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CALCULATION OF OVERVOLTAGES CAUSED BY LIGHTNING STROKES IN THE TOWER OF AN OVERHEAD LINE

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Abstract

On the bases of the field theory a calculation method is elaborated with respect to the grounding of the tower, a lightning model with finite lightning velocity, the retardation of the electromagnetic wave and the effect of the tower as well. In the calculation Fourier-transform for the time dependence and Hankel- transform for the radial variable is used. Numerical results computed on the basis of this new method are presented and discussed.

Keywords: overvoltage, lightning impulse current, tower, vector potential.

1. Introduction

When lightning stroke current enters a tower top of an overhead line, the impulse current creates a voltage across the tower impedance, and it can cause a flashover to the phase conductor. As these flashovers are sources of operating troubles, the determination of these over-voltages is of great importance.

To handle this problem there are two methods in the literature: in one of them the phenomena are modelled by concentrated circuit elements, and the tower is substituted by a distributed parameter line, in the other method electromagnetic field calculation is used. Some outstanding representation of this second possibility are the publications of WAGNER and HILEMAN (1956, 1959, 1960), where detailed calculations are made, taking the exact geometry of the tower into consideration. Recently NUCCI (1995) have made a review on the research in this field.

The aim of the present publication is to use – by simplification of the tower construction – a field calculation method and a computer program, which is suitable to investigate the main influencing parameters, and to draw consequences. As a starting step in this investigation, in 1988 the author with associate author (TEVAN and PETRI, 1988) published a method for calculation of the grounding impedance of the tower in case of lightning impulse current, but the real travelling speed of the lightning was not taken into consideration, and the retardation of the electromagnetic wave was neglected as well. In 1991 an article was presented by the author (TEVAN) on the ‘7th International Symposium on High Voltage Engineering’ in Dresden, which takes the retardation into consideration as well, and the calculation was evaluated with a finite speed lightning model.

The present publication proposes a calculation method and a computer program, which additionally considers *the influencing factor of the tower as well*. The method is described in four sections, then the calculation results are presented and discussed.

2. The Electromagnetic Field of an Elemental Vertical Current Phasor

Using the Fourier-transform the time dependence is taken into account, therefore the field calculation can be made with phasor quantities. Let us consider an elemental vertical complex current filament above the earth surface with the circular frequency w in a cylindrical co-ordinate system (*Fig. 1*).

The phasor of the vector potential $\mathbf{A} = \mathbf{A}_z$ has only z component and depends only on the r and the z co-ordinates in consequence of the cylindrical symmetry, and satisfies the following partial differential equation (the bold-face characters denote phasors, and not vectors in the whole article; further for the simplicity $\mathbf{A}$ is used instead of the more correct d$\mathbf{A}$)

$$\frac{\partial^2 \mathbf{A}}{\partial r^2} + \frac{1}{r}\frac{\partial \mathbf{A}}{\partial r} + \frac{\partial^2 \mathbf{A}}{\partial z^2} - \mathbf{k}^2 \mathbf{A} = \mathbf{0}. \tag{1}$$

Here $\mathbf{k} = \mathbf{k}_1$ for the earth ($z < 0$) or $\mathbf{k} = \mathbf{k}_2$ for the air ($z > 0$), where

$$\mathbf{k}_1 = \frac{1+j}{\delta}, \quad \mathbf{k}_1^2 = \frac{2j}{\delta^2}, \quad \delta = \sqrt{\frac{2}{\omega\gamma\mu_0}}, \tag{2a}$$

$$\mathbf{k}_2 = j\frac{\omega}{c}, \quad \mathbf{k}_2^2 = -\frac{\omega^2}{c^2}, \tag{2b}$$

and c is the velocity of the light in the vacuum, γ is the conductivity, and δ is the penetration depth in the earth.
The displacement current in the earth is neglected. The field quantities expressed in terms of the vector potential are ($\mathbf{H}_\varphi = \mathbf{H}$)

$$\mu_0 \mathbf{H} = -\frac{\partial \mathbf{A}}{\partial r}, \quad \mathbf{E}_r \frac{j\omega}{\mathbf{k}^2}\frac{\partial^2 \mathbf{A}}{\partial r \partial z}, \quad \mathbf{E}_\varphi = \mathbf{0},$$
$$\mathbf{E}_z = -\frac{j\omega}{\mathbf{k}^2}\left(\frac{\partial^2 \mathbf{A}}{\partial r^2} + \frac{1}{r}\frac{\partial \mathbf{A}}{\partial r}\right) \equiv j\omega\left(\frac{1}{\mathbf{k}^2}\frac{\partial^2 \mathbf{A}}{\partial z^2} - \mathbf{A}\right). \tag{3}$$

The interface conditions are at $z = 0$: (the field quantities $\mathbf{H}$ and $\mathbf{E}_r$ are continuous)

$$(\mathbf{A}_1)_{z=0} = (\mathbf{A}_2)_{z=0}, \quad \frac{1}{\mathbf{k}_1^2}\left(\frac{\partial \mathbf{A}_1}{\partial z}\right)_{z=0} = \frac{1}{\mathbf{k}_2^2}\left(\frac{\partial \mathbf{A}_2}{\partial z}\right)_{z=0}. \tag{4}$$

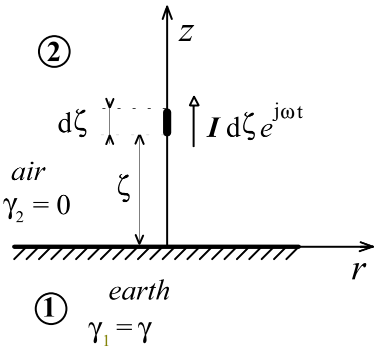

Fig. 1. Model for the Field of an Elemental Vertical Current Phasor

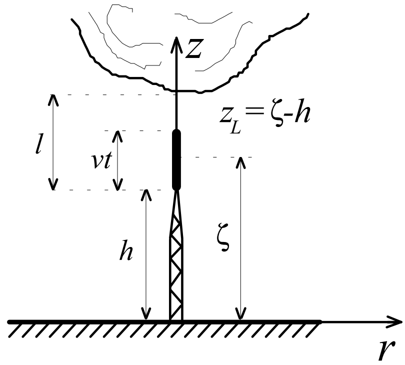

Fig. 2. Model for the Lightning Discharge

The boundary conditions at the infinity are:

$$\lim_{z \to \pm\infty} \mathbf{A} = \mathbf{0}, \qquad \lim_{r \to \infty} \mathbf{A} = \mathbf{0}. \tag{5}$$

