POSSIBILITIES OF ROTOR ECCENTRICITY DIAGNOSTICS FOR CAGE INDUCTION MOTORS – NUMERICAL STUDIES

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Abstract

In this paper, two methods of quantitative estimation of eccentricity degree for induction motors on the basis of numerical analysis of standard Fourier spectrum are presented. These spectra are results of the solution of so called circuit models of machines described in symmetrical components based on balance harmonic method. The first method is based on separation spectrum current to four characteristic sets of frequency. In the second method, the spectrum is not separated. It is estimated by using artificial neural network. To illustrate the above methods, exemplary computation results of stator currents for chosen motors are presented.

In the paper the package of computer programs for numerical computation, as well as initial verification of the obtained solutions are presented.

Keywords: diagnostic, eccentricity, numerical computing, multi-harmonic machine, harmonic balance method, current spectrum method separation, Fourier spectrum, neural networks.

1. Introduction

Two main trends of detection and estimation of the machine states are principally observed in electrical machine diagnostics.

The first general way is an empiric search for the relation between the measured values and machine condition and attributing to them quantitative values to estimate the machine condition. This way requires carrying out many measurements for the tested object, starting from normal to damage states of the machine. This method is only valid for testing a finite number of cases. Used in industry, online diagnostic systems confirmed enough efficiency of that approach. (An example is MotorMonitor™ system for estimation of cage condition and rotor eccentricity for induction motors by harmonic analysis of stator spectrum.) The disadvantage of this approach is the impossibility to separate quantitative influence of each damage cause. The measured effects are the results of electromechanical interactions. For example, high resistance damages of rotor are connected with dynamic eccentricity in the same frequencies.

The second way of analysis based on extended mathematical models and advanced computation techniques leads to creation of standard Fourier spectra for diagnostic estimation of motor. These standard spectra allow to precisely determine

quantitative and qualitative changes in measured available signals caused by rotor damage. This approach is easy to algorithmize and permits continuous generation of patterns as well as separate analysis of each damage type.

The disadvantage of this method is naturally limitation, by simplification of mathematical models and capabilities of computer hardware, leading to quantitative differences.

The present paper is based on the second way and has a study character. As a diagnostic signal, Fourier spectra analysis for stator phase currents was taken. The authors make use of the circuit model of a multi-harmonic machine, described in symmetrical components by the harmonic balance method. This method, proposed by Prof. Tadeusz J. Sobczyk, confirmed its efficiency in the parasite phenomena analysis and non-symmetrical states of induction machines.

This method is especially effective for quantitative and qualitative analysis of currents. Changes of spectra generated by rotor eccentricity are obtained in a wide range of frequencies, and at the same time this problem becomes more interesting for harmonic analysis than pure damage of rotor cage.

2. Two Proposals of Current Spectrum Estimation

The result of a recent research has been a software package enabling creation of arbitrary patterns of current spectra for induction motors. The authors propose two ways of quantitative estimation of the Fourier spectra of stator currents:

- by 'assignment function' proposed in [1], [2]
- by artificial neural networks.

In the first case the following steps should be distinguished:

- division of spectrum into harmonic sets for each kind of eccentricity (static, dynamic, mixed and symmetric)
- creation of an 'assignment function' corresponding to each relative eccentricity state and load level with the sets of harmonics chosen above.

It leads to forming the database of 'assignment function' and seeking relative eccentricity levels with satisfactory accuracy.

In the second case, by using artificial neural networks, it is necessary to:

- take specific class number, each of them having suitable cases
- create training vectors for suitable cases
- train the network by prepared data
- test the network by not presented data.

Neural network does not require selection of Fourier spectrum of stator current on particular harmonic sets corresponding to the suitable case of eccentricity.

The input signal is the vector of harmonic magnitudes related to slot harmonic.

This method does not require any assignment functions for eccentricity estimation. Determination of eccentricity level is done by neural network.

