).
- Step 4: Add the reference filters RF-1 and RF-2 to the control system structure. One example of such filters with the t.fs $H_{RF1}(s)$ and $H_{RF2}(s)$ is [13]:

$$H_{RFi}(s) = 1/(1 + \beta_i T_{\Sigma i} s), \ i = \overline{1, 2}, \tag{11}$$

which suppressed the action of the zero in (8).

- Step 5: Set the small sampling period h, accepted by quasi-continuous digital control, take into account the presence of zero-order hold blocks, discretize the two continuous-time PI controllers and compute the parameters $\{K_{P1}, K_{I1}, \delta_1\}$ and $\{K_{P2}, K_{I2}, \delta_2\}$ by (10).
- Step 6: Set the values of the parameters m_i, n_i, p_i and B_{ei} in accordance with the experience of the CS designer, and apply the modal equivalence principle [6] to obtain the values of $B_{\Delta ei}$:

$$B_{\Delta ei} = \delta_i B_{ei}, \ i = \overline{1, 2}. \tag{12}$$

The sensitivity study performed in the following Section offers useful information to the parameter setting of m_i, n_i, p_i and B_{ei} to obtain the desired FCS performance.

The FCS performance can be improved by the adequate choice of the inference method and of the defuzzification method as well.

4. Sensitivity Study

As it has been shown in [16], the variation of CP parameters (k_{Pi} and $T_{\Sigma i}$, $i = \overline{1, 2}$, for the considered servo systems) due to the change of the steady-state operating points or to other conditions leads to additional motion of the CSs. This motion is usually undesirable under uncontrollable parametric variations. Therefore, for the development of the FCs to alleviate the effects of parametric disturbances it is necessary to perform the sensitivity study with respect to the parametric variations of the CP.

The sensitivity models (SMs) enable the sensitivity study of the FCSs accepted, as mentioned in Section 1, to be approximately equivalent with the linear CSs (with the continuous-time PI controllers). In this context, it is necessary to obtain the SMs of the two linear CSs, the inner control loops in *Fig. 1.c* with the PI controllers (6) used as the controllers C1 and C2. The SMs of the FCSs will be identical to the SMs of the linear CSs. The only difference between these SMs is in the generation of the nominal trajectory of the CSs involved, by using either the Takagi-Sugeno PI-FCs or the linear PI controllers.

To derive the SMs it is necessary to obtain firstly the state mathematical models (MMs) of the CP. By considering the state variables x_{1i} and x_{2i} corresponding to $H_{Pi}(s)$ as in *Fig. 1.b*, the state MMs of the CP will result as:

$$\begin{aligned}
&\dot{x}_{1i}(t) = x_{2i}(t),\\
&\dot{x}_{2i}(t) = -(1/T_{\Sigma i})x_{2i}(t) + (k_{Pi}/T_{\Sigma i})u_i(t) + (k_{Pi}/T_{\Sigma i})d_i(t),\\
&y_i(t) = x_{1i}(t),\ i = \overline{1,2}\ .
\end{aligned} \tag{13}$$

Then, the state MMs of the linear PI controllers can be expressed in their parallel forms (14):

$$\begin{aligned}
&\dot{x}_{3i}(t) = (1/T_{ci})e_i(t),\\
&u_i(t) = k_{Ci}(x_{3i}(t) + e_i(t)),\ i = \overline{1,2}\ ,
\end{aligned} \tag{14}$$

where x_{3i} are the outputs of the integral components of the PI controllers (in parallel form).

The original linear PI controllers are tuned in terms of (7) by considering the nominal values of controlled plant parameters, k_{Pi0} and $T_{\Sigma i0}$, $i = \overline{1,2}$. So, the state MMs of the PI controllers (14) will be transformed into:

$$\begin{aligned}
&\dot{x}_{3i}(t) = [(1/\beta_i T_{\Sigma i0})]e_i(t),\\
&u_i(t) = [1/(\sqrt{\beta_i}k_{Pi0}T_{\Sigma i0})](x_{3i}(t) + e_i(t)),\ i = \overline{1,2}\ .
\end{aligned} \tag{15}$$

