PERIODICA POLYTECHNICA SER. EL. ENG. VOL. 46, NO. 3–4, PP. 195–208 (2002)

SWITCHING ROBUST ADAPTIVE CONTROL BASED ON RBF NEURAL NETWORKS

Lőrinc MÁRTON and Béla LANTOS

Department of Control Engineering and Information Technology
Budapest University of Technology and Economics
Magyar Tudósok körútja 2, H-1117 Budapest, Hungary
e-mail: martonl@seeger.iit.bme.hu, lantos@iit.bme.hu

Received: April 8, 2003; Revised: September 4, 2003

Abstract

The paper deals with robust adaptive control of a class of single-input single-output nonlinear system, in which robustness is guaranteed by switching control algorithm and adaptation law using smooth gradient projection. It is discussed the behavior of the control system, when the nonlinear part in the model of the controlled system is not known exactly. A modified control law is proposed that assures the boundedness of all the signals of the control system even if the nonlinear model contains unmodelled disturbance.

Keywords: Adaptive control, nonlinear control, switching functions, Lyapunov stability.

1. Introduction

The design of a stable control algorithm for nonlinear systems with unknown parameters is a challenging problem. Many of these plants have slowly varying or uncertain parameters and unknown disturbances. The adaptive control is a popular approach to the control of such systems.

If the mathematical model of the plant is known but the parameters in the nonlinearities are unknown, in order to assure precise tracking, adaptive control schemes can be used that estimate the value of the parameters based on input-output measurements. In the case of *linearly parameterized nonlinear systems*, which are discussed in this paper, gradient method or its modified versions can be used to estimate the unknown parameters of the controlled system. The linear parametrization of the nonlinearities is not a strong assumption, because if this nonlinearities are smooth well known universal approximation theorems guarantee that the nonlinearities can be approximated with neural or neuro-fuzzy systems with desired precision [4]. Many of these models have linearly parameterized forms.

Control laws depending on the estimated parameters can guarantee good transient performances and the stability of the closed loop system. But these algorithms show very poor robustness proprieties even in the case of small disturbances or small model uncertainties. One way to avoid the lack of robustness is the modification of the control law with discontinuous switching functions that depend on the maximum values of the unmeasurable disturbances [3]. Using Lyapunov method it can be shown that these modified discontinuous control laws guarantee the stability of the control system in the presence of bounded disturbances.

However, the robustified adaptive control leaves many open questions to be answered, some of them are enumerated as follows:

1. If we have *a priori* information on the parameters of the controlled plant, how can it be introduced in the adaptive law to obtain better performances?
2. How can the stability of the control system be guaranteed when the universal approximator describing the behavior of the nonlinear part of the model has a considerable approximation error?
3. If the adaptation law doesn't work properly in every time instant – for example at the beginning of the entire control process or at an abrupt change of the parameters of the plant, causing oscillatory behavior of the adaptation law – how can the boundedness of the control signal and the stability simultaneously be guaranteed?

The present study deals with these questions of the robust adaptive control schemes. To approximate the nonlinearities, Radial Basis Function (RBF) Networks were used. The robustness of the adaptation law is assured by smoothed gradient projection and saturation type switching functions.

The remaining part of the paper is organized as follows: Section 2 presents the standard problem of the robust adaptive control solved using soft gradient projection algorithms. Section 3 introduces the problem of the uncertain control algorithms in which the dynamic model of the plant is not known precisely and proposes a modified algorithm that guarantees bounded tracking error and the stability of the closed loop system. Simulation results are presented in Section 4, Section 5 summarizing the conclusions of this paper.

2. Robust Adaptive Control

Let us consider the following class of single-input single-output (SISO) nonlinear system written in phase variable form:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_n &= f(\underline{x}) + g(\underline{x})u + d \\ y &= x_1, \end{aligned} \tag{1}$$

where y denotes the output, u the input, $\underline{x} = (x_1 \;\; x_2 \ldots x_n)^T$ the states of the system and d is unmeasurable disturbance.

To apply earlier results in robust adaptive control some assumptions should be introduced:

(A1) The nonlinear function multiplying the input $g(\underline{x})$ is nonzero, moreover $0 < g_m \leq g(\underline{x})$, which guarantees that the plant is controllable for any $t \geq 0$.
(A2) The disturbance d is bounded, i.e. $|d| < D_M$.
(A3) The nonlinear functions $f(\underline{x})$ and $g(\underline{x})$ can be written in a linearly parameterized form according to it:

$$f(\underline{x}) = \underline{\theta}_f^T \underline{\xi}_f(\underline{x}), \qquad g(\underline{x}) = \underline{\theta}_g^T \underline{\xi}_g(\underline{x}). \tag{2}$$

In this paper linear parameterization is assured by approximating the functions f and g by *RBF neural networks* [4]. This popular neural model is widely used in adaptive control strategies because of its relatively simple structure and good approximation proprieties. It has two layers. The output of a neuron from the first, hidden layer can be written as:

$$z_i = R_i(\underline{x}) = R_i(\|\underline{x} - \underline{c}_i\|/\sigma_i), \quad i = 1 \ldots H, \tag{3}$$

where the vector $\underline{x}$ contains the input of the model, H is the number of neurons in the hidden layer and R_i is a radial basis function, typically of Gaussian type.