The solution of the differential equations (2a and 2b) is found in the following form:

$$\text{for } z \geq 0 \quad \mathbf{A}_2 = \mathbf{A}_{2,\infty} + \mathbf{A}_{2,c}, \tag{6}$$

$$\text{for } z \leq 0 \quad \mathbf{A}_1 = \mathbf{A}_{1,\infty} + \mathbf{A}_{1,c} = \mathbf{A}_{1,c}, \tag{7}$$

where $\mathbf{A}_\infty$ is the solution of the problem, if the earth is a perfect conductor, that is $\gamma = \infty$. In the earth $\mathbf{A}_{1,\infty} = \mathbf{0}$, in the air $\mathbf{A}_{2,\infty}$ can be obtained by image to the surface of the earth with retardation

$$e^{j\omega t}\mathbf{A}_{2,\infty} = \frac{\mu_0 \mathbf{I}\,\mathrm{d}\zeta}{4\pi}\left\{\frac{e^{j\omega\left[t-\frac{1}{c}\sqrt{r^2+(\zeta-z)^2}\right]}}{\sqrt{r^2+(\zeta-z)^2}} + \frac{e^{j\omega\left[t-\frac{1}{c}\sqrt{r^2+(\zeta+z)^2}\right]}}{\sqrt{r^2+(\zeta+z)^2}}\right\}, \tag{8}$$

where ζ is the vertical distance of the element dz from the earth surface. The solution for finite earth conductivity takes the following expression based on Hankel transform:

$$\mathbf{A}_1 = \int_0^\infty \mathbf{f}_1(s) e^{z\sqrt{s^2+\frac{2j}{\delta^2}}} J_0(sr)\,\mathrm{d}s, \qquad \text{for } z < 0, \tag{9}$$

$$\mathbf{A}_2 = \mathbf{A}_{2\infty} + \int_0^\infty \mathbf{f}_2(s) e^{-\sqrt{s^2-\frac{\omega^2}{c^2}}} J_0(sr)\,\mathrm{d}s, \qquad \text{for } z > 0, \tag{10}$$

where $J_0(..)$ is the zero order Bessel function of first kind, and s is a certain real variable. The *Eqs.* (9) and (10) satisfy the differential equation (1), and the boundary conditions (5), as it could be seen by substitution. If the functions $\mathbf{f}_1(s)$ and $\mathbf{f}_2(s)$ are

$$\mathbf{f}_1(s) = \frac{\mu_0 \mathbf{I}\,\mathrm{d}\zeta}{2\pi} \frac{s e^{-\zeta\sqrt{s^2-\frac{\omega^2}{c^2}}}}{\sqrt{s^2-\frac{\omega^2}{c^2}} + j\kappa\sqrt{s^2+\frac{2j}{\delta^2}}},$$

$$\mathbf{f}_2(s) = -j\kappa \frac{\sqrt{s^2+\frac{2j}{\delta^2}}}{\sqrt{s^2-\frac{\omega^2}{c^2}}} \mathbf{f}_1(s), \tag{11}$$

and

$$\kappa = \frac{\mathbf{k}_2^2}{j\mathbf{k}_1^2} = \frac{\delta^2}{2}\frac{\omega^2}{c^2} = \frac{\omega}{c^2\gamma\mu_0}, \tag{12}$$

then the interface conditions (4) are also satisfied, as it can be seen by some calculations (see Appendix).

The electromagnetic field is determined by the *Eqs.* (3), (9), (10) and (11).

3. Discharge Model of the Lightning

This model was published by the author on the '7th International Symposium on High Voltage Engineering' in Dresden (1991), and this model is a simplified variation of DIENDORFER and UMAN'-s model (DU-model, 1990).

The main discharge wave of positive lightning travels upward with constant velocity v in vertical direction (*Fig.* 2). The following expression is established for this current:

$$i(t, z_L) = \varepsilon\left(t - \frac{z_L}{v}\right) I_1 \left[e^{-b_1 v t} - e^{-b_2 v t} - e^{-a\left(t-\frac{z_L}{v}\right)}(e^{-b_1 z_L} - e^{-b_2 z_L})\right] \tag{13}$$

where t is the time, and $\varepsilon(t)$ is the unity step function, a, b_1, b_2 are positive parameters and $b_1 < b_2$. This model has the properties

a.) at any fixed height $z_L = \zeta - h$

$$\lim_{\Delta t \to 0} i\left(\frac{z_L}{v} + \Delta t, z_L\right) = 0 \text{ for } \Delta t > 0, \quad \text{and } \; i(t, z_L) = 0 \text{ for } t < \frac{z_L}{v},$$

that is the current wave started from zero at $t = z_L/v$.

b.) $i(t, 0) = I_1(e^{-b_1 v t} - e^{-b_2 v t})\varepsilon(t)$,
that is the time function of the lightning current at the top of the tower consists of two exponential terms for $t \geq 0$.

c.) from the time $t \geq z_L/v$ a charge of

$$\begin{aligned} \mathrm{d}q &= \int_t^\infty [i(\tau, z_L) - i(\tau, z_L + \mathrm{d}z_L)]\,\mathrm{d}\tau \\ &= -\mathrm{d}z_L \int_t^\infty \frac{\partial i(\tau, z_L)}{\partial z_L}\,\mathrm{d}\tau \\ &= \frac{I_1}{a} e^{-a\left(t-\frac{z_L}{v}\right)} \left[\left(\frac{a}{v} - b_1\right) e^{-b_1 z_L} - \left(\frac{a}{v} - b_2\right) e^{-b_2 z_L}\right] \mathrm{d}z_L \end{aligned}$$

is recombined in the space interval $(z_L,\ z_L + dz_L)$. The total charge – the charge before the main discharge – is:

$$\mathrm{d}q_t = (\mathrm{d}q)_{t=\frac{z_L}{v}} = \frac{I_1}{a}\left[\left(\frac{a}{v} - b_1\right) e^{-b_1 z_L} - \left(\frac{a}{v} - b_2\right) e^{-b_2 z_L}\right] \mathrm{d}z_L.$$

This means that $\mathrm{d}q_t$ has a distribution of two exponential functions of z_L.

d.) The charge decreases during the time $t = z_L/v$ to $t - z_L/v$ from $\mathrm{d}q_t$ on

$$\mathrm{d}q = e^{-a\left(t-\frac{z_L}{v}\right)}\,\mathrm{d}q_t,$$

i.e. the a is the recombination parameter.

Here v corresponds to the v^* in the DU-model, but the z/c value of the time delay in the DU-model was neglected here beside t. The approximation function of $i(t, 0)$ differs in the two models as well.

Fig. 3 shows the lightning current $i(t, z_L)$ against the time t at different height z_L at the parameters $a/v = 0.002/m$, $b_1 = 0.00015/m$, $b_2 = 0.025/m$. Otherwise the parameters $b_1 v$ and $b_2 v$ can be determined from the front time and half-time value of the lightning current at $z_L = 0$ (on the top of the tower).
The spectrum of the lightning current is as follows

$$\mathbf{I}(j\omega, z_L) = \int_{-\infty}^{\infty} i(t, z_L) e^{-j\omega t}\,\mathrm{d}t = \int_{\frac{z_l}{v}}^{\infty} i(t, z_L) e^{-j\omega t}\,\mathrm{d}t.$$

After substitution the expression (13) has the form

$$\begin{aligned} \mathbf{I}(j\omega, z_L) =& I_1 \left[\left(\frac{1}{b_1 v + j\omega} - \frac{1}{a + j\omega}\right) e^{-(b_1 v + j\omega)\frac{z_L}{v}} \right. \\ & \left. - \left(\frac{1}{b_2 v + j\omega} - \frac{1}{a + j\omega}\right) e^{-(b_2 v + j\omega)\frac{z_L}{v}}\right] \end{aligned} \tag{14}$$

and this phasor has to be substituted in the expression (11) instead of $\mathbf{I}$.