3. Equations of 3-Phase Cage Induction Motors after Transformation of the Stator and Rotor Variables to Their Symmetrical Components

Using the symmetrical components of voltages and currents defined by the matrices

$$[T_s] = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}; \qquad a = e^{j2\pi/3}, \qquad (1)$$

$$[T_r] = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & \ldots & 1 \\ 1 & b & \ldots & b^{N-1} \\ \vdots & \vdots & & \vdots \\ 1 & b^{N-1} & \ldots & b^{(N-1)(N-1)} \end{bmatrix}; \qquad b = e^{j2\pi/N}. \qquad (2)$$

The voltage equations take the form

$$\begin{bmatrix} [u^s] \\ [0] \end{bmatrix} = \begin{bmatrix} [R^s] & [0] \\ [0] & [R^r] \end{bmatrix} \begin{bmatrix} [i^s] \\ [i^r] \end{bmatrix} + \begin{bmatrix} [L_\sigma^s] & [0] \\ [0] & [L_\sigma^r] \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} [i^s] \\ [i^r] \end{bmatrix}$$
$$+ \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} [L^{ss}] & [L^{sr}] \\ [L^{rs}] & [L^{rr}] \end{bmatrix} \begin{bmatrix} [i^s] \\ [i^r] \end{bmatrix}. \qquad (3)$$

Equation set (3) together with detailed descriptions of inductance matrices is very suitable for the analysis of steady states. For diagnostic purposes the most interesting is a steady state when the motor is running with constant speed at a balanced sinusoidal voltage set. The Fourier spectrum of phase current is then a very good diagnostic signal.

At constant speed ($\varphi = \omega t + \varphi_o$) all matrices become periodic in time and can be expanded into Fourier series

$$[L_\sigma^s] + [L^{ss}] = \sum_k [L_k^s] e^{jk\omega t}, \qquad (4)$$

$$[L_\sigma^r] + [L^{rr}] = \sum_k [L_k^r] e^{jk\omega t}, \qquad (5)$$

$$[L^{sr}] = [\overset{*}{L}{}^{rs}]^T = \sum_k [M_k] e^{jk\omega t} \qquad (6)$$

and *Eqs.* (3) constitute a set of linear differential equations with periodically varied coefficients. For mono-harmonic symmetrical phase voltages a forced vector in (3) has the form

$$[u^s] = \sqrt{\frac{3}{2}} U([\, 0 \quad 1 \quad 0 \,]^T e^{j\omega_0 t} + [\, 0 \quad 0 \quad 1 \,]^T e^{-j\omega_0 t}). \qquad (7)$$

Then, a vector of symmetrical components of phase currents should be foreseen in the form

$$[i^s] = \sum_{\eta=\pm 1} \sum_k [I^s_{\eta,k}] \, e^{j(\eta\omega_0 + k\omega)t} \quad \text{where} \quad [I^s_{\eta,k}] = [I^{s,0}_{\eta,k} \, I^{s,1}_{\eta,k} \, I^{s,2}_{\eta,k}]^T \qquad (8)$$

and a vector of symmetrical components of rotor cage meshes should be foreseen in the form

$$[i^r] = \sum_{\eta=\pm 1} \sum_k [I^r_{\eta,k}] \, e^{j(\eta\omega_0 + k\omega)t} \quad \text{where} \quad [I^r_{\eta,k}] = [I^{r,0}_{\eta,k} \, I^{r,1}_{\eta,k} \, \cdots \, I^{r,N-1}_{\eta,k}]^T. \quad (9)$$

Using these equations, it is possible to predict qualitatively and quantitatively the Fourier spectra of all currents at the steady-state condition. The amplitude values of the terms of current series are calculated from an infinite of algebraic equation set, according to the harmonic balance method:

$$
\begin{bmatrix}
\vdots \\ [0] \\ [U^s_\eta] \\ [0] \\ \vdots \\ \vdots \\ [0] \\ [0] \\ [0] \\ \vdots
\end{bmatrix}
= \left\{
\text{diag}
\begin{bmatrix}
\vdots \\ [R^s] \\ [R^s] \\ [R^s] \\ \vdots \\ \vdots \\ [R^r] \\ [R^r] \\ [R^r] \\ \vdots
\end{bmatrix}
+ \text{diag}
\begin{bmatrix}
\vdots \\ j(\eta\omega_0 + \omega)[E_s] \\ j\omega_0[E_s] \\ j(\eta\omega_0 - \omega)[E_s] \\ \vdots \\ \vdots \\ j(\eta\omega_0 + \omega)[E_r] \\ j\omega_0[E_r] \\ j(\eta\omega_0 - \omega)[E_r] \\ \vdots
\end{bmatrix}
\right.
$$

