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COMPUTATION OF NON-LINEAR CHARACTERISTICS FOR WINDINGS OF SATURATED INDUCTION SLIP RING MOTOR

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Abstract

Analytical expressions of relationships between linked fluxes and currents of all individual windings are very important in modelling of electrical machines with saturated magnetic circuit. For induction machines the model can operate by magnetising current of the first and the higher harmonics instead of the circuit currents. In the paper the first and the third harmonics are taken into account. The non-linear characteristics based on the co-energy function were approximated by power series and transformed to the symmetrical components. The general properties of the co-energy function for a cylindrical machine were formulated and used in calculations of the linked fluxes by FEM method. The terms of the simplest series were calculated from the magnetic field computed for a slip ring induction motor. As a result, the particular terms of the series were determined, and the saturation effects generated by the positive and zero symmetrical components of the currents were described.

Keywords: AC electrical machine, induction motor, co-energy of magnetic field.

1. Introduction

The relations between armature currents and fluxes linked with each of the circuits located in the stator and rotor are necessary to modelling electrical machines in saturation conditions. They are called the non-linear characteristics of windings and should be treated as multivariable functions of the magnetising current of many harmonics. These functions are used in two-harmonic model of induction machine proposed in [1], and based on the co-energy function. Field calculations of the co-energy and its partial derivatives defining linked fluxes were presented in [2]. It turns out that accuracy of a numerical differentiation of the co-energy is not sufficient enough. To improve accuracy, another direct method of calculation is necessary. In this paper a method based on the energy supplied into the magnetic core by individual circuits was applied. The detailed calculations were executed for a small slip ring induction motor. The model was formulated taking into account the following assumptions:

- leakage fluxes and main flux may be treated separately;
- leakage magnetic circuits are linear;
- stator and rotor winding are symmetrically designed;
- windings produce the first and the third harmonics of MMF only.

The co-energy supplied by MMF into the main magnetic circuit is a function of the magnetising current, its harmonics may be expressed by the form

$$\underline{i}^{\rho}_{\mu} = \underline{i}^{\rho}_{\mu_s} + \underline{i}^{\rho}_{\mu_r} = \nu_\rho \left[1 \; a_\rho \; a^2_\rho\right] \left\{ \begin{bmatrix} i_{s_1} \\ i_{s_2} \\ i_{s_3} \end{bmatrix} + \begin{bmatrix} i'_{r_1} \\ i'_{r_2} \\ i'_{r_3} \end{bmatrix} e^{j\rho p\varphi} \right\}, \tag{1}$$

$$a_\rho = e^{j\rho\frac{2\pi}{3}}; \qquad i'_{r_n} = i_{r_n}/K^\rho; \qquad K^\rho = \frac{w_s k^\rho_s}{w_r k^\rho_r}; \qquad \nu_\rho = \frac{2 w_s k^\rho_s}{\pi_\rho}$$

and the co-energy may be written as an approximating Fourier series:

$$E_m\left(I^1_\mu, I^3_\mu, \alpha\right) = E_0\left(I^1_\mu, I^3_\mu\right) + E_3\left(I^1_\mu, I^3_\mu\right)\cos 3\alpha + \ldots \tag{2}$$

$$I^\rho_\mu = \text{abs}\;\left(i^\rho_\mu\right); \alpha^\rho = \arg\left(i^\rho_\mu\right); \alpha = \alpha^1 - \alpha^3.$$

Then the electric equations for the quantities 0fb in a stationary reference frame system have a form:

$$\begin{bmatrix} \underline{u}^f_s \\ \underline{u}^f_r \end{bmatrix} = \begin{bmatrix} R_s \underline{i}^f_s \\ R'_r \underline{i}^f_r \end{bmatrix} + \begin{bmatrix} L_{\sigma_s} & 0 \\ 0 & L'_{\sigma_r} \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \underline{i}^f_s \\ \underline{i}^f_r \end{bmatrix}$$
$$+ \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \underline{\psi}^f_s \\ \underline{\psi}^f_r \end{bmatrix} + \begin{bmatrix} 0 \\ -jp\omega_m \left(L'_{\sigma_r} \underline{i}^f_r + \underline{\psi}^f_r\right) \end{bmatrix},$$
$$\begin{bmatrix} \underline{u}^0_s \\ \underline{u}^0_r \end{bmatrix} = \begin{bmatrix} R^0_s \underline{i}^0_s \\ R^0_r \underline{i}^0_r \end{bmatrix} + \begin{bmatrix} L_{\sigma_s} & 0 \\ 0 & L'_{\sigma_r} \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \underline{i}^0_s \\ \underline{i}^0_r \end{bmatrix} + \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \underline{\psi}^0_s \\ \underline{\psi}^0_r \end{bmatrix}. \tag{3}$$

The '*f*' and '0' sequences of main fluxes are expressed by the derivatives of the co-energy versus absolute values of the magnetic current harmonics and versus angle between them:

$$\underline{\psi}^f_s = \underline{\psi}^f_r = \underline{\psi}^f_\mu = \frac{\sqrt{3}}{2}\left[\frac{\partial E_m}{\partial I^1_\mu} + j\frac{\partial E_m}{\partial \alpha}\frac{1}{\left(I^1_\mu\right)}\right] e^{j\alpha^1},$$
$$\psi^0_s = \frac{\sqrt{3}}{2}\frac{\partial E_m}{\partial I^3_\mu}\frac{1}{I^3_\mu}\left(\underline{i}^3_\mu + \underline{i}^{3*}_\mu\right) = \sqrt{3}\frac{\partial E_m}{\partial I^3_\mu}\cos\alpha^3,$$
$$\psi^0_r = \frac{\sqrt{3}}{2}\frac{\partial E_m}{\partial I^3_\mu}\frac{1}{I^3_\mu}\left(\underline{i}^3_\mu e^{-j3p\varphi} + \underline{i}^{3*}_\mu e^{j3p\varphi}\right) = \sqrt{3}\frac{\partial\, E_m}{\partial I^3_\mu}\cos\left(\alpha^3 - 3p\varphi\right). \tag{4}$$

Including the relations between the magnetising currents of ρ harmonic and the sequences of winding currents

$$\underline{i}^1_\mu = \sqrt{3}\underline{i}^f_\mu = \sqrt{3}\left(\underline{i}^f_s + \underline{i}^f_r\right) \qquad \underline{i}^3_\mu = \sqrt{3}\left(i^0_s + i^0_r e^{j3p\varphi}\right) = \sqrt{3}\underline{i}^0_\mu \tag{5}$$

is obtained

$$
\begin{aligned}
\underline{\psi}_{\mu}^{f} &= \frac{3}{2}\left[\frac{\partial E_m}{\partial I_{\mu}^{1}}\frac{1}{I_{\mu}^{1}} + j\frac{\partial E_m}{\partial \alpha}\frac{1}{\left(I_{\mu}^{1}\right)^{2}}\right]\underline{i}_{\mu}^{f} = \underline{L}_{\mu}^{f}\left(I_{\mu}^{1} I_{\mu}^{3}, \alpha\right)\underline{i}_{\mu}^{f}, \\
\psi_s^0 &= 3\frac{\partial E_m}{\partial I_{\mu}^{3}}\frac{1}{I_{\mu}^{3}}\mathrm{Re}\,\left(i_{\mu}^{0}\right) = L_{\mu}^{0}\left(I_{\mu}^{1}, I_{\mu}^{3}, \alpha\right)\left(i_s^0 + i_r^0\cos 3p\varphi\right), \\
\psi_r^0 &= 3\frac{\partial E_m}{\partial I_{\mu}^{3}}\frac{1}{I_{\mu}^{3}}\mathrm{Re}\,\left(i_{\mu}^{0}e^{-jp\varphi}\right) = L_{\mu}^{0}\left(I_{\mu}^{1}, I_{\mu}^{3}, \alpha\right)\left(i_s^0\cos 3p\varphi + i_r^0\right).
\end{aligned} \tag{6}
$$

