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VOLTAGE SPECTRA OF TWO-PHASE PWM TECHNIQUE WITH 120° CYCLE

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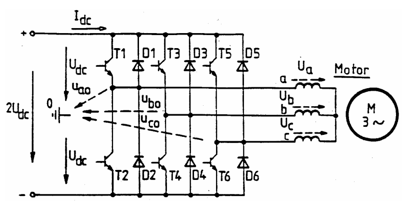
Abstract

This paper deals with a kind of two-phase PWMs when during each 1/3 of the period one of the motor phases is connected to the positive (or negative) dc bar and only the remaining two motor phases are switched between the dc bars. The novel accurate and the simpler approximated analytical equations of motor voltage harmonics are given and it is shown that the approximated equations provide a good precision of calculations. In comparison with three-phase PWMs this two-phase PWM – only for the same inverter commutation frequency – provides better quality of PWMs for high value of fundamental voltage. The realization of the two-phase PWM with 120° cycle is simpler than the other types of two-phase modulation but the transistor-diode load of the upper and bottom parts of the inverter bridge differs.

Keywords: voltage source inverter, two phase PWM, voltage spectra.

1. Introduction



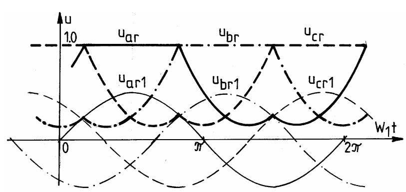


Fig. 1. Configuration of the inverter (a) and phase reference waves (b)

The carrier-based two-phase PWM techniques with 120° cycle are possible because each sinusoidal reference wave has the interval of 120° of the period when its value is higher (or lower) than the other ones (*Fig. 1*). Thus e.g. when the a phase reference signal is higher than the b and the c ones, then the phase sinusoidal reference signals can be modified by the addition of zero-sequence components in a manner that the a phase reference signal at $\pi/6 \le W_1 t \le 2\pi/3$ will be at its maximum: $u_{ar} = 1$. The reference waves of the phases in this interval will be as follows (where A is the modulation index, W_1 is angular frequency and t is the time):

$$\begin{aligned} u_{ar} &= 1 - A\sin W_1 t + A\sin W_1 t = 1, \\ u_{br} &= 1 - A\sin W_1 t + A\sin\left(W_1 t - \frac{2\pi}{3}\right) = 1 - \sqrt{3}A\sin\left(W_1 t + \frac{\pi}{6}\right), \\ u_{cr} &= 1 - A\sin W_1 t + A\sin\left(W_1 t - \frac{4\pi}{3}\right) = 1 - \sqrt{3}A\sin\left(W_1 t - \frac{\pi}{6}\right). \end{aligned} \tag{1}$$

This means that the motor phase a is connected on 120° of the period to the positive dc bus-bar ($u_{a0} = U_{dc}$, see *Fig. 1a*). For the time $5\pi/6 \le W_1 t \le 3\pi/2$ the phase b is switched to the positive dc bar, hence $u_{br} = 1$, therefore:

$$u_{ar} = 1 + \sqrt{3}A\sin\left(W_1 t + \frac{\pi}{6}\right), \tag{2}$$

while in $3\pi/2 \le W_1 t \le 13\pi/6$ the c phase is connected to the dc bar and $u_{cr} = 1$, hence:

$$u_{ar} = 1 + \sqrt{3}A\sin\left(W_1 t - \frac{\pi}{6}\right). \tag{3}$$

The a phase reference wave and its fundamental component are presented in *Fig.2*. The different types of two-phase PWMs with 60° or 120° cycle are investigated in [1]–[7]. The two-phase PWM with 120° cycle was examined in [4], but without the analytical investigation of the motor voltage spectra, in [9] only a simplified deduction of the voltage spectra is given.

This paper mainly deals with the analytical determination of the motor voltage spectra, the other characteristics of this type of PWM will only be shortly given.

2. Voltage Spectra

The order of the harmonics can be given as follows [5, 8]:

$$\nu = Km \pm n', \tag{4}$$

where $m = f_1/f_c$ and f_1 is the fundamental, f_c is the carrier (sampling) frequencies, K and n' are positive integers.

If the synchronized carrier wave is used, the u_{a0} motor voltage to the middle point of dc voltage (*Fig. 1a*) can be decomposed in Fourier series:

$$u_{a0} = a_0 + \sum_{\nu=1}^{\infty} (a_\nu \cos\nu\alpha + b_\nu \sin\nu\alpha), \tag{5}$$

where $\alpha = W_1 t$ and

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} u_{a0}\,\mathrm{d}\alpha,$$

$$U_\nu^* = a_\nu + jb_\nu = \frac{1}{\pi}\int_0^{2\pi} u_{a0}e^{j\nu\alpha}\,\mathrm{d}\alpha; \tag{6}$$

or:

$$U_0 = \frac{a_0}{U_{dc}} = \frac{1}{2\pi}\sum_i(\alpha_i - \alpha_{i-1})\ \mathrm{sign}\ U_{dc};$$

$$U_\nu = \frac{U_\nu^*}{U_{dc}} = \frac{2}{j\pi\nu}\sum_i\left(e^{j\nu\alpha_i} - e^{j\nu\alpha_{i-1}}\right)\mathrm{sign}\ U_{dc}, \tag{7}$$

where α_i and α_{i-1} are the modulation angles between $0 \le \alpha \le 2\pi$.

