EFFICIENT RESOURCE USAGE TECHNIQUES IN MULTISERVICE NETWORKS

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Abstract

This paper is concerned with the saturation probability of aggregate traffic data arrival rate on a communication link. This performance metric, also referred to as tail distribution of aggregate traffic, is essential in traffic control and management algorithms of high speed networks including future QoS Internet. In this paper efficient closed-form upper bounds have been derived for the saturation probability when very few information is available on the aggregate traffic. The performance of these estimators is also performed via numerical examples.

Keywords: Chernoff bound, Hoeffding inequality, traffic overflow, QoS parameters.

1. Introduction

The task of quantifying resource usage in broadband QoS guaranteed networks is closely related to the performance analysis of resource utilization. A basic problem in performance analysis of networks with statistical multiplexing feature is to efficiently estimate or bound the overflow probability of aggregate traffic on a single link with buffer. The question can be asked how to increase the buffer or link capacity in order to decrease the overflow probability while keeping the queuing delay at a reasonable level [1].

The performance of statistical multiplexing greatly depends on the multiplexing strategy. Probably, one of the most understood strategies is Rate Envelope Multiplexing (REM) [4] often referred to as bufferless statistical multiplexing [2, 13]. In this case the task is to ensure that the combined instantaneous arrival rate of multiplexed traffic sources (under the continuous fluid assumption) does not exceed the link capacity C. For this the saturation probability of aggregate traffic arrival rate on a single link is an important performance parameter. This probability can be used e.g. for calculating the probability of time-congestion of resources, i.e. the fraction of time when the traffic offered to a transmission link exceeds the capacity of that link. Moreover, it is also useful for the estimation of the traffic congestion (traffic blocking) probability. In recently developed measurement based admission control algorithms estimators for effective bandwidth [12] are often used which is also closely related to the overflow probability of aggregate traffic [2, 3, 5, 6, 8, 13].

A general widely accepted and used method for overflow probability estimation of aggregate traffic is the Chernoff's bounding method for upper tail distribution of sums of random variables. Although the use of the Central Limit Theorem (CLT) can be applied to similar problems [9], for service level guarantees upper bounds are more appropriate. Moreover the CLT is efficient only in case of 'large' number of independent traffic sources.

In this paper first we derive a new upper bound for the overflow probability of aggregate traffic which only requires the mean arrival rate of aggregate traffic, the individual peak rates and the number of traffic sources. Although the optimal solution of the new formula obtained cannot be drawn in closed form, we present simplification to resolve this difficulty, resulting, however, in weaker upper bounds. Even these bounds, by means of extensive numerical investigations, seem to be better than the well-known Hoeffding inequality which uses the same information on the traffic sources.

The rest of the paper is organized as follows. In the next section first we enlighten the idea behind the Chernoff bounding method, then present the use of this method for computing overflow probability of aggregate traffic on a communication link. In subsequent sections we perform the derivation of our new bound and its closed form approximations. The performance evaluation of these bounds are also shown through numerical examples.

2. Tail Estimates Via Basic Inequalities

Let us start with the introduction of the Markov inequality.

Lemma 1 *Let X be a non-negative random variable. Then, for any $C > 0$,*

$$P(X \geq C) \leq \frac{M}{C}, \tag{1}$$

where $M = E[X]$, the expectation value of X.

A well-known upper bound for the tail distribution is Chernoff's bound, i.e. for $C > M$,

$$P(X \geq C) \leq \inf_{s, s>0} \frac{E(e^{sX})}{e^{sC}}, \tag{2}$$

which comes from the Markov inequality if we apply Lemma 1 for the random variable e^{sX} with parameter e^{sC}. Now it can be clearly seen that Chernoff's idea was to compute the moment generation function $E(e^{sX})$ and optimizing the formula above with respect to s. Although this approach has turned out to provide good estimates for several problems, in many cases the exact distribution of X is not known, hence the formula in the equation above should be approximated.

2.1. Two Essential Theorems Providing Bounds for the Sum of Random Variables

In this subsection we present fundamental results on bounding the tail distribution of sum of bounded and independent random variables using only reasonable information on them.

