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A NEW CLUSTERING METHOD FOR PARTITIONING PRICE ZONE IN POWER MARKET ENVIRONMENT

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Abstract

In order to reduce the difficulties in using nodal pricing, the notion of price zone is widely adopted in actual pricing scheme. The key for establishing simple and efficient zonal pricing scheme is to accurately partition transmission network in the presence of congestion. Unfortunately, in actual power market operation, operators usually establish price zones based on their experience, considering the locations of congested lines, without mathematical analysis. In order to achieve accurate price zone partition without any a priori partition knowledge, this paper firstly extracts the sensitivities of nodal power injections to power flows on all congested lines as cluster features of nodal price. Secondly, the scale-space theory simulating human visual system is introduced, and further a scale-space-based hierarchical clustering method for price zone partition is proposed. Finally, test results on IEEE 118-node system show the validity and feasibility of this proposed method.

Keywords: hierarchical clustering, price zone, power market, scale-space.

1. Introduction

In 1988, SCHWEPPE et al. first proposed the nodal pricing (i.e. spot pricing) theory, which has become the chief instrument of electricity pricing in many countries or areas, such as Argentina, Chile, New Zealand, PJM (Pennsylvania-New Jersey-Maryland) and NY (New York) in USA. Nodal price reflects the marginal cost of supplying the next kwh load at a particular node. According to microeconomic price theory, it can provide right price signals for power market [1].

Transmission congestion is an operating condition where there is no sufficient network transfer capacity to simultaneously deliver all traded transactions owing to network constraints. When there is no congestion in transmission (nodal prices are solved based on DC power-flow-model implemented in many actual power markets), all nodal prices are identical. During congestion (transmission constrains are active in the model), different nodes have obviously different prices, that is, the nodal prices at the sending end area of a power-flow on congested line, where abundant generation resources are available, are lower than those at the receiving end area where there is a shortage of generation resources [2].

In actual power system operation, transmission congestion usually frequently occurs between some zones, while within these zones congestion is relatively in-frequent and of low cost to relieve, and the difference between nodal prices is small. Therefore, the zonal pricing scheme is proposed, in which every node within a zone has the same price, while nodes belonging to different zones are priced differently [3]. This pricing scheme is a simple method of the nodal pricing, it can simplify congestion charge settlement, improve market liquidity encouraging bilateral/multilateral trades and increase market price certainty on the premise of providing efficient and accurate price signals. Up to now, the zonal pricing has been widely adopted in many actual power markets, such as Norway, Australia, California, ERCOT and PJM in USA [4].

Price zone partition is the important first step to establish the zonal pricing scheme. The transmission network should be divided into some zones in the presence of congestion based on the principle of nodes with the same or similar prices clustered into a zone. In the actual power market, the number of zones and size of each zone (number of nodes within a zone) are determined based on the topology of transmission network, the frequency and degree of congestion. In a power market where congestion is slight and not frequent, there are a few zones, each of which includes many nodes, for example, the number of zones ranges from two to five in Norwegian transmission system [5]. In contrast in a power market where congestion is severe and frequent, there are many zones, each of which includes a few nodes, even if each node close to congested lines is an individual zone, for example, there are some zones with few nodes (such as hubs and load zones) and a lot of individual nodes in PJM and NY pricing scheme [4].

The key for establishing a simple and efficient pricing scheme is to accurately partition the price zone in the presence of congestion. Inaccurate price zone partition will distort electricity prices and increase the frequency of congestion. For example, in the Californian power market, there are a series of problems, electricity prices are distorted and intrazonal congestion is severe due to the use of three invariable zones in the long term. For currently operated power markets, price zone partition is usually established based on the operator's experience and judgment, without analytical tools [5, 6]. However, because transmission network is large and complex, it is difficult for the operator to establish an accurate partition only by experience. Reference [7] described a partition method where zonal boundaries are defined by congested lines in a radially connected network. However, in practice, transmission network is never truly radial, using congested lines as zonal boundaries alone does not allow to achieve network partition. In reference [8], a price zone partition idea was proposed based on the similarity of nodal prices, but the detailed algorithm was not reported to put it into practice.