There is no zero sequence component of the rotor current in slip ring motor. So, the below identities are

$$
\begin{aligned}
i_r^0 = 0 \quad &\Rightarrow \quad \alpha^3 = \arg\left(i_s^0\right) = \begin{cases} 0 & i_s^0 \geq 0 \\ \pi & i_s^0 < 0 \end{cases} \quad \Rightarrow \quad e^{j\alpha^3} = e^{-j\alpha^3} \\
&= \begin{Bmatrix} +1 & i_s^0 \geq 0 \\ -1 & i_s^0 < 0 \end{Bmatrix} = \frac{i_s^0}{I_s^0}
\end{aligned} \tag{7}
$$

and the flux sequence formulas assume final simplified forms:

$$
\begin{aligned}
\underline{\psi}_{\mu}^{f} &= \frac{\sqrt{3}}{2}\frac{\partial E_0}{\partial I_{\mu}^{1}}e^{j\alpha^1} + \frac{\sqrt{3}}{2}\left(\frac{1}{2}\frac{\partial E_3}{\partial I_{\mu}^{1}} + \frac{3}{2}\frac{E_3}{I_{\mu}^{1}}\right)\frac{i_s^0}{I_s^0}e^{-j2\alpha^1} \\
&\quad + \frac{\sqrt{3}}{2}\left(\frac{1}{2}\frac{\partial E_3}{\partial I_{\mu}^{1}} - \frac{3}{2}\frac{E_3}{I_{\mu}^{1}}\right)\frac{i_s^0}{I_s^0}e^{j4\alpha^1}, \\
\psi_s^0 &= \sqrt{3}\left(\frac{\partial E_0}{\partial I_{\mu}^{3}}\frac{i_s^0}{I_s^0} + \frac{\partial E_3}{\partial I_{\mu}^{3}}\cos 3\alpha^1 + \ldots\right);\ \psi_r^0 = \psi_s^0\cos 3p\varphi.
\end{aligned} \tag{8}
$$

2. General Properties of the Co-energy Components for a Cylindrical Machine

When one of the two harmonics of magnetising current equals zero, the co-energy does not depend on the angle a. Then the second term of the co-energy series equals zero

$$E_3\left(I_{\mu}^{1}, 0\right) = 0, \qquad E_3\left(0, I_{\mu}^{3}\right) = 0. \tag{9}$$

As a consequence its derivatives also equal zero

$$\frac{\partial}{\partial I_{\mu}^{1}}E_3\left(I_{\mu}^{1}, 0\right) = 0, \qquad \frac{\partial}{\partial I_{\mu}^{3}}E_3\left(0, I_{\mu}^{3}\right) = 0. \tag{10}$$

On the other hand, the first term of the co-energy has to fulfil the conditions

$$\frac{\partial}{\partial I_\mu^1} E_0\left(I_\mu^1, I_\mu^3\right)\bigg|_{I_\mu^1=0} = 0, \qquad \frac{\partial}{\partial I_\mu^3} E_0\left(I_\mu^1, I_\mu^3\right)\bigg|_{I_\mu^3=0} = 0 \tag{11}$$

arising from the assumption that the positive and zero sequential components of the fluxes equal zero when this component of the currents does not exist. Based on the above conditions the following formulas may be written:

$$\underline{\psi}_\mu^f\left(I_\mu^1, 0\right) = \frac{\sqrt{3}}{2}\frac{\partial E_0}{\partial I_\mu^1} e^{j\alpha^1}, \tag{12}$$

$$\psi_s^0\left(I_\mu^1, 0\right) = \sqrt{3}\left(\frac{\partial E_3}{\partial I_\mu^3}\cos 3\alpha^1 + \ldots\right), \tag{13}$$

$$\underline{\psi}_\mu^f\left(0, I_\mu^3\right) = \frac{\sqrt{3}}{2}\frac{\partial E_3}{\partial I_\mu^1}\frac{i_s^0}{I_s^0}, \tag{14}$$

$$\psi_s^0\left(0, I_\mu^3\right) = \sqrt{3}\frac{\partial E_0}{\partial I_\mu^3}\frac{i_s^0}{I_s^0}. \tag{15}$$

