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PROPERTIES OF THE CO-ENERGY FUNCTION FOR AC MACHINES WITH NON-LINEAR MAGNETIC CIRCUIT

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Abstract

To create equations of AC machines with non-linear magnetic circuit Lagrange's formalism can be used. A starting point for this is the co-energy function. In the paper properties of the co-energy function of most important types of AC machines are discussed. Introducing equivalent magnetising currents as substitute variables, the co-energy function can be rather easily foreseen. Qualitative analysis of physical phenomena in a given type of machine is a base of it. An approximation of the co-energy function for non-linear magnetic circuit is also presented.

Keywords: co-energy function, energy-based approach, magnetic non-linearity, AC machine equations.

1. Introduction

Magnetic non-linearity is still a problem when analysing any kind of electrical machines. 'Classic' models operating with inductances of machine windings are valid under the assumption of magnetic linearity. Any improvement of such models, even if leading to acceptable results, is theoretically non-correct. To create equations of AC machines accounting for magnetic non-linearity, the way has to be correct from very beginning. The theory of electromechanical energy conversion [1] is an excellent base to do it. The Lagrange's equations of AC machines in a general form

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial E_{\mathrm{co}}}{\partial i_n} = u_n - R_n i_n; \qquad \text{for} \qquad n = 1, \ldots, N, \tag{1a}$$

$$J\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} = \frac{\partial E_{\mathrm{co}}}{\partial \varphi} + T_m \tag{1b}$$

require the co-energy function E_{co} only. The magnetic co-energy of an AC machine is a function of $N+1$ variables, i.e. currents of machine windings $i_1, i_2, \ldots, i_N$ and a rotation angle φ

$$E_{\mathrm{co}}(\varphi, i_1, i_2, \ldots, i_N). \tag{2}$$

Then, it is a multi-variable function and may become very complicated when nonlinearity is taken into account. Considerations simplify significantly if two magnetic circuits in electrical machines for leakage fluxes and for the main flux are treated separately. In fact, these two circuits are mutually dependent, however, their separation gives acceptable results. From this assumption follows that leakage fluxes of individual coils depend nonlinearly on their own currents but the main flux has to be considered as a result of all coil currents. The saturation of a magnetic circuit of the main flux is then an effect of all currents. The co-energy function necessary to create machine's equations can also be divided into two parts: stored in the leakage magnetic circuits and in the main magnetic circuit

$$E_{\text{co}} = E_m(\varphi, i_1, \ldots, i_N) + \sum_{n=1}^{N} E_{\sigma n}(i_n). \tag{3}$$

Introducing equivalent magnetising currents can reduce the number of variables of the first term in this sum [8], [11].

An idea of equivalent magnetising currents follows from the formula for the total MMF magnetising the main magnetic circuit of a machine given below

$$\Theta(x,t) = \sum_{n=1}^{N} \sum_{\rho=1}^{\infty} i'_{n\rho}(t) \cos \rho(x - x_n) = \sum_{\rho=1}^{\infty} i_{\mu,\rho}(t) \cos \rho(x - \alpha_\rho),$$
$$i'_{n\rho} = i_n(t) \frac{2}{\pi} \left(\frac{w_n k_{n\rho}}{\rho} \right),$$

where w_n is a turn number and $k_{n\rho}$ is a winding coefficient of an individual machine winding and x_n is an angular position of its symmetry axis. An equivalent magnetising current of ρ-harmonic $i_{\mu,\rho}(t)$ is determined by the expression

$$\left(i_{\mu,\rho}(t)\right)^2 = \sum_{n=1}^{N} \sum_{k=1}^{N} i'_{n\rho} i'_{k\rho} \cos \rho(x_n - x_k) \tag{4}$$

and an angle of its angular position α_ρ follows from the formula

$$\rho\alpha_\rho = \arctan \left(\frac{\sum_{n=1}^{N} i'_{n\rho} \sin(\rho x_n)}{\sum_{n=1}^{N} i'_{n\rho} \cos(\rho x_n)} \right). \tag{5}$$