If we consider that the model has single output, then:

$$r(\underline{x}) = \sum_{i=1}^{H} \theta_i R_i(\underline{x}) + b. \tag{4}$$

Approximating nonlinear functions by RBF neural model, it yields:

$$r(\underline{x}) = \underline{\theta}^T \underline{\xi}(\underline{x}) \quad \text{where} \quad \underline{\theta} = (\theta_1 \ldots \theta_H\ b)^T; \quad \underline{\xi}(\underline{x}) = (R_1(\underline{x}) \ldots R_H(\underline{x})\ 1)^T \tag{5}$$

If we have *a priori* measurements on $f(\underline{x})$ and $g(\underline{x})$ separately, the design parameters of the functions R_i can be determined using clustering methods [4] or if not, they can be distributed uniformly in the input field.

The presented assumptions (controllability, bounded disturbance and linear parametrization) are generally enough for the development of stable adaptive control algorithms. The control problem can be formulated as follows: let us design a control law u in a way that the output $y(t)$ defined in (1) tracks a desired trajectory $y_d(t)$, which is a smooth, n times differentiable function of time.

To solve this problem, let us define the tracking error $e(t) = y(t) - y_d(t)$ and the tracking error metric $S(t) = (\frac{d}{dt} + \lambda)^{(n-1)} e(t)$ with $\lambda \geq 0$. Differentiating $S(t)$ with respect to time and taking into account the assumption **(A3)**, we obtain:

$$\begin{aligned} \dot{S}(t) &= e^{(n)} + \underline{k}^T \underline{e} = f(\underline{x}) + b(\underline{x})u + d - y_d^{(n)} + \underline{k}^T \underline{e} \\ &= \underline{\theta}_f^T \underline{\xi}_f(\underline{x}) + \underline{\theta}_g^T \underline{\xi}_g(\underline{x})u + d - y_d^{(n)} + \underline{k}^T \underline{e}, \end{aligned} \tag{6}$$

where $\underline{k} = (k_{n-1} \ldots k_1)$ is a vector that contains the coefficients in the expansion of $S(t)$ and $\underline{e} = (e^{(n-1)} \ldots e)^T$.

If the disturbance d was not present and the nonlinearities were known functions, then the control signal assuring the convergence of the tracking error to zero could easily be developed, see for example [9]. With f and g unknown, the control law can be developed by using estimated parameters that are generated on-line. Let us denote the estimated parameters with $\widehat{\underline{\theta}}$ and the estimation error by $\widetilde{\underline{\theta}} = \underline{\theta} - \widehat{\underline{\theta}}$. The control law can be written as function of the estimated parameters:

$$u = \frac{1}{\widehat{\underline{\theta}}_g^T \underline{\xi}_g}\left(-\widehat{\underline{\theta}}_f^T \underline{\xi}_f + y_d^{(n)} - \underline{k}^T \underline{e} - k_S S_\Delta(t) - D_M \operatorname{sat}(S/\Phi)\right), \tag{7}$$

where k_S and Φ are positive design constants and $\operatorname{sat}(\cdot)$ denotes the well known saturation function, and $S_\Delta = S - \operatorname{sat}(S/\Phi)$. It can easily be verified that S_Δ has the following useful property:

$$\dot{S}_\Delta = \dot{S} \text{ for } |S_\Delta| \geq \Phi \text{ and } \dot{S}_\Delta = 0 \text{ otherwise.} \tag{8}$$

With this control law the behavior of the closed loop system becomes:

$$\dot{S} = \widetilde{\underline{\theta}}_f^T \underline{\xi}_f(\underline{x}) + \widetilde{\underline{\theta}}_g^T \underline{\xi}_g(\underline{x})u - k_S S_\Delta(t) + d - D_M \operatorname{sat}(S/\Phi). \tag{9}$$

The most prevalent on-line estimation law used in adaptive control systems is the gradient algorithm because it can be relatively simply implemented and, together with control algorithm (7), can guarantee the stability of the control system. In robust adaptive control schemes a modified version of this algorithm can be used such as the gradient projection algorithm. Let us assume that the bounds of the parameters appearing in (2) are known. This can be formulated as an additional assumption:

(A4) All the parameters are in known interval, i.e. for any element of the parameter vectors $\underline{\theta}_{fi}$ and $\underline{\theta}_{gi}$ we have $\theta_{fmi} \leq \theta_i \leq \theta_{fMi}$ and $\theta_{gmi} \leq \theta_{gi} \leq \theta_{gMi}$ respectively with $\theta_{fmi}, \theta_{fMi}$ $\theta_{gmi}, \theta_{gMi}$ known.

