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ON A SEMIANALYTIC APPROACH FOR CAPACITANCE CALCULATION OF INTERCONNECTS IN HIGH SPEED INTEGRATED CIRCUITS

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Abstract

This paper describes a fast and accurate semi-analytical procedure for determining capacitance and inductance of multilayer structures with multiple conductors with zero thickness in the top layer. The technique uses the quasi-analytic electrostatic Green's function of multilayer structures, which is integrated to a series expansion valid for uniform charge distributions. The quasi-analytical evaluation of the entries of the Galerkin matrix leads to a very efficient and accurate computer code. Computed results are given for some cases of integrated circuit interconnects to show the advantages and simplicity of our procedure as compared to the methods available in the literature.

Keywords: interconnects, IC circuits, capacitance, semi-analytical procedure, Green function.

1. Introduction

The performance of interconnects is becoming one of the main limitations in high speed digital circuits and microwave networks. It is important to be able to characterise interconnects and predict their effects in circuits for such applications. Transmission properties of interconnects such as signal delay, reflection, attenuation, dispersion and crosstalk must be taken into consideration in the analysis and design procedures of high speed integrated circuits and microwave systems. The parasitic parameters of interconnects have a significant impact on the electrical performance of high-speed integrated circuits. For the evaluation of the circuit parameters, in the form of capacitance and inductance matrices, propagation velocity, and so on, several methods are useful. Some of them are the integral equation method implemented numerically by the method of moments [1], partial equivalent circuit method [2], the finite difference method [3], the finite element method [4], and the spectral domain Green's function approach [5]. All techniques suppose a quasi-TEM mode of propagation. The main disadvantages of these procedures are the fine mesh structure required not only in conducting regions but also around the conductor surfaces in the non-conducting region (finite difference and finite element methods), transformation of the main field variables from space domain in the Fourier domain (the method used in the spectral domain), and for good numerical accuracy, heavy memory storage and high CPU time in the field calculations is needed.

To overcome the difficulties mentioned above, a simple and accurate approach is proposed [7, 8] to analyse the EM field in complex geometries where wire conductors are embedded in a multilayer dielectric medium. In this paper, the capacitance matrices of the structures will be found by using the boundary integral equation method. This involves the solving of the appropriate quasi-static integral equation using the multilayer dielectric Green's function approach. Solutions based on the boundary integral equation formulation seem to be very well suited for many practical cases – specifically for application in microelectronic interconnect structures – if the goal is to get high accuracy with low computational cost. Our approach is based upon a quasi-TEM solution because the computational accuracy and efficiency are the major concerns from the point of view of the applications.

2. The Method of Analysis

The modelling of IC high speed interconnects, as compared to the modelling of microwave multistrip multidielectric circuits, is complicated by the following:

- multilayer very thin dielectric substrates.
- The substrates can be very lossy in Si structures (large loss tangent).
- The physical size of interconnect metalizations are very small, and have a high aspect ratio. A typical cross-sectional size is 1 μm × 1 μm or less.
- A large number of dielectric layers and conductors, respectively.

To overcome these problems a new semi-analytical space-domain Green function method has been created to model accurately the electromagnetic fields of such structures (in the direction of dielectric discontinuities the Green's function is integrated in analytical form) and yield the efficient solver for extracting the transmission line parameters.

Let us consider an arbitrary number N of metallic strips which are embedded in the top layer of a medium which consists of L dielectric layers, as shown in *Fig. 1*a. The length of the conductors is considered to be large compared to their cross-sectional dimensions, so the treated problem is a two-dimensional one. The permittivity of the dielectric layers is $\varepsilon_l = \varepsilon_0 \varepsilon_{rl}$ $(l = 1, \ldots, L)$ where ε_0 is the permittivity of free space and ε_{rl} is the relative permittivity of the dielectric layers. The whole structure is bound by a rectangular box defined by planes $x = 0$, $x = a$, $z = 0$ and $z = d_L$. The $z = 0$ plane is a perfectly conducting ground plane and the sidewalls and the top surface are open boundaries (Neuman type boundary conditions).

If the conductivity is small enough or the frequency is high enough but still well below the quasi-stationary frequency limit, the conduction current density produced by the quasi-static field strength $\mathbf{E}$ is negligible compared to the quasi-static displacement current density $j\omega\mathbf{D}$. Along with $\sigma\mathbf{E}$ ($\sigma \ll \omega\varepsilon$), the conduction current density $\sigma\mathbf{E}_i$ produced by the induced field strength $\mathbf{E}_i$ is also negligible since the fact that $\sigma \ll \omega\varepsilon$ ($\sigma\mathbf{E}_i \ll j\omega\varepsilon\mathbf{E}_i$). The contribution of $j\omega\varepsilon\mathbf{E}_i$ is already neglected in the quasi-stationary approximation because of $\|j\omega\varepsilon\mathbf{E}_i\| \ll \|\mathbf{J}^s + j\omega\varepsilon\mathbf{E}\|$.