The state MMs of the closed-loop systems can be obtained by connecting the MMs presented in (13) and (15):

$$\begin{aligned}
&\dot{x}_{1i}(t) = x_{2i}(t),\\
&\dot{x}_{2i}(t) = -[k_{Pi}/(\sqrt{\beta_i}k_{Pi0}T_{\Sigma i0}T_{\Sigma i})]x_{1i}(t) -\\
&(1/T_{\Sigma i})x_{2i}(t) + [k_{Pi}/(\sqrt{\beta_i}k_{Pi0}T_{\Sigma i0}T_{\Sigma i})]x_{3i}(t) +\\
&\ + [k_{Pi}/(\sqrt{\beta_i}k_{Pi0}T_{\Sigma i0}T_{\Sigma i})]r_i(t) + (k_{Pi}/T_{\Sigma i})d_i(t),\\
&\dot{x}_{3i}(t) = -[1/(\beta_i T_{\Sigma i0})]x_{1i}(t) + [1/(\beta_i T_{\Sigma i0})]r_i(t),\\
&y_i(t) = x_{1i}(t),\ i = \overline{1,2}\ .
\end{aligned} \tag{16}$$

For the MMs (16) there can be derived the state sensitivity functions $\{\lambda_{1i}, \lambda_{2i}, \lambda_{3i}\}$ and the output sensitivity functions, σ_i:

$$\begin{aligned}
&\lambda_{ji}(t) = [\partial x_{ji}(t)/\partial\alpha]_{\alpha 0},\\
&\sigma_i(\mathrm{t}) = [\partial y_i(t)/\partial\alpha]_{\alpha 0},\ j = \overline{1,3}\ ,\ i = \overline{1,2}\ ,
\end{aligned} \tag{17}$$

where the subscript "0" stands for the nominal values of the CP parameters, and $\alpha \in \{k_{Pi}, T_{\Sigma i}\}$.

Accepting the dynamic regimes characterized by the step modifications of the reference inputs r_i for $d_i(t)$=0, or the step modifications of the disturbance inputs d_i for $r_i(t)$=0, there will result four sets of SMs by computing the partial derivatives with respect to k_{Pi} and $T_{\Sigma i}$ in (16). Two of these SMs are presented as follows, firstly with respect to the variations of k_{Pi}, the step modifications of $r_i(t)$ and $d_i(t)$=0:

$$
\begin{aligned}
&\dot{\lambda}_{1i}(t) = \lambda_{2i}(t),\\
&\dot{\lambda}_{2i}(t) = -[1/(\sqrt{\beta_i}T^2_{\Sigma i0})]\lambda_{1i}(t) - (1/T_{\Sigma i0})\lambda_{2i}(t) + [1/(\sqrt{\beta_i}T^2_{\Sigma i0})]\lambda_{3i}(t) -\\
&\quad - [1/(\sqrt{\beta_i}k_{Pi0}T^2_{\Sigma i0})]x_{1i0}(t) + [1/(\sqrt{\beta_i}k_{Pi0}T^2_{\Sigma i0})]x_{3i0}(t) + [1/(\sqrt{\beta_i}k_{Pi0}T^2_{\Sigma i0})]r_{i0}(t),\\
&\dot{\lambda}_{3i}(t) = -[1/(\beta_i T_{\Sigma i0})]\lambda_{1i}(t),\\
&\sigma_i(t) = \lambda_{1i}(t),\ i = \overline{1,2}\,.
\end{aligned}
\tag{18}
$$

Secondly, the SMs with respect to the variation of $T_{\Sigma i}$, the step modification of $r_i(t)$ and $d_i(t)$=0, are presented in (19):