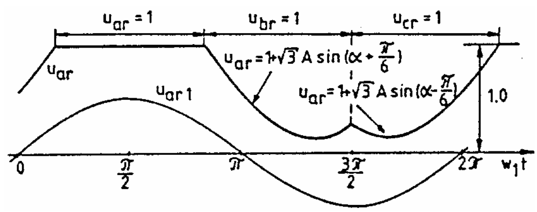


Fig. 2. The *a* phase reference wave and its fundamental component

The modulation processes for different initial phase angles of the carrier waves are presented in *Figs. 3a* and *3b*. Since PWM is synchronized with the continuous carrier wave and the control of the three phases is symmetrical, the value of $m/3$ must be an integer. The order of non-zero sequence harmonics can be $\nu^* = 1 \pm 3K'$ ($K' = 0, 1, 2 \ldots$) and the sign of ν^* shows the sequence of harmonics (plus means the positive one).

Thus, both even and odd order harmonics arise. According to the initial phase of the carrier wave two different motor voltage spectra are obtained.

2.1. *PWM according to Fig. 3a*

With the coordinate system according to *Fig. 3a* and taking (2) into account the intersection points of the carrier and reference waves for natural sampling during $0 \le \alpha \le 2\pi/3$ will be [8]:

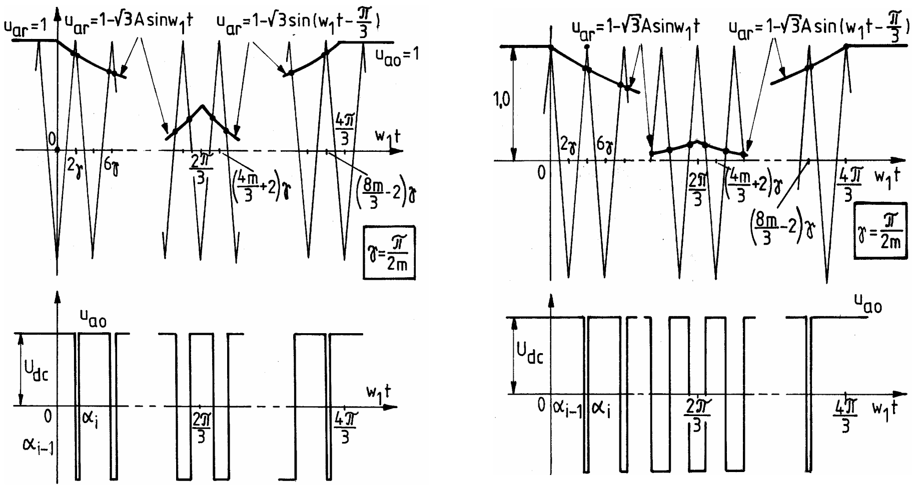
$$\left.\begin{aligned} \alpha_{i-1} &= \gamma i - \gamma + \gamma\left[1 - \sqrt{3}A\sin(\alpha_{i-1})\right] \\ \alpha_i &= \gamma i + \gamma - \gamma\left[1 - \sqrt{3}A\sin(\alpha_i)\right] \end{aligned}\right\}, \tag{8}$$

where $\gamma = \dfrac{\pi}{2m}$ and $i = 2, 6, 10\ldots$ (*Fig. 3a*).

According to (3) and coordinate system of *Fig. 3a* the intersection points will be:

$$\left.\begin{aligned} \alpha_{i-1} &= \gamma i - \gamma + \gamma\left[1 - \sqrt{3}A\sin(\alpha_{i-1} - \pi/3)\right] \\ \alpha_i &= \gamma i + \gamma - \gamma\left[1 - \sqrt{3}A\sin(\alpha_i - \pi/3)\right] \end{aligned}\right\} \tag{9}$$

and $i = 4m/3 + 2, 4m/3 + 6, 4m/3 + 10, \ldots$



a) b)

Fig. 3. Modulation processes for two-phase PWM with 120° and different initial phase angle of the carrier wave

According to Appendix *a*) the harmonic amplitudes e.g. of $\nu = Km - 1$ order will belong to $p_1 = Km$, $p_2 = Km + 2$, and $p_3 = Km - 4$ (neglecting Bessel functions of order $|\nu - p| \geq 9$):

$$\begin{aligned} |U_{Km-1}| = \Bigg| &\frac{4}{\pi\sqrt{3}K} J_1\left(KA\frac{\pi}{2}\sqrt{3}\right) - \frac{6}{\pi(Km+2)} \cdot J_3\left[\left(K + \frac{2}{m}\right) \cdot A\frac{\pi}{2}\sqrt{3}\right] \\ &\frac{1}{\sin 2\pi/m} - \frac{6}{\pi(Km-4)} \cdot J_3\left[\left(K - \frac{4}{m}\right) A\frac{\pi}{2}\sqrt{3}\right] \frac{1}{\sin 4\pi/m}\Bigg| \end{aligned} \tag{10}$$

while for $\nu = Km + 1$ order ($p_1 = Km$, $p_2 = Km - 2$ and $p_3 = Km + 4$):

$$|U_{Km+1}| = | \frac{4}{\pi\sqrt{3}K} J_1\left(KA\frac{\pi}{2}\sqrt{3}\right) - \frac{6}{\pi(Km-2)} \cdot J_3\left[\left(K-\frac{2}{m}\right)\cdot A\frac{\pi}{2}\sqrt{3}\right]$$
$$\times\frac{1}{\sin 2\pi/m} - \frac{6}{\pi(Km+4)} \cdot J_3\left[\left(K+\frac{4}{m}\right) A\frac{\pi}{2}\sqrt{3}\right] \frac{1}{\sin 4\pi/m}\Bigg|. \qquad (11)$$