When the stator and rotor windings are symmetrically designed, the zero sequential component of the currents cannot generate the positive component of the fluxes. From this the additional condition arises

$$\underline{\psi}_\mu^f\left(0, I_\mu^3\right) = 0 \quad \Rightarrow \quad \frac{\partial E_3}{\partial I_\mu^1}\bigg|_{I_\mu^1=0} = 0. \tag{16}$$

3. Approximation of Co-energy Components

The first two Taylor's series terms of a function of two variables have a known form:

$$\begin{aligned} E_k\left(I_\mu^1, I_\mu^3\right) = {} & \frac{1}{2!}\left[A_{20}^k\left(I_\mu^1\right)^2 + A_{02}^k\left(I_\mu^3\right)^2 + 2A_{11}^k\left(I_\mu^1\right)\left(I_\mu^3\right)\right] \\ & + \frac{1}{4!}\Big[A_{40}^k\left(I_\mu^1\right)^4 + A_{04}^k\left(I_\mu^3\right)^4 \\ & + 4\left(A_{31}^k\left(I_\mu^1\right)^3\left(I_\mu^3\right) + A_{13}^k\left(I_\mu^1\right)\left(I_\mu^3\right)^3\right) + 6A_{22}^k\left(I_\mu^1\right)^2\left(I_\mu^3\right)^2\Big]. \end{aligned} \tag{17}$$

Each of the above co-energy functions has 8 coefficients

From the conditions (9) arises: $A_{20}^3 = A_{02}^3 = A_{40}^3 = A_{04}^3 = 0$,
from the (11): $A_{11}^0 = A_{31}^0 = A_{13}^0 = 0$,
and from the (16): $A_{11}^3 = A_{13}^3 = 0$.

These identities reduce the number of unknown coefficients from 16 to 7 only

$$E_0\left(I_\mu^1, I_\mu^3\right) = \frac{1}{2!}\left[A_{20}^0\left(I_\mu^1\right)^2 + A_{02}^0\left(I_\mu^3\right)^2\right] + \frac{1}{4!}\left[A_{40}^0\left(I_\mu^1\right)^4 + A_{04}^0\left(I_\mu^3\right)^4 + 6A_{22}^0\left(I_\mu^1\right)^2\left(I_\mu^3\right)^2\right], \tag{18}$$

$$E_3\left(I_\mu^1, I_\mu^3\right) = \frac{1}{4!}\left[4\left(A_{31}^3\left(I_\mu^1\right)^3\left(I_\mu^3\right)\right) + 6A_{22}^3\left(I_\mu^1\right)^2\left(I_\mu^3\right)^2\right] \tag{19}$$

and the flux sequence components may be written as:

$$\begin{aligned}\underline{\psi}_\mu^f = {} & \frac{3}{2}\left(A_{20}^0 + \frac{1}{6}A_{40}^0\left(I_\mu^1\right)^2 + \frac{1}{2}A_{22}^0\left(I_\mu^3\right)^2\right)\underline{i_\mu^f} \\ & + \frac{3}{2}\left(\frac{1}{2}A_{31}^3\left(I_\mu^1\right)^2 + \frac{5}{8}A_{22}^3\left(I_\mu^1\right)\left(I_\mu^3\right)\right)i_s^0 e^{-j2\alpha^1} \\ & + \frac{3}{2}\left(-\frac{1}{8}A_{22}^3\left(I_\mu^1\right)\left(I_\mu^3\right)\right)i_s^0 e^{j4\alpha^1},\end{aligned} \tag{20}$$

$$\begin{aligned}\psi_s^0 = {} & 3\left(A_{02}^0 + \frac{1}{6}A_{04}^0\left(I_\mu^3\right)^2 + \frac{1}{2}A_{22}^0\left(I_\mu^1\right)^2\right)i_s^0 \\ & + \sqrt{3}\left(\frac{1}{6}A_{31}^3\left(I_\mu^1\right)^3 + \frac{1}{2}A_{22}^3\left(I_\mu^1\right)^2\left(I_\mu^3\right)\right)\cos 3\alpha^1 + \ldots\end{aligned} \tag{21}$$

The positive sequence component of the fluxes has three terms. In steady state, the first expresses the fundamental harmonic of the fluxes. The second and the third terms express the high harmonics of the fluxes, which can only be generated when both the sequences of the currents are not zero.

