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A NEW PERMEABILITY FOR PERMANENT MAGNETS AND ANOTHER THEOREM OF REFRACTION IN ISOTROPIC MATERIALS WITH PERMANENT MAGNETIZATION

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Abstract

A new relative magnetic permeability is defined for permanent magnets, which advantageously allows to approach the non-linearity of demagnetization curve of permanent magnets. Also, using the defined quantity, we have demonstrated another form of the theorem of refraction for the surface of separation between two isotropic materials with permanent magnetization. A practical example where the defined quantities are used is presented.

Keywords: new permeability, permanent magnet, refraction.

1. Introduction

Taking into account the relation law between flux density $\overline{B}$, magnetic field intensity $\overline{H}$ and magnetization $\overline{M}$, also considering the temporary magnetization law, in the case of an isotropic material *with permanent magnetization*, we could write the following relation

$$\overline{B} = \mu_0\overline{H} + \mu_0\chi_{mp}\overline{H} + \mu_0\overline{M}_p, \tag{1}$$

where χ_{mp} is the material's magnetic susceptivity and μ_0 is magnetic permeability of the vacuum. The separation in temporary ($\overline{M}_\tau = \chi_{mp}\overline{H}$) and permanent ($\overline{M}_p$) components is unique if $\overline{M}_p$ is independent of $\overline{H}$, and $\overline{M}_\tau$ is null at the same time with $\overline{H}$. The value of $\overline{B}$ for $\overline{H} = 0$ represents the remanent flux density, that is:

$$\overline{B}_r = \overline{B}\big|_{\overline{H}=0} = \mu_0\overline{M}_p. \tag{2}$$

From *Eq.* (1) follows that for materials with $\overline{M}_p \neq 0$ (permanent magnets), the quantity $\overline{B}/\overline{H}$ (which for materials with $\overline{M}_p = 0$ represents the classic magnetic permeability $\mu = \overline{B}/\overline{H}$) is ambiguously determined by the material, because $\overline{M}_p$ can have more values, for the same material (for diverse minor cycles of hysteresis which are possible, $\overline{B}_r = \mu_0\overline{M}_p$ can have more values). In this context it is useful to define another magnetic permeability for permanent magnets, which helps overcome the above mentioned difficulty.

2. A New Permeability for Permanent Magnets

Defining the calculation quantity

$$\overline{B}_p = \overline{B} - \overline{B}_r, \tag{3}$$

Eq. (1) becomes

$$\overline{B}_p = \mu_0(1 + \chi_{mp})\overline{H} = \mu_p\overline{H}, \tag{4}$$

where the calculation permeability of the permanent magnet is

$$\mu_p = \frac{\overline{B}_p}{\overline{H}}. \tag{5}$$

It is known that *in the permanent magnets field lines of $\overline{B}$ and field lines of $\overline{H}$ are different* [1, 2, 3].

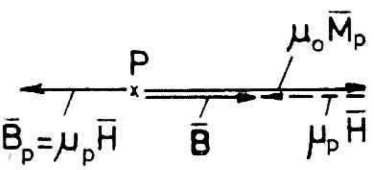

Fig. 1. The position of the vectors

If the magnetic field is produced only by permanent magnets, vector $\overline{H}$ is, in fact, practically opposite to vector $\overline{M}_p$, and to vector $\overline{B}$. For example, in the uniformly magnetized ellipsoid, $\overline{H}$ and $\overline{M}_p$ form together an angle which is about π and in the uniformly magnetized sphere, $\overline{H}$ and $\overline{M}_p$ are anti-parallel. It means that vector $\overline{B}$ – although parallel with $\overline{H}$ – is opposite to $\overline{H}$ [3]. For such a situation, the relative position of the vectors in a point P in the permanent magnet is presented in *Fig. 1*. Consequently, if $\overline{B}$ and $\overline{H}$ can be approximated or even are anti-parallel, the result is that $\overline{B}_p$ and $\overline{H}$ have the same direction and the same sense. *Eq.* (5) shows that μ_p is a positive scalar quantity. In other words, in these circumstances *defined quantity $\overline{B}_p$ and magnetic field intensity $\overline{H}$ have the same field lines*. Magnetic permeability μ_p and relative magnetic permeability μ_{rp} for permanent magnet become:

$$\mu_p = \frac{B_p}{H} = \frac{B - B_r}{H}; \qquad \mu_{rp} = \frac{B - B_r}{\mu_0 H}. \tag{6}$$