$$
\begin{aligned}
&\dot{\lambda}_{1i}(t) = \lambda_{2i}(t),\\
&\dot{\lambda}_{2i}(t) = -[1/(\sqrt{\beta_i}T^2_{\Sigma i0})]\lambda_{1i}(t) - (1/T_{\Sigma i0})\lambda_{2i}(t) + [1/(\sqrt{\beta_i}T^2_{\Sigma i0})]\lambda_{3i}(t) +\\
&\quad + [1/(\sqrt{\beta_i}T^3_{\Sigma i0})]x_{i10}(t) + (1/T^2_{\Sigma i0})x_{2i0}(t) - [1/(\sqrt{\beta_i}T^3_{\Sigma i0})]x_{3i0}(t) -\\
&\quad - [1/(\sqrt{\beta_i}T^3_{\Sigma i0})]r_{i0}(t),\\
&\dot{\lambda}_{3i}(t) = -[1/(\beta_i T_{\Sigma i0})]\lambda_{1i}(t),\\
&\sigma_i(t) = \lambda_{1i}(t),\ i = \overline{1,2}\,.
\end{aligned}
\tag{19}
$$

The following notations were used in these SMs: $\{x_{1i0}, x_{2i0}, x_{3i0}\}$ – nominal values of the state variables, r_{i0} – nominal values of the reference inputs, and d_{i0} – nominal values of the disturbance inputs, $i = \overline{1,2}$.

5. Case study. Simulation and Experimental Results

To validate all approaches, a case study is applied that corresponds to integral-type servo systems with the simplified transfer functions (1), generally used in real-world applications [4, 8, 10]. The case study accepted here corresponds to the speed control of a servo system with DC motor used as actuator in mobile robot control.

The hardware modules of the experimental setup, presented in *Fig. 4*, ensure the following specific tasks: ACT – actuator, it is a power amplifier (integrated amplifier and two power transistors), MB – measurement block, μC – microcontroller, local system for control signal generation, data acquisition and interfacing the computer, C – (personal) computer, it is a control unit dedicated to control, monitoring and supervision. The μC is a C517a (Siemens) type microcontroller, very cheap and having enough resources to perform the experiment. The software contains two programs: the main program (the control program), running on the personal computer, and a program to generate the control signal and to perform the data acquisition (interface).

The dedicated control programs were implemented as Matlab files and Simulink models corresponding to the implementation of the control algorithms and to the analysis of experimental and simulated results: a Simulink model of the digital

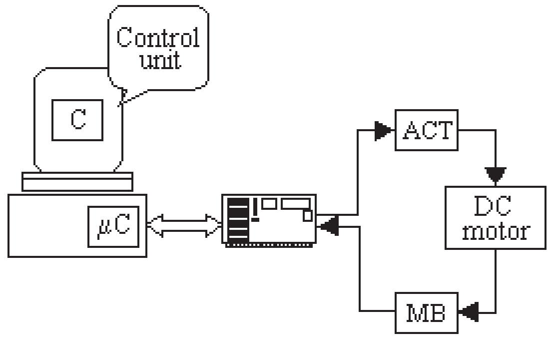

Fig. 4. Structure of experimental setup

controller (PI / PID or PI-FC), a Simulink model of the control system, two Matlab files containing the dataset saved by the control program and processing this dataset, and a Matlab file to compare the experimental and simulated results. All these programs are running on the personal computer as well.

The user interface was designed to fulfill general technical requirements such as control mode selection and parameter setting [14].

The CP parameters are the followings in the conditions of this case study (in relation with (3)):$k_{P1} = k_{P2} = 1$ and $T_{\Sigma 1} = T_{\Sigma 2} = 1$ s. It is accepted that the simplified dynamic models (2) characterize well the considered class of wheeled mobile robots of tricycle type with two degrees of freedom.

Applying the steps 1 … 6 of the development method presented in Section 3 involves the choice of the design parameters $\beta_1 = \beta_2 = 4$ (in Kessler's case [1]), and the parameters of the linear PI controllers will obtain the values $k_{c1} = k_{c2} = 0.125$ and $T_{c1} = T_{c2} = 4$ s. For h=0.1 s, the parameters of the PI quasi-continuous digital controllers will obtain the values $K_{P1} = K_{P2} = 0.4938$, $K_{I1} = K_{I2} = 0.0125$, $\delta_1 = \delta_2 = 0.0253$. Setting B_{e1}= 0.3, B_{e2}= 0.45, $m_1 = m_2 = 1$, $n_1 = n_2 = 2$ and $p_1 = p_2 = 2.4$, the values of the other parameters of the PI-FCs will be $B_{\Delta e1}$= 0.0076, $B_{\Delta u1}$= 0.0037, $B_{\Delta e2}$= 0.0114 and $B_{\Delta u2}$= 0.0056.

