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ROBUST STATIC OUTPUT FEEDBACK FOR DISCRETE-TIME SYSTEMS – LMI APPROACH[1]

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Abstract

Two novel linear matrix inequality (LMI) based procedures to receive a stabilizing robust output feedback gain are presented, one of them being a modification of previous results of OLIVEIRA et al., [5]. The proposed robust control law stabilizes the respective uncertain discrete-time system described by a polytopic model with guaranteed cost. The obtained results are compared with other LMI results from literature and illustrated on an example.

Keywords: robust stability, static output feedback, discrete-time system, polytopic uncertainties, parameter-dependent Lyapunov function, guaranteed cost, LMI approach.

1. Introduction

Robust control of linear systems has attracted considerable interest lately, and various aspects and approaches for analysis and control design for uncertain linear systems have been investigated (e.g. OLIVEIRA et al., [5]; CRUSIUS and TROFINO, [2]; HENRION et al., [4]; TAKAHASHI et al., [9]). This paper deals with robust control design in discrete-time domain via static output feedback using LMI approach.

The studied problem comprises two issues: *robust stabilization* and *static output feedback* design. Though the latter belongs to the 'classic problems' of control theory and considerable efforts have been made to develop efficient procedures to design output feedback controller (KUCERA and de SOUZA, [11]; CRUSIUS and TROFINO, [2]; VESELÝ, [10]; ROSINOVA et al., [7]) there still remain open questions. The major problem follows from non-convexity of output feedback problem. In general, the solution of the non-convex problem requires non-polynomial (NP) hard algorithms as it is using bilinear matrix inequalities (BMI). To avoid computational complexity, another approach resorts to solutions based on convex optimization where a solution can be found using standard software tools as LMI

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approach (BOYD et al., [1]) that provide results in polynomial time. This is done either using iterative procedures where the question of convergence remains open or adding supplementary conditions to the output feedback problem so that it is restricted to convex problem formulation. In the latter case the space for the solution is reduced that it yields more or less conservative results, however, it is computationally attractive (CRUSIUS and TROFINO, [2]; VESELY, [10]; ROSINOVA et al., [7]; HENRION et al., [4]). The present effort in this field focuses on finding the ways to relax the conservatism and to develop simple, computationally efficient algorithms based on standard software tools like LMI solvers to obtain the required result – stabilizing output feedback gain matrix.

The frequent approach used to study *robust stabilization* of uncertain systems is based on quadratic stability notion. To reduce the conservatism of quadratic stability approach the parameter-dependent Lyapunov function has been introduced and the respective stability condition has been developed in different forms (OLIVEIRA et.al., [5]; HENRION et al., [4]; TAKAHASHI et al., [9]; PEAUCELLE and ARZELIER, [6]). The LMI condition and the respective design procedure have been proposed for robust state feedback control, however, in the case of output feedback the problems mentioned above still remain.

In this paper several methods of robust stabilizing control design are compared after some modification with the novel robust output feedback control design procedure provided by the authors. The respective methods are briefly characterized and their properties are demonstrated on an illustrative example. The stability margin is considered as well as performance index (in the sense of guaranteed cost).

2. Problem Formulation and Preliminaries

Consider a linear discrete-time uncertain dynamic system

$$\begin{aligned} x(k+1) &= (A+\delta A)x(k) + (B+\delta B)u(k) \\ y(k) &= Cx(k), \end{aligned} \tag{1}$$

where $x(k) \in R^n$, $u(k) \in R^m$, $y(k) \in R^l$ are state, control and output vectors respectively; A, B, C are known constant matrices of appropriate dimensions and δA, δB are matrices of uncertainties of appropriate dimensions. Uncertainties are considered to be of the affine type

$$\delta A = \sum_{j=1}^{p} \varepsilon_j \overline{A}_j, \qquad \delta B = \sum_{j=1}^{p} \varepsilon_j \overline{B}_j, \tag{2}$$

where $\underline{\varepsilon}_j \leq \varepsilon_j \leq \overline{\varepsilon}_j$ are unknown parameters; A_j, B_j, $j = 1, 2, \ldots, p$ are constant matrices of the corresponding dimensions. The affine parameter dependent model (1), (2) can be readily converted into a polytopic one and described by a list of its vertices

$$\{(A_1, B_1, C), (A_2, B_2, C), \ldots, (A_N, B_N, C)\} \quad N = 2^p. \tag{3}$$