The zero sequence component of the fluxes has two terms. The first disappears when the zero sequence of the currents equals zero. The second expresses the third harmonic of the fluxes generated by the positive component of the currents.

The coefficients A_{22}^0, A_{31}^3,A_{22}^3 express the mutual linkage between the positive and zero sequence components which may appear in the saturation conditions.

When the highest harmonic is omitted, the coefficient $A_{22}^3 = 0$, and the formulas expressing sequences of the fluxes are analogous to the series defined for three phase stator winding. These series have been described in the previous paper.

$$\begin{aligned}\underline{\psi}_\mu^f &= \left(C_l + C_1\left(I_\mu^f\right)^2 + C_2\left(i_s^0\right)^2\right)\underline{i_\mu^f} + C_3 i_s^0\left(\underline{i_\mu^{f*}}\right)^2, \\ \psi_s^0 &= \left(C_0 + \frac{1}{3}C_4\left(i_s^0\right)^2 + 2C_2\left(I_\mu^f\right)^2\right)i_s^0 + \frac{1}{3}C_3\left[\left(\underline{i_\mu^f}\right)^3 + \left(\underline{i_\mu^{f*}}\right)^3\right].\end{aligned} \tag{22}$$

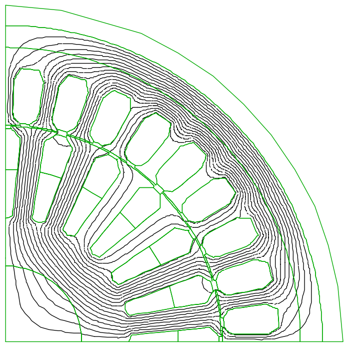


Fig. 1. One pitch pole of the modelled motor 1.1 [kW]. The field was computed for the original width of the stator yoke and for winding supplied only by the positive sequence component of the currents. $I_1 = -3.5A$, $I_2 = I_3 = -1.75A$

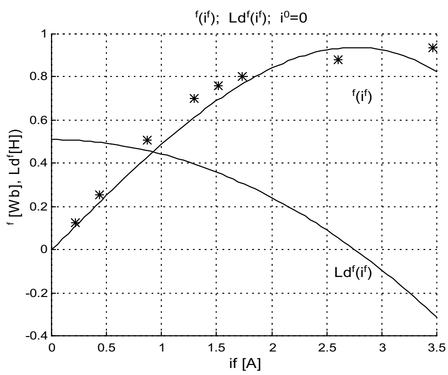


Fig. 2. Non-linear characteristic and its derivative for positive sequence component of the currents. The approximating functions lost their physical correctness for $i^f > 2.5$ A, $A_{20}^0 = +0.34$; $A_{40}^0 = -0.03$

4. Identification of the Linked Fluxes Based on Modelling of Magnetic Field

Searched non-linear characteristics

$$\psi_\mu^f \left(I_\mu^1, I_\mu^3\right); \qquad \psi_s^0 \left(I_\mu^1, I_\mu^3\right) \tag{23}$$

may be determined by modelling the magnetic field in the machine.

There are four methods:

1^0 Basing on calculation of the global co-energy values by FEM for the chosen values of the currents and approximation of the results by Taylor's series together with the derivatives

$$E_{co} = \int_V \left[\int B(H) \mathrm{d}H \right] \mathrm{d}V,$$
$$\psi_n = \frac{\partial}{\partial i_n} E_{co} \left(\varphi, i_1, \ldots\ i_N \right). \tag{24}$$

On account of a numerical differentiation, the correctness of the results is sufficient only for the major terms of the fluxes.