Defining vector $\overline{B}_p$ (*Eq.* (3)) and new permeability μ_p (*Eq.* (5)) we get a similar expression – but with another significance – with that of the classic relation $\mu = \overline{B}/\overline{H}$ for materials without permanent magnetization. Furthermore, as μ_p

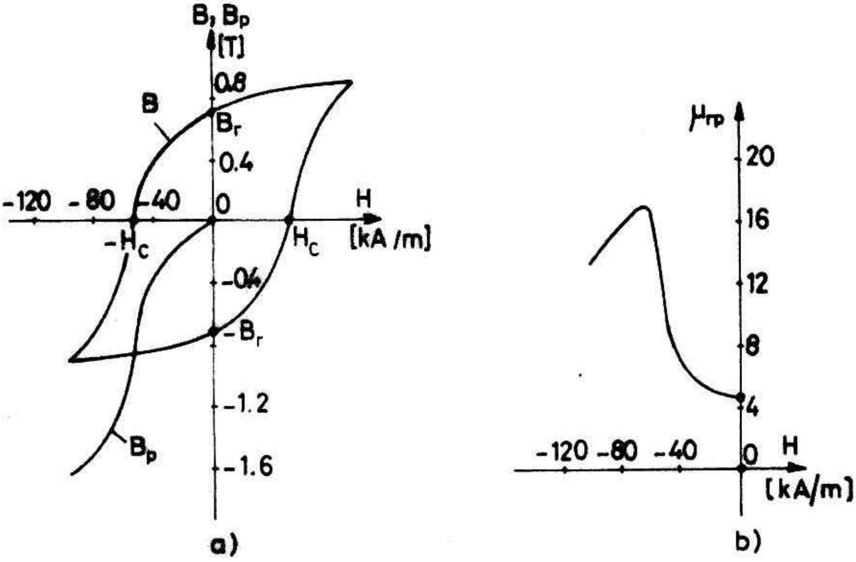

Fig. 2. Examples for $B_p(H)$ and $\mu_{rp}(H)$

is dependent on permanent magnetization, with this new quantity we can advantageously consider the nonlinear demagnetization curves of the permanent magnets. Taking into account that for the operating point of permanent magnet $B < B_r$ and $H < 0$, μ_p and μ_{rp} are positive quantities. If the hysteresis cycle for the material of permanent magnet is known, we can determine the diagram of function $B_p(H)$.

After that, from the second *Eq.* (6), we have deduced nonlinear function $\mu_{rp}(H)$. For example, in *Fig. 2* nonlinear functions $B_p(H)$ and $\mu_{rp}(H)$ are presented, for ALNICO 13/5, considering the major curve of demagnetization for this material [5]. We have observed that nonlinear curve $\mu_{rp}(H)$ has a similar form with classic relative magnetic permeability $\mu_r(H)$ (in the first quadrant) for materials without permanent magnetization, while $\mu_{rp}(H)$ is in the second quadrant.

If term $\mu_0 \chi_{mp} \overline{H} = \mu_0 \overline{M}_\tau$ is negligible, from *Eqs.* (1) and (6) follows that $\mu_{rp} = 1$. In this case, the nonlinear demagnetization curve 1 (*Fig. 3*) is replaced by straight line 2, where

$$k_s \mathrm{tg}\, \varphi = \mu_0 \tag{7}$$

(k_s – scale coefficient of the diagram).

If demagnetization curve 1 is approximated with straight line 3, temporary magnetization $\overline{M}_\tau$ is not neglected, but μ_{rp} is approximated as being constant. Angle φ' is to be given by *Eq.* (8)

$$k_s \,\mathrm{tg}\, \varphi' = \frac{B_r}{H_C} = \frac{B_r}{B_r/\mu_0\mu_{rp}} = \mu_0\mu_{rp}. \tag{8}$$

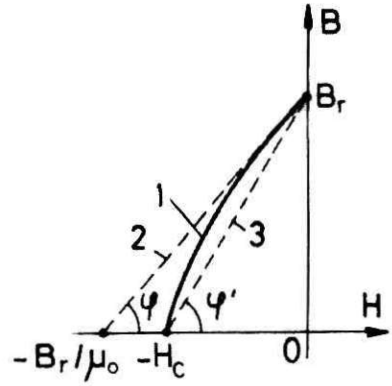

Fig. 3. Demagnetization curve

That is to say that temporary magnetization of the permanent magnet can only be neglected ($\mu_{rp} \approx 1$) if the following approximation is admitted

$$\operatorname{tg}\varphi \approx \operatorname{tg}\varphi' \approx \mu_0/k_s. \tag{9}$$