The considered control law is static output feedback

$$u(k) = KCx(k). \tag{4}$$

Then the uncertain closed-loop polytopic system is described by

$$x(k+1) = A_C(\alpha)x(k), \tag{5}$$

where

$$A_C(\alpha) \in \left\{ \sum_{i=1}^{N} \alpha_i (A_i + B_i KC), \quad \sum_{i=1}^{N} \alpha_i = 1, \quad \alpha_i \geq 0 \right\}. \tag{6}$$

Consider a quadratic cost function associated with the uncertain system (1), (2), (4) or, alternatively, (5), (6)

$$J = \sum_{k=0}^{\infty} \left[x(k)^T Q x(k) + u(k)^T R u(k) \right], \tag{7}$$

where Q, R are symmetric positive definite matrices, $Q \in R^{n \times n}$, $R \in R^{m \times m}$.

The major aim is to determine conditions and the corresponding controller design procedure for static output feedback stabilization of the uncertain system (1), (2), (4) (or alternatively (5), (6)) with guaranteed cost (i.e. to guarantee the performance index upper limit).

Firstly, several notions are specified that are used in the following. The *quadratic stability* is equivalent to the existence of one Lyapunov function for the whole set of system models that describes the uncertain system. The polytopic system is quadratically stable if and only if there exists a Lyapunov function for all vertices of the respective polytope describing the uncertain system. The following lemma summarizes the quadratic stability condition for a closed loop uncertain polytopic system with output feedback.

LEMMA 1 *System* (1) *with uncertainties* (2) *and control law* (4), *or equivalently the polytopic system* (5), (6) *is quadratically stable if and only if there exists a symmetric positive definite matrix P such that*

$$(A_i + B_i KC)^T P (A_i + B_i KC) - P < 0 \tag{8}$$

for $i = 1, 2, \ldots, N$.

The following equivalence provides a very useful tool to transform Lyapunov-type matrix inequality into LMI with dilation, i.e. avoiding the products of the respective matrices P and A. This is achieved by the introduction of an additional matrix G without restricting it to any special form.

LEMMA 2 (OLIVEIRA et al., [5]) *The following conditions are equivalent:*

(i) *There exists a symmetric matrix* $P > 0$ *such that*

$$A^T PA - P < 0 \tag{9}$$

(ii) *There exist a symmetric matrix* P *and a matrix* G *such that*

$$\begin{pmatrix} -P & A^T G^T \\ GA & -G - G^T + P \end{pmatrix} < 0. \tag{10}$$

The above lemma can be readily used also when a parameter-dependent Lyapunov function is applied in order to reduce the conservatism of the quadratic stability approach. The *parameter-dependent Lyapunov function* $P(\alpha)$ and the respective stability condition is considered in compliance with (OLIVEIRA et al., [5]).

DEFINITION 1 (according to OLIVEIRA et al., [5]) System (5) is robustly stable in the convex uncertainty domain (6) with parameter-dependent Lyapunov function if and only if there exists a matrix $P(\alpha) = P(\alpha)^T > 0$ such that

$$A_C(\alpha)^T P(\alpha) A_C(\alpha) - P(\alpha) < 0 \tag{11}$$

for all α such that $A_C(\alpha)$ is given by (6).

Now let us introduce several notions concerning the concept of *guaranteed cost*, that is considered here in the sense respective to LQ approach.