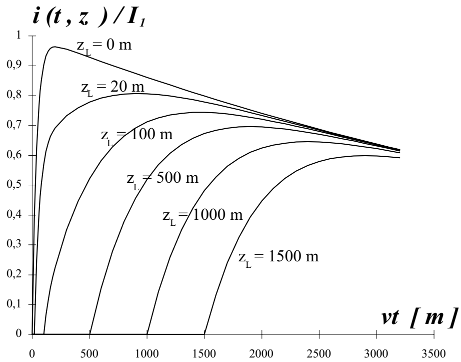

Fig. 3. The Lightning-current versus the Time at Different Height

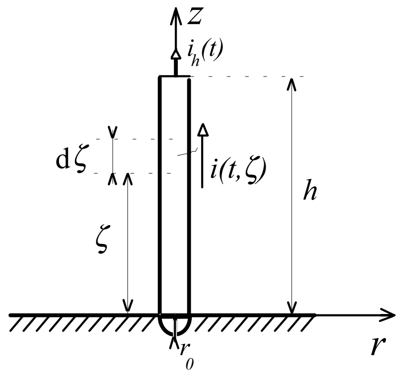

Fig. 4. Model for the Tower-current

4. The Model of the Current in the Tower

The electromagnetic field generated by the current in the tower is approximated by the field of the current of a cylindrical metallic tube of perfect conductivity (*Fig. 4*).The cylindrical tube has an external radius r_0. The field of the current of the metallic tube is considered to be valid for $r \geq r_0$. Introducing the Fourier transform, the current $i(t, \zeta)$ at the height ζ, and the current $i_h(t) = i(t, h)$ at the height h are substituted by their spectrum:

$$\mathbf{I}(\zeta) = \int_{-\infty}^{\infty} i(t, \zeta) e^{-j\omega t}\, \mathrm{d}t, \qquad \mathbf{I}(h) = \mathbf{I}_h = \int_{-\infty}^{\infty} i_h(t) e^{-j\omega t}\, \mathrm{d}t.$$

This latter is the current phasor of the lightning current in the striking point, which can be gained from (14) by substitution $z_L = 0$ $(\zeta = h)$, so

$$\mathbf{I}_h = I_1 \left(\frac{1}{b_1 v + j\omega} - \frac{1}{b_2 v + j\omega} \right). \tag{15}$$

We can see in the following, that the current of the cylindrical metallic tube is totaly defined by the behaviour of the electromagnetic field at small neighbourhood of $r = r_0$ in the air and in the surface of the earth. This does not mean, that the far-field of the tower-current is not taken into account, because the final field of this current is calculated on the base of the expressions (9), (10) and (11).

In the proximity of the metallic tube-tower, owing to the Ampere's law, the magnetic field can be calculated at the height of ζ as

$$\mathbf{H} = \frac{\mathbf{I}(\zeta)}{2\pi r} \tag{16}$$

because of the displacement current can be neglected in the metallic tube of perfect conductivity and in its small proximity. This formula ensures, that at the small neighbourhood of $r = r_0$ the tangential component of the electric field becomes zero, since according to the first an third formulas in the group (3)

$$\mathbf{E}_z = \frac{j\omega\mu_0}{\mathbf{k}^2}\left(\frac{\partial\mathbf{H}}{\partial r} + \frac{1}{r}\mathbf{H}\right),$$

and substituting (16) into this expression, $\mathbf{E}_z$ results $\mathbf{0}$. With derivation of (1) by r, and using the first formula of (3), the differential equation for $\mathbf{H}$ is

$$\frac{\partial}{\partial r}\left(\frac{\partial\mathbf{H}}{\partial r} + \frac{1}{r}\mathbf{H}\right) + \frac{\partial^2\mathbf{H}}{\partial\zeta^2} - \mathbf{k}^2\mathbf{H} = \mathbf{0},$$

where z is changed for ζ. Substituting (16) into this differential equation, the following differential equation is derived

$$\mathbf{I}''(\zeta) - \mathbf{k}^2\mathbf{I}(\zeta) = \mathbf{0},$$

the solution of which, taking (2b) into consideration as well, is

$$\mathbf{I}(\zeta) = \mathbf{K}_1 e^{-j\frac{\omega}{c}\zeta} + \mathbf{K}_2 e^{j\frac{\omega}{c}\zeta}. \tag{17}$$

The derived time function $\mathbf{I}(\zeta)e^{jwt}$ shows, that in the metallic conductor (with perfect conductivity) there are two current waves, one in $+z$, the other in $-z$ direction. (In the reality, because of the finite conductivity, the velocities are somewhat below the velocity of the light, but this approximation yields a negligible error.) The constants $\mathbf{K}_1$ and $\mathbf{K}_2$ are given by the boundary conditions. One of them is at $\zeta = h$ by formula (15):

$$\mathbf{K}_1 e^{-j\frac{\omega}{c}} + \mathbf{K}_2 e^{j\frac{\omega}{c}h} = \mathbf{I}_h. \tag{18}$$

The other boundary condition is the equivalence of the radial components of the electric field in the earth and in the air at $r = r_0$, and $\zeta = 0$. In the area of the grounding the electromagnetic field in the earth can be considered to be independent of the field in the air, and the radial component of the electric field, according to the formula of BILINSKY (1938), yields

$$(\mathbf{E}_r)_{\zeta=0,\ r=r_0} = -\frac{\mathbf{I}(0)}{2\pi r_0\gamma}\left(\frac{1+j}{\delta} + \frac{1}{r_0}e^{-\frac{1+j}{\delta}r_0}\right).$$

However, in the air, at the same location, according to (3), (2b) and (16), when substituting z for ζ

$$\zeta(\mathbf{E}_r)_{\zeta=0,\ r=r_0} = \frac{j\omega\mu_0 c^2}{\omega^2}\frac{\partial\mathbf{H}}{\partial\zeta} = j\frac{c^2\mu_0}{\omega}\frac{\mathbf{I}'(0)}{2\pi r_0}.$$

Because these two last values are equal, using notation (12) we can obtain

$$\mathbf{I}'(0) = j\kappa\left(\frac{1+j}{\delta} + \frac{1}{r_0}e^{-\frac{1+j}{\delta}r_0}\right)\mathbf{I}(0).$$

Substituting the expression (17) we can get the relation

$$\mathbf{K}_2 - \mathbf{K}_1 = \mathbf{q}(\mathbf{K}_1 + \mathbf{K}_2),$$

where

$$\mathbf{q} = \frac{\kappa c}{r_0\omega}\left[\frac{r_0}{\delta} + e^{-\frac{r_0}{\delta}}\cos\frac{r_0}{\delta} + j\left(\frac{r_0}{\delta} - e^{-\frac{r_0}{\delta}}\sin\frac{r_0}{\delta}\right)\right]. \tag{19a}$$

Comparing the expression (19a) with the formula (18), the constants of formula (17) are

$$\begin{aligned}\mathbf{K}_1 &= \frac{(1-\mathbf{q})\mathbf{I}_h}{2\left[\cos\left(\frac{\omega}{c}h\right) + j\,\mathbf{q}\sin\left(\frac{\omega}{c}h\right)\right]},\\ \mathbf{K}_2 &= \frac{(1+\mathbf{q})\mathbf{I}_h}{2\left[\cos\left(\frac{\omega}{c}h\right) + j\,\mathbf{q}\sin\left(\frac{\omega}{c}h\right)\right]},\end{aligned} \tag{19b}$$

The expression (17) has to be substituted into the phasor **I** in formulas (8) and (11).