In order to achieve an accurate and rational price zone partition without any a priori partition knowledge, this paper firstly extracts the sensitivities of nodal power injections to power flows on congested lines as cluster features of nodal price, and then forms cluster samples. Secondly, the scale-space theory simulating human visual system is introduced, and further a scale-space-based hierarchical clustering method for price zone partition is proposed. In this method, as scale ranges from small to large, the sample point set is merged gradually, and consequently a hierarchical structure of clusters in the scale-space is generated, and then based on human visual perception, a set of clustering selection rules are proposed by integrating price zone partition problem.

2. Cluster Features of Nodal Price

Price zone partition is essential to cluster nodes according to the similarity of nodal prices, i.e., nodal price used as cluster feature to cluster nodes. However, directly using nodal price as cluster feature has the following disadvantages:

1. The calculation of nodal prices is complicated because there are a great lot of nodes in the transmission network.
2. The time-varying characteristics of nodal prices lead to the instability of zonal boundaries, which damage the stability of the power market.

Therefore, it is not perfect to use nodal price as cluster feature. So based on the DC power-flow-model, we obtain cluster features that not only reflect nodal prices, but also have relatively stable (not time-varying) characteristics.

Based on DC power-flow, the optimal dispatch problem can be set up as follows:

$$\min e^T \cdot c(p) + c_N(p_N) \quad \text{s.t.} \quad H \cdot p \leq z_{\max}, \qquad e^T \cdot p + p_N = 0, \tag{1}$$

where p denotes a $(N-1) \times 1$ nodal active power injection vector (excluding the reference node N) and H denotes the transfer admittance matrix (dimension $L \times (N-1)$) that represents the sensitivities of the nodal power injections to line power flows; e is a vector of all ones; $z_{\max}$ is the $L \times 1$ vector of power flow limits; c_i is the net cost/benefit function at node i that can be obtained by the bidding curves of market participants; N is the total number of nodes, L is the total number of lines.

The Lagrangian for the optimization problem (1) can be written as:

$$L = \sum_i c_i(p_i) - \lambda(e^T \cdot p + p_N) - \mu^T(H \cdot p - z_{\max})$$

Differentiating the Lagrangian with respect to the power injections, that is, $\partial L / \partial p = 0$, the nodal prices are given:

$$\rho_N = c'_N = \lambda, \tag{2}$$

$$\rho = c' = \lambda \cdot e + H^T \mu, \tag{3}$$

where ρ_N is the nodal price at the reference node N, ρ is the $(N-1) \times 1$ nodal price vector (excluding the reference node N).

If line l is congested, the inequality constraint becomes active, whose Lagrangian multiplier $\mu_l \neq 0$. If line l is not congested, the inequality constraint is not active, whose Lagrangian multiplier $\mu_l = 0$. From (3), the nodal prices relate the marginal cost (i.e. the nodal price) of the reference node, the shadow prices (i.e. the Lagrangian multiplier) for congested lines and the sensitivities of the nodal power injections to power flows on congested lines. The difference between nodal prices at any two nodes is:

$$\rho_i - \rho_j = \sum_{l \subset \Omega} \mu_l h_{li} - \sum_{l \subset \Omega} \mu_l h_{lj} = \sum_{l \subset \Omega} \mu_l (h_{li} - h_{lj}) \quad (i \neq j), \tag{4}$$

where Ω denotes the set of congested lines, h_{li}, h_{lj} are the $li-, lj$-th terms of the matrix H.

From (4), the difference between nodal prices is proportional to the difference between the sensitivities of the nodal power injections to power flows on congested lines. As long as the difference of the sensitivities between the nodes is very small, the nodal prices are always similar no matter what the shadow prices for congested lines are, though the similarity of nodal prices changes like the different operating conditions of the system and the different shadow prices for congested lines. Thus, the sensitivities of the nodal power injections to power flows on congested lines as cluster features can effectively lead to aggregate nodes with similar nodal prices. In addition, the sensitivities do not vary with the operating condition of the system, and are only sensitive to major changes in topology of transmission network. Thus, using the sensitivities of the nodal power injections to power flows on congested lines as cluster features can provide relatively stable price zone partition in a period of time.