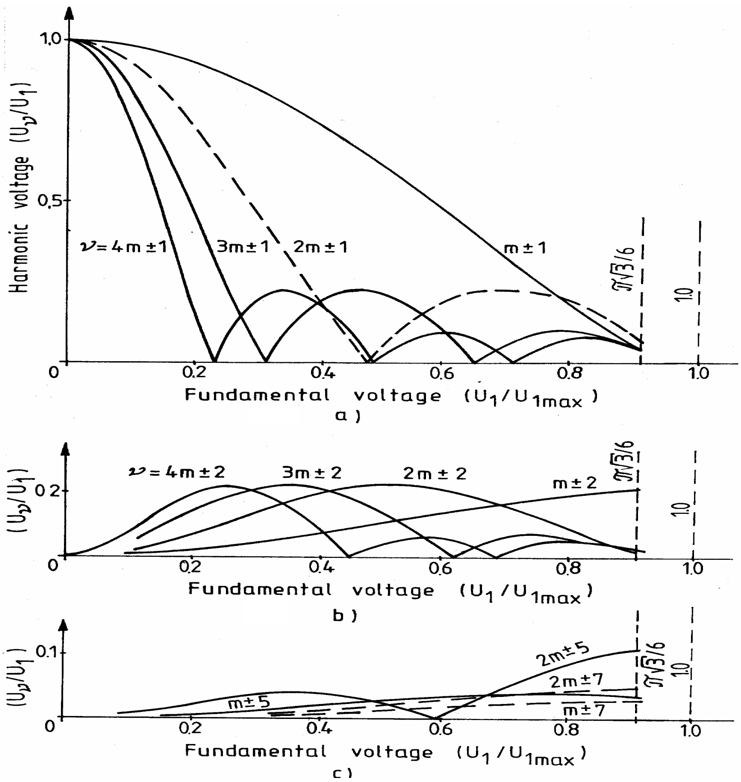


Fig. 4. Relative amplitudes of voltage harmonics

Similar expressions can be given for $\nu = Km \pm 5$, $\nu = Km \pm 7$ etc. For $m \to \infty$ and neglecting the Bessel functions with order $|\nu - p| \geq 15$ for the motor voltage harmonics considerably simpler equations are valid:

$$|U_\nu| = \left|\frac{4}{\sqrt{3}\pi K} J_{n'}\left(KA\frac{\pi}{2}\sqrt{3}\right) - \frac{9}{\alpha\pi^2 K} J_3\left(KA\frac{\pi}{2}\sqrt{3}\right)\right.$$
$$\left. - \frac{27}{\beta\pi^2 K} J_9\left(KA\frac{\pi}{2}\sqrt{3}\right)\right|, \qquad (12)$$

where $\alpha = 2$ and $\beta = 20$ if $n' = 1$, while $\alpha = 4$ and $\beta = -14$ if $n' = 5$, then $\alpha = -10$ and $\beta = 8$ if $n' = 7$.

The voltage harmonic of order $\nu = Km + 2$ (see App.) will belong to $p_1 = Km - 1$, $p_2 = Km + 5$ (for $p - \nu \leq 3$ only):

$$|U_{Km+2}| = \left| \frac{6}{\pi(Km-1)} \cdot J_3 \left[\left(K - \frac{1}{m} \right) \cdot A \frac{\pi}{2} \sqrt{3} \right] \right. \\ \left. \times \frac{1}{\sin \pi/m} + \frac{6}{\pi(Km+5)} \cdot J_3 \left[\left(K + \frac{5}{m} \right) A \frac{\pi}{2} \sqrt{3} \right] \frac{1}{\sin 5\pi/m} \right| \tag{13}$$

or with $m \to \infty$ for the harmonics of the order $\nu = Km \pm 2$ and $\nu = Km \pm 4$ the voltage amplitudes are expressed by the following expression:

$$|U_\nu| = \left| \frac{36}{\alpha \pi^2 K} J_3 \left(K A \frac{\pi}{2} \sqrt{3} \right) + \frac{108}{\beta \pi^2 K} J_9 \left(K A \frac{\pi}{2} \sqrt{3} \right) \right|, \tag{14}$$

where $\alpha = 5$ and $\beta = 77$ if $n' = 2$ ($\nu = Km \pm 2$), while $\alpha = 7$ and $\beta = -6.5$ if $n' = 4$ ($\nu = Km \pm 4$).

The relative amplitudes of the important voltage harmonics for $m \to \infty$ as a function of the fundamental one are presented in *Fig.4*. In *Fig. 5* the voltage spectra of the three-phase PWM (space vector method) and two-phase ones are compared for $A = 0.1$ and $A = 1.0$. It can be seen that for low modulation indices there are significant differences in amplitudes of harmonics of order $m+1, 3m+1, \ldots$, while for high modulation indexes the differences in amplitudes of important harmonics are very small.

2.2. PWM according to Fig. 3b

In this case the equations of voltage harmonics are more complicated than before (See App.). But for $m \to \infty$ the results are the same. Virtually, in dependence on the initial phase angle of the carrier wave for $m \geq 36$ there are no significant differences in the harmonic amplitudes.

2.3. Results

The voltage spectra investigations have shown that:

1. The voltage harmonics for the low fundamental voltage region – in comparison to three-phase PWMs – vary considerably in magnitude and in order as well. E.g. for $A \to 0$ the harmonics of order $Km \pm 1$ become equal to the fundamental voltage ($K = 1.2, 3, \ldots$). For three-phase PWMs this was valid

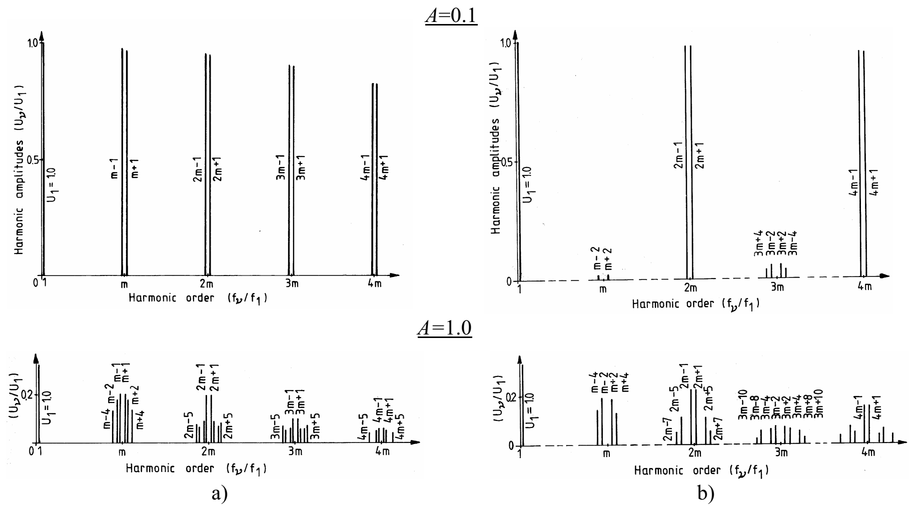


Fig. 5. Voltage spectra of two-phase (a) and three-phase PWMs (space vector method) (b) for $A = 0.1$ and $A = 1.0$