Hence, the field equations for the quasi-TEM approximation are

$$\begin{aligned} \nabla \times \mathbf{E} &= 0, \\ \nabla \cdot \mathbf{D} &= \rho, \\ \mathbf{D} &= \varepsilon_0 \varepsilon_r \mathbf{E}, \end{aligned}$$

just as they are in electrostatics. Thus, with known charge density distribution, the quasi-static electric field can be calculated independently of the magnetic field. If in addition the magnetic field quantities are also required, they can subsequently be determined by $\nabla \times \mathbf{H} = \mathbf{J}^s + j\omega\mathbf{D}$, the solenoidal (non-irrotational) field $\mathbf{B}$, and the corresponding constitutive relation $\mathbf{B} = \mu_0\mu_r\mathbf{H}$.

Because of the *total neglection of the induced field strength* $\mathbf{E}_i$, the law of induction is of no significance to quasi-static fields. (In modern microelectronic devices with linear dimensions in the microrange, induced voltages and currents are usually negligible, except for the larger electrical interconnects between microelectronic components, e.g. at the board and system level).

Relation $\nabla \times \mathbf{E} = 0$ implies that the field $\mathbf{E}$ can be written as the gradient of a potential function φ, i.e.,

$$\mathbf{E} = -\nabla\varphi.$$

Bearing in mind the above mentioned equations, the Poisson equation for potential function can be viewed as

$$\nabla^2\varphi = -\rho/\varepsilon.$$

Here, we consider the problem of finding multilayer dielectric Green's function for the interconnects with rectangular boundary surfaces (planar layered structures).

The Green function of the medium $G(\mathbf{r}_p; \mathbf{r}_s)$ (see *Fig. 1*b) is the potential at any point $\mathbf{r}_p$ due to a Dirac charge placed at point $\mathbf{r}_s$ in the top layer and is given as a solution of the Poisson equation:

$$\nabla^2 G(\mathbf{r}_p; \mathbf{r}_s) = -\frac{\delta(\mathbf{r}_p - \mathbf{r}_s)}{\varepsilon_L}. \tag{1}$$

Using the method of separation of variables [7, 8] solution of G can be expressed as

$$G(\mathbf{r}_p; \mathbf{r}_s) = \frac{1}{a\varepsilon_L C_L}(C_p z_p + D_p) + \sum_{m=0}^{\infty} H_m f_m \cos(m\pi x_p/a)\cos(m\pi x_s/a), \tag{2}$$

where f_m is given by

$$f_m = \frac{(C_p \exp(t_m z_p) + D_p \exp(-t_m z_p))(\exp(t_m(d_L - z_s)) + \exp(-t_m(d_L - z_s)))}{a t_m \varepsilon_L (C_s \exp(t_m d_L) - D_s \exp(-t_m d_L))}, \tag{3}$$

and where $t_m = m\pi/a$, $H_m = 1$, for $m \neq 0$ and $H_m = 0$ for $m = 0$. The constants C_k and D_k are determined recursively from

$$\begin{aligned} C_{k+1} &= \frac{1}{2}\left(1 + \frac{\varepsilon_k}{\varepsilon_{k+1}}\right) C_k + \frac{1}{2}\left(1 - \frac{\varepsilon_k}{\varepsilon_{k+1}}\right) \exp(-2t_m d_k) D_k, \\ D_{k+1} &= \frac{1}{2}\left(1 - \frac{\varepsilon_k}{\varepsilon_{k+1}}\right) \exp(2t_m d_k) C_k + \frac{1}{2}\left(1 + \frac{\varepsilon_k}{\varepsilon_{k+1}}\right) D_k \qquad \text{for } m \neq 0, \\ C_{k+1} &= \frac{\varepsilon_k}{\varepsilon_{k+1}} C_k, \qquad D_{k+1} = \left(1 - \frac{\varepsilon_k}{\varepsilon_{k+1}}\right) d_k C_k + D_k \qquad \text{for } m = 0. \end{aligned}$$

The constants C_1 and D_1 respectively, for $m = 0$ and $m \neq 0$, are determined using the boundary conditions on the bottom of the first dielectric layer (the bottom plane can be electric wall, magnetic wall or semi-space ‘radiation’ type of boundary condition).