3. The Conditions of Continuity in the Separation Surface between Two Materials with Permanent Magnetization

There are two different materials, at rest, with permanent magnetization and without conduction currents (two permanent magnets), separated from surface S_{12} and where the field lines of vectors $\overline{B}_p$ and $\overline{H}$ can be considered identical (*Fig. 4*). *Eq.* (5) written for the points belonging to the two zones, which are on separation surface S_{12}, leads to:

$$\overline{B}_{p1} = \mu_{p1}\overline{H}_1, \qquad \overline{B}_{p2} = \mu_{p2}\overline{H}_2. \tag{10}$$

The angle between the normal direction in any point of the surface and vector $\overline{B}_p$, and the angle between the normal direction and vector $\overline{H}$ is the same for both materials. From *Fig. 4a* results:

$$\frac{\operatorname{tg}\alpha_1}{\operatorname{tg}\alpha_2} = \frac{B_{p1t}}{B_{p2t}} \cdot \frac{B_{p2n}}{B_{p1n}}. \tag{11}$$

Taking into account the local magnetic circuit law for the considered conditions ($H_{1t} = H_{2t}$), from *Fig. 4b* also results:

$$\frac{\operatorname{tg}\alpha_1}{\operatorname{tg}\alpha_2} = \frac{H_{2n}}{H_{1n}}. \tag{12}$$

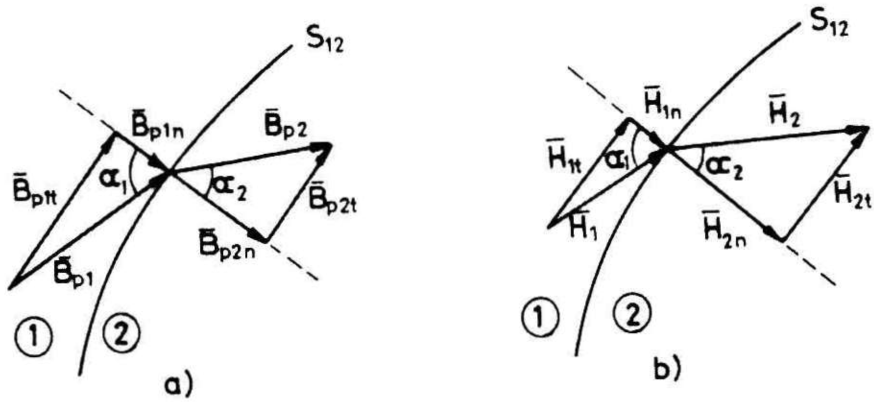

Fig. 4. Continuity conditions for $\overline{B}_p$ and $\overline{H}$

In *Eqs.* (11) and (12), the tangential components are noted with index 't' and the normal ones with index 'n'. Therefore, from *Eqs.* (11) and (12), we have obtained:

$$\frac{B_{p1n}}{B_{p2n}} = \frac{B_{p1t}}{B_{p2t}} \cdot \frac{H_{1n}}{H_{2n}}. \tag{13}$$

Generally, this expression (13) emphasizes that the normal components of vector $\overline{B}_p$ are not continuous. If we write *Eq.* (3) for the normal components of the vectors, for points included in separation surface S_{12} in both materials, we get

$$B_{1pn} = B_{1n} - \mu_0 M_{p1n}, \qquad B_{p2n} = B_{2n} - \mu_0 M_{p2n}. \tag{14}$$

As the local form of the magnetic flux law is $B_{1n} = B_{2n}$ and, generally, $M_{p1n} \neq M_{p2n}$, we reach the same conclusion ($B_{p1n} \neq B_{p2n}$) from *Eq.* (14). The normal components of vector $\overline{B}_p$ are equal only in the particular case when the normal components of the permanent magnetization are equal ($M_{p1n} = M_{p2n}$).