only for even K. Owing to this fact the loss-factor of a two-phase PWM for the same m will be ($Km \pm 1 \cong Km$):

$$K_\psi = \sum_{K=1}^{\infty} \left[\frac{1}{(Km-1)^2} + \frac{1}{(Km+1)^2} \right] \cong \frac{2}{m^2} \sum_{1}^{\infty} \frac{1}{K^2} = \frac{\pi^2}{3m^2},$$

consequently by four times more than that for three-phase PWMs and the same carrier period or by (4*4/9) times more than that in the case of the same switching frequency since the switching frequency of two-phase PWMs is 2/3 of that for three-phase ones [7].

2. The motor harmonic losses – for the same switching frequency of the inverter – are considerably higher in the low voltage region than those for a three-phase PWM, and about from $A > 0.82$, lower than those in the high voltage region [4]–[7]. For $A = 1.0$ e.g. $K_\Psi = 0.156/\text{m}^2$ if three-phase PWMs are used and $K_\Psi = 0.103/\text{m}^2$ if two-phase PWMs are used [7].

In *Fig. 6* the motor phase voltage, stator flux, harmonic current and torque pulsations are presented for A= 0.5 and A= 1.0 (simulation results). It is seen that in the case of $A = 0.5$ the three-phase PWM with $m = 48$ produces smaller current and torque pulsations than two-phase PWMs with $m = 72$ (hence, for the same commutation frequency). In the case of $A = 1.0$ the two-phase PWM with $m = 36$ and 120° cycle obtains the smallest current and about the same torque pulsations as the two-phase PWM with 60° cycle. And confirming the theoretical results, the three-phase PWM with $m = 24$

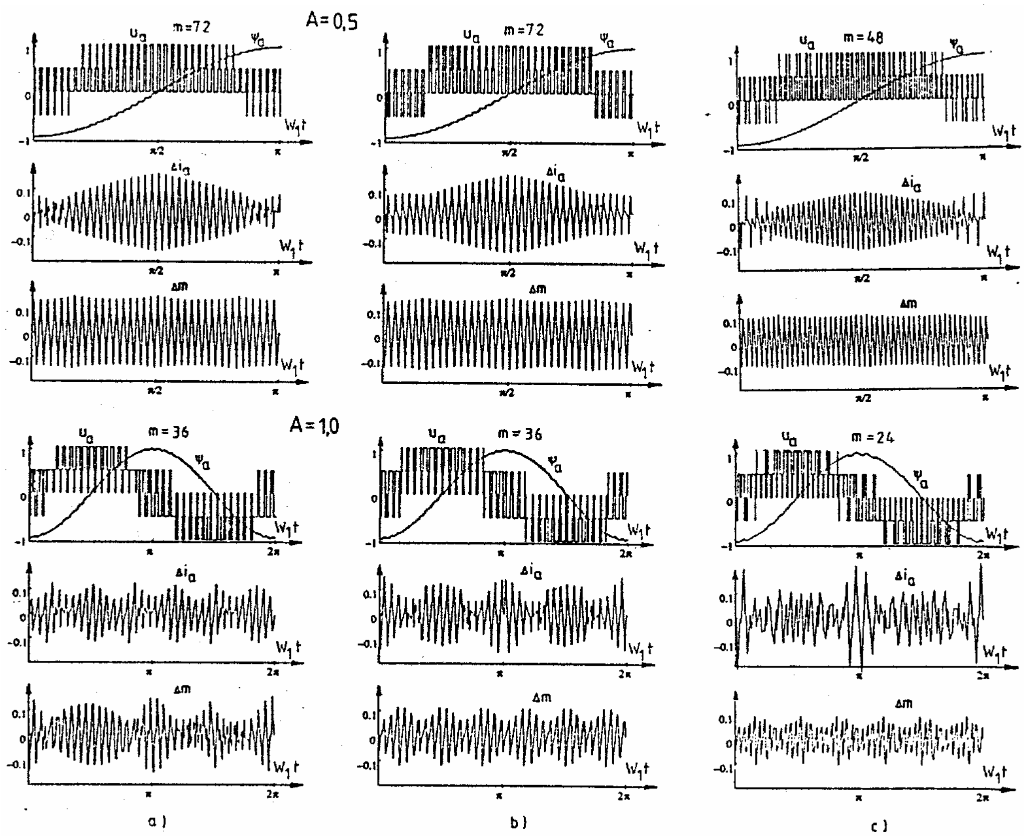


Fig. 6. Motor phase voltage, flux and current and torque pulsations vs time, two-phase PWM, 120° cycle (a) and 60° cycle (b), three-phase PWM with space vector method (c)

(for the same commutation frequency) gives the highest current pulsations. But the torque ripples of the two-phase PWM are higher than those of the three-phase space vector modulation (and will be lower than those only for the three-phase PWM without third harmonic injection in the reference wave).