The notion of guaranteed cost J_0 represents the cost function value for the closed loop system $J \leq J_0$ for all admissible uncertainties and considered initial conditions.

The following result provides the basis for further developments in the next section.

LEMMA 3 *Consider the nominal system* (1) *without uncertainties, and cost function* (7). *The following statements are equivalent:*

(i) *System* (1) *without uncertainties is a static output feedback quadratically stabilizable by* (4) *with guaranteed cost*

$$J \leq J_0 = x_0^T P x_0, \tag{12}$$

where P *is a real symmetric positive definite matrix,* $x_0 = x(0)$ *is the initial value of the state vector* $x(k)$.

(ii) *The following inequality holds for some real symmetric positive definite matrix* P *and a matrix* K

$$(A + BKC)^T P(A + BKC) - P + C^T K^T RKC + Q \leq 0. \tag{13}$$

Frequently, the maximal eigenvalue of P, denote it $\lambda_M(P)$, is considered to evaluate the right hand side of (12).

Considering the parameter-dependent Lyapunov function in the form $P(\alpha) = \sum_{i=1}^{N} \alpha_i P_i$ for uncertain system we will use $\max_i \{\lambda_M(P_i)\}$ to evaluate the cost function. (Obviously $\lambda_M P(\alpha) \leq \max_i \{\lambda_M(P_i)\}$.)

3. Robust Output Feedback Control Design – Main Result

In this section LMI-based design procedures are presented and compared with other existing LMI control design methods (CRUSIUS and TROFINO, [2]; HENRION et al., [4]). Firstly, the original procedure developed by the authors is given in 3.1. Secondly, in 3.2, the iterative procedure of so-called V-K iteration type is proposed (GHAOUI and BALAKRISHNAN, [3]). This V-K iteration procedure is based on the results of OLIVEIRA et al. [5], see Lemma 2, and modify them to receive stabilizing *output* feedback. Thirdly, in 3.3, the procedures of CRUSIUS and TROFINO, [2] and HENRION et al., [4] are considered for comparison, the latter slightly modified to the iterative one to relax the necessity of appropriate input data choice.

3.1. Output Feedback Stabilization Procedure with Guaranteed Cost

A novel condition for output feedback stabilization of uncertain polytopic system (5), (6) and the respective LMI-based control design procedure is presented. To obtain LMI formulation the sufficient condition is considered and the parameter-dependent Lyapunov function is applied including extra degree of freedom to avoid too conservative results.

We start with several results useful for further developments.

LEMMA 4 *The following two statements are equivalent for the given system matrices A, B, C of the corresponding dimensions.*

(i) *There exist a matrix $P = P^T > 0$ and K of appropriate dimension so that*

$$(A + BKC)^T P(A + BKC) - P + Q + C^T K^T RKC < 0 \tag{14}$$

(ii) *There exist a matrix $P = P^T > 0$ and K of appropriate dimension so that*

$$\begin{pmatrix} -P + Q & (A + BKC)^T & C^T K^T \\ A + BKC & -P^{-1} & 0 \\ KC & 0 & -R^{-1} \end{pmatrix} < 0. \tag{15}$$

The above equivalence is received applying Schur complement formula. Notice that (14) and (15) consist of stability condition and condition for guaranteed cost (adding Q and the last row and column of the left hand side of (15) to include a cost factor).

The form of (15) was chosen to avoid the product of P and the system matrix. Since (15) is not in the LMI form due to P^{-1} we will use the following inequality to substitute for P^{-1}

$$-P^{-1} \leq -\frac{2}{\rho}D + \frac{1}{\rho^2}DPD \tag{16}$$

for any matrices $P = P^T > 0$, $D = D^T$ and real scalar $\rho > 0$.

Inequality (16) follows from inequality

$$(D - \rho P^{-1})^T \frac{P}{\rho}(D - \rho P^{-1}) \geq 0 \tag{17}$$

that generally holds for any matrices $P = P^T > 0$, $D = D^T$ and real scalar $\rho > 0$. Free scalar parameter ρ is introduced to reduce the conservatism of the sufficient condition given below.

