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FINITE ELEMENT CALCULATION OF DOUBLE-CAGE SKIN FACTORS

Károly NÉMETH

Department of Electrical Machines and Drives
Budapest University of Technology and Economics
H–1521 Budapest, Hungary
T: (36) 1 463-3608
e-mail: nemeth@goliat.eik.bme.hu

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Abstract

Resistance and inductance skin factors for a double-cage induction motor are calculated as the ratio of AC and DC values in the 0-70Hz rotor frequency range by the help of a finite element analysis software. In this way, the parameters of a complete or correct equivalent circuit are determined which allows for the skin-effect in the outer and inner bars themselves. Results of calculation from the correct and the generally used approximate equivalent circuit are compared. If finite element analysis is available, the use of the correct equivalent circuit is proposed.

Keywords: double-cage induction motor, skin effect.

Electromagnetic design of induction machines even today is based on classic methods and the well-known equivalent circuit. The needed great amount of calculation is performed by computers so electromagnetic design is quicker and easier. The 2D finite element method is generally used for the calculation of the elements of the equivalent circuit or for the calculation of a particular detail. At starting of a double-cage induction motor, variation of rotor resistances and inductances will occur due to skin effect and the saturation of the leakage magnetic circuit. For finite element calculation it is better to separate these effects and treat them independently. Otherwise, owing to the magnetic saturation, currents and voltages are not sinusoidal quantities so analysis becomes very complex. For the determination of skin factors iron is considered ideal with infinite permeability. In this case, currents and voltages in the rotor bars are sinusoidal in time. Saturation of leakage magnetic circuit could be calculated as a nonlinear magnetostatic problem. In this case, currents in the rotor are DC currents and magnetic nonlinearity of iron could be taken into account. In this article we deal with the first problem only.

Our aim was to use a professional software package available in the market instead of home-made programs as a design and simulation tool. With this choice we accept all the mathematical methods and calculation processes that are applied in the software and we adapt ourselves to the capabilities of the program.

1. Machine Data and Selected Software

The test machine is the product of GANZ-ANSALDO Electric Ltd. (Budapest). The details are: power 2500 kW, line voltage 6300/3600 V, connection wye/delta, line current 258/452 A, frequency 50 Hz, number of poles 4, speed 1489 rpm, number of stator/rotor slots 72/80. Outline of a rotor slot and the embedded outer and inner bars can be seen in *Fig. 1*. The selected software is the QUICK FIELD finite element analysis system, the product of Tera Analysis Company (USA). This program is easy to learn, easy to use, very fast and efficient. The model editor allows to define the model and the mesh quickly and easily. The postprocessor unit makes it easy to analyze the results in different graphical forms as colour maps or plots of field lines, and it can calculate various design parameters, surface and volume integrals in arbitrary regions.

2. DC Resistances and Inductances

The outer bar is made of an alloy of increased resistivity. For 2D plane-parallel problems all integral quantities are considered per unit (1 m) length in z (perpendicular to the plane) direction. For this reason resistances will be given as Ω/m, inductances as H/m, flux as Wb/m etc. Actually, the ideal length of the machine is 0.912 m, so the actual values of the machine are a bit less. The calculated cross-sections and DC resistances are in *Table 1*. We can find the outer and inner bar self-inductances and the mutual inductance between them by solving magnetostatic 2D plane parallel problems where the field source is DC current in the outer or inner bar or DC currents in both bars described by current densities. The magnetic circuit is linear because a Neumann boundary condition is supposed around the slot iron contour, and a Dirichlet boundary condition is supposed at the top of the slot. Inductances can be calculated either from the flux linkages or from the magnetic field energy. Flux linkages are computed by the post-processor unit as a surface integral of the magnetic vector potential. The magnetic energy is calculated as volume integral of the product of the magnetic field intensity and the flux density values [4].

Table 1. DC resistance and inductance per bar

	σ (20 ° C) MS/m	S mm^2	DC resistance (20 °C) $\mu\Omega$/m	DC inductance μH/m		
Outer bar	14.92	319.8	209.5	2.332	Mutual	2.688
Inner bar	57.14	253.3	69.07	6.503		

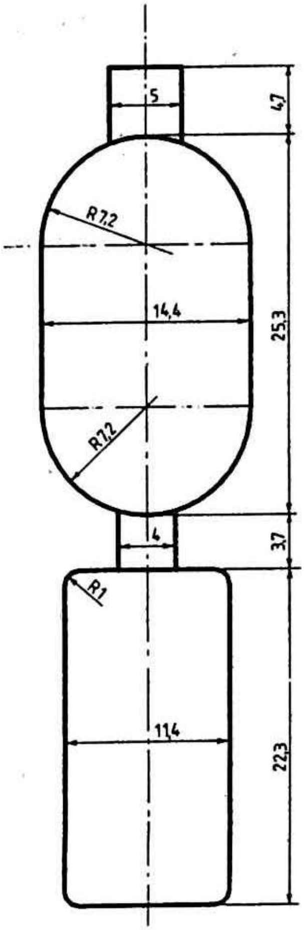

Fig. 1. Outline of rotor slot, outer and inner bar. All dimensions are in millimetres.