This assumption can be explored if we use as adaptation algorithm the gradient projection method instead of classical gradient method. In this paper we use the *smooth gradient projection algorithm proposed* in [5, 8] that overcomes the discontinuous behavior of the adaptation which is the main disadvantage of these algorithms. Generally, let us assume the following parameter sets $\Omega = \{\underline{\theta} \mid a_i \leq \theta_i \leq b_i \; \forall i\}$ and $\Omega_\delta = \{\underline{\theta} \mid a_i - \delta \leq \theta_i \leq b_i + \delta \; \forall i\}$, where $\delta > 0$. If the adaptation step size for each parameter θ_i is denoted with $\gamma_i > 0$, the strictly positive diagonal matrix $\Gamma = \operatorname{diag}(\gamma_1, \gamma_2, \ldots, \gamma_p)$ can be introduced. The smooth gradient projection algorithm can be written as:

$$\dot{\underline{\widehat{\theta}}} = \mathrm{Proj}(S, \underline{\widehat{\theta}}, \underline{\xi}) \quad \text{where} \tag{10}$$

$$\mathrm{Proj}(S, \underline{\widehat{\theta}}, \underline{\xi})_i = \begin{cases} \gamma_i \xi_i, & \text{if} \quad a_i \leq \widehat{\theta}_i \leq b_i \\ & \text{or if} \quad \widehat{\theta}_i > b_i \text{ and } \xi_i \leq 0 \\ & \text{or if} \quad \widehat{\theta}_i < a_i \text{ and } \xi_i \geq 0 \\ \gamma_i (1 + \frac{b_i - \widehat{\theta}_i}{\delta}) \xi_i, & \text{if} \quad \widehat{\theta}_i > b_i \text{ and } \xi_i > 0 \\ \gamma_i (1 + \frac{\widehat{\theta}_i - a_i}{\delta}) \xi_i, & \text{if} \quad \widehat{\theta}_i < a_i \text{ and } \xi_i < 0 \end{cases} \tag{11}$$

It is known [8], [5] that this rule guarantees the following propriety: $\underline{\widehat{\theta}}(0) \in \Omega \Rightarrow \underline{\widehat{\theta}} \in \Omega_\delta$ for any $t \geq 0$. Moreover, the smooth gradient projection has the following useful propriety:

$$\underline{\widetilde{\theta}}^T S_\Delta \Gamma \underline{\xi} \leq \underline{\widetilde{\theta}}^T \mathrm{Proj}(S, \underline{\widehat{\theta}}, \underline{\xi}). \tag{12}$$

By (2) the adaptation rule can be written as follows:

$$\dot{\underline{\widehat{\theta}}}_f = \mathrm{Proj}(S, \underline{\widehat{\theta}}_f, \underline{\xi}_f) \qquad \dot{\underline{\widehat{\theta}}}_g = \mathrm{Proj}(S, \underline{\widehat{\theta}}_g, \underline{\xi}_g). \tag{13}$$

This algorithm and the assumption **(A4)** guarantees that the estimated parameters also remain bounded with known bounds introduced in **(A4)**.

It can be shown that control law (7) with the adaptation law (13) guarantees the boundeness of all signals in the closed loop system. The proof will be omitted because this affirmation is a special case of the more general theorem that will be presented in the next section.

3. Robust Adaptive Control with Uncertain Control Law

In the implementation of a model based control law, modelling errors should be taken into account. These errors could appear when some terms in the mathematical model of the plant are neglected, or when the nonlinear part is approximated by using neural or fuzzy models. We can disregard the uncertainty of the model when it does not affect the stability of the control system, or the requirements for the performances of the control are not too high. Otherwise, it should be taken into consideration at the development of the control algorithms.

Robust modifications of adaptive control schemes were treated by many authors. A first approach could be the design of the linear part of the controller using H_∞ control theory. These types of control algorithms were introduced in [1] and [2] for robotic systems. In these papers it was shown that if the disturbance d is quadratically integrable, then H_∞ performance can be guaranteed. Another approach could be the introduction of an additive supervisory control signal in the control law that guarantees the boundeness of all the signals in the closed loop system. This part of the control law is switched on only when a known performance measure of the control system (for example the tracking error metric) leaves a prescribed domain.

These types of control laws are described in [7] [6]. Note that all these algorithms use some types of gradient projection algorithms as adaptation law.

In the adaptive control schemes the model uncertainty is generally represented by an additive bounded disturbance, i.e.

$$f(\underline{x})_{\text{applied}} = f(\underline{x})+d_f(\underline{x}) = \underline{\theta}_f^T\underline{\xi}_f(\underline{x})+d_f(\underline{x}), \quad \text{where} \quad |d_f(\underline{x})| \leq D_{fM}. \quad (14)$$

Let us consider the case when the nonlinearity is approximated by using RBF networks presented in Section 2. Since separate measurements for $f(\underline{x})$ and $g(\underline{x})$ are not available, we use a finite number of fixed shape, fixed center basis functions. The modelling errors are also influenced by regressor vectors $\underline{\xi}(\underline{x})$. For example, in the case of $f(\underline{x})$ the modelling error can be written as:

$$f(\underline{x})_{\text{applied}} = \underline{\theta}_f^T\underline{\xi}_{f\ \text{applied}}(\underline{x}) = \underline{\theta}_f^T(\underline{\xi}_f(\underline{x}) + d_{\xi f}(\underline{x})), \quad \text{where} \quad |d_{\xi f}(\underline{x})| \leq D_{fM}. \quad (15)$$

Thus, the problem cannot be treated just as an additive modelling error because *it also affects the adaptation rule that can influence the stability of the closed loop system.* In this section a new algorithm is introduced that assures the stability in the presence of uncertainly modelled regressors.