Theorem 2 *(Hoeffding [10]) Let $X_1, \ldots, X_n$ be independent random variables with unit peaks, i.e. $0 \leq X_k \leq 1$ for each k. Let $S = \sum_k X_k$, $\bar{X} = \frac{1}{n}\sum_k X_k$, $M = E[\sum_k X_k]$, $p = E[\bar{X}]$ and $q = 1 - p$, that is $S = n\bar{X}$ and $C = n(p + t)$. Then for $0 \leq t < q$,*

$$P(\bar{X} - p \geq t) \leq \left(\left(\frac{p}{p+t}\right)^{p+t}\left(\frac{q}{q-t}\right)^{q-t}\right)^n \tag{3}$$

or equivalently

$$P(S \geq C) \leq \left(\frac{M}{C}\right)^C\left(\frac{n-M}{n-C}\right)^{n-C}. \tag{4}$$

To achieve this result it is enough to realize that substituting the original random variables X_k with Bernoulli distributed ones having the same mean values maximizes the left-hand side of (3), i.e. the tail probability.

Note that the optimal parameter s^* in the transformed formula comes out to be

$$s^* = \log\frac{C\,(n-M)}{M\,(n-C)}. \tag{5}$$

For detailed proof see [10].

The subsequent inequality is concerned with X_k random variables having different upper bounds, i.e. $0 \leq X_k \leq p_k$. A reformulated version of the original inequality states

Theorem 3 *(Hoeffding 2 [10])*[1]

$$P(S \geq C) \leq \exp\left(-2\frac{(C-M)^2}{\sum_k p_k^2}\right). \tag{7}$$

Remarks: This latter bound cannot be reduced to the first Hoeffding inequality when $p_k = 1, \forall k$. It is due to the different approximation technique used in the derivations. It can also be shown that for this case the first Hoeffding bound is never worse than the second one.

[1] The original inequality was formulated as

$$P(S \geq C) \leq \exp\left(-2\frac{(C-M)^2}{\sum_k (b_k - a_k)^2}\right), \tag{6}$$

where for the random variables $a_k \leq X_k \leq b_k$ holds.

At this state, the obvious question arises how to develop an upper bound for the case of different p_k's which can be the real counterpart of the first Hoeffding inequality in the sense that it would give back this bound for the case $p_k = 1, \forall k$. In the next section we derive this bound for random variables with different peaks, and also closed-form approximations are performed.

3. Improved Resource Utilization Formulae

3.1. Modeling Principles

Let us suppose that a multiservice communication network serves N users belonging to one and only one of J different sets of type, that is J service class is required to be defined, each of them having $n_1, n_2, \ldots, n_J$ users, so that

$$N = \sum_{j=1}^{J} n_j. \tag{8}$$

The only traffic descriptor of a service class is the maximum instantaneous data arrival rate (peak rate). Each source in a service class has identical peak rates.

We model the generation of information of traffic sources in the service classes by stationary stochastic processes $X_{11}(t), \ldots, X_{1n_1}(t), \ldots, X_{J1}(t), \ldots, X_{Jn_J}(t)$, where $X_{ji}(t)$ denotes the instantaneous arrival rate of the ith traffic source of type j at time t. We also assume that the arrival rate $X_{ji}(t)$ varies between 0 and p_j the peak rate of the traffic sources in class j, i.e. $0 \le X_{ji}(t) \le p_j, \forall t, j$. The sources are regarded as independent random variables for $\forall t$.

Let $S(t) = \sum_{j=1}^{J} \sum_{i=1}^{n_j} X_{ji}(t)$ and $M(t) = E[S(t)]$ where E[.] denotes the expectation value operator. Due to stationarity we can neglect the time dependency, so the tail distribution of the aggregate traffic can simply be $P(S \ge C)$.

3.2. Improved Bound for Guaranteeing Link Saturation Probability

The following contribution introduces a new formula that can be used for bounding the tail probability of the link saturation of (bufferless) statistically multiplexed traffic:

Theorem 4 *Let X_{ji} $(j = 1, \ldots, J; i = 1, \ldots, n_J)$ be independent (not necessarily identically distributed) random variables where $0 \le X_{ji} \le p_j$ then*

$$P(S \ge C) \le e^{-s^* C} \left(\frac{M + \sum_{j=1}^{J} n_j \frac{p_j}{e^{s^* p_j} - 1}}{N} \right)^N \prod_{j=1}^{J} \left(\frac{e^{s^* p_j} - 1}{p_j} \right)^{n_j}. \tag{9}$$

where s^* is the solution of the following equation,

$$\sum_{j=1}^{J} \frac{n_j p_j}{1-e^{-sp_j}} - \frac{N\sum_{j=1}^{J} n_j p_j^2 \frac{e^{sp_j}}{(e^{sp_j}-1)^2}}{M+\sum_{j=1}^{J} n_j \frac{p_j}{e^{sp_j}-1}} - C = 0. \tag{10}$$