In order to determine the capacitance of the structure, a unit charge is distributed uniformly on conductor j. The potential on any conductor (including j), resulting from this charge distribution is computed using a convolution of the form

$$\varphi(\mathbf{r}_p) = \int_{V_s} \rho(\mathbf{r}_p) G(\mathbf{r}_p; \mathbf{r}_s)\, \mathrm{d}V_s. \tag{4}$$

From this integral the coefficients of the potential matrix $[P]$ are found straightforwardly, i.e., the set of coefficients p_{ij}, which linearly relate the potential V_i of any conductor to the free charges Q_j on all of the conductors:

$$V_i = \sum_{j=1}^{N} p_{ij} Q_j \qquad (i = 1, \ldots, N). \tag{5}$$

From the matrix $[P]$ all the transmission line parameters can be found, since $[C] = [P]^{-1}$ and $[L] = \varepsilon_0 \mu_0 [C_0]^{-1}$, where $[C_0]$ is the capacitance matrix which would result if all dielectric layers were replaced by free space. Since we can calculate the capacitance and the inductance matrices very easily with the new procedure, all quasi-TEM propagation parameters of multiconductor transmission lines may be obtained.

3. Numerical Results

In order to demonstrate the accuracy and efficiency of our quasi-analytical multilayered Green’s function approach some practical interconnect structures with a variety of dimensions and number of dielectric layers are analysed. In all cases, the calculated results have agreed well with the results of previously published papers. This comparison clearly demonstrates a greater degree of accuracy for the method presented in this paper, particularly for strong coupled structures with a number of dielectric layers (more than five layers, for example, where other methods are heavily time consuming).

This modelling technique assumed dielectric layers to be lossless and conductors infinite thin and lossless too. With small modifications in the part of relative dielectric constants our procedure can be extended to consider the cases of dielectrics and conductors as lossy media.

4. Example 1

A pair of coupled interconnect lines on a two-layer substrate with air above it is sandwiched between two conducting ground planes as shown in *Fig.2*. If we take a constant distribution of charge density on the strip conductors, the results obtained by using our method agree with those of [1], as shown in *Table 1* (differences within 0.3%). Numerical results for self and mutual capacitances are presented as a function of the spacing between lines s for different values of the two-layer dielectric substrate height d_2. As we can see from *Table 1*, the mutual capacitance was found to decrease at faster rate than the mutual inductance as a result of increasing spacing of the lines, which concluded that the coupling was primarily inductive (the numerical results are not presented for inductance here, and for this will be reported in another paper).

Table 1. Self- and mutual capacitance for the structure in *Fig. 2*

s/d_2	$\varepsilon_{r1} = 4$, $\varepsilon_{r2} = 2.5$, $\varepsilon_{r3} = 1$			
	$C_{11} = C_{22}$ (10 pF/m) this paper	$C_{11} = C_{22}$ (10 pF/m) [1]	$C_{12} = C_{21}$ this paper (10 pF/m)	$C_{12} = C_{21}$ (10 pF/m) [1]
0.1	4.064	4.059	−1.621	−1.618
0.2	3.802	3.797	−1.208	−1.205
0.3	3.664	3.660	−0.9500	−0.9478
0.4	3.583	3.579	−0.7698	−0.7677
0.5	3.531	3.528	−0.6342	−0.6331
0.7	3.472	3.471	−0.4443	−0.4443
1.0	3.367	3.364	−0.3268	−0.3272