3. Price Zone Partition Based on Scale-Space Hierarchical Clustering

3.1. Scale-Space Theory [9]

In recent years, physiological discoveries and researches on computer-aided neuroanatomy have developed several quite accurate computational models of primary visual system, each modelling some parts of the human visual system at a particular level of details. Among them is the scale-space theory, which models the blurring effect of lateral retinal interconnection.

In the process of human perception, human eye images an outer scene onto retina, then transmits it to visual ganglia in brain through retina ganglion cells with multi-scale receptive fields (called the front-end visual system). The image perceived by the receptive fields with different scales is equivalent to that perceived by receptive field with certain scale in different distances. So the image perceived in the brain is a multi-scale representation.

The image perceived can be regarded as the set of light points in the space. As scale increases, the image is blurred gradually, each light point merging into smaller blobs, and then into larger ones until the whole image contains only one light blob at a large enough scale. The blobs obtained for the image at different scales form a hierarchical structure: large blobs are made up of small blobs; each blob has its own survival range of scale, which will split into multiple smaller blobs if scale is smaller than this range and merges into a larger blob with other blobs if scale is greater than this range.

For a given d-dimensional light point set:

$$X = (x_i \in R^d : i = 1, \cdots, N).$$

A light point x_i can be mathematically expressed as a δ function, that is,

$$\delta(x - x_i) = \begin{cases} 0, & x \neq x_i \\ +\infty, & x = x_i, \end{cases} \qquad \int_{-\infty}^{+\infty} \delta(x - x_i)\,\mathrm{d}x = 1$$

So the image $p(x)$ formed by the light point set is:

$$p(x) = \frac{1}{N} \sum_{i=1}^{N} \delta(x - x_i)$$

Based on the scale-space theory in the front-end visual system, the multi-scale presentation $P(x, \sigma)$ for image $p(x)$ is the convolution of $p(x)$ with the Gaussian kernel [9], i.e.

$$P(x, \sigma) = p(x) * g(x, \sigma) = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{2\pi\sigma^2} e^{-\frac{\|x - x_i\|^2}{2\sigma^2}}, \tag{5}$$

where $g(x, \sigma)$ is the Gaussian function $g(x, \sigma) = \frac{1}{2\pi\sigma^2} e^{-\frac{\|x\|^2}{2\sigma^2}}$, σ is the scale parameter, (x, σ)-plane is the scale-space.

At a given scale σ, the blob center x^* is defined as the local maximum of $P(x, \sigma)$ with respect to σ, the corresponding light blob as being a region specified as follows:

$$B(x^*) = \left\{ x_0 \in R^d : \lim_{t \to \infty} x(t, x_0) = x^* \right., \tag{6}$$

where $x(t, x_0)$ is the solution of the gradient dynamic system:

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = \nabla_x P(x, \sigma) \\ x(0, x_0) = x_0. \end{cases} \tag{7}$$

At a given scale σ, testing whether x_0 belongs to the light blob $B(x^*)$ can be accomplished by solving (7).

Recently, some scholars regarded the clustering process as an analogy to the way human eye perceives objects, and by using the scale-space theory of front-end visual system, they proposed a scale-space-based hierarchical clustering algorithm [10]. This algorithm has a series of advantages: the initial partition and global minimum are not needed and the optimal number of clusters and cluster centers can be determined efficiently, and so on. It overcomes the shortcomings of partitional clustering algorithms like k-means and fuzzy c-means clustering algorithm, such as difficulty of deciding initial price zone partition, inability to find a global minimum and difficult of estimating the validity of clusters. Therefore, the scale-space-based hierarchical clustering algorithm provides a new approach to efficient solution of the price zone partition problem.