If we write *Eqs.* (10) for the normal components and the tangential components, in the case of isotropic materials, the result is:

$$B_{p1n} = \mu_{p1} H_{1n}; \qquad B_{p2n} = \mu_{p2} H_{2n}, \tag{15}$$

$$B_{p1t} = \mu_{p1} H_{1t}; \qquad B_{p2t} = \mu_{p2} H_{2t}. \tag{16}$$

From these, we deduce:

$$\frac{H_{2n}}{H_{1n}} = \frac{\mu_{p1}}{\mu_{p2}} \cdot \frac{B_{p2n}}{B_{p1n}}, \qquad \frac{B_{p1t}}{B_{p2t}} = \frac{\mu_{p1}}{\mu_{p2}}. \tag{17}$$

From *Eq.* (12) and the first expression (17) for $\overline{H}$ refraction lines we can write:

$$\frac{\operatorname{tg} \alpha_1}{\operatorname{tg} \alpha_2} = \frac{\mu_{p1}}{\mu_{p2}} \cdot \frac{B_{p2n}}{B_{p1n}}. \tag{18}$$

The same *Eq.* (18) for the refraction of $\overline{B}_p$ lines also results from *Eq.* (11) and the last *Eq.* (17). That means *vectorial quantity $\overline{B}_p$, defined in Eq. (3), is refracting in the same way as magnetic field intensity $\overline{H}$*. The simple *Eq.* (18) and the last observation are possible after using permeability μ_p defined in *Eq.* (5) and flux density $\overline{B}_p$ defined in *Eq.* (3). *Eq.* (18) will be named the *refraction theorem of magnetic field lines in materials with permanent magnetization.*

We have noticed that in [4] refraction theorems were demonstrated in materials with permanent magnetization, *using the classic quantities* and there the following results have been obtained:

- for $\overline{H}$

$$\frac{\operatorname{tg}\alpha_1}{\operatorname{tg}\alpha_2} = \frac{\mu_1}{\mu_2} \cdot \frac{1 + \dfrac{\mu_0}{\mu_1}\dfrac{M_{p1t}}{H_t}}{1 + \dfrac{\mu_0}{\mu_2}\dfrac{M_{p2t}}{H_t}}, \tag{19}$$

- for $\overline{B}$

$$\frac{\operatorname{tg}\alpha_1'}{\operatorname{tg}\alpha_2'} = \frac{\mu_1}{\mu_2} \cdot \frac{1 - \mu_0\dfrac{M_{p2n}}{B_n}}{1 - \mu_0\dfrac{M_{p1n}}{B_n}}. \tag{20}$$

The notations in *Eqs.* (19) and (20) are common and quantities $\overline{B}_p$ and μ_p defined in this paper do not appear there. As the relations for $\overline{H}$ and for $\overline{B}$ are different and $\alpha_1 \neq \alpha_1'$ and $\alpha_2 \neq \alpha_2'$, flux density $\overline{B}$ (not $\overline{B}_p$) and magnetic field intensity $\overline{H}$ are refracting differently, and have different field lines.

The particularization of *Eq.* (18) in the case of $B_{p1n} = B_{p2n}$, namely $M_{p1n} = M_{p2n}$ is worth mentioning, resulting in:

$$\frac{\operatorname{tg}\alpha_1}{\operatorname{tg}\alpha_2} = \frac{\mu_{p1}}{\mu_{p2}}, \tag{21}$$

which is *a similar form* of the theorem of refraction for materials which have no permanent magnetization, without imposing particular values of tangential components M_{p1t} and M_{p2t}. Of course, if the materials in contact are without permanent magnetization ($M_{p1n} = M_{p2n} = M_{p1t} = M_{p2t} = 0$), *Eqs.* (18), and (21) lead to the classic expression $\operatorname{tg}\alpha_1/\operatorname{tg}\alpha_2 = \mu_1/\mu_2$, because $\overline{B}_p \equiv \overline{B}$ and μ_p becomes μ.

For the comparison of the magnetic field intensity values in both materials, considering *Eqs.* (15), the following expressions can be written:

$$H_1 = \sqrt{H_{1t}^2 + \left(\frac{B_{p1n}}{\mu_{p1}}\right)^2}, \qquad H_2 = \sqrt{H_{2t}^2 + \left(\frac{B_{p2n}}{\mu_{p2}}\right)^2}, \tag{22}$$

namely relations which have *similar forms* with the relations for materials without permanent magnetization. But, *Eqs.* (22) have another significance, because they were written with quantities B_p and μ_p. In [4] the fact was proved that the analysis of refraction with vectors $\overline{B}$ and $\overline{H}$ (not $\overline{B}_p$ and $\overline{H}$) does not result in the similar expressions for the theorem of refraction in media with permanent magnetization. Only in the particular case when $B_{p1n} = B_{p2n}$ (namely $M_{p1n} = M_{p2n}$) the result is that the magnetic field intensity is higher in the medium where permeability μ_p is lower.