$$
\times
\begin{bmatrix}
\ddots & \ddots & \ddots & & \ddots & \ddots & \ddots & \\
\ddots & [L^s_0] & [L^s_1] & [L^s_2] & \ddots & [M_0] & [M_1] & [M_2] \\
\ddots & [L^s_{-1}] & [L^s_0] & [L^s_1] & \ddots & \ddots & [M_{-1}] & [M_0] & [M_1] & \ddots \\
& [L^s_{-2}] & [L^s_{-1}] & [L^s_0] & \ddots & [M_{-2}] & [M_{-1}] & [M_0] & \ddots \\
& & \ddots & \ddots & \ddots & & \ddots & \ddots & \ddots \\
\ddots & \ddots & \ddots & & \ddots & \ddots & \ddots & \\
\ddots & [M_0]^{*T} & [M_{-1}]^{*T} & [M_{-2}]^{*T} & \ddots & [L^r_0] & [L^r_1] & [L^r_2] \\
\ddots & [M_1]^{*T} & [M_0]^{*T} & [M_{-1}]^{*T} & \ddots & \ddots & [L^r_{-1}] & [L^r_0] & [L^r_1] & \ddots \\
& [M_2]^{*T} & [M_1]^{*T} & [M_0]^{*T} & \ddots & [L^r_{-2}] & [L^r_{-1}] & [L^r_0] & \ddots \\
& & \ddots & \ddots & \ddots & & \ddots & \ddots & \ddots
\end{bmatrix}
\right\}
$$

$$\times \begin{bmatrix} \vdots \\ [I^s_{\eta,1}] \\ [I^s_{\eta,0}] \\ [I^s_{\eta,-1}] \\ \vdots \\ \vdots \\ [I^r_{\eta,1}] \\ [I^r_{\eta,0}] \\ [I^r_{\eta,-1}] \\ \vdots \end{bmatrix}. \tag{10}$$

The matrices of inductance contain all features of electric and magnetic circuits of a motor, and their structures depend on the type of eccentricity.

Structures of matrices of inductance were introduced in detail in [2], [3], [4].

4. Numerical Computing Algorithm and Its Automation

Diagnostics analysis for cage induction motors with rotor eccentricity based on the solution model (10) presented previously is possible after fulfilling the following conditions:

- precise determination of mathematical model equations parameters
- effective derivation of a huge number of mathematical model equations
- precise estimation of the obtained results
- full automation of computation.

At the stage of calculating the mathematical model parameters, it is most important to precisely determine the winding inductance coefficients. For this purpose, a method of determining permeance for the non-uniform air gap described previously in [2], [3] was applied. In the next step, the balance harmonic method for determined inductance coefficients of windings was used. At this stage also leakage inductance and resistance of windings and rotor bars were determined.

The number of MMF harmonics taken into account determines the number of mathematical model equations. For a hypothetical motor (with $p = 2$ and $N = 22$) analyzed for studying the Fourier spectrum of current in the range to 2500 Hz 100 MMF harmonics shall be taken into consideration. It leads to the necessity of solving a complex equations set of 5025×5025 dimensions. For effective solution of mathematical model equations algorithms based on sparse matrices were applied. Additionally, some properties of block matrices were used. Because discussed matrices are sparse the effectiveness of solutions strongly depends on the degree of matrix filled by nonzero elements.