3.2. Price Zone Partition Based on Scale-Space Hierarchical Clustering

3.2.1. Cluster Sample

Before partitioning price zone, all potentially congested lines are firstly determined in a period of time based on actual operating conditions of the system (these congested lines can be determined by using the Monte Carlo simulation method to analyse the congestion probability of transmission network considering uncertainty in the power market, the detailed procedure is not described here). Secondly, the sensitivities of nodal power injections to power flows on congested lines, i.e. the cluster features of nodal price are calculated. Then, the features for all nodal prices are respectively mapped into high-dimensional space (dimension is the number of congested lines), forming a sample point set for clustering. By clustering the sample point set, nodes are aggregated effectively according to the similarity of nodal prices, and further to achieve accurate price zone partition.

3.2.2. Scale-Space-Based Hierarchical Clustering

Introducing the scale-space theory to clustering nodes of the transmission network, the sample point set is considered as an image with each light point located at a sample point position, that is,

$$X = (x_i \in R^d : i = 1, \cdots, N)$$

where $x_i = (h_{1i}, \cdots, h_{di})$, $h_{li}(l = 1, \cdots, d)$ denotes the sensitivities of the power injections at node i to power flows on line l; d is the total number of congested lines.

As scale increases, the light points merge gradually into blobs until all light points are contained only by one light blob at a large enough scale, and each blob is equated with a cluster, which consists of all light points in the blob and is characterized by the blob center. So, the merging process for the sample points generates a family of clusters along the hierarchy.

The blob center or cluster center is characterized by the local maximum of $P(x, \sigma)$ with respect to σ and the membership of each sample point is determined by the gradient dynamical system in (7). Since the solution of the initial problem of (7) cannot be found analytically, some numerical methods should be used. If the

Euler difference method is used to the solution of (7), $x(t, x_0)$ is then approximated by the sequence $\{x(n)\}$ and the magnitude of $P(x, \sigma)$ is scaled by the logarithmic function, the iteration formula for the corresponding blob center or cluster center is expressed as follows:

$$\begin{cases} x(n+1) = x(n) + h \cdot \nabla_x \ln P(x(n), \sigma) = x(n) + \frac{h}{\sigma^2} \frac{\sum_{i=1}^{N} (x_i - x(n)) e^{-\frac{\|x(n)-x_i\|^2}{2\sigma^2}}}{\sum_{i=1}^{N} e^{-\frac{\|x(n)-x_i t\|^2}{2\sigma^2}}} \\ x(0) = x_0 \end{cases} \quad (8)$$

where h is the step length, and is usually taken as σ^2 in the proposed scale-space hierarchical clustering algorithms so that the iteration process has better convergence characteristic [10].

Integrating the above merging procedure for sample clustering, the scale-space-based hierarchical clustering algorithm is described as:

step 1) Let the iteration number $i = 1$. Initialize σ to a small enough value σ_0, let each sample point be a cluster and its cluster center is itself.

step 2) Find the new cluster center at σ_i for each cluster center obtained at scale σ_{i-1} by the iterative scheme in (8). Merge the clusters whose cluster centers arrive at the same cluster center into a new cluster.

step 3) If there is more than one cluster, increase σ by a constant factor, i.e. $\sigma_{i+1} = k\sigma_i$, let $i = i + 1$, go to step 2).

step 4) Stop when there is only one cluster.

3.2.3. *Cluster Validity*

In the hierarchical clustering process for the sample point set, a hierarchical structure of clusters is generated. In the different scale, different clusters are formed, and further a series of price zones with different sizes is obtained. In order to obtain the best price zone, the cluster validity problem is solved on the basis of human visual perception and actual price zone partition.