To solve this problem additional assumptions will be introduced.

(A5) The applied values of the regressor vectors $\underline{\xi}_f(\underline{x})$ in the control law can be written in function of the real regressor vectors as $\underline{\xi}_{f\ \text{applied}} = \underline{\xi}_f(\underline{x}) + \underline{d}_{\xi f}$ with $|(\underline{d}_{\xi f})_i| \leq D_f\ \forall i$.

(A6) The applied values of the regressor vectors $\underline{\xi}_g(\underline{x})$ in the control law can be written in function of the real regressor vectors as $\underline{\xi}_{g\ \text{applied}} = \underline{\xi}_g(\underline{x}) + \underline{d}_{\xi g}$ with $|(\underline{d}_{\xi g})_i| \leq D_g\ \forall i$.

(A7) In the assumption **(A4)** $\text{sign}(\theta_{gmi}) = \text{sign}\ (\theta_{gMi})\ \forall i$ and $|\theta_{gmi}| > 0$. Here $\xi_{f\ \text{applied}}$ and $\xi_{g\ \text{applied}}$ represent the gaussian membership functions that we use in the control law, while $\underline{\xi}_f$ and $\underline{\xi}_g$ represent the best regression vectors that describe the unknown nonlinearities of the plant for prescribed precision.

The tracking error behavior for the closed loop system with the proposed control law and estimation dynamics is described in the following theorem:

THEOREM 1 *Consider the system* (1). *If the assumptions* **(A1)** – **(A7)** *hold, then for a given* Φ *the control law*

$$u = u_c + \{|S(t)| > S_{\text{LIM}}\}u_{sw}, \quad (16)$$

with the adaptation law defined in (10)*:*

$$\dot{\widehat{\underline{\theta}}}_f = \text{Proj}\left(S_\Delta, \widehat{\underline{\theta}}_f, \underline{\xi}_{f\ \text{applied}}\right) \quad (17)$$

$$\dot{\widehat{\underline{\theta}}}_g = \text{Proj}\left(S_\Delta, \widehat{\underline{\theta}}_g, \left(\underline{\xi}_{g\ \text{applied}} + \underline{C}\right) u_c\right)$$

guarantees asymptotically the boundeness of all signals in the control system, where:

$$u_c = \frac{1}{\underline{\widehat{\theta}}_g^T \left(\underline{\xi}_{g\,\text{applied}}(\underline{x}) + \underline{C}(\underline{\widehat{\theta}}_g, \underline{x})\right)} \left(-\underline{\widehat{\theta}}_f^T \underline{\xi}_{f\,\text{applied}}(\underline{x}) - k_S S(t) - \underline{k}^T \underline{e}(t) + y_d^{(n)} - D\,\text{sat}\,(S/\Phi)\right), \tag{18}$$

$$u_{sw} = -\frac{1}{g_m}\left(D_f \sum_i |\theta_{fMi}| + D_g \sum_i |\theta_{gMi}|\,|u_c|\right) \text{sat}(S/\Phi), \tag{19}$$

g_m is known as minimal value of $g(\underline{x})$ defined in the assumption **(A1)**, *$\underline{C}(\underline{\widehat{\theta}}_g, \underline{x})$ is a correction vector whose elements are:*

$$C(\underline{\widehat{\theta}}_g, \underline{x})_i = \rho(\underline{\widehat{\theta}}_{gi}) \frac{g_m - \widehat{\theta}_{gi} \xi_{g\,\text{applied}\,i}(\underline{x})}{\widehat{\theta}_{gi}} \tag{20}$$

with:

$$\rho(\underline{\widehat{\theta}}_{gi}) = \begin{cases} 1 & \text{if} \quad \widehat{\theta}_{gi} \xi_{g\,\text{applied}\,i} \le g_m \\ 0, & \text{otherwise.} \end{cases} \tag{21}$$

The notation $\{|S(t)| > S_{LIM}\}$ means that the switching term u_{sw} acts only when the absolute value of tracking error metric $S(t)$ is higher than a prescribed limit $S_{LIM} > \Phi$.

Because of the assumptions **(A4)** and **(A7)** the denominator in the expression of $\underline{C}(\underline{\widehat{\theta}}_g, \underline{x})_i$ will never be zero. It can easily be seen that with this modification the denominator of u_c will always be greater than or equal to g_m, which is a necessary condition for the feasibility of the control law.

Proof. Let us consider the following Lyapunov like function:

$$V(t) = \frac{1}{2} S_\Delta^2(t) + \frac{1}{2} \underline{\widetilde{\theta}}_f^T \Gamma_f^{-1} \underline{\widetilde{\theta}}_f + \frac{1}{2} \underline{\widetilde{\theta}}_g^T \Gamma_g^{-1} \underline{\widetilde{\theta}}_g, \tag{22}$$

where Γ_f and Γ_g are diagonal matrices with positive adaptation step sizes in the diagonal.