As to the values of B_p in both media, considering the *Eqs.* (16) the relations hold:

$$B_{p1} = \sqrt{(\mu_{p1}H_{1t})^2 + B_{p1n}^2}, \qquad B_{p2} = \sqrt{(\mu_{p2}H_{2t})^2 + B_{p2n}^2}. \tag{23}$$

Also for this quantity, only in the particular case in which $M_{p1n} = M_{p2n}$ we obtain a similar formulation as in the case of media without permanent magnetization: flux density B_p is higher in the medium where permeability μ_p is higher. It is obvious that the demonstrated theorems can be particularized for the cases when one of the materials has permanent magnetization and the other one does not (for example: permanent magnet – air gap, permanent magnet – common ferromagnetic material).

4. Practical Examples

The advantages of defining $\overline{B}_p$ and μ_p previously introduced in this paper have got both theoretical (mentioned in paragraph 3) and practical aspects. The practical advantages have been used by the author for designing and realizing an optimum variant of relay with cylindrical permanent magnet, which is an important component with good reliability in control circuits (protection of power systems, other automatic equipment).

In order to establish the components of the relay and their optimum dimensions we need to analyze the field problem. A cylindrical geometry has been chosen for the relay (*Fig. 5*) where: 1-ferromagnetic material, 2-coil, 3-the permanent magnet, 4-fixing piece and permanent magnetization $\overline{M}_p$ in the magnet has the direction and sense of Oy axis.

The computation of the magnetic field in the relay has been realized with a numerical program (MEFMAG08) designed by the author, based on the finite element method and using quantities $\overline{B}_p$ and μ_p. After computing $\overline{B}_p$, using *Eq.* (3) quantity $\overline{B}$ can be determined, taking a given permanent magnetization $\overline{M}_p$ into consideration. For establishing the optimum variant (a relay with high sensitivity and a low price) more variants of dimensions and materials have been calculated. The global electric quantity that we have had in view is the working current of the relay which was determined from the electromagnetic torque working on the coil of the relay; the torque was computed on the basis of the flux density distribution in the relay. *Fig. 6* shows an example of such a distribution.

As the relay has got a symmetry, vectors $\overline{B}$ are represented only for a quarter of the section (the first quadrant in *Fig. 5b*). Each vector $\overline{B}$ is represented by a scale