The following theorem (Rosinova and Vesely, [8]) provides a way to LMI-based output feedback design.

Theorem 1 *Uncertain system* (5) *is static output feedback robustly stabilizable with guaranteed cost with respect to cost function* (7)*, if for some* $D_i = D_i^T$ *there exist a feedback gain matrix* K*, symmetric matrices* P_i *and a matrix* Z *satisfying LMI*

$$\begin{pmatrix} -P_i + Q & (A_i + B_i KC)^T & 0 & C^T K^T \\ A_i + B_i KC & -\dfrac{2}{\rho} D_i & \dfrac{1}{\rho} D_i^T Z & 0 \\ 0 & \dfrac{1}{\rho} Z^T D_i & -Z - Z^T + P_i & 0 \\ KC & 0 & 0 & -R^{-1} \end{pmatrix} < 0$$

$$\textit{for } i = 1, \ldots, N. \tag{18}$$

Proof. The proof is based on:

- inequality (16) that is used to substitute for $-P^{-1}$ in (15),
- Lemma 2 that allows 'to separate' matrices appearing in the product.

Firstly we will show that if (18) holds for one certain index i then (15) holds for the same i and the respective A_i, B_i and P_i. Denote $A_{ci} = A_i + B_i KC$ to simplify the reading of the respective formulas. Owing to (16) the inequality

$$\begin{pmatrix} -P_i + Q & A_{ci}^T & C^T K^T \\ A_{ci}^T & -\dfrac{2}{\rho} D_i + \dfrac{1}{\rho^2} D_i^T P_i D_i & 0 \\ KC & 0 & -R^{-1} \end{pmatrix} < 0 \quad \text{for some } D_i = D_i^T \tag{19}$$

implies that (15) holds for the respective i, A_i, B_i.

Analogously to Oliveira, [5], (Lemma 2 above) we prove that

$$\begin{pmatrix} -P_i + Q & A_{ci}^T & 0 \\ A_{ci} & -\dfrac{2}{\rho} D_i & \dfrac{1}{\rho} D_i^T Z \\ 0 & \dfrac{1}{\rho} Z^T D_i & -Z - Z^T + P_i \end{pmatrix} < 0 \tag{20}$$

implies

$$\begin{pmatrix} -P_i + Q & A_{ci}^T \\ A_{ci} & -\dfrac{2}{\rho} D_i + \dfrac{1}{\rho^2} D_i^T P_i D_i \end{pmatrix} < 0. \tag{21}$$

The implication (20) $\Rightarrow$ (21) is obtained multiplying (20) by

$$T = \begin{pmatrix} I & 0 & 0 \\ 0 & I & \dfrac{1}{\rho} D^T \end{pmatrix}$$

on the left and by T^T on the right. Inequality (19) is obtained from (21) by adding the last row and column to comply with the form (18) or (15). Hence the proof of (18) $\Rightarrow$ (15) for one index i is completed. It remains to prove the robust stability of the overall uncertain system (5). Due to linearity of (18) with respect to indexed matrices P_i, A_i, B_i, D_i and to Z it can be shown that if (18) holds for all $i = 1, \dots, N$ then there exists a parameter-dependent Lyapunov function $P(\alpha) = \sum_{i=1}^{N} \alpha_i P_i$, $\sum_{i=1}^{N} \alpha_i = 1$ for which (18) holds with $A_i + B_i K C = A_{ci} \to A_C(\alpha) = \sum_{i=1}^{N} \alpha_i A_{ci}$ and $D_i \to D(\alpha) = \sum_{i=1}^{N} \alpha_i D_i$. Since in the previous step of proof it has been shown that for one index i inequality (18) implies that (15) holds with the respective values of A, B, C, K and P, applying this for $A_C(\alpha)$ and $P(\alpha)$, we receive from Lemma 4 that $A_C(\alpha)^T P(\alpha) A_C(\alpha) - P(\alpha) + Q + C^T K^T R K C < 0$ and therefore the considered system is robustly stable (see Definition 1), that completes the proof. □