Structure matrices of impedance for individual type of eccentricity and for symmetry air gap were presented for illustration. Figures show (for value parameter $k = 2$ according to formula (4-9)) how changes of eccentricity type influence the appearance of nonzero elements in impedance matrices.
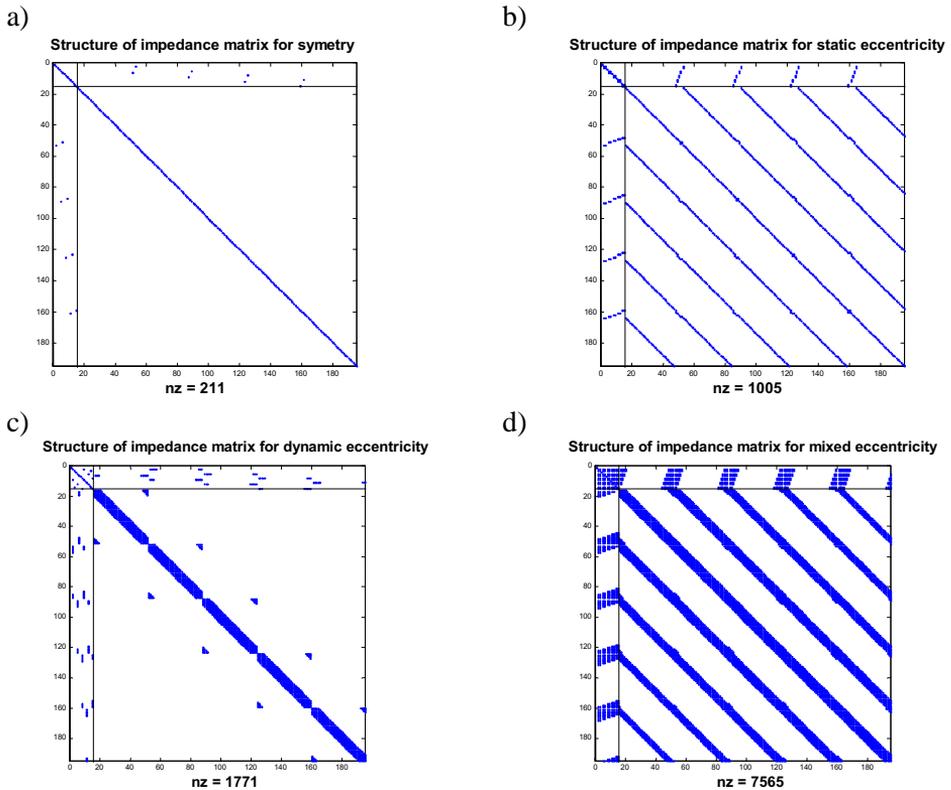
a)

Structure of impedance matrix for symetry

b)

Structure of impedance matrix for static eccentricity

c)

Structure of impedance matrix for dynamic eccentricity

d)

Structure of impedance matrix for mixed eccentricity

Fig. 1. Structure of matrix impedance for equation set (10) and $k = 2$, for each eccentricity and symmetry type

Qualitative analysis of the mathematical model solutions enables us to obtain precise rates of quantitative analysis. Determination of indexes for diagnostics evaluation, their variety due to changes in factors influencing them required automation of the computation. It was done by writing individual solutions to dynamically named files. Special functions for string variables were applied here.

A computer program package has been prepared using the MATLAB 5.2 software and applying basic and also special *m*-files included in Toolboxes. For computation needs many own *m*-files were created.

5. Current Spectrum Method Separation

Analysis results of the equations set (10) for stator phase currents for all types of rotor eccentricity are given below in *Table 1*.

Table 1. Frequencies of stator phase currents for all types of rotor eccentricity

	Symmetry $f_{sym,k}$	Static eccentricity $f_{sta,k}$	Dynamic eccentricity $f_{dyn,k}$	Mixed eccentricity $f_{mix,k}$
Frequency	$\dfrac{\|\omega_0 + kgN\omega\|}{2\pi}$	$\dfrac{\|\omega_0 + kN\omega\|}{2\pi}$	$\dfrac{\|\omega_0 + k2p\omega\|}{2\pi}$	$\dfrac{\|\omega_0 + k\omega\|}{2\pi}$

where: p – pole-pair number;
        N – number of rotor bars, $k \in \{\ldots -2, -1, 0, 1, 2, \ldots\}$.

From *Table 1* follows that each type of eccentricity has a different set of frequencies of the stator currents.

Then, theoretically the type of eccentricity can be uniquely recognized. These frequencies depend on the pole-pair number p and the number of rotor bars N. For fully symmetrical motors additional frequencies (called slots harmonics) depend on a characteristic parameter $g(1 \leq g \leq 2p)$ equal to a minimal integer number satisfying for $l = 0, 1, 2, \cdots$ the condition

$$gN \in \{\{(3l + 0)2p\} \cup \{(3l + 1)2p\} \cup \{(3l + 2)2p\}\}. \tag{11}$$

It means that a stator phase current is characterized by a set of pairs $F = \{f_k, I_k\}$ (frequency–amplitude), which is different for each type of eccentricity. These sets are:

- for symmetry
$$F_{sym} = \{f_{sym,k}, I_{sym,k}\} \tag{12}$$
- for static eccentricity
$$F_{stat} = \{f_{sta,k}, I_{sta,k}\} \tag{13}$$
- for dynamic eccentricity
$$F_{dyn} = \{f_{dyn,k}, I_{dyn,k}\} \tag{14}$$
- for mixed eccentricity
$$F_{mix} = \{f_{mix,k}, I_{mix,k}\} \tag{15}$$