5. Calculation of the Voltage on the Insulator String

The overvoltage on the insulators of overhead line can be calculated on the basis of *Fig.* 5. The points D, E, F are as far as from the tower that the electromagnetic wave of the lightning does not reach these points in the investigated duration. We write the Faraday's induction law on the loop $AGFEDCBA$:

$$\int_{\mathbf{A}}^{G} E_l\,\mathrm{d}l + \int_{G}^{F} E_l\,\mathrm{d}l + \int_{F}^{E} E_l\,\mathrm{d}l + \int_{E}^{D} E_l\,\mathrm{d}l + \int_{D}^{C} E_l\,\mathrm{d}l + \int_{C}^{B} E_l\,\mathrm{d}l + \int_{B}^{A} E_l\,\mathrm{d}l = -\frac{\mathrm{d}\phi}{\mathrm{d}t}.$$

The line integrals GF, CB, and BA are practically zeros, because the resistance of the tower and the conductor are very small. The line integrals ED and FE are zeros as well, because these lines are far from the tower. Therefore the last expression takes the following form:

$$u_{AG} \equiv \int_{\mathbf{A}}^{\mathbf{G}} E_l\,\mathrm{d}l = \int_{D}^{C} E_l\,\mathrm{d}l + \frac{\mathrm{d}\phi}{\mathrm{d}t}.$$

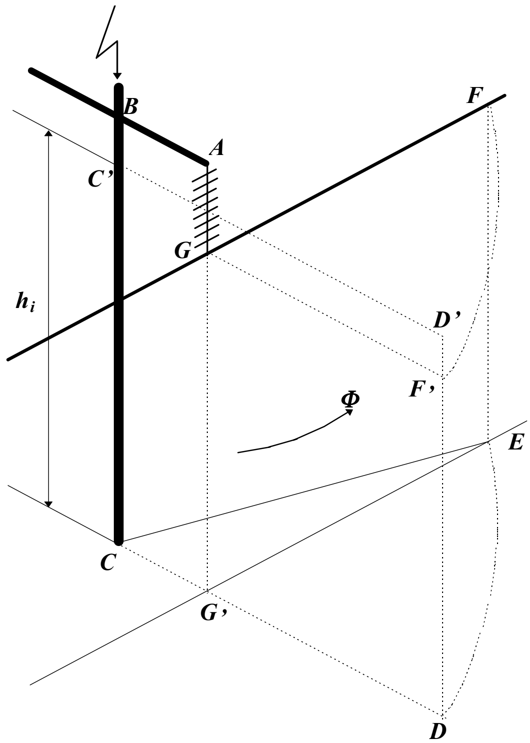

Fig. 5. Model for the Calculating of the Overvoltage on the Insulator String

We can see that the magnetic flux through the loop $GF'DG'G$ is the same as through the loop $GFEG'G$, because the magnetic field strength has only φ – component. Therefore the flux through the loop $AGFEDCBA$ is the same as through the loop $AGF'DCBA$. This letter one can be approximated by the flux φ' through the loop $CC'D'DC$: $\phi' \cong \phi$. The induction law for this loop is:

$$\int_{C'}^{D'} E_l \, \mathrm{d}l + \int_{D}^{C} E_l \, \mathrm{d}l = -\frac{\mathrm{d}\phi}{\mathrm{d}t},$$

(the line integrals $D'D$ and CC' are zeros). The (approximate) formula of the insulator voltage follows from the two last equations and from the approximate equation: $\phi' \cong \phi$:

$$u_{AG} \cong \int_{D'}^{C'} E_l \, \mathrm{d}l. \tag{20}$$

To constitute the Fourier-transform of this voltage the horizontal component phasor of the electric field strength at height $z = h_i$ has to be integrated from infinity to the radius r_0 of the tower model. Based on the relations

$$\frac{\partial H}{\partial z} = -\varepsilon_0 \frac{\partial E_{r2}}{\partial t}, \qquad \mu_0 H = -\frac{\partial A_2}{\partial r} \qquad (A_2 = A_{z2},\ H = H_\varphi),$$

connection between the horizontal component of the electric field strength E_r and vector potential A in the air is (c is the velocity of the light in vacuum):

$$E_{r2} = c^2 \int_{\tau=-\infty}^{t} \frac{\partial^2 A_2}{\partial r \partial z}\, \mathrm{d}\tau.$$

The phasor $\mathbf{E}_{r2}$ is obtained by Fourier-transform of this expression:

$$\mathbf{E}_{r2} = \frac{c^2}{j\omega} \frac{\partial^2 \mathbf{A}_2}{\partial r \partial z} + c^2 \pi \delta(\omega) \left(\frac{\partial^2 \mathbf{A}_2}{\partial r \partial z} \right)_{\omega=0},$$

here $\delta(\omega)$ is the Dirac – function.
[The formula (3) with formula (2b) is valid, only if $\omega \neq 0$.]

First the elemental $\mathrm{d}\mathbf{U}$ voltage phasor is calculated, which is the consequence of the elemental current phasor discussed in chapter 2. According to the last equation and expression (20) this voltage phasor is:

$$\mathrm{d}\mathbf{U} \int_{\infty}^{r_0} (\mathbf{E}_{r2})_{z=h_i}\, \mathrm{d}r = -j \frac{c^2}{\omega} \left(\frac{\partial \mathbf{A}_2}{\partial z} \right)_{z=h_i,\ r=r_0} + c^2 \pi \delta(\omega) \cdot \left(\frac{\partial \mathbf{A}_2}{\partial z} \right)_{z=h_i,\ r=r_0,\ \omega=0};$$

namely $\dfrac{\partial \mathbf{A}_2}{\partial z} \to \mathbf{0}$ if $r \to \infty$.
With the notation

$$\mathrm{d}\mathbf{V} = c^2 \left(\frac{\partial \mathbf{A}_2}{\partial z} \right)_{z=h_i,\ r=r_0}, \tag{21}$$

$$\mathrm{d}\mathbf{U} = -\frac{j}{\omega} [\mathrm{d}\mathbf{V} - (\mathrm{d}\mathbf{V})_{\omega=0}] + (\mathrm{d}\mathbf{V})_{\omega=0} \left[\frac{1}{j\omega} + \pi \delta(\omega) \right];$$