3. The motor harmonic losses and torque pulsations are mainly determined by the harmonic currents of the order $m \pm 1$, $2m \pm 1$, $3m \pm 1$ and $m \pm 2$, $2m \pm 2$, $m \pm 4$. At the same time the amplitudes of order $Km \pm 4$, $Km \pm 5$ and $Km \pm 7$ are not significant (*Fig. 4* note that the scale of *Fig. 4c* is twice as big as that of *Figs. 4a–4b*).
4. The motor voltage amplitudes were derived for natural sampling, but the equations for $m \to \infty$ are valid for the regular sampling [9] and for the two-phase space vector methods too.

5. In the motor voltage spectrum – in comparison with three-phase PWMs without third harmonic injection – a lot of new components must be taken into consideration. With a good approximation the harmonic pairs of the important orders $Km \pm n'$ have the same values (for $m > 18$).
6. Several remarks must be made:
   a) the zero sequence components of the motor voltage can also be calculated by means of the analytical equations (they contain the terms with Bessel functions of orders 1, 5, 7, 11...),
   b) all the harmonics amplitudes only depend on m owing to the dependence of the sum of the series similar to (A5) on m which for $m > 36$ becomes negligibly small,
   c) harmonics for $K = 0$ consist of the fundamental one (which is equal to A with an acuracy better than 0.1% when $m > 36$) and zero sequence harmonics of the reference wave with the orders 0, 3, 9, 15... and amplitudes:

$$U_0 = 1 - \frac{3\sqrt{3}}{2\pi}A; \qquad U_\nu = \frac{3\sqrt{3}}{\pi\left(\nu^2 - 1\right)}A. \tag{15}$$

7. Although the equations were derived for synchronous modulation technique, the numerical computations have shown that the results are valid for asynchronous PWMs too. This statement is confirmed by [7] where the motor voltage harmonics of two-phase PWMs with 120° cycle are determined as usually using three-phase PWMs with the decomposed in Fourier series reference wave with harmonics according to (15). In this case the voltage spectra are independent of the fact whether the synchronous or asynchronous modulation techniques are used.

3. Transistor Load

The conduction losses of the transistors and diodes on the bridge side to which the motor phases are connected on 120° will be higher than on the other side, especially for the low voltage region. In *Fig. 7* for $A = 0.1$ and $A = 2/\sqrt{3}$ the transistor and diode currents are drawn on a period for the positive (+) and negative (−) inverter sides (about rated load condition, $m = 36$). The average current and square of the rms current of both sides as functions of the fundamental voltage are presented in *Fig. 8*. It is seen that the load of the two bridge sides of the inverter considerably differ in the first case and have about the same value for the second one.

The different load of the transistors is a little balanced by higher switching losses of the negative transistors. At the same time the load of negative diodes will be very small in comparison with the load of positive diodes.

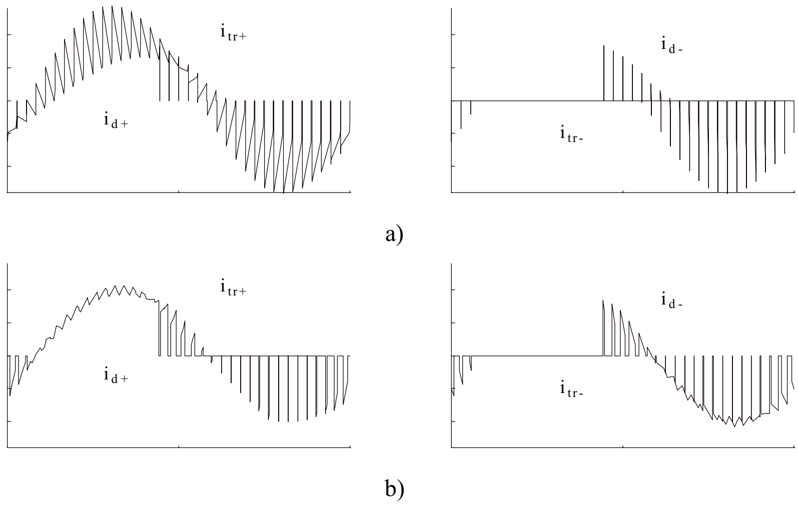


Fig. 7. Transistor and diode currents vs time for $m = 36$ a) $A = 0.1$ b) $A = 2/\sqrt{3}$

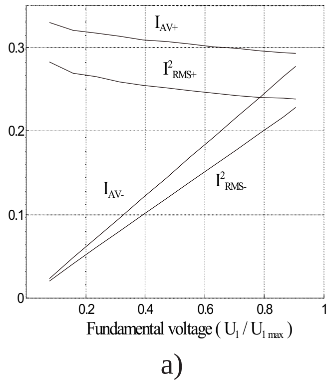
4. Verification of the Results

The theoretical calculations were verified by straight computer Fourier analysis of $u_{a0}(t)$ function and the accurate equations provide 100% accuracy of calculations both for harmonic amplitudes and its phase angles, but only in the case if all the terms with Bessel function of different orders with sensitive values are taken into account. The Bessel function variable is $(Km + n)\sqrt{3}A\pi/(2m)$. For $A \leq 2/\sqrt{3}$ and $K \leq 3$ its highest value is about $3\pi = 9.42$ since all the terms with Bessel functions of orders over 15 can be omitted. At the same time, e.g., for $K = 1$ one can neglect Bessel functions of orders over 7. The accurate calculation especially for PWMs according to *Fig. 3b* is complicated, too, since the terms with Bessel functions of even orders have to be taken into account.

The approximated equations provide a simpler calculation of harmonic amplitudes for any initial phase angle of the carriers. It was shown (See App.) that for PWMs in *Fig. 3a* the deviations of estimated amplitudes were for $K \leq 3$ less than 1% if $m > 36$. For PWMs in *Fig. 3b* it is valid if $m > 66$.