Using the equivalent magnetising currents and their angular positions $i_{\mu,1}, \alpha_1, \cdots, i_{\mu,\rho}, \alpha_\rho, \cdots, i_{\mu,R}, \alpha_R$ as substitutional variables, the co-energy function can be expressed as

$$E_m(\varphi, i_1, \ldots, i_N) \approx E_m(i_{\mu,1}, \alpha_1, \cdots, i_{\mu,\rho}, \alpha_\rho, \cdots, i_{\mu,R}, \alpha_R).$$

The Lagrange's equations of AC machines can be then rewritten into the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial E_{\sigma n}}{\partial i_n} + \sum_{r=1}^{R}\left(\frac{\partial E_m}{\partial i_{\mu,\rho}}\frac{\partial i_{\mu,\rho}}{\partial i_n} + \frac{\partial E_m}{\partial(\rho\alpha_\rho)}\frac{\partial(\rho\alpha_\rho)}{\partial i_n}\right) = u_n - R_n i_n;$$
$$\text{for} \qquad n = 1, \ldots, N, \tag{6a}$$

$$J\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} = \sum_{r=1}^{R}\left(\frac{\partial E_m}{\partial i_{\mu,\rho}}\frac{\partial i_{\mu,\rho}}{\partial \varphi} + \frac{\partial E_m}{\partial(\rho\alpha_\rho)}\frac{\partial(\rho\alpha_\rho)}{\partial \varphi}\right) + T_m. \tag{6b}$$

It should be noticed that derivatives

$$\frac{\partial i_{\mu,\rho}}{\partial i_n}, \frac{\partial i_{\mu,\rho}}{\partial \varphi} \text{ and } \frac{\partial(\rho\alpha_\rho)}{\partial i_n}, \frac{\partial(\rho\alpha_\rho)}{\partial \varphi}$$

can be enveloped from expressions for equivalent magnetising currents and respective angular positions. Then, in the Lagrange's equations the derivatives

$$\frac{\partial E_m}{\partial i_{\mu,\rho}}, \frac{\partial E_m}{\partial(\rho\alpha_\rho)}$$

are only unknown. They follow from the co-energy function determined by the new substitutional variables. Reducing the number of MMF harmonics to the most important ones, general properties of that co-energy function can be predicted by physical considerations.

The main aim of this paper is to create the co-energy functions for typical AC machines taking into account a few important MMF harmonics. The co-energy function depends on geometry of the main magnetic circuit and the number of considered MMF harmonics. AC machines can be divided into two classes: those having smooth air-gap and those with a salient-pole rotor. The most significant are p, $3p$ and at least $5p$ MMF harmonics, where p is the pole-pair number.

2. The Co-Energy Functions for Machines with a Smooth Air-Gap

Idealising geometry of machine's magnetic circuit to the coaxial cylinders with smooth air-gap it should be noticed that there is not any preferable symmetry axis in radial direction. Then, magnetising such a circuit by a total MMF of a sinusoidal form with p pole-pair number, the magnetic co-energy stored in it cannot depend on a position of the MMF with respect to machine's circumference but on the amplitude of a total MMF only. This is valid for any machine with an arbitrary number of independent windings if an individual winding produces sinusoidal MMF. It means that the co-energy function depends on an equivalent magnetising current of p-harmonic $i_{\mu,p}$ only. In conclusion, considering the p MMF harmonic only the co-energy of any machine with smooth air-gap takes the form

$$E_m = E_m(i_{\mu,p}). \tag{7}$$

The conditions change when a third with respect to the p-harmonic is also taken into account. In *Fig. 1* mutual positions of these two harmonics are schematically shown, where the angles α_p, α_{3p} denote angular positions of maxima of respective MMFs. The shape of a total MMF depends on an angle of position of the $3p$-harmonic with respect to the p-harmonic $(\alpha_p - \alpha_{3p})$ and repeats $3p$ times per revolution (with a period $2\pi/3p$). There are two equivalent magnetising currents $i_{\mu,p}$ and $i_{\mu,3p}$, then the co-energy function depends on three independent variables $i_{\mu,p}$, $i_{\mu,3p}$, $\alpha_p - \alpha_{3p}$. Because it is periodic and even with respect to the angle $(\alpha_p - \alpha_{3p})$ it can be expanded into a Fourier series in which cosines terms appear only. The coefficients of this series depend on two variables $i_{\mu,p}$, $i_{\mu,3p}$.