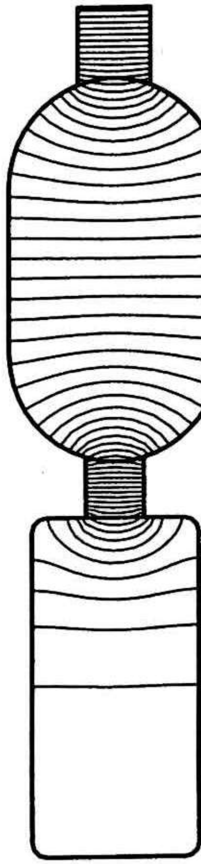

Fig. 2. Field lines at phase 0° if I_i = 1000 A(peak)/0°, $I_o = 0$ and $f = 50$ Hz. Each flux tube contains 0.1 mWb/m.

3. AC Resistances and Inductances

These values can be found if we solve time harmonic magnetic field problems where the field source is AC current or voltage with ω angular frequency and φ phase angle. U and I are zero to peak values (amplitudes). The boundary conditions are the same as in the previous case. Instead of the mutual inductance, there is a mutual impedance between the two bars so the following voltage equations can be written [2], [3]:

$\mathbf{U}_o = \mathbf{I}_o \cdot \mathbf{Z}_o + \mathbf{I}_i \cdot \mathbf{Z}_m$	$\mathbf{Z}_o = R_o + j\omega \cdot L_o$	outer cage leakage impedance
$\mathbf{U}_i = \mathbf{I}_i \cdot \mathbf{Z}_i + \mathbf{I}_o \cdot \mathbf{Z}_m$	$\mathbf{Z}_i = R_i + j\omega \cdot L_i$	inner cage leakage impedance
$\mathbf{U}_o = \mathbf{U}_i = \mathbf{U}$	$\mathbf{Z}_m = R_m + j\omega \cdot L_m$	mutual leakage impedance

Here we supposed the parallel connection of the two bars and $\mathbf{U}$ is the common voltage induced by the main flux of the machine. Resistance and inductance of the bar can be calculated from the complex values of $\mathbf{U}$ and $\mathbf{I}$ as:

$$R = (U/I) \cdot \cos(\varphi_u - \varphi_i) \quad \text{and} \quad L = U/(\omega \cdot I) \cdot \sin(\varphi_u - \varphi_i),$$

where φ_u and φ_i are the voltage and current phase angles and $\omega = 2 \cdot \pi \cdot f$ [4]. Complex value of the voltage is computed by the post-processor unit if the current is given as a field source. The complex value of the resultant current is calculated if the field source is the voltage. For check, R can be calculated from the time average of the Joule heat and L from time average of magnetic field energy. Both values are given by the post-processor unit. First we suppose 1000 A/50 Hz current in the inner bar and $\mathbf{I}_o = 0$ resultant current in the outer bar, i.e. we suppose the outer bar is temporarily broken off the short circuit ring. The voltage equations in this case are $\mathbf{U}_i = \mathbf{I}_i \cdot \mathbf{Z}_i$, $\mathbf{U}_o = \mathbf{I}_i \cdot \mathbf{Z}_m$ and from the equations $\mathbf{Z}_i$ and $\mathbf{Z}_m$ can be calculated. From the shape of the field lines in *Fig. 2* can be seen that eddy currents will be induced in the outer bar. In this case, $\mathbf{U}_o$ voltage leads less than 90° to the $\mathbf{I}_i$ current that results in $\mathbf{Z}_m$ instead of $j\omega \cdot L_m$. In *Fig. 3* the current density distribution along the symmetry axis of the slot from the top to the bottom can be seen. Skin effect is very intensive in the inner bar. Current density distribution in the outer bar is symmetrical and the direction in the upper half and the lower half is opposite, so the resultant current is zero. Second we suppose 1000 A/50 Hz current in the outer bar and $\mathbf{I}_i = 0$ resultant current in the inner bar, i.e. we suppose the inner bar is temporarily broken off the short circuit ring. In *Fig. 4* and *Fig. 3* we can see that there are no induced eddy currents in the inner bar. Flux lines produced by the outer current close around the inner bar and induce a voltage in it. This voltage leads less than 90° to the outer current and the calculation gives the same R_m and L_m values as in the first case. It is evident that if ω approaches zero, R_m will approach zero as well and L_m will approach its DC value.