5. Conclusion

The novel accurate and approximated equations of motor voltage harmonics for two-phase PWMs with 120° cycle are given and it is shown that this PWM can be realized by non-synchronized modulation technique. The motor voltage spectra contain harmonics of order $\nu = Km \pm n'$ where K and n are positive integers and m is the relation of the carrier and the fundamental frequencies. The main harmonics



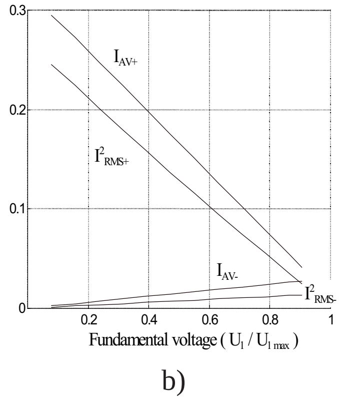


Fig. 8. Average current and square of rms current values of transistors (a) and diodes (b) of the positive $(+)$ and negative $(-)$ sides of the inverter bridge

have the orders $m \pm 1$, $2m \pm 1$, $3m \pm 1$ and $m \pm 2$, $2m \pm 2$ (hence n' can be even and odd). Non-zero sequence harmonic amplitude equations consist of the sum of the terms with Bessel functions of the orders n', 3, 9, 15 etc. if n' is odd but only order of 3, 9, 15 etc. if n' is even, while the zero sequence harmonic amplitude equations contain Bessel functions of orders 1, 5, 7, 11, 13 etc. The motor harmonic losses and torque pulsations are about the same as those for the other types of two-phase PWMs. The values of these losses – for the same switching frequency of the inverter – are considerably higher in the low voltage region and lower in the high one than those for three-phase PWMs. But the torque-ripples are higher than those for three-phase space vector PWMs in the whole voltage control region. The transistor and diode conduction and switching losses are different on the positive and negative sides of the inverter.

Acknowledgement

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Appendix

a) The intersection point *Eqs*. (8) can be transformed as follows:

$$\gamma i + \pi = \alpha_{i-1} + \pi - \sqrt{3} A \gamma \sin\left(\alpha_{i-1} + \pi\right),$$
$$\gamma i = \alpha_i - \sqrt{3} A \gamma \sin \alpha_i, \qquad \text{(A1)}$$

while taking into account (9) the intersection point equations will be:

$$\gamma i + 2\pi/3 = \alpha_{i-1} + 2\pi/3 - \sqrt{3}A\gamma \sin(\alpha_{i-1} + 2\pi/3),$$
$$\gamma i - \pi/3 = \alpha_i - \pi/3 - \sqrt{3}A\gamma \sin(\alpha - \pi/3), \tag{A2}$$

according to WATSON G.N. (A Treatise on the Theory of Bessel Functions, Cambridge, 1966, pp. 553–554). If

$$M = E - \varepsilon \sin E, \tag{A3}$$

then

$$e^{j\nu E} = \nu \sum_{p=1}^{\infty} \frac{1}{p} \left[J_{p-\nu}(p\varepsilon) \cdot -e^{jpM} - J_{p+\nu}(p\varepsilon) \cdot e^{-jpM} \right], \tag{A4}$$

where $J_{p-\nu}(p\varepsilon)$ and $J_{p+\nu}(p\varepsilon)$ are first kind Bessel functions at $p-\nu$ and $p+\nu$ orders, respectively.

In our application for the important harmonics $p-\nu$ will have small values but $p+\nu$ high ones, therefore the values of $J_{p+\nu}(p\varepsilon)$ can be neglected. With (A1), (A4) and (7) the following equation is valid (sign $U_{dc} = -1$):

$$U_\nu = -\frac{2}{j\nu\pi} \sum_{i=2}^{4m/3-2} \left[e^{j\nu\alpha_i} - e^{-j\nu\pi} e^{j\nu(\alpha_{i-1}+\pi)} \right]$$
$$= -\frac{2}{j\pi} \sum_{i=2}^{4m/3-2} \sum_{p=1}^{\infty} \frac{1}{p} J_{p-\nu}\left(p\sqrt{3}A\gamma\right) \left[e^{p\gamma i} - e^{-j\nu\pi} e^{jp(\gamma i+\pi)} \right].$$

After the change of order of the summation and taking into account that $|\nu - p|$ must be odd:

$$U_\nu = -\frac{4}{j\pi} \sum_{p=1}^{\infty} \frac{1}{p} J_{p-\nu}\left(p\sqrt{3}A\gamma\right) \sum_{i=2}^{4m/3-2} e^{p\gamma i}.$$

Let be $p = Km + n$. In this case the last summation is equal to

$$\sum_{i=2}^{4m/3-2} e^{j(Km+n)\frac{\pi}{2m}i} = (-1)^K \frac{\sin n\pi/3}{\sin n\pi/m} e^{jn\frac{\pi}{3}}.$$

With that:

$$U'_\nu = -\frac{4}{j\pi} \sum_{p=1}^{\infty} J_{p-\nu}\left(p\sqrt{3}A\gamma\right) (-1)^K \frac{\sin n\pi/3}{\sin n\pi/m} e^{jn\pi/3}.$$

After similar computation ($\nu = Km + n'$):

$$U''_\nu = -\frac{2}{j\pi}\sum_{i=4m/3+2}^{8m/3-2}\sum_{p=1}^{\infty}\frac{1}{p}J_{p-\nu}\left(p\sqrt{3}A\gamma\right) \times \left[e^{j\nu\pi/3}e^{jp(\gamma i-\pi/3)} - e^{-j\nu 2\pi/3}e^{jp(\gamma i+2\pi/3)}\right] = -\frac{4}{j\pi}\sum_{p=1}^{\infty}J_{p-\nu}\left(p\sqrt{3}A\gamma\right)e^{j(n'+n)\pi/3}\cdot(-1)^K\frac{\sin n\pi/3}{\sin n\pi/m}e^{jn\pi/3}.$$