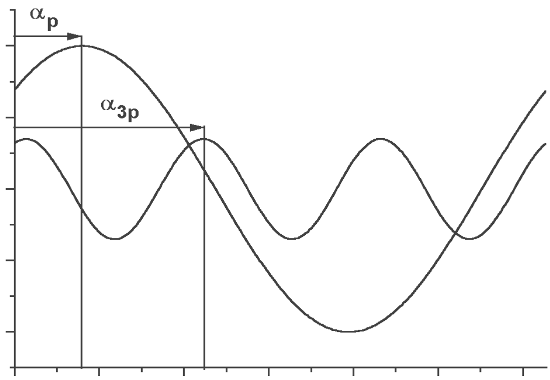

Fig. 1. The p- and the $3p$- harmonics of a total MMF

Finally, the co-energy function can be written in the form

$$E_m = E_m(i_{\mu,p}, i_{\mu,3p}, \alpha_p - \alpha_{3p}) = \sum_{k=0}^{\infty} E_{k3p}(i_{\mu,p}, i_{\mu,3p}) \cos k3p(\alpha_p - \alpha_{3p}). \quad (8)$$

It should be noticed that it has to reduce to the function (7) at $i_{\mu,3p} \equiv 0$.
For its simplest approximation

$$E_m = E_0(i_{\mu,p}, i_{\mu,3p}) + E_{3p}(i_{\mu,p}, i_{\mu,3p}) \cos 3p(\alpha_p - \alpha_{3p}) \quad (9)$$

two functions $E_0(i_{\mu,p}, i_{\mu,3p})$, $E_{3p}(i_{\mu,p}, i_{\mu,3p})$ of two variables $i_{\mu,p}$, $i_{\mu,3p}$have to be determined.

When harmonics p and $5p$ are most important, the co-energy function of the form

$$E_m = E_0(i_{\mu,p}, i_{\mu,5p}) + E_{3p}(i_{\mu,p}, i_{\mu,5p}) \cos 5p(\alpha_p - \alpha_{3p}) \quad (10)$$

can be predicted in the same way.

Considering the MMF harmonics of orders p, $3p$ and $5p$, an expression for the co-energy becomes more complicated because the number of independent variables increases to five. There are three equivalent magnetising currents $i_{\mu,p}$, $i_{\mu,3p}$, $i_{\mu,5p}$ and two relative angles: an angular position of the third harmonic with respect to the main harmonic $(\alpha_p - \alpha_{3p})$ and an angular position of the fifth harmonic with respect to the main $(\alpha_p - \alpha_{5p})$. The co-energy has to be periodic with respect to each of those angles and repeats $3p$ or $5p$ times per revolution, respectively. Then, the co-energy function in that case can be foreseen in the form

$$\begin{aligned} E_m &= E_m(i_{\mu,p}, i_{\mu,3p}, i_{\mu,5p}, \alpha_p - \alpha_{3p}, \alpha_p - \alpha_{5p}) \\ &= \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} E_{k3p,l5p}(i_{\mu,p}, i_{\mu,3p}, i_{\mu,5p}) e^{jk3p(\alpha_p - \alpha_{3p})} e^{jl5p(\alpha_p - \alpha_{5p})}. \end{aligned} \tag{11}$$