The above Theorem 1 provides sufficient condition for robust stability with guaranteed cost, however, it can be supposed as not being too restrictive since there is certain degree of freedom in matrix Z and free scalar parameter ρ that can be appropriately tuned. If (18) provides feasible solution for unknown P_i, Z and K, the resulting output feedback control guarantees the robust stability of uncertain system (1), (2) and the value of cost function limited by $\lambda_M P(\alpha)$.

3.2. V-K Iterative Procedure for Output Feedback Stabilizing Control

In Oliveira et al., [5] the sufficient stability condition (see Lemma 2 above) developed using parameter-dependent Lyapunov function and the respective LMI formulation to find state feedback stabilizing control was provided. In this section we use the former result to build an iterative procedure to receive *output* feedback control, the stability condition is extended to include performance index (7). The respective sufficient stability condition with guaranteed cost is then in the form

$$\begin{pmatrix} -P_i + Q & (A_i + B_i K C)^T G^T & C^T K^T \\ G(A_i + B_i K C) & -G - G^T + P_i & 0 \\ K C & 0 & -R^{-1} \end{pmatrix} < 0. \tag{22}$$

Since besides P_i both matrices K and G are unknown, we propose the iterative procedure to convert the problem into LMI formulation. The V-K iteration approach is used that was declared to have good convergence properties (GHAOUI and BALAKRISHNAN, [3]).

Algorithm V–K

1. Initialization:
   - stability test for vertex system matrices A_i,
   - for unstable A_i choose multiplier α_i so that $A_{pi} = \alpha_i A_i$ is stable, for stable A_i set $A_{pi} = A_i$.
   - set maximal number of iterations *max* and prescribed error ε.
2. Set $j \leftarrow 0$. Compute initial value G_0 from the following LMI (with unknown P_i and G_0)

$$\begin{pmatrix} -P_i + Q & A_{pi}^T G_0^T & 0 \\ G_0 A_{pi} & -G_0 - G_0^T + P_i & 0 \\ 0 & 0 & -R^{-1} \end{pmatrix} < 0. \tag{23}$$

3. Set $j \leftarrow j + 1$. Compute $K^{(j)}$ and $P_i^{(j)}$ from LMI (24):

$$\begin{pmatrix} -P_i^{(j)} + Q & (A_i + B_i K_{(j)} C)^T G_{(j-1)}^T & C^T K_{(j)}^T \\ G_{(j-1)}(A_i + B_i K_{(j)} C) & -G_{(j-1)} - G_{(j-1)}^T + P_i^{(j)} & 0 \\ K_{(j)} C & 0 & -R^{-1} \end{pmatrix} < 0 \tag{24}$$

   compute $G_{(j)}$ from LMI (25) (with unknown $G_{(j)}$ and $P_i^{(j)}$):

$$\begin{pmatrix} -P_i^{(j)} + Q & (A_i + B_i K_{(j)} C)^T G_{(j)}^T & C^T K_{(j)}^T \\ G_{(j)}(A_i + B_i K_{(j)} C) & -G_{(j)} - G_{(j)}^T + P_i^{(j)} & 0 \\ K_{(j)} C & 0 & -R^{-1} \end{pmatrix} < 0 \tag{25}$$

4. Check the terminal conditions ($j < \max$) and ($\|G_{(j)} - G_{(j-1)}\| / \|G_{(j)}\| < \varepsilon$), if they do not hold, repeat step 3, else end.

Though Algorithm V–K is iterative, it provides good qualities in practical examples.