It should be noticed that the frequency sets of different eccentricities have common elements. Frequencies at symmetry are common elements of frequency sets for static and dynamic eccentricities. Frequencies for mixed eccentricity contain both frequencies at static and at dynamic eccentricities. Then, it is possible to define disjoint sets characterizing each type of eccentricity. A detailed analysis leads to the following definitions

- for symmetry

$$M_{\text{sym}} = F_{\text{sym}} \backslash F_{50 \text{ Hz}} \tag{16}$$

- for static eccentricity

$$M_{\text{sta}} = F_{\text{sta}} \backslash F_{\text{sym}} \tag{17}$$

- for dynamic eccentricity

$$M_{\text{dyn}} = F_{\text{dyn}} \backslash F_{\text{sym}} \tag{18}$$

- for mixed eccentricity

$$M_{\text{mix}} = F_{\text{mix}} \backslash F_{\text{dyn}} \backslash F_{\text{sta}} \tag{19}$$

Based on these four sets some assignment functions characterizing each type of eccentricity can be defined. Because static and dynamic eccentricities are independent, their values per unit:

$$\varepsilon_s = \frac{d_s}{\delta} \ , \ \varepsilon_d = \frac{d_d}{\delta}, \tag{20}$$

can be treated as two independent variables of functions. As dependent variables of such assignment functions, norms of harmonic amplitudes of each set, defined as $\|I\| = \sum_k |I_k|$, can be taken. Then, at least four functions can be created

$$f_{\text{type}}(\varepsilon_s, \varepsilon_d) = \frac{\left\| I_{\text{type}}(\varepsilon_s, \varepsilon_d) \right\|}{\left\| I_{\text{sym}}(0, 0) \right\|}, \tag{21}$$

where

- index type $\in$ {sym,sta,dyn,mix}
- values $\left\| I_{sym}(\varepsilon_s, \varepsilon_d) \right\|$, $\|I_{sta}(\varepsilon_s, \varepsilon_d)\|$, $\left\| I_{dyn}(\varepsilon_s, \varepsilon_d) \right\|$, $\|I_{mix}(\varepsilon_s, \varepsilon_d)\|$ are respective norms.

Because the basic slot harmonic characteristic for symmetrical motor corresponds to the number of rotor bars N by parameter g, the investigated motors required division into two groups: when $g = 1$ and when $g > 1$.

In the case of motors with static eccentricity, each time slot harmonics will appear in current spectrum. Hence, it is easier to detect and estimate static eccentricity for motors with $g > 1$. In most induction motors, the stator windings are connected in star. This fact causes modification of the current spectrum for symmetrical motors by eliminating harmonics corresponding to '0' symmetrical current component. Static eccentricity is the reason of appearance of these harmonics for motors with $g = 1$ as well. This is an additional precious diagnostic signal.

Exemplary spectra of stator currents (presented below) were computed for chosen eccentricity levels and a constant value of slip (close to nominal slip).

Relative levels of eccentricity (reference to value of air gap length for symmetrical motors) are each time determined by coefficients 'ε_s' for static eccentricity and 'ε_d' for dynamic eccentricity.

Presenting spectra in logarithmic scale facilitates estimation. Reference level for computing magnitudes of phase current is the same for all presented cases.

Data necessary for computation was fixed based on design data of a cage motor SzJe 14b produced in Poland. It has pole-pair number $p = 2$ and number of slots on a rotor $N = 22$ (non-skewed). The rated values of this motor are:

$P_N = 1.1 \, \text{kW}, U_N = 380 \, \text{V(Y)}, I_N = 2.8 \, \text{A}, n_N = 1390 \, \text{rpm}, \cos \varphi_N = 0.77,$
$\eta_N = 0.765.$