(1) *Lifetime*

Each cluster has its own survival range of scale. When scale goes beyond this range, it will merge or split into another cluster. The Witkin's empirical observation 'that survive over a broad range of scale tend to leap out at the eye…' [9], leads us to adopt the notion of lifetime of a cluster as its validity criterion: A cluster with a longer lifetime is preferred to one with a short lifetime. Lifetime of a cluster is defined to the range of logarithmic scales over which the cluster survives, i.e. the logarithmic difference between the point when the cluster is formed and the point when the cluster is absorbed into or merged with other clusters.

$$\text{life} = \ln \sigma_2 - \ln \sigma_1,$$

where life is the lifetime of a cluster; σ_1 is the scale when the cluster is formed; σ_2 is the scale when the cluster disappears.

(2) *Compactness and Isolation*

Intuitively, a cluster is good if the distances between the sample points inside the cluster are small and those outside are large. To make this ideal operational, two measures for the identification of good clusters, i.e. compactness and isolation of a cluster are defined. For a cluster C_i, they are given as follows:

$$\text{compactness} = \frac{\sum_{x \in C_i} e^{-\|x - x_i^*\|^2/2\sigma^2}}{\sum_{x \in C_i} \sum_j e^{-\|x - x_j^*\|^2/2\sigma^2}}, \quad \text{isolation} = \frac{\sum_{x \in C_i} e^{-\|x - x_i^*\|^2/2\sigma^2}}{\sum_x e^{-\|x - x_i^*\|^2/2\sigma^2}},$$

where x_i^* is the cluster center of cluster C_i; For a good cluster, the compactness and isolation are close to one.

Hierarchical clustering provides a sequence of clusters. Using the above three validity criteria and integrating the actual price zone partition, the following procedure is given to choose good clusters (i.e. optimal price zone) from the sequence of clusters in hierarchy.

step 1) Let $C = \{C_1, \ldots, C_K\}$ be the set of all clusters in hierarchy which has the following properties:

a. Its compactness and isolation is greater than a certain threshold;
b. It has at least n sample points (i.e. nodes);
c. The distance between its sample points is less than a certain value, that is, the price difference at any two nodes in the cluster is less than a certain value, which is expressed as:

$$|\rho_i - \rho_j| = \left| \sum_{l=1}^{d} \mu_l (h_{li} - h_{lj}) \right| \leq \varepsilon$$

where μ_l is the expected value of the shadow price for congested line l; ε is the highest acceptable price difference within a zone. In the actual power market operation, μ_l is taken as the average of the shadow prices in all possible operating conditions; ε can be determined by consultation between the system operator and market participants according to the marginal cost of the system.

step 2) Initialize the set of real clusters U to be an empty set.
step 3) Pick the cluster C_k in C with the longest lifetime and put it into U. Remove C_k itself plus the clusters in C that are contained in C_k and the clusters in C that contain C_k.
step 4) Go to step 3) until C is empty.

4. Numerical Results

Two examples are provided on the IEEE 118-node system to test the proposed price zone partition method based on scale space hierarchical clustering.

4.1. Case 1

Suppose lines 64–65, 69–77 possibly congested in the period of time, the sensitivities of the power injections at each node to power flows on these two lines are calculated, and further mapped into two-dimensional space, generated a sample point. The plot of the 118 sample points is shown in *Fig. 1*.

The scale-space-based hierarchical clustering algorithm clusters this sample set with 118 two-dimensional points, where $\sigma_0 = 0.01$, $k = 1.025$. *Fig. 2* shows the evolutionary plot of the cluster centers in the scale-space. By using the clustering selection procedure, we pick good clusters from a sequence of clusters, and then obtain 6 good clusters (shown in *Table 1*), i.e. 6 price zones (shown in *Fig. 3*). The corresponding parameters for the clustering selection procedure are taken as follows: the threshold of the compactness and isolation is 0.9; the cluster has at least 5 sample points; the shadow prices for two congested lines are 10 \$/MWh and 5 \$/MWh, respectively; the highest acceptable price difference is 5 \$/MWh.