The second term of the last relation is the Fourier-transform of a step-function, which does not go to zero if the time goes to infinity, so this would give false result. This term compensates the voltage caused by the charge procedure before discharge, they were not taken into account up to this point. Therefore this term must be left:

$$\mathrm{d}\mathbf{U} = -\frac{j}{\omega} [\mathrm{d}\mathbf{V} - (\mathrm{d}\mathbf{V})_{\omega=0}]. \tag{22}$$

According to *Equ.* (8) and ref. ERDÉLYI, MAGNUS, ..., $\mathbf{A}_{2\infty}$ at $r = r_0$ can be transferred by Hankel-transform in the following form:

$$(\mathbf{A}_{2\infty})_{r=r_0} = \frac{\mu_0 \mathbf{I}\,\mathrm{d}\zeta}{4\pi} \int_0^\infty \frac{s}{\sqrt{s^2 - \frac{\omega^2}{c^2}}} \times \left[e^{-|\zeta - z|\sqrt{s^2 - \frac{\omega^2}{c^2}}} + e^{-|\zeta + z|\sqrt{s^2 - \frac{\omega^2}{c^2}}} \right] J_0(r_0 s)\,\mathrm{d}s,$$

and therefore

$$\left(\frac{\partial \mathbf{A}_{2\infty}}{\partial z}\right)_{r=r_0,\ z=h_i} = -\frac{\mu_0 \mathbf{I}\,\mathrm{d}\zeta}{4\pi} \int_0^\infty s \times \left[\operatorname{sgn}(h_i - \zeta) e^{-|\zeta - h_i|\sqrt{s^2 - \frac{\omega^2}{c^2}}} + e^{-|\zeta + h_i|\sqrt{s^2 - \frac{\omega^2}{c^2}}} \right] J_0(r_0 s)\,\mathrm{d}s.$$

Further based on expressions (10) and (11)

$$\left(\frac{\partial \mathbf{A}_{2}}{\partial z}\right)_{r=r_0,\ z=h_i} = \left(\frac{\partial \mathbf{A}_{2\infty}}{\partial z}\right)_{r=r_0,\ z=h_i} + \frac{\mu_0 \mathbf{I}\,\mathrm{d}\zeta}{2\pi} j\kappa \int_0^\infty \frac{s\sqrt{s^2 + \frac{2j}{\delta^2}}}{\sqrt{s^2 - \frac{\omega^2}{c^2}} + j\kappa\sqrt{s^2 + \frac{2j}{\delta^2}}} e^{-(\zeta + h_i)\sqrt{s^2 - \frac{\omega^2}{c^2}}} J_0(r_0 s)\,\mathrm{d}s.$$

In both models of the lightning current and of the current in tower, the current time-function has the form as follows, or can be calculated as the linear combination of similar forms:

$$\mathbf{I} = \mathbf{I}_0 e^{-(b + jw)\zeta}.$$

Substituting the last three expressions into *Equ.* (21) we obtain:

$$\mathrm{d}\mathbf{V} = -\frac{\mu_0 c^2 \mathbf{I}_0}{4\pi}\,\mathrm{d}\zeta \int_0^\infty s \times \left\{ \frac{\sqrt{s^2 - \frac{\omega^2}{c^2}} - j\kappa\sqrt{s^2 + \frac{2j}{\delta^2}}}{\sqrt{s^2 - \frac{\omega^2}{c^2}} + j\kappa\sqrt{s^2 + \frac{2j}{\delta^2}}} e^{-h_i\sqrt{s^2 - \frac{\omega^2}{c^2}}} e^{-\left(b + jw + \sqrt{s^2 - \frac{\omega^2}{c^2}}\right)\zeta} + \operatorname{sgn}(h_i - \zeta) e^{-\left[|\zeta - h_i|\sqrt{s^2 - \frac{\omega^2}{c^2}} + (b + jw)\zeta\right]} \right\} J_0(r_0 s)\,\mathrm{d}s.$$

Integrating the last relation with respect to ζ and introducing the following notations,

$$\mathbf{G}_1(s) = \frac{\sqrt{s^2 - \frac{\omega^2}{c^2}} - j\kappa\sqrt{s^2 + \frac{2j}{\delta^2}}}{\sqrt{s^2 - \frac{\omega^2}{c^2}} + j\kappa\sqrt{s^2 + \frac{2j}{\delta^2}}},$$
$$\mathbf{G}_2(s) = \frac{s}{b + jw + \sqrt{s^2 - \frac{\omega^2}{c^2}}}, \tag{23a}$$
$$\mathbf{G}_3(s) = \frac{s}{b + jw - \sqrt{s^2 - \frac{\omega^2}{c^2}}},$$

we get for the primitive functions:

if $\zeta > h_i$;

$$\mathbf{V}(\zeta) = \frac{c^2\mu_0}{4\pi}\mathbf{I}_0 \int_0^\infty \mathbf{G}_2(s)\left[\mathbf{G}_1(s)e^{-h_0\sqrt{s^2-\frac{\omega^2}{c^2}}} - e^{h_i\sqrt{s^2-\frac{\omega^2}{c^2}}}\right] e^{-\left(b+jw+\sqrt{s^2-\frac{\omega^2}{c^2}}\right)\zeta} J_0(r_0 s)\,\mathrm{d}s$$

if $\zeta < h_i$:

$$\mathbf{V}^*(\zeta) = \frac{c^2\mu_0}{4\pi}\mathbf{I}_0 \int_0^\infty \left[\mathbf{G}_2(s)\mathbf{G}_1(s)e^{-\left(b+jw+\sqrt{s^2-\frac{\omega^2}{c^2}}\right)\zeta} + \mathbf{G}_3(s)e^{-\left(b+jw-\sqrt{s^2-\frac{\omega^2}{c^2}}\right)\zeta}\right] + e^{-h_i\sqrt{s^2-\frac{\omega^2}{c^2}}} J_0(r_0 s)\,\mathrm{d}s \tag{23b}$$