The simulated and the experimental responses of the control system controlling the y-axis (with the developed Takagi-Sugeno PI-FC) with respect to a step modification of the reference input corresponding to the rated value of the motor speed are presented in *Fig. 5*. These results were obtained in the conditions of the feedforward filter (11), and the controlled output (the motor speed) is represented by its measured value, the tachometer voltage.

To study the behaviour of the mobile robot, the obstacles are placed in the points having the following coordinates: (3, 3), (9, 3), (6, 7) with the potentials 0.3, the initial position of the robot is in the point (10, 4), and the goal, representing the desired / final position, is placed in the point (6, 11) with the potential –1. In these conditions, the application of the artificial potential method leads to the reference trajectory (x_r, y_r) presented in *Fig. 6.a* (with dotted line).

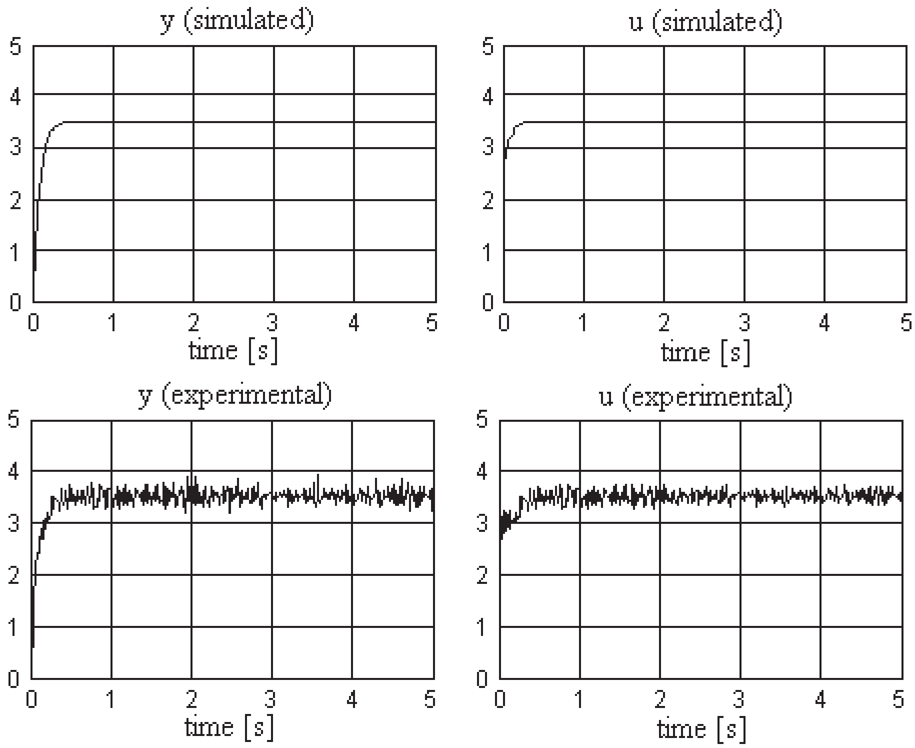

Fig. 5. Simulation and experimental results for the y-axis

To perform the sensitivity study regarding developed FCS there is considered a simulation scenario with the disturbance inputs $d_1(t)$ and $d_2(t)$:

$$\begin{aligned} d_i(t) &= f_i\sigma(t-110) - f_i\sigma(t-220) - f_i\sigma(t-440) + f_i\sigma(t-550) + \\ &+ f_i\sigma(t-770) - f_i\sigma(t-880),\ t \in [0,\ 1100]\,s\ ,\ i = \overline{1,2}\ , \end{aligned} \quad (20)$$

where: σ – unit step signal; f_1=0.1, f_2=0.01; these variations are acceptable to model the contact of the robot with the environment. In this scenario, the actual trajectory of the mobile robot is illustrated in *Fig. 6.a* (with continuous line). By setting the initial conditions $\lambda_{11}(0) = \lambda_{12}(0) = 0.2$, $\lambda_{21}(0) = \lambda_{22}(0) = 0.1$, $\lambda_{31}(0) = \lambda_{32}(0) = -0.2$, the behaviour of the SM (18) is illustrated for the first 50 s in *Fig. 6.b*. The behaviour of the SMs (19) and of the other three SMs is similar to that presented in *Fig. 6.b*.