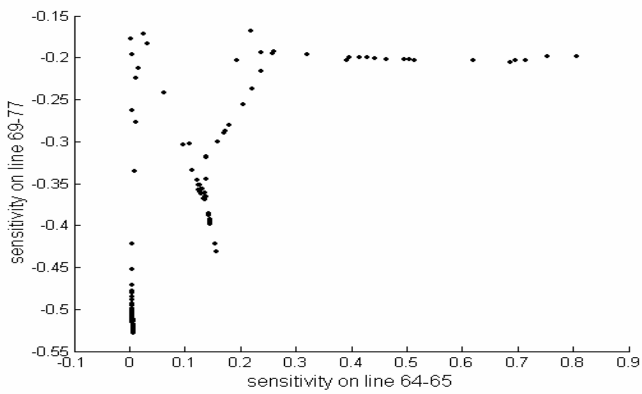

Fig. 1. Plot of sample point set

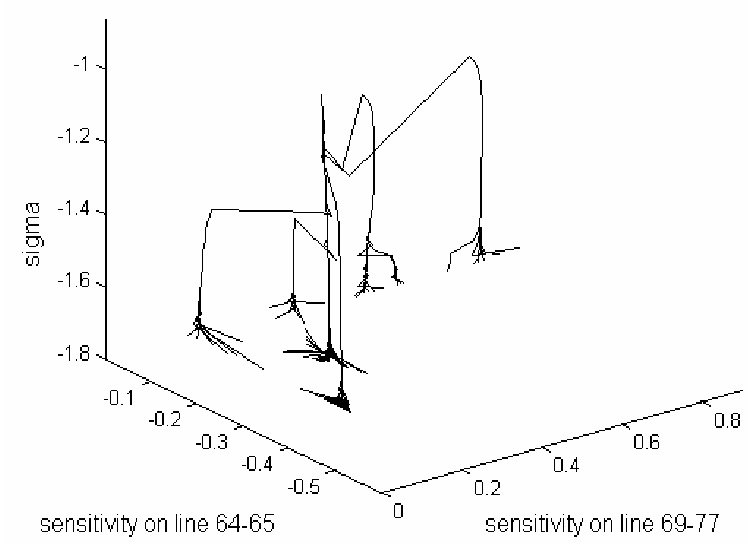

Fig. 2. Evolutionary plot of cluster centers

Table 1. Clusters of nodes

Cluster	Nodes in cluster	Cluster center
1	1–43, 113–115, 117	(0.1356, −0.3689)
2	44–50	(0.2362, −0.2028)
3	51–58, 67	(0.4317, −0.2010)
4	59–64	(0.7050, −0.2034)
5	65–68, 68-76, 81, 116, 118	(0.0166, −0.1922)
6	77–80, 82–112	(0.0059, −0.5078)

4.2. Case 2

If congestion is more severe compared with that of Case 1, lines 64–65, 69–77, 37–38 and 69–70 are congested, and their shadow prices are 10 \$/MWh, 5 \$/MWh, 10 \$/Mw and 5 \$/MWh, respectively. The sensitivities of the nodal power injections to power flows on these four lines are mapped into four-dimensional space and further 118 sample points are generated. The scale-space-based hierarchical clustering algorithm clusters this sample set. The parameters for the cluster selection procedure are set the same as those in Case 1 (excluding the shadow prices for congested lines).

By using the clustering selection procedure, we obtain 5 efficient clusters and a lot of individual nodes (shown in *Table* 2 and *Fig.* 3). From *Table* 2 and *Fig.* 3,