Computations have been made assuming that stator phases are connected in star without neutral, are supplied by balanced mono-harmonic voltages and the motor works at slip $s = 0.07$. This motor has a parameter $g = 2$, meaning that the first slot harmonic in stator currents has a frequency of $(23 - 22 \, s) * 50 \, \text{Hz}$ – for symmetry case. In order to consider at least two slot harmonics, the Fourier spectrum till 2500 Hz was investigated for each type of eccentricity.
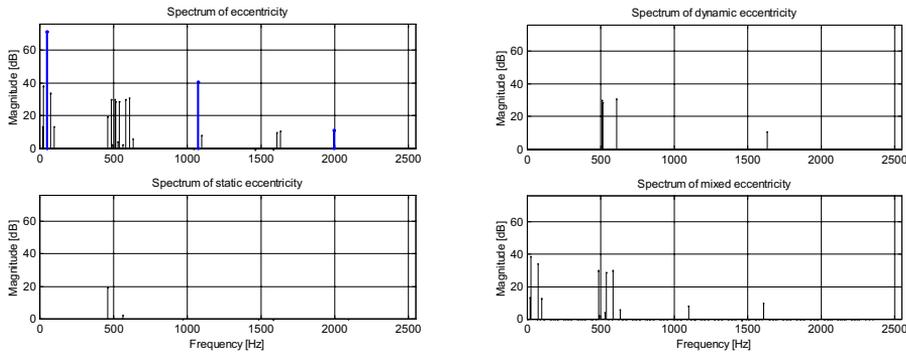

Fig. 2. Fourier spectrum of the phase stator current for $\varepsilon_s = 0.20$, $\varepsilon_d = 0.40$, $s = 0.07$

Depending on phase in current spectrum there may occur differences of magnitude values from a collection characteristic of static, dynamic and mixed eccentricity.

It means that the right estimation of eccentricity levels requires determination of the current spectrum in three phases. To simplify the comparison of the spectrum analysis method only one chosen phase was estimated.

Changes in relative levels of eccentricity (static and dynamic) have influence on value assignment function, according to formula (21).

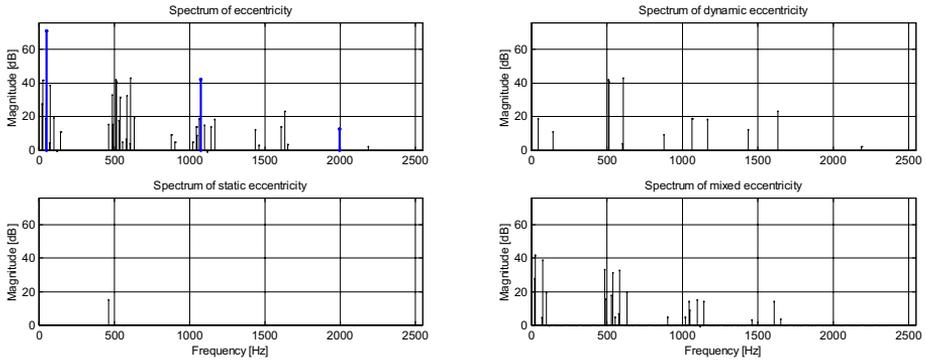Values of these functions are presented in *Table 2*.

Fig. 3. Fourier spectrum of the phase stator current for $\varepsilon_s = 0.10$, $\varepsilon_d = 0.70$, $s = 0.07$
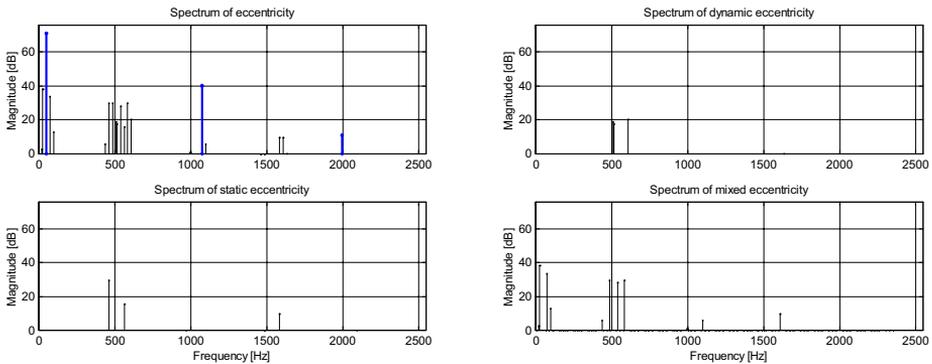

Fig. 4. Fourier spectrum of the phase stator current for $\varepsilon_s = 0.40$, $\varepsilon_d = 0.20$, $s = 0.07$

Table 2. Values of assignments functions for chosen levels eccentricities

ε_s	ε_d	$f_{\text{sym}}(\varepsilon_s, \varepsilon_d)$	$f_{\text{sta}}(\varepsilon_s, \varepsilon_d)$	$f_{\text{dyn}}(\varepsilon_s, \varepsilon_d)$	$f_{\text{mix}}(\varepsilon_s, \varepsilon_d)$
0.2	0.4	1.068	0.246	0.982	2.483
0.4	0.2	1.060	0.825	0.286	2.411
0.1	0.7	1.277	0.151	4.236	4.339