3.3. *Existing Procedures for Output Feedback Stabilizing Gains – (Crusius and Trofino, Henrion et al.)*

To compare our results from sections 3.1. and 3.2. with the existing ones we briefly recall the respective results of CRUSIUS and TROFINO, [2] and HENRION et al.,

[4]. We have turned the latter method into iterative form to relax its sensitivity to the initial input data.

Algorithm C–T (CRUSIUS and TROFINO, [2])

- Solve the following LMI for unknown matrices F and W of appropriate dimensions, with W being symmetric (corresponding to P^{-1})

$$\begin{pmatrix} -W & WA_i^T - C^T F^T B_i^T \\ A_i W - B_i FC & -W \end{pmatrix} < 0, \qquad i = 1, \dots, N,$$

$$W > 0, \tag{26}$$

$$MC = CW. \tag{27}$$

- Compute the corresponding output feedback gain matrix

$$K = FM^{-1}. \tag{28}$$

The above algorithm can be used under the assumption that the *Eq.* (28) can be met what is not always the case and this limits the use of it. Algorithm C–T is computationally rather efficient and does not require any iteration or initial data choice. However, it is based on quadratic stability and since feedback gain matrix K is computed from (27) and (28), there is no obvious way to use the parameter-dependent Lyapunov function – the varying W_i would be required for A_i in such case.

The other way to compute stabilizing output feedback gain was developed in HENRION et al., [4], based on the sufficient LMI stability condition. In a discrete-time case it is described by

$$\begin{pmatrix} F^T A_c + A_c^T F + P_i & -A_c^T - F^T \\ -A_c - F & 2I - P_i \end{pmatrix} > 0$$

$$i = 1, 2, \dots, N, \qquad A_c = A_i + B_i KC \tag{29}$$

for a given stable matrix F and unknown K and P_i of appropriate dimensions.

Since it is not clear how to choose the stable matrix F while this choice is important for the result, we propose the following iterative variant of (29).

Algorithm H

1. Initialization – choice of initial stable matrix F_0:
   - test a stability of mean value of vertex system matrices A_i, $A_0 = \sum_{i=1}^N A_i/N$, for unstable A_0 choose multiplier α_0 so that $F_0 = \alpha_0 A_0$ is stable, for stable A_0 set $F_0 = A_0$;
   - determine maximal number of iterations *iter* and prescribed error ε; $j \leftarrow 0$
2. Compute K and P_i from (29) having $F = F_j$;
   Set $F_{j+1} = \frac{1}{N} \sum_{i=1}^N (A_i + B_i KC)$.

3. $j \leftarrow j+1$;
   repeat step 2. until $(\|F_{j+1} - F_j\|/\|F_j\| < \varepsilon)$ or $j > iter$.

Condition (29) can be extended to include cost function matrices Q and R, analogously to the way (10) was extended to (22). However, in examples it provides much more difficulty to obtain a feasible solution in comparison to algorithms described in sections 3.1. and 3.2. even in the case when the iterative variant has been adopted for the extended condition, analogous to the Algorithm H. Therefore in the illustrative example we tested the mere stabilizing output design method given in Algorithm H.

The algorithms C–T and H provide the stabilizing output feedback gain (if the respective LMIs are feasible), however, they do not involve the performance index. Therefore, to evaluate and compare the quality of the obtained results we consider Lemma 3 together with (10) and (11). The guaranteed cost for closed loop system is then evaluated using parameter-dependent Lyapunov function $P(\alpha) = \sum_{i=1}^{N} \alpha_i P_i$, where P_i are solutions of LMI (unknown P_i and G)

$$\begin{pmatrix} -P_i + Q + C^T K^T RKC & A_c^T G^T \\ GA_c & -G^T - G + P_i \end{pmatrix} < 0,$$
$$i = 1, 2, \ldots, N, \quad A_c = A_i + B_i KC \tag{30}$$

Then $\max_i\{\lambda_M(P_i)\}$ provides the upper bound on cost function (see last paragraph in Section 2).

4. Example

The results developed in the previous section are illustrated on the following example.