2^0 Basing on the average magnetic potential value in the slots. This may be written:

$$\psi_n = \pm 2plw \sum_{k=1}^{K} A_n^k. \tag{25}$$

A – magnetic potential in a slot
K – number of the slots for one phase and one pole
The correctness of this method depends on the precision of reading the magnetic vector potential values.

3^0 Basing on the energy supplied by individual circuits into the magnetic core

$$E_{\text{linear}} = \sum_{n=1}^{N} E_{\text{linear}_n} = 2p \sum_{n=1}^{N} \sum_{k=1}^{K} \frac{1}{2} \int_{V_n^k} A_n^k J_n^k \mathrm{d}V, \tag{26}$$

where
J – current density on a slot.
The energy is calculated by postprocessor as the energy of the system linearized in the final point. Taking into account the formula

$$\sum_{n=1}^{N} \psi_n i_n = E_{\text{co}} + E = 2E_{\text{linear}} \tag{27}$$

the flux linked with the n-coil equals

$$\psi_n = \frac{2E_{\text{linear}_n}}{i_n} = \frac{2p2 \sum_{k=1}^{K} E^k_{\text{linear}_n}}{i_n} \tag{28}$$

The results obtained in this way are more correct but it is obvious that the linked flux may only be calculated for circuits with current not equalling zero. All of the methods calculate the global flux as a sum of a main flux and a part of leakage fluxes. To separate those two the calculation of leakage magnetic field in one slot pitch is necessary.

4^0 Basing on the magnetic flux density in air-gap

$$\psi_n = \pm w_n \sum_{k=1}^{K} rl \int_{\alpha_n^k}^{\alpha_n^k+\beta} B(\alpha)\, \mathrm{d}\alpha. \tag{29}$$

The results do not include the leakage fluxes.

The computations have been done for a small slip ring induction motor 1.1 kW. The diameter of the stator was increased and decreased to observe the influence of the yoke width on the saturation effects.

Supplying the winding by:

- positive sequence component of currents,
- zero sequence of the currents,
- positive sequence component of currents at the zero sequence equals constant,

the searched characteristics were determined:

The coefficients values were estimated using a discrete least squares approximating method with the weight coefficients chosen experimentally.

When the windings are supplied only by the positive sequence component of currents, the absolute values of the positive flux sequence versus positive sequence of currents is

$$\psi_\mu^f = \frac{3}{2}\left(A_{20}^0 + \frac{1}{6}A_{40}^0\left(I_\mu^1\right)^2\right) i_\mu^f; \qquad I_\mu^1 = \sqrt{3} i_\mu^f. \tag{30}$$

It can be seen that the approximating functions lost their physical correctness at maximum point, were the derivative equals zero. The zero sequence of fluxes equalled zero, even at the high saturation conditions,

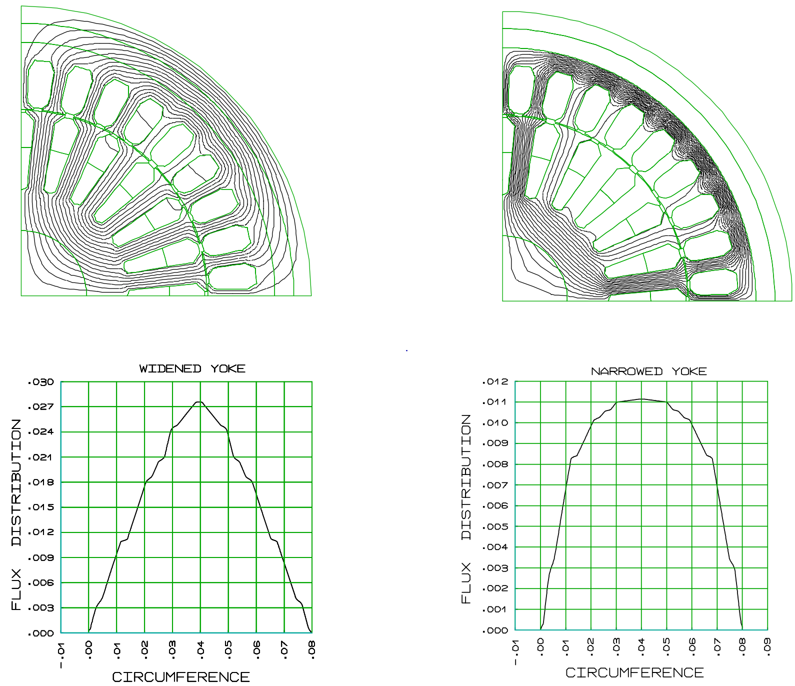
$$\psi_s^0 = \sqrt{3}\left(\frac{1}{6}A_{31}^3\left(I_\mu^1\right)^3\right) = 0, \tag{31}$$