For the calculation of the full voltage phasor, the formula (23a) and (23b) must be applied many times:

$$\left.\begin{array}{l}
\bullet \text{ in the lightning model, substituting } z_L = \zeta - h \text{ in } Eq. \text{ (14), and at} \\
\quad \text{at the first term} \\
\quad b = b_1, \quad w = \dfrac{\omega}{v}, \qquad \mathbf{I}_0 = \mathbf{I}_1 \left(\dfrac{1}{b_1 v + j\omega} - \dfrac{1}{a + j\omega} \right) e^{\left(b_1 + j\frac{\omega}{v}\right)h}; \\
\quad \mathbf{V_{L1}} = \mathbf{V}(h+l) - \mathbf{V}(h), \\
\bullet \text{ at the second term} \\
\quad b = b_2, \quad w = \dfrac{\omega}{v}, \qquad \mathbf{I}_0 = \mathbf{I}_1 \left(\dfrac{1}{a + j\omega} - \dfrac{1}{b_2 v + j\omega} \right) e^{\left(b_2 + j\frac{\omega}{v}\right)h}; \\
\quad \mathbf{V_{L2}} = \mathbf{V}(h+l) - \mathbf{V}(h), \\
\bullet \text{ in the tower current model, according } Eqs. \text{ (17) and (19b), and at} \\
\quad \text{the first term} \\
\quad b = b_0, \quad w = \dfrac{\omega}{c}, \qquad \mathbf{I}_0 = \mathbf{K}_1 \\
\quad \mathbf{V}_{T1} = \mathbf{V}(h) - \mathbf{V}(h_i + 0) + \mathbf{V}^*(h_i - 0) - \mathbf{V}^*(0), \\
\bullet \text{ at the second term} \\
\quad b = b_0, \quad w = -\dfrac{\omega}{c}, \quad \mathbf{I}_0 = \mathbf{K}_2 \\
\quad \mathbf{V}_{T2} = \mathbf{V}(h) - \mathbf{V}(h_i + 0) + \mathbf{V}^*(h_i - 0) - \mathbf{V}^*(0).
\end{array}\right\} \tag{24}$$

Finally, based on *E.* (22) the voltage spectrum is calculated as follows:

$$\mathbf{U}(j\omega) = -\frac{j}{\omega}\left[(\mathbf{V}_{L1} + \mathbf{V}_{L2} + \mathbf{V}_{T1} + \mathbf{V}_{T2}) - (\mathbf{V}_{L1} + \mathbf{V}_{L2} + \mathbf{V}_{T1} + \mathbf{V}_{T2})_{\omega=0}\right]. \tag{25}$$

The momentary value of the voltage is:

$$u(t) = \frac{1}{\pi}\Re e\left[\int_0^\infty \mathbf{U}(j\omega)e^{j\omega t}\,\mathrm{d}\omega\right]. \tag{26}$$

If there is ground wire on the tower then the following (approaching) procedure can be used. The ground wire and the earth form a transmission line being in short circuit at the next tower. We can define an input impedance of this transmission line at the tower entered by lightning stroke. This impedance is:

$$\mathbf{Z}_g = \frac{1}{2}\mathbf{Z}_0 \tanh\left(\mathbf{g}l_t\right). \tag{27}$$

Here $\mathbf{Z}_0$ is the surge impedance, $\mathbf{g}$ is the propagation coefficient of the transmission line, l_t is the distance from the next tower and the multiplier $\frac{1}{2}$ follows from the fact, that ground wire exists in both sides of the tower. Knowing the series impedance $\mathbf{Z}_s$ and the shunt reactance X_c per unit length the following relations are valid:

$$\mathbf{Z}_0 = \sqrt{-jX_c\mathbf{Z}_s}, \qquad \mathbf{g} = \sqrt{j\frac{\mathbf{Z}_s}{X_c}}, \tag{28}$$

further

$$X_c = \frac{c^2 \mu_0}{2\pi\omega} \ln \frac{2h}{r_W}, \tag{29}$$

and using the ‘complex image’ approximate formula (see in references DÉRI et al.)

$$\mathbf{Z}_s = \frac{\omega\mu_0}{2\pi}\left[a \tan\left(\frac{\delta}{2h+\delta}\right) + j \ln \frac{\sqrt{(2h+\delta)^2+\delta^2}}{r_w}\right], \tag{30}$$

where the resistance of the ground wire and of the grounding of the towers are neglected; r_w is the radius of the ground wire.

The impedance $\mathbf{Z}_g$ is parallel connected to the examined tower with the entering current $\mathbf{I}_h$, therefore

$$\mathbf{U}_r = \frac{\mathbf{Z}_g \frac{\mathbf{U}}{\mathbf{I}_h}}{\mathbf{Z}_g + \frac{\mathbf{U}}{\mathbf{I}_h}} \mathbf{I}_h \equiv \frac{\mathbf{Z}_g \mathbf{U}}{\mathbf{Z}_g + \frac{\mathbf{U}}{\mathbf{I}_h}}. \tag{31}$$

In the case of towers with ground wire the voltage $\mathbf{U}_r$ has to be substituted into the relation (26) instead of $\mathbf{U}$.

Based on the principles above computer program has been produced.

6. Results of the Program

Input data for the program are:

length of the lightning channel, l
recombination factor, a
front time and time to the half value, t_f, t_h
velocity of the main discharge, v
per unit resistance of the soil, ρ
height of the tower, h.
average height of the isolator, h_i,
radius of the tower model, r_0.

If exists ground wire, then

distance from the next tower, l_t,
radius of the ground wire, r_w.

For the parameter γ in previous formulae $\gamma = 1/r$ is valid. The parameters b_1 and b_2 of the lightning model are calculated based on the input data, where the calculation methods are not detailed here. Based on preliminary results it was detected that input parameters l, a and v, within the interval published in the literature of HORVÁTH (1965) have only a slight influence on the voltage time function. So results described here are based on the following average values $l = 400$ m; $a = 2.25 \cdot 10^5$/sec; $v = 7.5 \cdot 10^7$ m/sec. The peak value of the lightning current at the bottom of the lightning channel (at the top of the tower) is I_p. Two kinds of towers are investigated. One of them has a height of $h = 45$ m, an average height of $h_i = 40$ m for the insulator string and an average radius of $r_0 = 0.5$ m. At the other tower these parameters are: $h = 100$ m, $h_i = 93$ m, $r_0 = 0.7$ m. The overvoltage is investigated with also two kinds of front time t_f and the time to the half value th of the lightning current at the top of the tower. In one case these parameters are: $t_f = 3$ μsec, $t_h = 40$ μsec, in the other case they are: $t_f = 0.3$ μsec, $t_h = 4$ μsec. *Fig.* 6 shows the quantity $\frac{u}{I_p}$ and i as a function of time, and shows $\frac{u}{I_p}$ versus i *without* ground wire as well. (Here u is the momentary voltage on the insulator string, i is the momentary value of the lightning current at the top of the tower.) *Fig.* 7 shows these functions at the second tower *without* ground wire. Finally *Fig.* 8 shows these functions at the first tower *with* ground wire.

7. Conclusions:

1. *The maximal value of the 'insulator string impedance'* $\frac{u}{I_h}$ *is many times greater in the case of without ground wire.* (Compare *Fig.* 6 with *Fig.* 8)
2. *The maximal value of the 'insulator string impedance' is greater, if the front time* t_h *of the lightning current is smaller, that is if the steep of the current is greater.* (Compare the first example with the second one in the *Figs.* 6 and 7, and the first with the third example in the *Fig.* 8). *This can be considered trivial.*
3. *In the case of towers with ground wire the 'impedance' is smaller, that is the effect of the ground wire is greater, if the distance between towers is smaller.* (Compare the first example with the second one in the *Fig.* 8.)