Example

Consider uncertain system (1), (2) with matrices

$$A = \begin{pmatrix} 0.7118 & 0.0736 & 0.1262 \\ 0.7200 & 0.6462 & 2.3432 \\ 0 & 0 & 0.6388 \end{pmatrix}, \qquad B = \begin{pmatrix} 0.0122 & 0.0412 \\ 0.3548 & 0.1230 \\ 0.2015 & 0.2301 \end{pmatrix}$$

– nominal model

$$\overline{A}_1 = \begin{pmatrix} 0.08 & 0.006 & 0.01 \\ 0.07 & 0.03 & 0.1 \\ 0 & 0 & 0.03 \end{pmatrix}, \qquad \overline{B}_1 = \begin{pmatrix} 0 & 0.001 \\ 0.012 & 0 \\ 0.007 & 0.004 \end{pmatrix}$$

$$\overline{A}_2 = \begin{pmatrix} 0 & 0.0004 & 0 \\ 0.1 & 0 & 0.12 \\ 0 & 0 & 0 \end{pmatrix} \qquad \overline{B}_2 = \begin{pmatrix} 0 & 0 \\ 0.02 & 0 \\ 0.014 & 0.02 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

with $-1 \le \varepsilon_i \le 1, i = 1, 2$.

The vertices of the corresponding polytopic model are

$$A_1 = \begin{pmatrix} 0.7918 & 0.0792 & 0.1362 \\ 0.6900 & 0.6762 & 2.3232 \\ 0 & 0 & 0.6688 \end{pmatrix} \quad B_1 = \begin{pmatrix} 0.0122 & 0.0422 \\ 0.3468 & 0.1230 \\ 0.1945 & 0.2141 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 0.6318 & 0.0680 & 0.1162 \\ 0.7500 & 0.6162 & 2.3632 \\ 0 & 0 & 0.6088 \end{pmatrix} \quad B_2 = \begin{pmatrix} 0.0122 & 0.0402 \\ 0.3628 & 0.1230 \\ 0.2085 & 0.2461 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 0.7918 & 0.0800 & 0.1362 \\ 0.8900 & 0.6762 & 2.5632 \\ 0 & 0 & 0.6688 \end{pmatrix} \quad B_3 = \begin{pmatrix} 0.0122 & 0.0422 \\ 0.3868 & 0.1230 \\ 0.2225 & 0.2541 \end{pmatrix}$$

$$A_4 = \begin{pmatrix} 0.6318 & 0.0672 & 0.1162 \\ 0.5500 & 0.6162 & 2.1232 \\ 0 & 0 & 0.6088 \end{pmatrix} \quad B_4 = \begin{pmatrix} 0.0122 & 0.0402 \\ 0.3228 & 0.1230 \\ 0.1805 & 0.2061 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

The spectral radii for A_1, A_2, A_3, A_4 are, $\rho_1 = 0.9748$; $\rho_2 = 0.8499$; $\rho_3 = 1.007$; $\rho_4 = 0.8164$, respectively; i.e. the vertex (A_3, B_3, C) without control corresponds to unstable system. The quadratic cost function matrices $Q = 0.1 * I_n$ and $R = 0.1 * I_m$ are considered.

The results using the stabilizing robust output feedback design methods described in Section 3 are summarized in *Table 1*.

What remains open is the choice of matrices D_i, the ideal value being $D_i = P_i^{-1}$. We obtained good results choosing $D_i = (A_i^T A_i + Q)^{-1}$. In the case where the LMI (18) does not provide any feasible solution, the computation can be repeated for another value of D_i, it is recommended to repeat the solution of (18) for $D_i \leftarrow P_i^{-1}$(from previous computation). We intentionally do not call this procedure iteration since in fact the resulting gain matrix K is obtained from the solution of LMI (18) in non-iterative way.