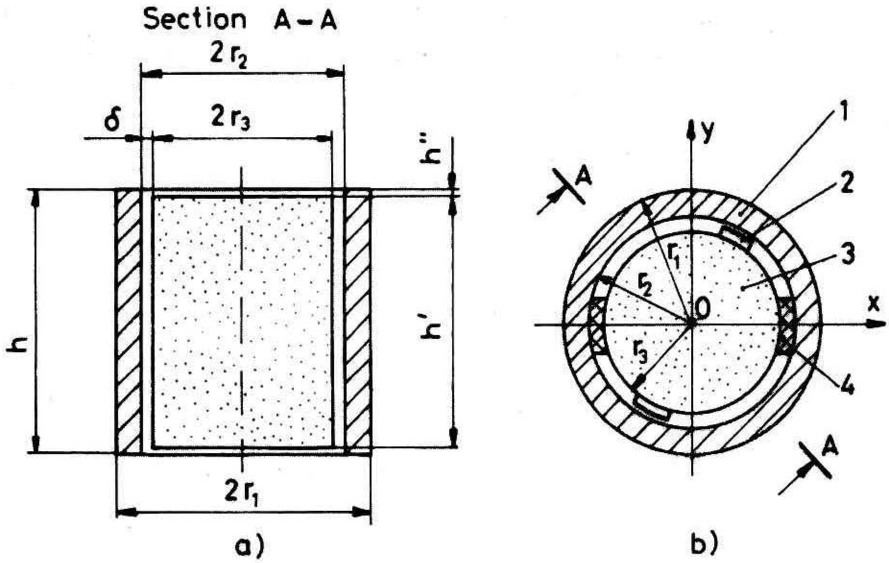

Fig. 5. Relay with permanent magnet

coefficient $k_B = 0.5$ T/1 cm, in the relevant finite element. This distribution has been found for the following conditions: nonlinear permanent magnet ALNICO 13/5, nonlinear ferromagnetic material OL 37, non-ferromagnetic fixing pieces, $h = 22$ mm, $h' = 20$ mm, $h'' = 1$ mm, $r_1 = 16.25$ mm, $r_2 = 12.75$ mm, $r_3 = 11$ mm. The experimental determinations of the flux density in the air gap of the relay have been made with a Hall teslameter. The errors between measured and calculated values are less than 2.4%. All the tests have proved the good accuracy of the results obtained with the numerical program and the advantages of using these quantities ($\overline{B}_p$ and μ_p defined in this paper).

Through the numerical determination of the flux density in the relay – for different variants – the direction of action for raising the sensitivity of the relay was found. Then, using common materials a relay with a higher sensitivity at a low price was built. Thus, in its best variant, the relay has got a working current of 10.1 μA, namely a current 4.95 times less than that of the initial variant (50 μA). The relay sensitivity increases approximately five times if the conditions of the rules in the domain are respected.

5. Conclusions

For materials with permanent magnetization, in which magnetic field intensity $\overline{H}$ is practically opposite to permanent magnetization $\overline{M}_p$, quantity $\overline{B}_p$ – defined in

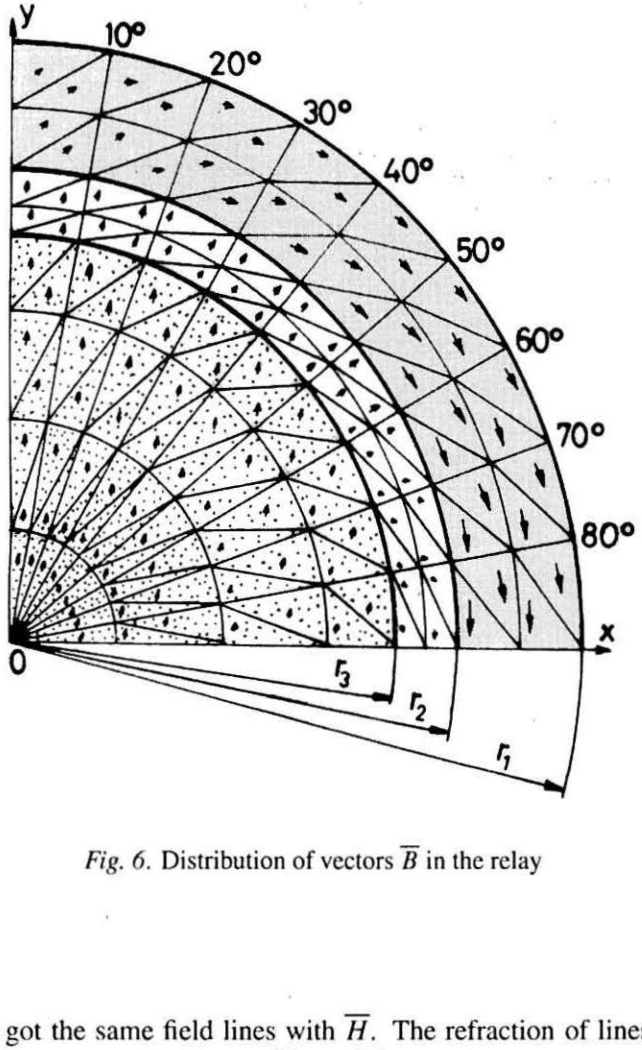

Fig. 6. Distribution of vectors $\overline{B}$ in the relay

Eq. (3) – has got the same field lines with $\overline{H}$. The refraction of lines $\overline{B}_p$ and $\overline{H}$ follow the same theorem (18), namely $\overline{B}_p$ and $\overline{H}$ refract the same way. For materials with permanent magnetization, expressions similar to those of refraction in media without permanent magnetization can only be formulated in particular cases.

Using $\overline{B}_p$ and μ_p, the quantities defined in this paper, we have advantageously taken into account the non-linearity of the demagnetization curves of permanent magnets. These advantages have been used in the computation of magnetic field distribution in a relay with permanent magnet, on the basis of a numerical program of the author. Following the analysis, an efficient product has been got with a reasonable price and which is made of common materials. Practical tests confirm the accuracy of the results, as well as the advantage of using the quantities defined by the author.

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