If the sums are limited to $(1, -1)$ the function takes the form

$$\begin{aligned} E_m = {} & E_0(i_{\mu,p}, i_{\mu,3p}, i_{\mu,5p}) + E_{3p}(i_{\mu,p}, i_{\mu,3p}, i_{\mu,5p}) \cos 3p[(\alpha_p - \alpha_{3p}) + \gamma_{3p}] \\ & + E_{5p}(i_{\mu,p}, i_{\mu,3p}, i_{\mu,5p}) \cos 5p[(\alpha_p - \alpha_{5p}) + \gamma_{5p}] \\ & + E_{3p,5p}(i_{\mu,p}, i_{\mu,3p}, i_{\mu,5p}) \cos 3p[(\alpha_p - \alpha_{3p}) + \delta_{3p}] \\ & \times \cos 5p[(\alpha_p - \alpha_{5p}) + \delta_{5p}] \end{aligned} \tag{12}$$

which can be reduced to (7) (9) or (10), respectively, according to the assumptions for the total MMF.

3. The Co-Energy Functions for Salient-Pole Machines

Magnetic circuit of AC machines having salient-pole rotor with $2p$ poles has the same number of preferable magnetising directions. Magnetising such a circuit by a total MMF repeated p-times sinusoidally on machine's circumference, magnetic co-energy depends on a position of the MMF with respect to a symmetry axis of that circuit. In order to find a formula for the co-energy for that class of machines the following, rather simple, reasoning is very useful.

For an elementary energy converter, shown schematically in *Fig.2*, under the assumption that the magnetic circuit is linear, the co-energy function has the form

$$E_{\mathrm{co}} = \frac{1}{2}(L_0 + L_2 \cos 2p\varphi + L_4 \cos 4p\varphi + \cdots) i^2,$$

where i is the winding current and φ is an angle of rotor position with respect to the symmetry axis of a phase, $L_0, L_2, L_4, \ldots$ are harmonic coefficients of the phase inductance. For non-linear magnetic circuit, harmonic coefficients $L_0, L_2, L_4, \ldots$ are nonlinearly dependent on the current but co-energy is still periodic with respect to the angle φ. Then, the co-energy function takes the form

$$E_{\mathrm{co}} = E_0(i) + E_2(i) \cos 2p\varphi + E_4(i) \cos 4p\varphi + \cdots .$$

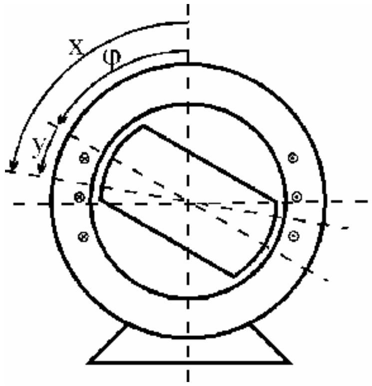

Fig. 2. An elementary one-phase converter

This simple statement is very useful to create the co-energy function of multi-winding machines with non-linear magnetic circuit if each winding produces sinusoidal p-harmonic MMF. The equivalent magnetising current $i_{\mu,p}$ and its angle position α_p with respect to a symmetry axis determine uniquely the total MMF. They could substitute a phase current i and a rotor position angle φ in the formula given above. Then, the co-energy function of a multi-winding salient-pole machine can be written in the form

$$E_m = E_m(i_{\mu,p}, \alpha_p) = \sum_{k=0}^{\infty} E_{k2p}(i_{\mu,p}) \cos k2p\alpha_p. \tag{13}$$

Its simplest approximation contains two terms only

$$E_m = E_0(i_{\mu,p}) + E_2(i_{\mu,p}) \cos 2p\alpha_p \tag{14}$$

and requires two non-linear functions $E_0(i_{\mu,p})$, $E_2(i_{\mu,p})$ of one variable $i_{\mu,p}$.