then $A_{31}^3 = 0.0000$.

The zero value of this coefficient is the result of zero values of the zero component of linked fluxes obtained for the symmetrical 3-phase supply condition even for high saturation. It is an effect of a compensation of the co-energy changes due to saturation of the teeth and the stator yoke for original geometry of the modelled motor. This effect was confirmed by the field computations for modified width of the stator yoke.

The shape of the flux distribution and the co-energy value do not change when the flux distribution rotates along the circumference. The deformations of the shape of the flux density distributions are inverted.

When the windings are supplied by the zero and the positive sequence component of currents, it is:



A^3_{31} = -0.0004 A^3_{31} = +0.0004

Fig. 3. The air-gap flux distributions along the circumference at widened and narrowed yoke of the stator. Winding supplied only by the positive sequence component of the currents. $I_1 = -7A$, $I_2 = I_3 = 3.5A$.

$$\psi_s^0 = \sqrt{3}\left(A^0_{02} + \frac{1}{6}A^0_{04}\left(I^3_\mu\right)^2 + \frac{1}{2}A^0_{22}\left(I^1_\mu\right)^2\right) I^3_\mu + \sqrt{3}\left(\frac{1}{6}A^3_{31}\left(I^1_\mu\right)^3\right)\cos 3\alpha^1 + \dots, \tag{32}$$

$$L^0_\mu = 3\left(A^0_{02} + \frac{1}{6}A^0_{04}\left(I^3_\mu\right)^2 + \frac{1}{2}A^0_{22}\left(I^1_\mu\right)^2\right).$$

In saturation conditions the influence of the positive sequence component of the currents on the inductance for the zero sequence component is apparent. The influence of the zero sequence component of the currents on the inductance for the positive sequence component is not observed.

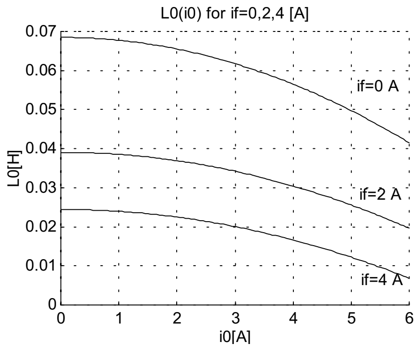


Fig. 4. Inductance of zero sequence component versus zero sequence of the currents at the positive sequence component equalling constant. $A^0_{02} = \pm 0.023$; $A^0_{04} = -0.0003$; $A^0_{22} = -0.001 = 3A^0_{04}$

5. Conclusions

The model of induction machine based on the co-energy, as a function of the magnetising current harmonics is useful for qualitative analysis of saturation phenomena. Its parameters can be calculated by modelling the magnetic field. In comparison with the well-known models taking into account only the positive sequence of the currents and linked fluxes, the following phenomena are considered:

- the generation of the zero sequence of phase linked fluxes by the positive sequence of the currents,
- the decrease of the inductance for zero sequence by the positive sequence of the currents,
- the generation of the higher harmonic of fluxes when the zero sequence of current is present.

The approximation by two Taylor's series terms is correct in the initial intervals of approximation only. If the terms of higher degree are considered, the intervals of the correct approximation are widened.

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