The time derivative of $V(t)$ is given by:

$$\dot{V}(t) = S_\Delta(t) \dot{S}_\Delta(t) + \underline{\widetilde{\theta}}_f^T \Gamma_f^{-1} \underline{\dot{\widehat{\theta}}}_f + \underline{\widetilde{\theta}}_g^T \Gamma_g^{-1} \underline{\dot{\widehat{\theta}}}_g. \tag{23}$$

According to the definition of adaptation law (17) and the propriety (8) of S_Δ we have $\dot{V}(t) = 0$ for $|S| \le \Phi$.

Therefore, the remaining part of the proof treats only the case when $|S| > \Phi$. By (8) $\dot{S}_\Delta = \dot{S}$. Applying (12) for the estimation law (17) and substituting it into the derivative of the Lyapunov function:

$$\dot{V}(t) \le S_\Delta(t) \dot{S}(t) - \underline{\widetilde{\theta}}_f^T S_\Delta(\underline{\xi}_f + d_{\xi f}) - \underline{\widetilde{\theta}}_g^T S_\Delta((\underline{\xi}_g + \underline{C}) u_c + d_{\xi g} u_c). \tag{24}$$

The tracking error dynamics can be written as follows:

$$\dot{S} = \underline{\theta}_f^T \underline{\xi}_f(\underline{x}) + \underline{\theta}_g^T \underline{\xi}_g(\underline{x})u + d - y_d^{(n)} + \underline{k}^T \underline{e}. \quad (25)$$

Let us consider that $|S(t)| \leq S_{LIM}$. In this case u can be replaced by u_c. Let us introduce the value of u_c into (25):

$$\dot{S} = -k_S S_\Delta + \underline{\widetilde{\theta}}_f^T \underline{\xi}_f(\underline{x}) + \underline{\widetilde{\theta}}_g^T (\underline{\xi}_g(\underline{x}) + \underline{C})u_c - \underline{\widehat{\theta}}_f^T \underline{d}_{\xi f}$$
$$- \underline{\widehat{\theta}}_g^T \underline{d}_{\xi g} u_c + d - DS_\Delta \operatorname{sat}(S/\Phi). \quad (26)$$

With (26) the relation (24) can be written as:

$$\dot{V}(t) \leq -k_S S_\Delta^2 - S_\Delta \underline{\theta}_f^T \underline{d}_{\xi f} - S_\Delta \underline{\theta}_g^T \underline{d}_{\xi g} u_c + S_\Delta d - S_\Delta D \ \operatorname{sat}(S/\Phi). \quad (27)$$

Taking into consideration that $|S| > \Phi$, we can replace $\operatorname{sat}(S/\Phi)$ with sign (S) or even with sign (S_Δ) in the expression of u_{sw}. It can easily be shown that if $d \leq D$, where $D > 0$ than for any S, we have $dS \leq D|S|$. From these inequalities it results:

$$\dot{V}(t) \leq -k_S S_\Delta^2 - S_\Delta \underline{\theta}_f^T \underline{d}_{\xi f} - S_\Delta \underline{\theta}_g^T \underline{d}_{\xi g} u_c. \quad (28)$$

Note that the sign of $\dot{V}(t)$ cannot be determined because of model uncertainties $d_{\xi f}$ and $d_{\xi g}$.

If the value of $S(t)$ reaches the value S_{LIM}, the value of u_{sw} switches on. In this case the relation (23) can be written as:

$$\dot{V}(t) \leq -k_S S_\Delta^2 - S_\Delta \underline{\theta}_f^T \underline{d}_{\xi f} - S_\Delta \underline{\theta}_g^T \underline{d}_{\xi g} u_c + \underline{\theta}_g^T \underline{\xi}_g(\underline{x}) S_\Delta u_{sw} \quad (29)$$

According to the assumption **(A1)** we also have $0 < g_m \leq \underline{\theta}_g^T \underline{\xi}_g(\underline{x})$, hence:

$$\dot{V}(t) \leq -k_S S_\Delta^2 - S_\Delta \underline{\theta}_f^T \underline{d}_{\xi f} - S_\Delta \underline{\theta}_g^T \underline{d}_{\xi g} u_c +$$
$$- \left(D_f \sum_i |\theta_{fMi}| + D_g \sum_i |\theta_{gMi}| \, |u_c| \right) |S_\Delta| . \quad (30)$$

With the same considerations as in the step from (27) to (28) it yields:

$$\dot{V}(t) \leq -k_S S_\Delta^2. \quad (31)$$

If $S(0)$ is bounded then if $S(t)$ leaves the limit S_{LIM}, then the boundedness of $V(t)$ is assured by (31), and so $S(t)$ is also bounded according to (22).

The boundedness of the estimated parameters is guaranteed by the estimation laws (17).