Then:

$$U_\nu = U'_\nu + U''_\nu = -\frac{4}{j\pi}\sum_{p=1}^{\infty}\frac{1}{p}J_{p-\nu}\left(p\sqrt{3}A\gamma\right)\left[1+e^{j(n'+n)\frac{\pi}{3}}\right] \times(-1)^K\frac{\sin n\pi/3}{\sin n\pi/m}e^{jn\pi/3} = \frac{8}{\pi}j\sum_{p=1}^{\infty}\frac{1}{p}J_{p-\nu}\left(p\sqrt{3}A\gamma\right)(-1)^K\frac{\sin n\pi/3}{\sin n\pi/m} \times\cos\left[(n'+n)\frac{\pi}{6}\right]e^{jn\frac{\pi}{2}}\,e^{jn'\frac{\pi}{6}}.$$

In the coordinate system fixed according to *Fig.* 2 $\alpha^* = \alpha + 5\pi/6$, therefore:

$$U_\nu = -\frac{8}{\pi}j\sum_{p=1}^{\infty}\frac{1}{p}J_{p-\nu}\left(p\sqrt{3}A\gamma\right)(-1)^K\frac{\sin n\pi/3}{\sin n\pi/m} \times\cos\left[(n'+n)\frac{\pi}{6}\right]e^{jn\frac{\pi}{2}}(-1)^{n'}e^{jkm5\pi/6}. \tag{A5}$$

Let be $\nu = Km + 1$, in this case for $K \le 2$ only Bessel functions of orders $|p-\nu| = 1$ and $|p-\nu| = 3$ have considerable values, hence only that $|p-\nu| = |n-1|$ must be taken into consideration, for which $n = 0$ ($p-\nu = -1$), $n = -2$ ($p-\nu = -3$) and $n = 4$ ($p-\nu = 3$).

The component with $|p-\nu| = 9$ only gives considerable value for $K > 3$. After the similar computations for $m \to \infty$:

$$\left|U'''_{Km+1}\right| = \left|\frac{27}{j\pi^2 K20}J_9\left(\sqrt{3}AK\frac{\pi}{2}\right)\right|.$$

For $\nu = Km \pm 2$ or $\nu = Km \pm 4$ the results have the same structure, but only the components with $n = 3, 9, 15$ differ from zero.

The dc component of u_{a0} can be computed as in [8]. The result is:

$$U_0 = 1 - \frac{8}{\pi}\sum_{1}^{\infty}\frac{1}{p}J_p(p\sqrt{3}A\gamma)\frac{\sin^2 p\pi/3}{\sin\ p\pi/m},$$

where p= 1, 5, 7, ... etc. For $m > 36$ only $p = 1$ is important:

$$U_0 = 1 - \frac{6}{\pi}J_1\left(\sqrt{3}A\gamma\right)\frac{1}{\sin\ \pi/m},$$

and for $m \to \infty$ with a very good approximation $J_1\left(\sqrt{3}A\gamma\right) = \sqrt{3}A\gamma/2$ the dc component of the reference wave (15) is obtained:

$$U_0 = 1 - \frac{6}{\pi}\frac{A\sqrt{3}\pi}{4m}\frac{m}{\pi} = 1 - \frac{3\sqrt{3}}{2\pi}A.$$

b) According to *Fig. 3b* the modified intersection point equations for $0 \le \alpha \le 2\pi/3$ will be:

$$\gamma i - 2\gamma = \alpha_{i-1} - \sqrt{3}A\gamma \sin\alpha_{i-1},$$
$$\gamma i + 2\gamma + \pi = \alpha_i + \pi - \sqrt{3}A\gamma \sin(\alpha_i + \pi)$$

and for $2\pi/3 \le \alpha \le 4\pi/3$:

$$\gamma i - 2\gamma - \pi/3 = \alpha_{i-1} - \pi/3 - \sqrt{3}A\gamma \sin\left(\alpha_{i-1} + -\pi/3\right),$$
$$\gamma i + 2\gamma + 2\pi/3 = \alpha_i + 2\pi/3 - \sqrt{3}A\gamma \sin\left(\alpha_i + 2\pi/3\right).$$

After the similar computations as in App. a) and with sign $U_{dc} = 1$ a more complicated expression is obtained (in the coordinate system fixed according to *Fig.2*):

$$U_\nu = \frac{4}{\pi\nu}\sin\nu\frac{\pi}{3}e^{-j\nu\frac{\pi}{2}} + U'_\nu + U''_\nu,$$

where for odd $|p-\nu|$:

$$U'_\nu = -\frac{8}{\pi}j\sum_{p=1}^{\infty}\frac{1}{p}J_{p-\nu}\left(p\sqrt{3}A\gamma\right)\frac{\sin n\pi/3}{\operatorname{tg} n\pi/m}\cdot\cos\left[(n'+n)\frac{\pi}{6}\right]e^{jn\frac{\pi}{2}}(-1)^{n'}e^{jkm5\pi/6},$$

while for even $|p-\nu|$:

$$U''_\nu = \frac{8}{\pi}\sum_{p=1}^{\infty}\frac{1}{p}J_{p-\nu}\left(p\sqrt{3}A\gamma\right)\sin n\pi/3\cdot\cos\left[(n'+n)\frac{\pi}{6}\right]e^{jn\frac{\pi}{2}}(-1)^{n'}e^{jkm5\pi/6}.$$

For high values of m only U'_ν produces considerable values, therefore for $m \to \infty$ the result will be the same as in App. a) and harmonic amplitudes can be written as in (12) and (14).

Examples. For m= 36 and A =0.5 the straight digital calculations for different initial phase angle of the carrier wave give: $U_{35} \cong U_{37} = 0.74310 \div 0.74331$, $U_{34} \cong U_{38} = 0.06653 \div 0.06833$, $U_{137} \cong U_{151} = 0.03967 \div 0.04260$ (related to fundamental component value). Using (12) and (14) gives:

$$U_{m\pm1} = \frac{1}{0.5}\left(\frac{4}{\sqrt{3}\pi}0.5345 - \frac{9}{2\pi^2}0.0466\right) = 0.7433$$

(maximum deviation: 0.15%).

$$U_{m\pm2} = \frac{1}{0.5}\left(\frac{36}{5\pi^2}0.0466\right) = 0.06806$$

(max. deviation: 2.2%, for $m = 66$ the deviation decreases to 0.3%).