6. Neural Networks for Diagnostic Estimation

After having created database of the harmonic spectra for each case of eccentricity, tests were started to apply artificial neural networks to quantitative and qualitative eccentricity estimation. Fifty-five characteristic classes for different eccentricity levels were taken into account. These all are available combinations of static and
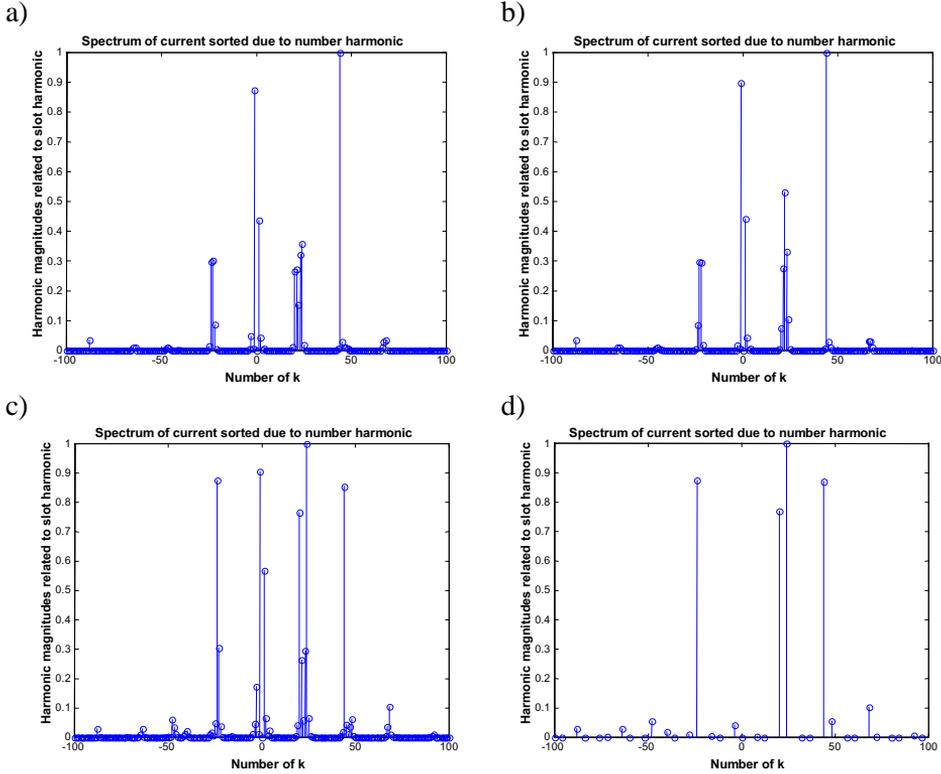
a)

b)

c)

d)

Fig. 5. Examples of spectrum current sorted due to number harmonic. a) $\varepsilon_s = 0.2$, $\varepsilon_d = 0.4$ (class number 24); b) $\varepsilon_s = 0.4$, $\varepsilon_d = 0.2$ (class number 37); c) $\varepsilon_s = 0.1$, $\varepsilon_d = 0.7$ (class number 18); d) $\varepsilon_s = 0.0$, $\varepsilon_d = 0.7$ (class number 8).

dynamic eccentricity levels (from 0% to 90% with division 10%). Every particular level has its own class, in its right order:

$$0.00.0(1); 0.00.1(2); \ldots 0.80.1(54); 0.90.0(55);$$

Training vector contains 275 sub-vectors with 201 elements (magnitude values for k from -100 to 100) each. Such vectors were created on the base of the numerical calculation for the presented mathematical model of the motor. For each class, five patterns for slips changing from $s = 0.05$ to $s = 0.09$ were taken into account.

For individual levels of eccentricity spectrum of stator currents was normalized relating all the values to basic slot harmonic.

All these frequencies of stator current belong to set:

$$\overline{\overline{\Omega}} = \left\{ \frac{|\omega_0 + k\omega|}{2\pi} \right\} \quad \text{where:} \quad k = 0, \pm 1, \pm 2, \ldots$$

Exemplary spectra characterizing chosen classes are presented below.

Additionally, the order of individual data in training vector was established at random. In this way the process of learning is accelerated and its quality is improved.