The boundedness of $S(t)$ and $x_d^{(i)}(t)$ results the boundedness of $x^{(i)}(t) \ \forall \ i$.

From the boundedness of the control signal u, the external disturbance and the modelling errors (assumptions **(A2)** **(A5)** and **(A6)**) results that $\dot{S}_\Delta$ is also bounded (see (26)).

REMARK 1 *The switching term represents a high gain term in the control scheme that acts only when the behavior of the control system is critical. Its introduction is necessary to guarantee the theoretical and practical stability of the control system. An important result that we can conclude from this approach of adaptive control law is related to the minimum value of the saturation limit of the applied actuator. The actuator should be able to reproduce control signals with amplitude at least* $S_{LIM} * |u_{sw}|$. *In this case the boundedness of all the signals in the closed loop system can be guaranteed according to the theorem.*

4. Simulation Results

To examine the performances of the previously presented control law, a nonlinear mass-spring damper system was considered [9]. The equation of motion for this mechanical system can be expressed as:

$$m\ddot{x} + b\dot{x}\,|x| + k_0 x + k_1 x^3 = ku \tag{32}$$

with $b\dot{x}\,|x|$ modelling the nonlinear dissipation and $k_0 x + k_1 x^3$ modelling the nonlinear spring term. m is the mass of the load and k is the gain of the drive, respectively. x denotes the position.

The functions f and g can be written as follows:

$$f(x) = -\frac{b}{m}\dot{x}\,|\dot{x}| - \frac{k_0}{m}x - \frac{k_1}{m}x^3 \qquad g(x) = \frac{k}{m}. \tag{33}$$

For simplicity it was considered that $k = 1$.

The nonlinear function f was approximated with an RBF Neural Network. Firstly, it was considered that the parameters of the nonlinear function are unknown so the neural model was trained with wrong initial parameters, namely:

$$b = 0.1 \qquad k_0 = 0.01 \qquad k_1 = 1 \qquad m = 1 \tag{34}$$

The resulted RBF network has 8 RBF type neurons in its hidden layer. Due to the low number of neurons and the absolute value function in the damping term there is a fitting error as it can be seen in *Fig. 1* and *Fig. 2*.

During the simulations the parameters of the controlled plant were:

$$b = 0.5 \qquad k_0 = 0.05 \qquad k_1 = 5 \qquad m = 0.1 \tag{35}$$

The controller parameters were chosen as $K_S = 20$, $\lambda = 10$, $\Phi = 0.01$, $g_m = 0.05$, $D = 0.01$, $D_f = 0.01$, $D_g = 10^{-4}$. The bound of the parameters were determined by using the results from the initial training of the RBF neural network.