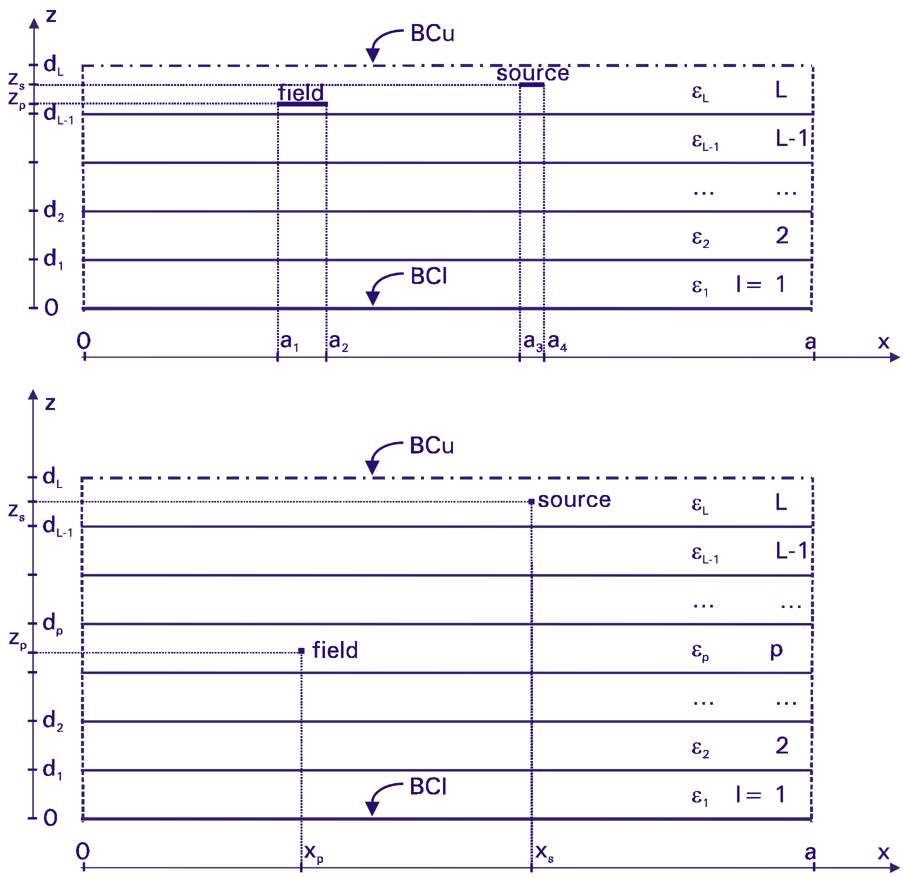

Fig. 1. Geometry of a multilayer structure a) with conductors in the top layer, b) for Green's function calculation

5. Example 2

The second structure with two dielectrics and three conductor interconnection lines is also found in [1] and is shown in *Fig. 3*. The capacitance matrix of this configuration is listed in *Table 2*, where the conductors are numbered from left to right as 1, 2 and 3 respectively. The results for capacitance are computed with differences within 0.4%. In the general case when full moment method is applied for determination of the matrix capacitance using the total charge free-space Green function approach as reported in [1], the maximum difference between our results and those of [1] is less than 4%.

From these examples we observe that there is a good agreement between our

Table 2. Self- and mutual capacitance for the structure in *Fig. 3*

Capacitance	$\varepsilon_{r1} = 4, \varepsilon_{r2} = 2.5$	
	Results of [1] (10 pF/m)	This paper (10 pF/m)
C_{11}	6.223	6.226
C_{12}	−5.580	−5.580
C_{13}	−1.097	−1.098
C_{21}	−5.580	−5.580
C_{22}	6.223	6.226
C_{23}	−1.097	−1.098
C_{31}	−1.097	−1.098
C_{32}	−1.097	−1.098
C_{33}	4.634	4.637

results and those available in the literature. The proposed procedure allows us to assess in an analytical and simple way the integral equations of the problem; it mainly results in a low CPU time and about 5 and more times faster than the total charge approaches.

6. Conclusion

This paper describes a simple and accurate procedure to compute the quasi-TEM transmission line interconnect parameters in multilayered dielectric media with infinitely thin conductors in the top layer. Accuracy and numerical efficiency are