In all the experiments feedforward neural networks on two hidden layers were used. Structure of the feedforward neural network is presented below in *Fig.6*.
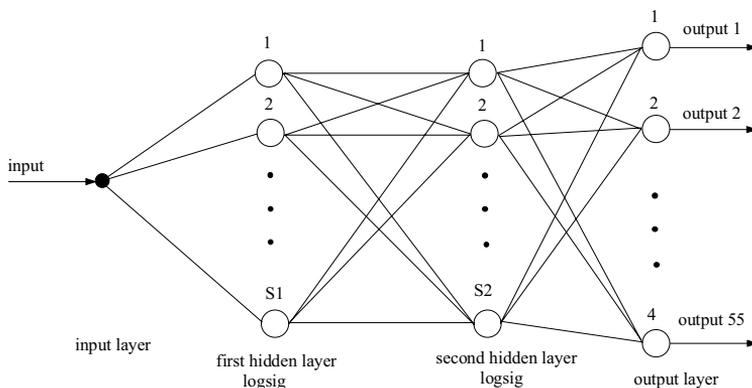

Fig. 6. Structure of the feedforward neural network

Neurons transfer functions in individual layers are logsig (Log-Sigmoid Transfer Function).

This transfer function was chosen because earlier the training vector values were normalized within the range values (0,1).

Number of neurons in individual hidden layers of neural network was:

S1 – first hidden layer – 69 neurons,
S2 – second hidden layer – 55 neurons.

In the learning process advantages were made of different modifications of gradient backpropagation method implemented in package MATLAB 5.2.

Apparently, the most effective learning algorithm in this case is the TRAIN-SCG method (Scaled conjugate gradient backpropagation method).

With the aid of the remaining methods it was not possible to achieve the assumed error level or the learning process was stopped automatically because of reaching gradient minimum.

Efficiency of spectrum estimation by neural network was tested from the outside of the learning vector. Results for chosen test data are presented in *Table3*.

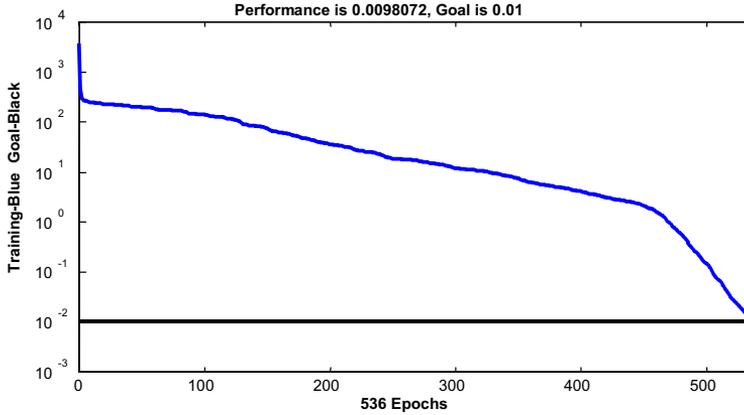These results are in agreement with the results of the numerical method of distribution and analysis (method 1).

Fig. 7. Course of learning process with TRAINSCG method

Table 3. Results of estimation test with neural network

Real levels of eccentricity		Eccentricity level estimation using neural network	
ε_s	ε_d	ε_s	ε_d
0.19	0.39	0.2	0.4
0.19	0.44	0.2	0.4
0.24	0.44	0.2	0.4
0.39	0.18	0.4	0.2
0.39	0.24	0.4	0.2
0.44	0.24	0.4	0.4
0.10	0.75	0.1	0.7
0.15	0.75	0.1	0.8
0.14	0.72	0.1	0.7

7. Conclusions

Both methods presented above are based on Fourier's spectrum analysis of phase current.

In both cases it is necessary to create a sizeable database with numerical computation. The basic difference between the presented methods resulted from making use of this database.

In the case of the method of spectrum separation, the correct arrangement of eccentricity levels requires each time determining (by a special computer program) the values corresponding to assignment functions for individual eccentricity types

and searching database in order to find the closest pattern value.

In the second way, making use of artificial neural networks, database is used only to prepare learning vectors. In this case it is enough to teach the neural network only once and it will recognize the levels of eccentricity.

For the sake of numerical difficulties in searching optimization and demands of sizeable database, the method of eccentricity estimation based on spectrum separation is less effective than the method taking advantage of artificial neural networks.

Summing up the conclusions it can be said that the method of using artificial neural networks provides better circumstances for technical application.

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