Considering the p and $3p$ MMF harmonics, there are two equivalent magnetising currents $i_{\mu,p}, i_{\mu,3p}$ and respective angles α_p, α_{3p}. Thus, the co-energy function depends on four variables and should be foreseen in the form

$$E_m = E_m(i_{\mu,p}, i_{\mu,3p}, \alpha_p, \alpha_{3p}) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} E_{kp,l3p}(i_{\mu,p}, i_{\mu,3p}) e^{jkp\alpha_p} e^{jl3p\alpha_{3p}}. \tag{15}$$

Its simplest approximation has to be reducible

- for $i_{\mu,3p} \equiv 0$ to the form $E_m = E_{0,0}(i_{\mu,p}) + E_{2,0}(i_{\mu,p}) \cos 2p\alpha_p$
- for $i_{\mu,p} \equiv 0$ to the form $E_m = E_{0,0}(i_{\mu,3p}) + E_{0,2}(i_{\mu,3p}) \cos 6p\alpha_{3p}$
- and to the form $E_m = E_{0,0}(i_{\mu,p}, i_{\mu,3p}) + E_{3,1}(i_{\mu,p}, i_{\mu,3p}) \cos 3p(\alpha_p - \alpha_{3p})$ for cylindrical rotor.

It is possible when limiting the sums in a series (15) as follows

$$E_m = \sum_{k=-3}^{3} \sum_{l=-2}^{2} E_{kp,l3p}(i_{\mu,p}, i_{\mu,3p}) e^{jkp\alpha_p} e^{jl3p\alpha_{3p}}. \tag{16}$$

4. The Co-Energy Functions for Machines with Cylindrical but Eccentrically Located Rotor

Due to a rather small air-gap in induction machines, an eccentricity of rotor position in a stator frame could be a serious problem. Unsymmetrical location of a rotor changes geometry of the machine's main magnetic circuit. In that case, only one symmetry axis exists in spite of pole-pair number of machine's windings. Then, it is interesting to determine the co-energy function for the case when modelling saturation effects and the eccentricity ones together.

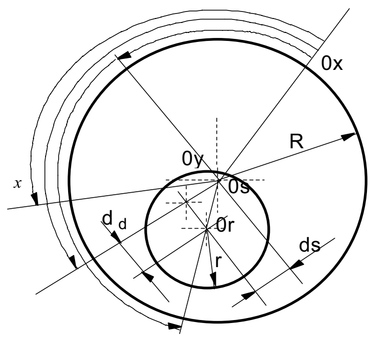

Fig. 3. Explanation of eccentricities: static, dynamic and mixed

Three types of eccentricity are schematically shown in *Fig. 3*:

- static, when the rotor rotation axis coincides with the rotor symmetry axis $0r$ but does not with the stator symmetry axis $0s$,
- dynamic, when the rotor rotation axis coincides with the stator symmetry axis $0s$ but does not with the rotor symmetry axis $0r$,
- mixed, when the rotor rotation axis does not coincide either with the rotor symmetry axis $0r$ or with the stator symmetry axis $0s$.

Under assumption that the total MMF in a machine is mono-harmonic of p order, there are two substitutional variables: an equivalent magnetising current – $i_{\mu,p}$ and an angular position of the MMF maximum with respect to a chosen reference axis – α_p. The air-gap geometry is determined by two values: a relative distortion of air-gap symmetry $\varepsilon = \frac{\delta_{\min}}{\delta}$ and an angular position of the air-gap minimum with respect to the same reference axis – η. The co-energy stored in the main magnetic circuit for all types of eccentricity depends on those four variables. Due to symmetry, the co-energy changes periodically with an angle $(p\alpha_p - \eta)$ twice per revolution.

$$
\begin{aligned}
E_m &= E_0(i_{\mu,p}, \alpha_p, \varepsilon, \eta) \\
&= E_0(i_{\mu,p}, \varepsilon) + E_2(i_{\mu,p}, \varepsilon)\cos 2(p\alpha_p - \eta) + E_4(i_{\mu,p}, \varepsilon)\cos 4(p\alpha_p - \eta) + \cdots .
\end{aligned}
\tag{17}
$$

For distinguished types of eccentricity, values ε and η are determined as follows