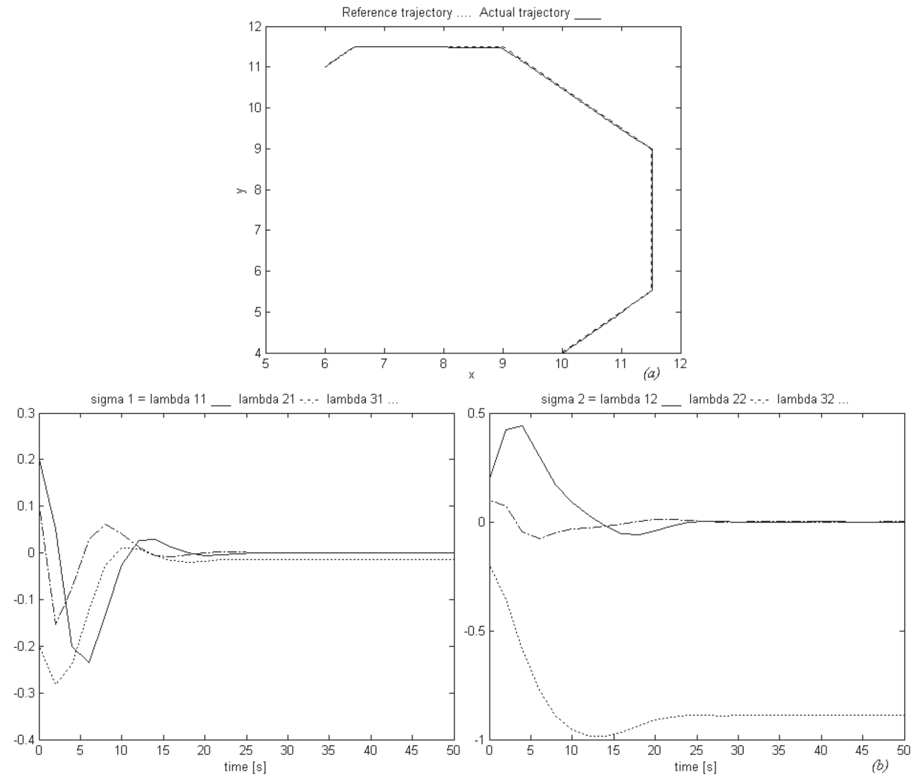

Fig. 6. Reference and actual trajectory (*a*), sensitivity functions versus time in case of SMs (18) (*b*)

6. Conclusions

The paper derives sensitivity models that enable the sensitivity study of FCSs dedicated to a class of servo systems with integral behaviour. In addition, the paper offers an attractive development method for Takagi-Sugeno PI-fuzzy controllers, expressed in terms of six development steps.

The sensitivity models and the development method are focused on a class of servo systems used in solving control problems dedicated to wheeled mobile robots of tricycle type with two degrees of freedom which can be applied in the framework of Intelligent Space in several applications [19].

The case study presented in the paper deals with fuzzy control systems with Takagi-Sugeno PI-FCs that can be implemented as tracking controllers in a cascaded CS structure using the off-line generation of the reference inputs by means of the artificial potential field method for obstacle avoidance.

The considered case study, accompanied by experimental and simulation results, validates the SMs and the development method.

The approaches presented in the paper deal with low-cost fuzzy control solutions. However, the goal of fuzzy control is not to control simple plants (the plants presented in the paper are simplified models of more complex ones), but to initially approach as a convenient and easy-to-understand nonlinear solution the control of plants with problems regarding their mathematical models or even the control problem setting. In this context, to improve the control system performance the presented approaches can be used successfully accompanied by other ones dedicated to Takagi-Sugeno fuzzy systems [3].

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