In the third case we suppose a voltage of $U = 0.1$ V/m(50 Hz) on the parallel connected bars. Field lines and current density distribution can be seen in *Fig. 5* and *Fig. 6*, respectively. The post-processing unit gives us the resultant $\mathbf{I}_o$ and $\mathbf{I}_i$ currents (see *Fig. 9*). We can get the same currents from the voltage equations using the previously calculated $\mathbf{Z}_o$, $\mathbf{Z}_i$, $\mathbf{Z}_m$ impedances that proves the validity of the voltage equations and the equivalent circuit based on them (*Fig. 7a*). This equivalent circuit is modified eliminating the coupling and we get the complete or correct equivalent circuit (*Fig. 7b*). Note frequency is the rotor frequency, and impedances are calculated pro bar and pro 1 m length bases and are not referred to the stator.

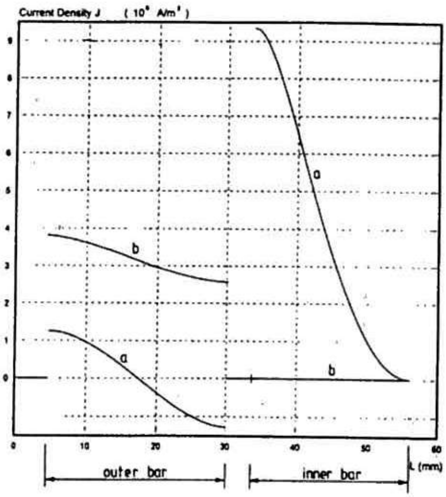

Fig. 3. Current density distribution along the symmetry axis ($f = 50$ Hz). Instantaneous value at phase 0° is shown. a.) $I_i = 1000$ A(peak)/0°, $I_o = 0$; b.) $I_i = 0$, $I_o = 1000$ A(peak)/0°

4. Skin Factors

Impedances were calculated in the 0–70 Hz frequency range and the skin factors were determined based on the following definitions where index 0 indicates the DC values:

Resistance	$k_{ro} = R_o/R_{o0}$	$k_{ri} = R_i/R_{i0}$	$k_{rm} = R_m/R_{i0} + 1$
Inductance	$k_x = L_o/L_{o0}$	$k_{xi} = L_i/L_{i0}$	$k_{xm} = L_m/L_{m0}$

Definition of k_{rm} is arbitrary. The calculated skin factors can be seen in *Fig. 8*. Considering the slot dimensions, we can find k_{ri} too big. This is owing to the eddy currents in the outer bar.

5. Approximate Equivalent Circuit

In the classic theory of double-cage induction motors the skin effect in the bars themselves is ignored, the DC values are used in the calculation and in the equivalent circuit ($k_r = k_x = 1$) and the resistive part of the mutual impedance is ignored as well ($R_m = 0$). Moreover, $L_{m0} = L_{o0}$ is supposed in order to simplify the equivalent circuit [1]. This results in the approximate equivalent circuit shown in *Fig. 7c*. To illustrate the difference in the results coming from the correct and

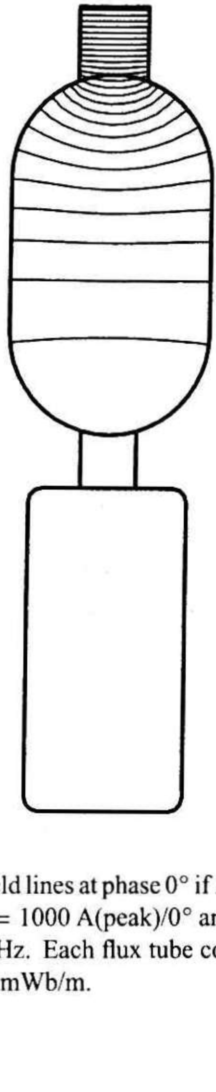

Fig. 4. Field lines at phase 0° if $I_i = 0$, $I_o = 1000$ A(peak)/0° and $f =$ 50 Hz. Each flux tube contains 0.1 mWb/m.