An other article is necessary for more detailed investigation.

Appendix

It is obvious from the *Eq.*(9), that

$$(\mathbf{A}_1)_{z=0} = \int_0^\infty \mathbf{f}_1(s) J_0(sr)\,\mathrm{d}s,$$

and

$$\left(\frac{\partial \mathbf{A}_1}{\partial z}\right)_{z=0} = \int_0^\infty \sqrt{s^2 + \frac{2j}{\delta^2}}\mathbf{f}_1(s) J_0(sr)\,\mathrm{d}s$$

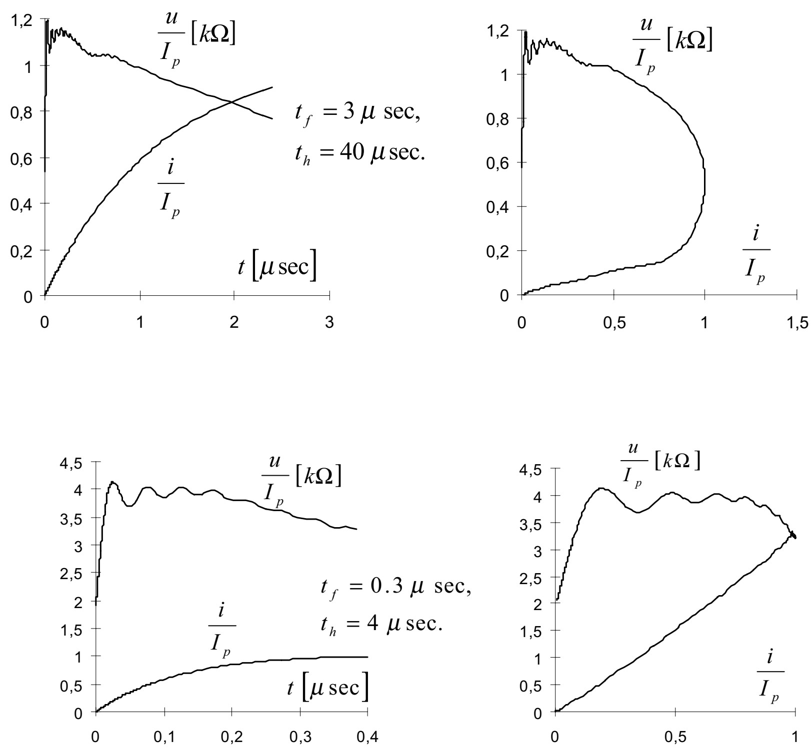

Fig. 6. Overvoltage on the Insulator String and Lightning-current versus Time (on left), Overvoltage versus Lightning-current (on right) *Without Ground Wire*.
$h = 45$ m, $h_i = 40$ m, $r_0 = 0.5$ m.

are. We obtain from the *Eq.* (8) and on the basis of ref. ERDÉLYI et al (1954).

$$(\mathbf{A}_{2\infty})_{z=0} = \frac{\mu_0 \mathbf{I}\,\mathrm{d}\zeta}{2\pi} \frac{e^{-j\frac{\omega}{c}\sqrt{r^2+\zeta^2}}}{\sqrt{r^2+\zeta^2}} \equiv \frac{\mu_0 \mathbf{I}\,\mathrm{d}\zeta}{2\pi} \int_0^\infty \frac{s}{\sqrt{s^2 - \frac{\omega^2}{c^2}}} e^{-\zeta\sqrt{s^2-\frac{\omega^2}{c^2}}} J_0(sr)\,\mathrm{d}s,$$

and

$$\left(\frac{\partial \mathbf{A}_{2\infty}}{\partial z}\right)_{z=0} = \mathbf{0}.$$

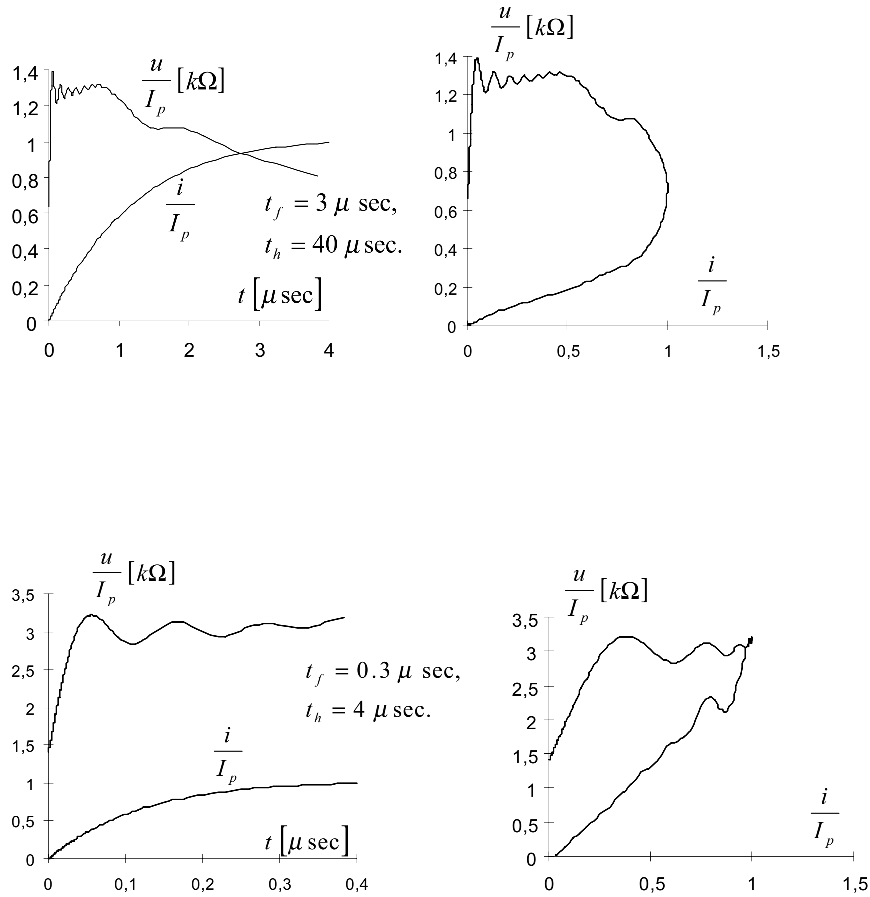

Fig. 7. Overvoltage on the Insulator String and Lightning-current versus Time (on left), Overvoltage versus Lightning-current (on right) *Without Ground Wire.*
$h = 100$ m, $h_i = 93$ m, $r_0 = 0.7$ m.