With Bessel functions of order 7 and 3:

$$U_{4m\pm7} = \frac{1}{0.5}\left(\frac{0.08215}{\sqrt{3}\pi} + \frac{9\cdot 0.2710}{40\pi^2}\right) = 0.04255$$

(max. deviation: 4.8%).

Taking into account the Bessel function of order $n = 9$:

$$U_{4m+7} = 0.04255 - \frac{27}{32\pi^2}\frac{0.0111}{0.5} = 0.04065$$

(max. deviation: 4.8 %, for $m = 66$ it decreases to 1.3%).

For $A = 1.0$ the digital calculation gives $U_{35} \cong U_{37} = 0.20214 \div 0.2023$ and (12) gives:

$$U_{35,37} = \frac{4}{\pi\sqrt{3}}\cdot 0.43521 - \frac{9}{2\pi^2}\cdot 0.2579 = 0.2023$$

(max. 0.08%).

Similarly $U_{71} \cong U_{73} = 0.18713 \div 0.18917$, but (12) gives:

$$U_{71,73} = \frac{2}{\sqrt{3}\pi}\cdot 0.34414 + \frac{9}{4\pi^2}\cdot 0.27104 = 0.1883$$

(max. 0.6%).

Or $U_{32} \cong U_{40} = 0.13405 \div 0.13506$, acccording to (14):

$$U_{32,40} = \frac{36}{7\pi^2}\,0.2579 = 0.1344,$$

(max. 0.5%).

The $\nu = 3$ zero sequence harmonic belongs to $p_1 = n = 2$ and $p_2 = n = 4$, therefore, for $m \to \infty$:

$$U_3 = \frac{3}{2\pi^2} m J_1\left(2\sqrt{3}A\frac{\pi}{2m}\right) - \frac{3}{8\pi^2} m J_1\left(4\sqrt{3}A\frac{\pi}{2m}\right) \cong \frac{3\sqrt{3}}{8\pi} = 0.2067.$$

Thus, it is equal to the third harmonic amplitude in the reference wave. For $A = 0.5$ and $m = 36$ the straight calculation gives $U_3 = 0.2095$ while for $m = 66$ it is $U_3 = 0.2075$. Similarly from (A4) e.g. the zero-sequence harmonic of order $m + 3$ is $U_{m+3} = 0.1620$ ($m \to \infty$). The computer gives 0.1640 if $m = 36$ (deviation 1.2%) and 0.1610 if $m = 66$ (deviation 0.6%).

References

[1] JENNI, F. – WUEEST, D., *Steuerverfahren für selbstgeführte Stromrichter*, Hochschuleverlag AG an der ETH, Zürich, 1995.

[2] HALÁSZ, S. – CSONKA, G. – HASSAN, A. A.M., Generalized Harmonic Loss Curves of ac Motors Fed from Two and Three Level Inverters, In *Proc. of the Int. Conf. on Ind. Electronics Control and Instrumentation*, August 5–9., 1996, Taipei, Taiwan, pp. 957–963.

[3] NIELSEN, P. E. – THOMSON, E. CH. – NIELSEN, M. T., Digital Voltage Vector Control with Adaptive Parameter Tuning, In *Proc. of 3rd European Conf. on Power Electronics and Applications*, Aachen, 1989, pp. 313–318.

[4] H. VAN DER BROECK, Analysis of the Harmonics in Voltage Fed Inverter Drives Caused by PWM Schemes with Discontinuous Switching Operation, In *Proc. of 3rd European Conf. on Power Electronics and Applications*, Firenze, 1991, pp. 3-261-266.

[5] HALÁSZ, S. – CSONKA, G. – HASSAN, A. A. M., Sinusoidal PWM Techniques with Additional Zero-Sequence Harmonics, In *Proceedings of the Int. Conf. on Ind. Electronics Control and Instrumentation*, Sept. 5–9. 1994, Bologna, Italy, pp. 273–289.

[6] KOLAR, J. W. – ERTL, H. – ZACH, F., Influence of the Modulation Method on the Conduction *l* and Switching Losses of a PWM Converter System, *IEEE Trans. Ind. Applications*, **27** No. 6 Nov.-Dec. (1991), pp. 1063–1075.

[7] HALÁSZ, S. – HUU, B. T., Two-Phase PWM Technique for Inverter Fed ac Drives, In *Proc. of the Int. Conf. on Ind. Electronics Control and Instrumentation*, Nov. 1997. New Orleans, USA, pp. 688–693.

[8] HALÁSZ, S., Voltage Spectrum of Pulse-Width-Modulated Inverters, *Periodica Polytechnica, El. Engineering*, **25** No. 2 (1981), pp. 135–145.

[9] HALÁSZ, S. – ZAHAROV, A., Two-Phase PWM Technique with 120° Dead Time in Inverter Fed ac Drives, In *Proc.of Symposium on Power Electron., Electr. Drives, Adv. Machines, Power Quality*, Sorrento, Italy, 3–5 June 1998, pp. P2-17-22.

[10] HALÁSZ, S. – ZAHAROV, A., Voltage Spectra of Two-Phase PWM Techniques in Inverter Fed ac Drives, In *Proc.of the IEEE International Symposium on Ind. Electronics*, Pretoria, South Africa, 7–10 July 1998.

[11] HALÁSZ, S. – ZAHAROV, A., Voltage Spectra of Two-Phase PWM Technique with 120° cycle, In *Proc.of 8th International Power Electronics and Motion Control Conference*, Prague, Czech Republic, 8–10 September 1998.