- static eccentricity $\varepsilon = \varepsilon_s = \frac{d_s}{\delta}$, $\eta = \gamma = \text{const.}$
- dynamic eccentricity $\varepsilon = \varepsilon_d = \frac{d_d}{\delta}$; $\eta = \varphi + \chi$; $\chi = \text{const.}$
- mixed eccentricity $\varepsilon = \sqrt{\varepsilon_s^2 + \varepsilon_d^2 + 2\varepsilon_s\varepsilon_d\cos(\varphi - \gamma)}$

$$
\eta = \arcsin\left(\frac{\varepsilon_d}{\varepsilon}\sin(\varphi - \gamma)\right) \text{ at } \varepsilon \neq 0.
$$

5. Approximation of Components of the Co-Energy Function

In all foreseen expressions for the co-energy functions, functions of one, two or three variables appear as Fourier coefficients mainly. They depend on the equivalent magnetising currents of respective MMF harmonics. It is difficult to find a general expression for these functions. However, using an approximation by one- or multivariable power series, some properties of each individual function can be foreseen.

The functions of one variable can be approximated by a power series of the form

$$
E_k(i_{\mu,1}) = \frac{1}{2}A_{k2}(i_{\mu,1})^2 + \frac{1}{4}A_{k4}(i_{\mu,1})^4 + \frac{1}{6}A_{k6}(i_{\mu,1})^6 + \cdots . \tag{18}
$$

It is commonly known that approximation by power series is not very efficient and needs numerous terms. However, these functions can have so different properties that till now a more effective approximation is not known. The co-energy function of machines with smooth air-gap only when p-harmonic is only considered can be described in a closed form based on the approximation of the 'classic' magnetisation curve $\psi = \psi(i)$ and using the relation

$$
E_m(i_\mu) = \int_0^{i_\mu} \psi(i)\,\mathrm{d}i. \tag{19}
$$

For the functions of two variables, the approximation done in [9] could be applied

$$E_k(i_{\mu,1}, i_{\mu,2}) = \frac{1}{2}\sum_{n=1}^{2}\sum_{m=1}^{2} A^k_{n,m} i_n i_m + \frac{1}{4}\sum_{n=1}^{2}\sum_{m=1}^{2}\sum_{r=1}^{2}\sum_{s=1}^{2} A^k_{n,m,r,s} i_n i_m i_r i_s + \cdots . \tag{20}$$

It seems to be rather complicated but practically it is the only possible way. A matrix form of those functions [9] can help to operate with them.

$$\begin{aligned} E_k(i_{\mu,1}, i_{\mu,2}) &= \frac{1}{2}\begin{bmatrix} i_{\mu,1} & i_{\mu,2} \end{bmatrix}\begin{bmatrix} A^k_{2,1} & A^k_{2,2} \\ A^k_{2,2} & A^k_{2,3} \end{bmatrix}\begin{bmatrix} i_{\mu,1} \\ i_{\mu,2} \end{bmatrix} \\ &+ \frac{1}{4}\begin{bmatrix} i_{\mu,1} & i_{\mu,2} \end{bmatrix}\begin{bmatrix} i_{\mu,1} & i_{\mu,2} & 0 & 0 \\ 0 & 0 & i_{\mu,1} & i_{\mu,2} \end{bmatrix} \\ &\times \begin{bmatrix} A^k_{4,1} & A^k_{4,2} & A^k_{4,2} & A^k_{4,3} \\ A^k_{4,2} & A^k_{4,3} & A^k_{4,3} & A^k_{4,4} \\ A^k_{4,2} & A^k_{4,3} & A^k_{4,3} & A^k_{4,4} \\ A^k_{4,3} & A^k_{4,4} & A^k_{4,4} & A^k_{4,5} \end{bmatrix}\begin{bmatrix} i_{\mu,1} & 0 \\ i_{\mu,2} & 0 \\ 0 & i_{\mu,1} \\ 0 & i_{\mu,2} \end{bmatrix}\begin{bmatrix} i_{\mu,1} \\ i_{\mu,2} \end{bmatrix} + \cdots . \end{aligned} \tag{21}$$