Therefore, using *Eq.* (10), the following expressions are valid:

$$(\mathbf{A}_2)_{z=0} = \int_0^\infty \left[\frac{\mu_0 \mathbf{I}\, \mathrm{d}\zeta}{2\pi} \frac{s}{\sqrt{s^2 - \frac{\omega^2}{c^2}}} e^{-\zeta\sqrt{s^2 - \frac{\omega^2}{c^2}}} + \mathbf{f}_2(s) \right] J_0(sr)\, \mathrm{d}s,$$

$$\left(\frac{\partial \mathbf{A}_2}{\partial z}\right)_{z=0} = -\int_0^\infty \sqrt{s^2 - \frac{\omega^2}{c^2}}\, \mathbf{f}_2(s)\, J_0(sr)\, \mathrm{d}s.$$

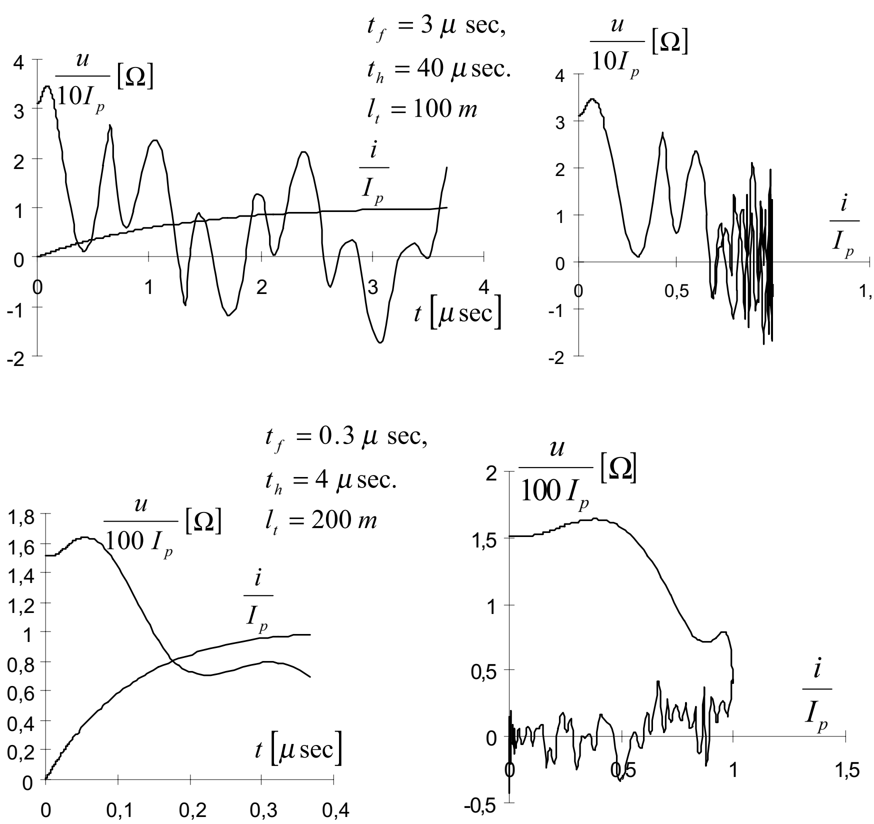

Fig. 8. Overvoltage on the Insulator String and Lightning-current versus Time (on left), Overvoltage versus Lightning-current (on right) *With Ground Wire*.
$h = 45$ m, $h_i = 40$ m, $r_0 = 0.5$ m, $r_w = 6$ mm.

Consequently, to satisfy the interface conditions (4) the following equations have to be fullfilled

$$\mathbf{f}_1(s) = \frac{\mu_0 \mathbf{I}\,\mathrm{d}\zeta}{2\pi} \frac{s}{\sqrt{s^2 - \frac{\omega^2}{c^2}}} e^{-\zeta\sqrt{s^2 - \frac{\omega^2}{c^2}}} + \mathbf{f}_2(s),$$

$$\frac{\mathbf{k}_2^2}{\mathbf{k}_1^2}\mathbf{f}_1(s)\sqrt{s^2 + \frac{2j}{\delta^2}} \equiv j\kappa \mathbf{f}_1(s)\sqrt{s^2 + \frac{2j}{\delta^2}} = -\mathbf{f}_2(s)\sqrt{s^2 - \frac{\omega^2}{c^2}}.$$

It is easy to see, that these *Eqs.* are satisfied by the expressions (11).

References

[1] BILINSKY, S., In the Field due to a Vertical Line Source of Current Grounded to earth. *Geophysics*, **3** (1938), pp. 58–62.

[2] DÉRI, A. – TEVAN, G. – SEMLYEN, A. – CASTANHEIRA, A., The Complex Grond Return Plane, a Simplified Model for Homogeneous and Multi-Layer Earth Return. *IEEE. Trans.* PAS 100 (1981), pp. 3686–3693.

[3] DIENDORFER, G. – UMAN, M. A., An Improved Return Stroke Model with Specified Chanell-Base Current. *J. Geophysics, Res.* **95** No. D9. (1990), pp. 13621–13644.

[4] ERDÉLYI, A. – MAGNUS, W. – OBERHETTINGER, F. – TRICOMI, F. G., *Tables of Integral Transformes*. Mc Graw-Hill, Book Company, 1954.

[5] HORVÁTH, T., *Lightning-Protection* (in Hungarian). Műszaki Könyvkiadó, Budapest, 1965.

[6] IVÁNYI, A., Transient Field of Lightning Impulse. 4-th Int. IGTE Symp. and European TEAM Workshop, Graz, 10–12.Oct. (1990), pp. 167–172.

[7] KOSZTALUK, R. – LOBODA, M. – MUKDEKAR, D., Experimental Study of Transient Ground Impedances. *IEEE* Vol. Pas-100. No.11, (1981), pp. 4653–4660.

[8] NUCCI, C. A., Lightning-Induced Voltages on Overhead Power Lines; Part I. and Part II. *Électra* 161 and 162 (1995), pp. 75–102, and 121–145.

[9] RUBINSTEIN, M. – UMAN, M. A., Methods for Calculating the Electromagnetic Fields from a Known Source Distribution: Application to Lightning. *IEEE Trans. on Electromagnetic Compatibility*, **31** No.2. (1989), pp. 183–189.

[10] SIMONYI, K., *Foundation of Electrical Engineering.* Oxford: Pergamon Press Ltd., 1963.

[11] SZUNYOGH, G., Exakte mathematische Untersuchung der Blitzschutzerder. *13. Internationale Blitzschutzkonferenz*, Venezia 1976. R-2.8.

[12] TEVAN, G. – PETRI, K., Calculation of the Voltage on Earth Impedance Caused by Lightning Impulse Current. *Archiv für Elektrotechnik*, **71** (1988), pp. 297–306.

[13] TEVAN, G., Analytical Calculation of the Voltage on Earth Caused by Lightning. *7-th International Symposium on High Voltage Engineering.* Technische Universität Dresden. 26–30. Aug. 1991. 82.-06

[14] VAJNER, A. L. – FLORU, V. N., Experimental Study and Calculation Method of Impulse Characteristic of Deep Grounding Electrodes (in Russian). *Elektrichestvo*, No. 5 (1971), pp. 18–22.

[15] WAGNER, C. F. – HILEMAN, A. R., New Approach to the Calculation of the Lightning Performance of ransmission Line. *Trans. AIEE*, I. **75** (1956), II. **78** (1959), III. **79** (1960).