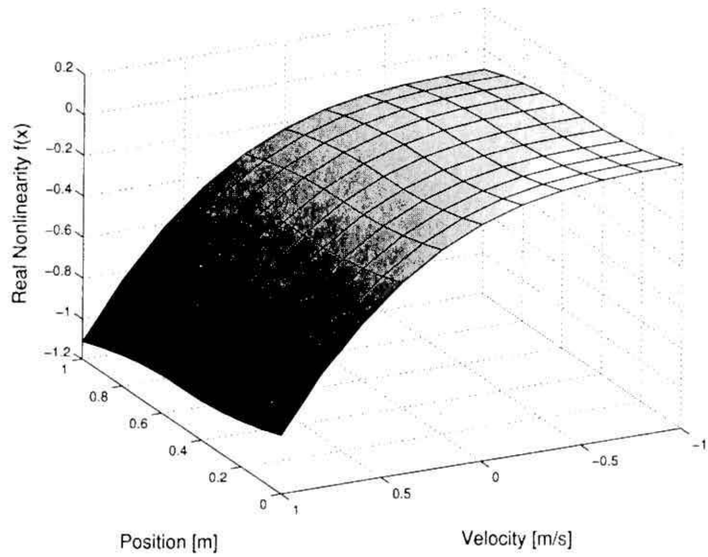

Fig. 1. Training Data for RBF

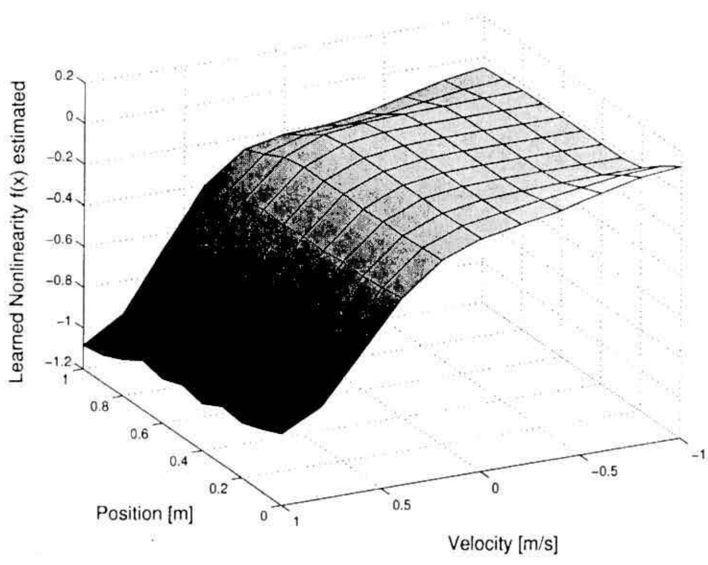

Fig. 2. Learned Data for RBF

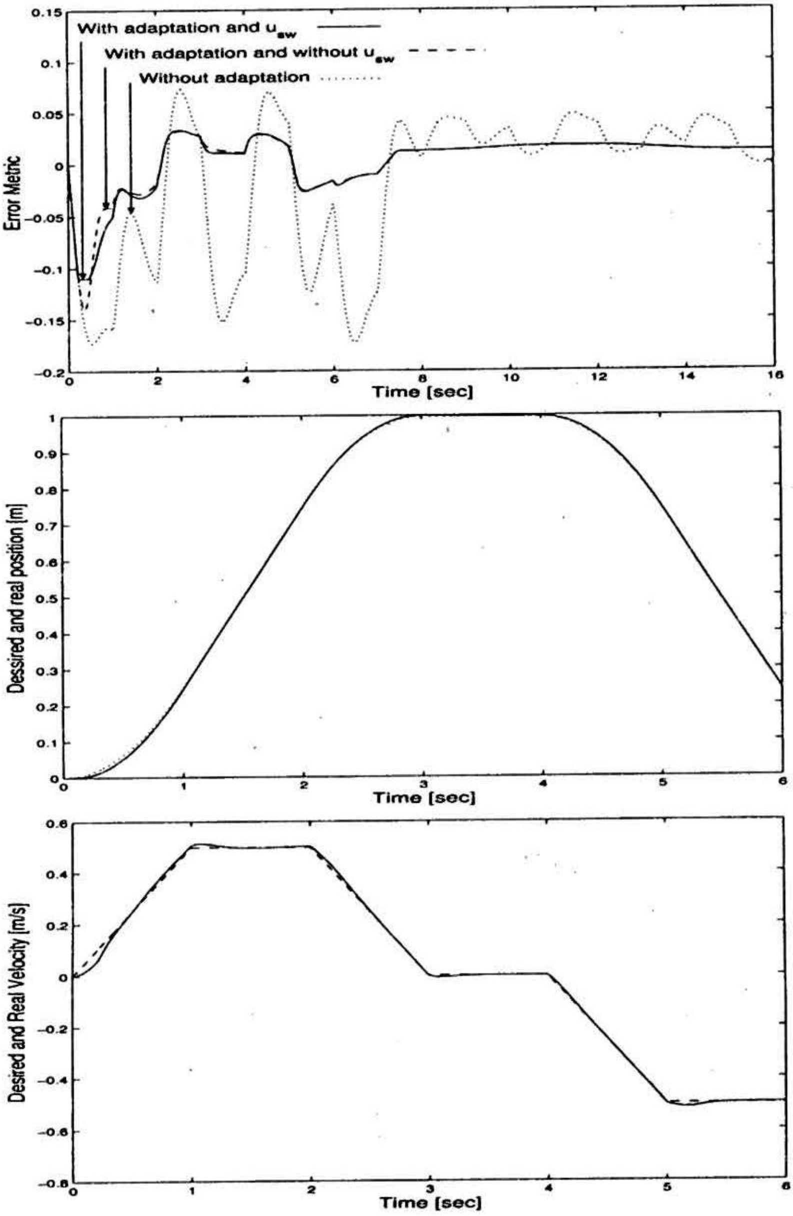

Fig. 3. Error metric convergence (top) and signal tracking proprieties (middle, bottom)