6. Examples of the Co-Energy Functions

The co-energy function (7) of an AC machine with smooth air-gap when only p-harmonic is taken into account depends on just one variable only. It can be done in the form of power series

$$E_m(i_{\mu,p}) = \sum_{k=1}^{K} \tfrac{1}{2k} A_{2k} (i_{\mu,p})^{2k}$$

which leads to the following relation between magnetising flux and the magnetising current

$$\Psi_m(i_{\mu,p}) = \sum_{k=1}^{K} A_{2k} (i_{\mu,p})^{2k-1}.$$

For technical application five to six terms are necessary for good approximation. Another form of the co-energy function can be obtained by integrating according to (19) the formula for magnetising flux. Taking the very popular approximation

$$\Psi_m(i_{\mu,p}) = \frac{A i_{\mu,p}}{1 + B\sqrt{(i_{\mu,p})^2}},$$

the co-energy function obtains the form

$$E_m(i_{\mu,p}) = \frac{A}{B}\left[\sqrt{(i_{\mu,p})^2} - \frac{1}{B}\ln\left(B\sqrt{(i_{\mu,p})^2} + 1\right)\right].$$

For the salient-pole AC machines with p-harmonic only two functions appearing in the formulas for co-energy

$$E_m(i_{\mu,p},\, p\alpha_p) = E_0(i_{\mu,p}) + E_2(i_{\mu,p})\cos 2p\alpha_p$$

should be foreseen in the forms

$$\begin{aligned}E_0(i_{\mu,p}) &= \frac{1}{2}A_2^0(i_{\mu,p})^2 + \frac{1}{4}A_4^0(i_{\mu,p})^4 + \frac{1}{6}A_6^0(i_{\mu,p})^6 + \cdots\,,\\ E_2(i_{\mu,p}) &= \frac{1}{2}A_2^2(i_{\mu,p})^2 + \frac{1}{4}A_4^2(i_{\mu,p})^4 + \frac{1}{6}A_6^2(i_{\mu,p})^6 + \cdots\end{aligned}$$

but each of them has quite different properties.

As examples of two variable functions, the components of the co-energy function for an AC machine with cylindrical rotor with p and $3p$ harmonics are discussed. In its simple form two such functions appear

$$E_m = E_0(i_{\mu,p}, i_{\mu,3p}) + E_{3p}(i_{\mu,p}, i_{\mu,3p})\cos 3p(\alpha_p - \alpha_{3p}).$$

The function $E_0(i_{\mu,p}, i_{\mu,3p})$ limited to the first two terms of a power series should be foreseen in the form

$$\begin{aligned}E_0(i_{\mu,p}, i_{\mu,3p}) &= \frac{1}{2}[A_{2,1}^0(i_p)^2 + A_{2,3}^0(i_{3p})^2]\\ &+\frac{1}{4}[A_{4,1}^0(i_p)^4 + 6A_{4,3}^0(i_p)^2(i_{3p})^2 + A_{4,5}^0(i_{3p})^4].\end{aligned}$$

The function $E_3(i_{\mu,p}, i_{\mu,3p})$ limited to the same terms of a power series has to be predicted in the form

$$E_3(i_{\mu,p}, i_{\mu,3p}) = \frac{1}{4}[4A_{4,2}^3(i_p)^3 i_{3p} + 6A_{4,3}^3(i_p)^2(i_{3p})^2].$$

7. Conclusions

In order to avoid difficulties in creation of equations of AC machines when magnetic non-linearity has to be considered, an energy-based approach can be applied. This eliminates the necessity to operate with self- and mutual inductances but all properties of magnetic circuit and machine windings have to be reflected in the magnetic co-energy function. Introducing the equivalent magnetising currents it is possible to create the expressions for the co-energy function for typical AC machines. The most important forms have been provided in this paper.

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