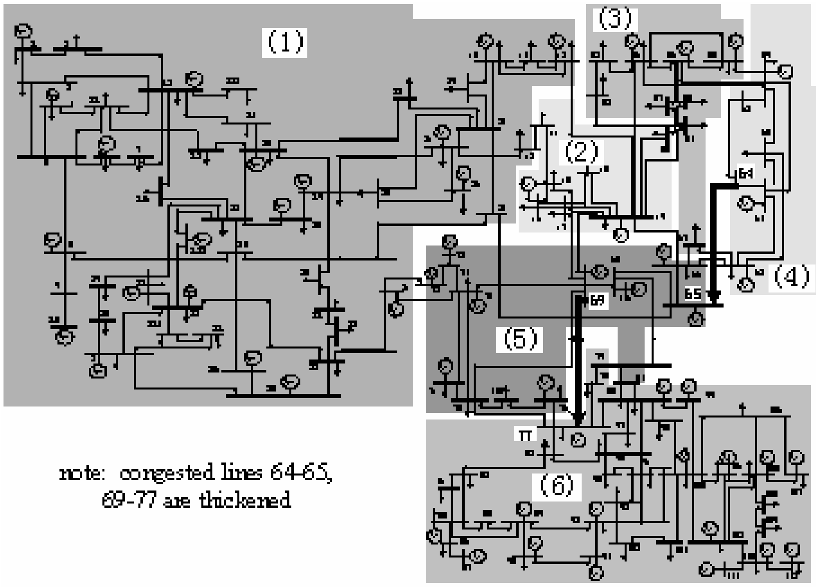

Fig. 3. Partitioning transmission network with 6 price zones

as the degree of congestion increases, the size of price zone is smaller than that of Case 1, the nodes far from these congested lines are clustered into 5 zones, and each node close to these congested lines becomes an individual zone so as to provide right price signals.

Table 2. Clusters of nodes

Cluster	Nodes in cluster
1	1–32, 113–115, 117
2	46–50
3	51–58, 67
4	59–64
5	77–80, 82–112
Remark	each other node is an individual cluster

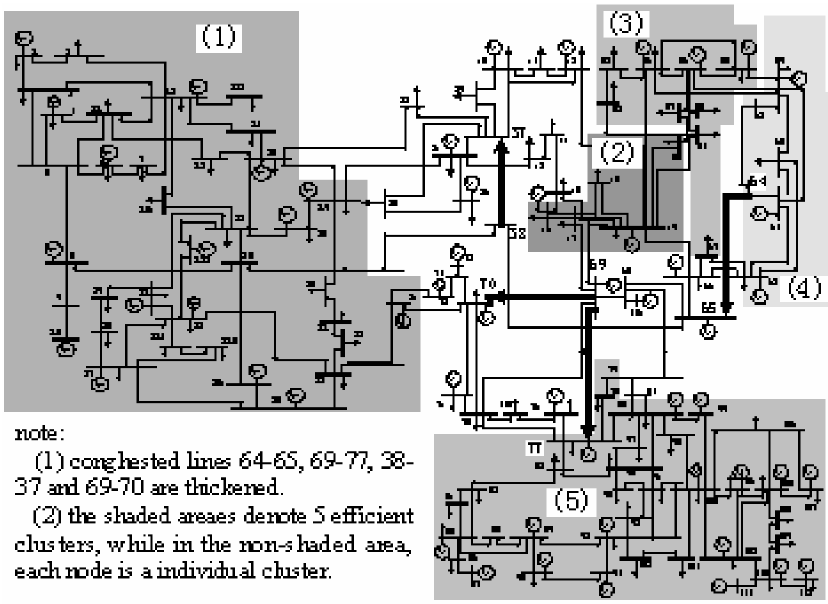

Fig. 4. Partitioning transmission network with 5 price zones and some independent nodes

5. Conclusion

The scale-space theory simulating human visual system is introduced, and further a scale-space-based hierarchical clustering method for price zone partition is proposed in this paper. This proposed method is successfully applied to the 118-node system in two congestion cases. As indicated above, the following conclusions are made:

1. The sensitivities of nodal power injections to power flows on congested lines are extracted as cluster feature, which can not only reflect nodal price but also do not vary with operating condition so as to provide relatively stable price zone partition in a period of time.
2. The scale-space-based hierarchical clustering simulates human visual system. With the range of scale from small to large, the sample points are merged gradually, and consequently a hierarchical structure of clusters in the scale-space is generated.
3. A set of clustering selection rules is proposed on the basis of human visual perception and actual network partition problem. In the application of this proposed method, the cluster selection rules are still modified according to knowledge about power market operation.

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