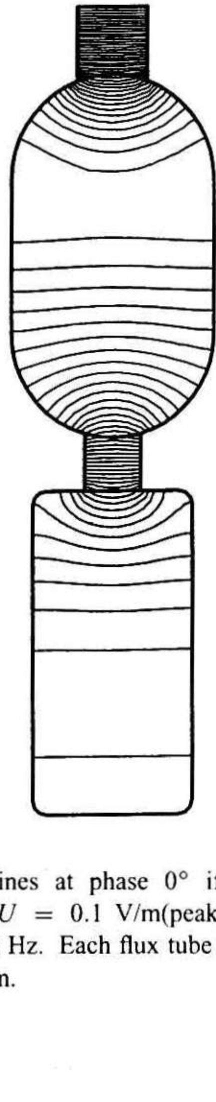

Fig. 5. Field lines at phase 0° if $U_i = U_o = U = 0.1$ V/m(peak)/0° and $f = 50$ Hz. Each flux tube contains 2μWb/m.

approximate equivalent circuits, the current phasor diagrams are drawn in *Fig. 9* for $U = 0.1$ V/m (50 Hz). The current phasors calculated from the approximate equivalent circuit are plotted with thin line. The approximate circuit is less resistive than the correct circuit, so the current component in phase with voltage is 25% less. We can improve this approximate equivalent circuit taking the skin effect into account but ignoring R_m further on. The skin factors in this case are: $k_{ri} = 2.36$, $k_{xi} = 0.955$, $k_{ro} = 1.2$, $k_{xo} = 0.98$. End point of the resultant I current phasor is indicated by a dot in *Fig. 9*. The difference is now reduced to 15%.

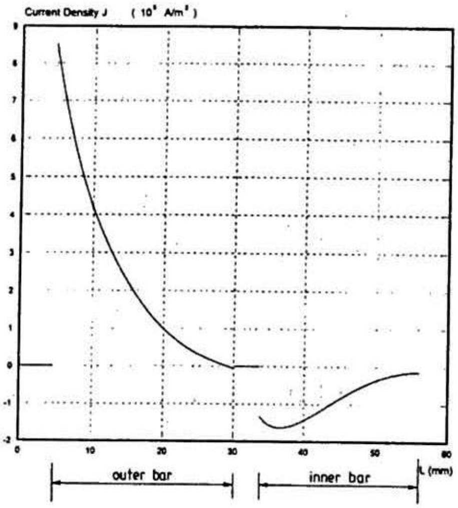

Fig. 6. Current density distribution along the symmetry axis ($f = 50$ Hz). Instantaneous value at phase 0° if $U_i = U_o = U = 0.1$ V/m(peak)/0° is shown.

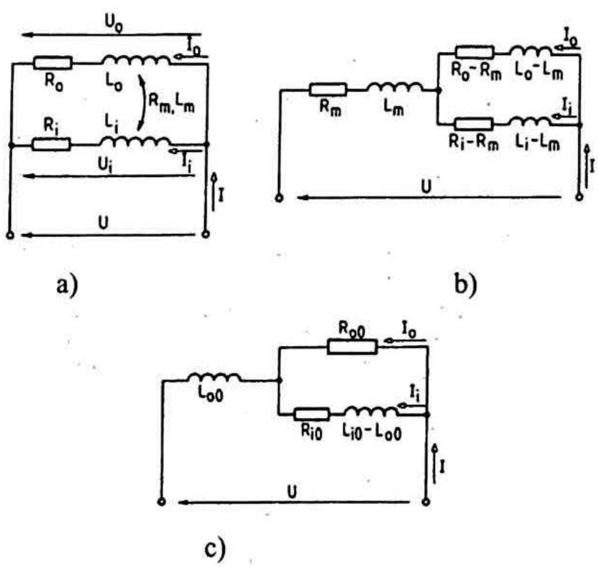

Fig. 7. Complete (a./) and modified complete (b./) equivalent circuits that include mutual impedance. Approximate equivalent circuit (c./). Values are unreferred to stator.

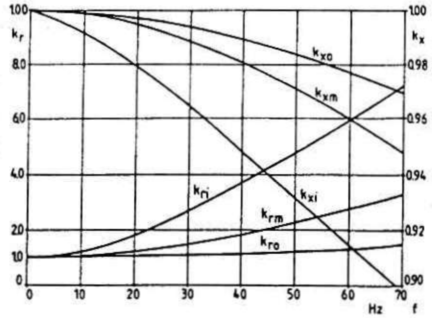

Fig. 8. Resistance and inductance skin factors of outer and inner bar

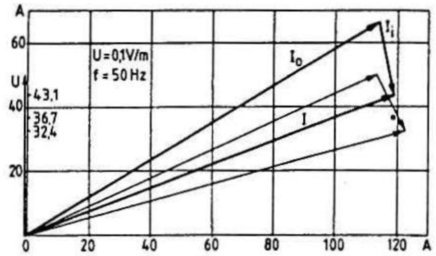

Fig. 9. Current phasors calculated from the correct and the approximate equivalent circuit

6. Conclusions

Professional finite element software packages can easily be used in the calculation of the rotor equivalent circuit elements of double-cage induction motors. It is not necessary to simplify the equivalent circuit because parameters of the so called correct or complete equivalent circuit, that is based on a more complex physical background, can be calculated. There is a significant difference in the results, therefore the use of the complete circuit is proposed.

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