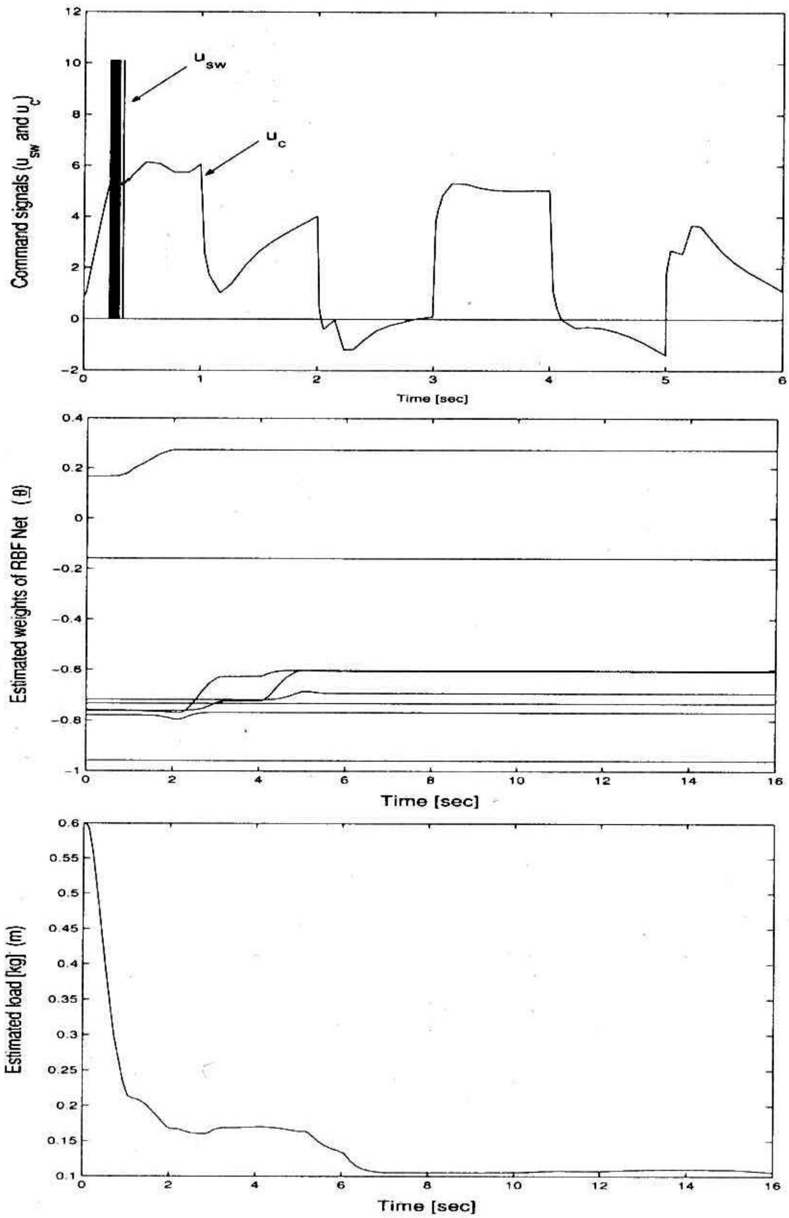

Fig. 4. Control signal components (top) and parameter convergence (middle, bottom)

Simulations were performed with the initial RBF network, with the trained RBF network and with the modified control law. The simulation results are presented in *Fig. 3* and *Fig. 4*.

The error metric (combined position and speed tracking error) shows poor convergence when the adaptation is not applied and the control law is used with the initially trained RBF network. When the adaptation is applied, the adaptation law re-tunes the parameters of the neural model and the estimated mass of the load hence the error metric convergence shows much better properties. Observe that when the error metric reaches the limit $S_{LIM} = 0.1$, the high gain term in the control law u_{sw} turns on and does not allow the error metric to be increased above S_{LIM}.

5. Conclusions

Robust modifications in the adaptive control schemes were introduced to solve the well known problems of the adaptive control schemes, such as high sensitivity on external disturbances or lack of robustness and stability when the model of the controlled system is not known exactly. When the system nonlinearities are modelled with neural or fuzzy systems, the modelling uncertainties should always be taken into consideration at the design of the control law. The present paper deals with the adaptive control schemes in the perspective of the implemented control law. The standard adaptive control algorithm was modified with a supervisory additive term that acts only when the tracking error metric leaves a predetermined limit. It was shown that this adaptive law can guarantee the boundedness of all the signals in the closed loop control system when the regressor vectors in the adaptation laws are not known exactly, and the control signal is perturbed by additive disturbances.

Acknowledgement

Support for the research of robust adaptive control was provided by the Hungarian National Research Program under grant No. OTKA T 042634 and by the Control System Research Group of the Hungarian Academy of Sciences.

References

[1] CHANG, Y-C, Robust Tracking Control for Nonlinear MIMO Systems Via Fuzzy Approaches, *Automatica*, **36** (2000), pp. 1535–1545.

[2] CHANG, Y.-C. – CHEN, B.-S., A Nonlinear Adaptive Hinf Tracking Control Design in Robotic Systems Via Neural Networks, *IEEE Trans. on Control Systems Technology*, **5** No. 1 (1997), 13–29.

[3] IOANNOU, P. A. – SUN, J., *Robust Adaptive Control*, Prentice Hall, Upper Slade River, NJ 1996.

[4] JANG, J.-S. R. – SUN, C. T. – MIZUTANI, E., *Neuro-Fuzzy and Soft Computing*, Prentice Hall, Inc, Upper Saddle River, NJ, 1997.

[5] KHALIL, H. K., Adaptive Output Feedback Control of Nonlinear Systems Represented by Input-Output Models, *IEEE Trans. on Automatic Control*, **41** No. 2 (1996), pp. 177–188.

[6] LANTOS, B., Some Application of Soft Computing Methods in System Modelling and Control, *Advanced Computational Intelligence*, **2** No. 3 (1998), pp. 82–87.

[7] LANTOS, B., *Fuzzy Systems and Genetic Algorithms*, Műegyetemi Kiadó, 2002.

[8] POMET, J.-B. – PRALY, L., Adaptive Nonlinear Regulation: Estimation from the Lyapunov Equation, *IEEE Trans. on Automatic Control*, **37** No. 6 (1992), pp. 729–739.

[9] SLOTINE, J.-J. E. – LI, W., Applied Nonlinear Control, Premtice Hall International, Inc., Englewood Cliffs, New Yersey, 1991.