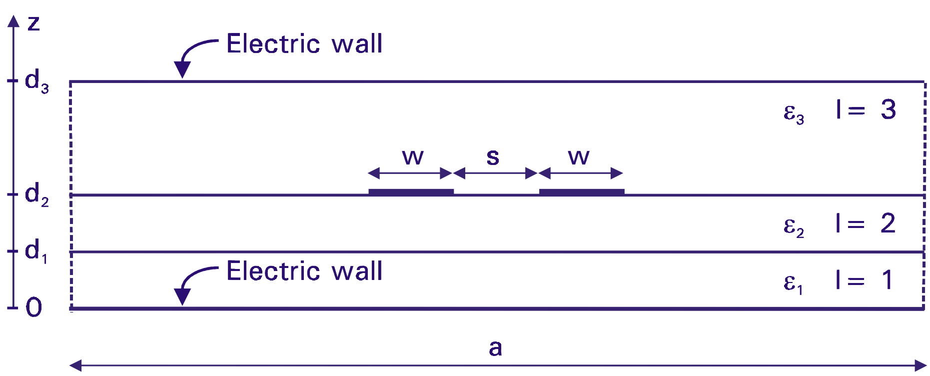

Fig. 2. Two coupled and shielded interconnect lines on a two-layer substrate

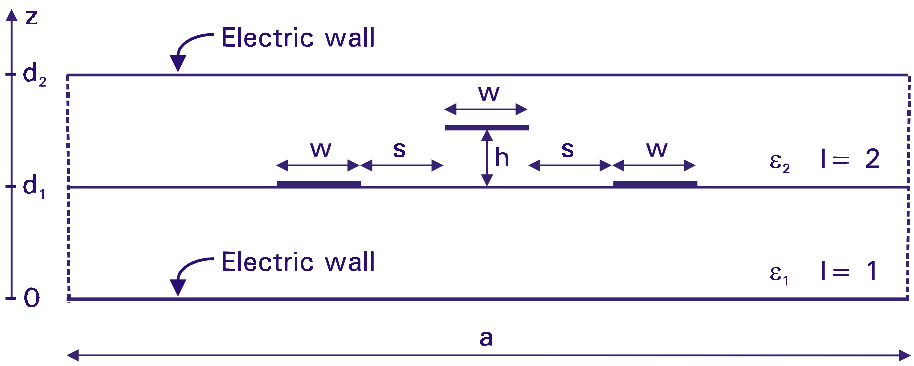

Fig. 3. Three coupled interconnect lines in an asymmetric stripline-like configuration

achieved by means of the following elements:

1. In that case when the conductivity is small enough or the frequency is high enough but still well below the quasi-stationary frequency limit and $\sigma \ll \omega\varepsilon$, we can neglect totally the induced field strength $\mathbf{E}_i$, the law of induction is of no significance and the quasi-static electric field **E** can be calculated independently of the magnetic field.
2. We use the constant charge distribution on the interconnect lines (the method has been developed specifically for application in microelectronic interconnect structures, where dimensions of the conductors are small). This fact allows us to keep the Galerkin matrix size small when compared with typical matrix sizes associated with the use of subsectional functions.
3. We use analytical calculation of the integrals defining the Galerkin matrix entries. This has been done by taking advantage of the use of the space-domain Green's function as the kernel of boundary integral equation formulation.

We can conclude that the method presented in this paper leads to an accurate and efficient computer code that permits one to analyze under quasi-TEM assumption a variety of interconnect multilines in the multilayered dielectric region. The method can be extended to structures with non-thin conductors in non-homogeneous layered dielectric media.

References

[1] WEI, C. – HARRINGTON, R. F. – MAUTZ, J. R. – SARKAR, T. K., Multiconductor Transmission Lines in Multilayered Dielectric Media, *IEEE Trans. Microwave Theory Techn.*, **MTT-32** (1984), pp. 439–449.

[2] RUEHLI, A. E. – BRENNAN, P. A., Efficient Capacitance Calculations for Three-Dimensional Multiconductor Systems, *IEEE Trans. Microwave Theory Techn.*, **MTT-21** (1973), pp. 76–82.

[3] JIN, R. – CAO, Y. – LI, Z. F., Fast Parameter Extraction for Multiconductor Interconnects in Multilayered Dielectric Media Using Mixture Method of Equivalent Source and Measured Equation of Invariance, *IEEE Trans. Components, Packaging, and Manufacturing Technology*, **CPMT-20** (1997), pp. 235–240.

[4] KONRAD, A. – CHARI, M. – CSENDES, Z., New Finite Element Techniques for Skin Effect Problems, *IEEE Trans. Magnetics*, **MAG-18** (1982), pp. 542–549.

[5] DELBARE, W. – ZUTTER, D. D., Space-Domain Green's Function Approach to the Capacitance Calculation of Multiconductor Lines in Multilayered Dielectrics with Improved Surface Charge Modelling, *IEEE Trans. Microwave Theory Techn*, **MTT-37** (1989), pp. 1562–1568.

[6] BAKOGLU, H. B., *Circuits, Interconnects and Packaging for VLSI*, Addison-Wesley, Reading, MA, 1990.

[7] YMERI, H. – NAUWELAERS, B. – MAEX, K. – DE ROEST, D., Fast and Accurate Analysis of Multiconductor Interconnects, *Microelectronic Engineering*, **55** (2001), pp. 37–45.

[8] YMERI, H. – NAUWELAERS, B. – MAEX, K. – DE ROEST, D. – STUCCHI, M., Effect of the Depletion Layer on the Capacitance and Inductance of MIS Interconnects, Accepted for the *8th International Conference Mixed Design of Integrated Circuits and Systems*, 21–23 June, Zakopane, Poland.