The results in *Table 1* show that the method 3.1. ‘tailors’ the output feedback gain to minimize the considered cost function value. Simulation results then show less oscillations than using other method results. The simulation results are depicted in *Fig. 1*. All the tested methods provide in this case stabilizing output feedback gains, the respective LMIs provide feasible solutions.

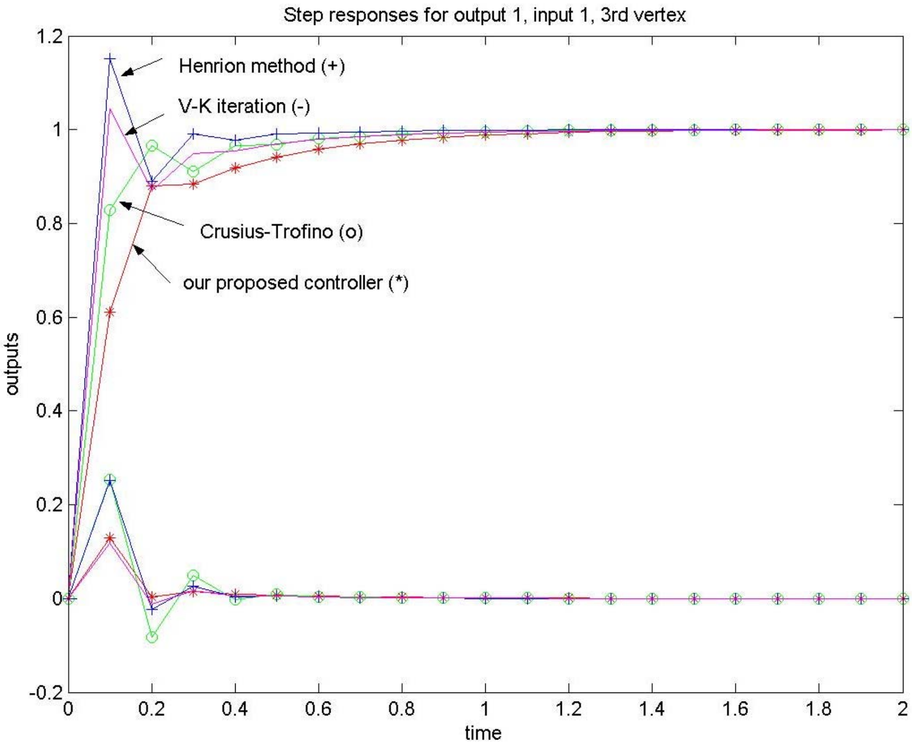

Fig. 1. Step responses (1st input step, 3rd vertex)

Table 1.

Method	Output feedback gain matrix K	Spectral radii	Bound on J
Our method	$\begin{pmatrix} -0.9638 & -4.1954 \\ 0.2867 & -0.4189 \end{pmatrix}$	0.8911; 0.8757 0.7222; 0.7059	13.251 (8.04)
Oliveira+ VK iter.	$\begin{pmatrix} -2.6665 & -7.4946 \\ 1.7799 & 3.6662 \end{pmatrix}$	0.8722; 0.8595 0.7025; 0.6891	35.745
Henrion (7 iter.)	$\begin{pmatrix} -2.6665 & -7.4946 \\ 1.7799 & 3.6662 \end{pmatrix}$	0.8111; 0.8089 0.6377; 0.6369	39.964
Crusius–Trofino	$\begin{pmatrix} -2.6665 & -7.4946 \\ 1.7799 & 3.6662 \end{pmatrix}$	0.8531; 0.8434 0.6849; 0.6747	19.898

5. Conclusion

Novel robust output feedback design procedures were proposed and studied in comparison with previous results. The proposed control design scheme includes the terms corresponding to performance index, therefore the resulting control law both robustly stabilizes the uncertain system and tends to minimize the chosen quadratic cost function. The obtained results indicate the potential qualities of the studied methods concerning both robust stability and performance measured by quadratic cost function.

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