Proof. Let us follow the same line of the derivation as in (3). The starting point is again the Chernoff formula:

$$P(S \geq C) \leq E[e^{s\sum_{j=1}^{J}\sum_{i=1}^{n_j} X_{ji}}]e^{-sC}.$$

Due to independence we obtain

$$P(S \geq C) \leq \prod_{j=1}^{J}\prod_{i=1}^{n_j} E[e^{sX_{ji}}]e^{-sC}.$$

The exponential function is strictly convex, thus, the expectation value of the random variables $e^{sX_{ji}}$ can be bounded as

$$E(e^{sX_{ji}}) \leq 1 + m_{ji}\frac{e^{sp_j}-1}{p_j},$$

where m_{ji} is the mean value of X_{ji}.

In this way, the tail distribution of the sum of the random variables can be bounded above by

$$P(S \geq C) \leq \prod_{j=1}^{J}\prod_{i=1}^{n_j}\left(1 + m_{ji}\frac{e^{sp_j}-1}{p_j}\right)e^{-sC}. \tag{11}$$

The product on the right-hand side can be reformulated as

$$\prod_{j=1}^{J}\left(\frac{e^{sp_j}-1}{p_j}\right)^{n_j}\prod_{j=1}^{J}\prod_{i=1}^{n_j}\left(m_{ji} + \frac{p_j}{e^{sp_j}-1}\right). \tag{12}$$

The second product can be further approximated by

$$\prod_{j=1}^{J}\prod_{i=1}^{n_j}\left(m_{ji} + \frac{p_j}{e^{sp_j}-1}\right) \leq \left(\frac{\sum_{j=1}^{J}\sum_{i=1}^{n_j}\left(m_{ji} + \frac{p_j}{e^{sp_j}-1}\right)}{N}\right)^N \tag{13}$$

because of the relation between geometric and arithmetic mean of non-negative real numbers. Now an upper bound of the tail distribution can be expressed as

$$\begin{aligned} P(S \geq C) &\leq e^{-sC}\left(\frac{M+\sum_{j=1}^{J} n_j \frac{p_j}{e^{sp_j}-1}}{N}\right)^N \prod_{j=1}^{J}\left(\frac{e^{sp_j}-1}{p_j}\right)^{n_j} \\ &\stackrel{\circ}{=} T(s, C, M, \mathbf{n}, \mathbf{p}), \end{aligned} \tag{14}$$

where $\mathbf{n}$ is the vector of n_j's and $\mathbf{p}$ comprises the peak rates.

In addition, we have to determine the optimal s, which minimizes the right-hand side of (14), in other words, which gives the lowest upper bound. Unfortunately, a closed form expression for the optimal s cannot be derived, however, an essentially non-algebraic equation can be formulated, of which solution serves as the optimal s. The upper bound in (14) is minimized if its logarithm is minimized, that is, the optimal s should minimize the function

$$\sum_{j=1}^{J} n_j \log \frac{e^{sp_j} - 1}{p_j} + N \log \left(\frac{M + \sum_{j=1}^{J} \frac{n_j p_j}{e^{sp_j} - 1}}{N} \right) - sC. \tag{15}$$

Taking the derivative of this with respect to s we get

$$\sum_{j=1}^{J} \frac{n_j p_j}{1 - e^{-sp_j}} - \frac{N \sum_{j=1}^{J} n_j p_j^2 \frac{e^{sp_j}}{(e^{sp_j} - 1)^2}}{M + \sum_{j=1}^{J} n_j \frac{p_j}{e^{sp_j} - 1}} - C = 0. \tag{16}$$

The solution of (16) gives the optimal s^*. □

Remarks: The optimal parameter s^* can be numerically computed in a straightforward manner using any standard root-finding algorithm. Note, that the equation above contains only the aggregate mean M, meaning that there is no need to know the mean values of the random variables (i.e. the mean arrival rates of the traffic flows).

The upper bound presented in Theorem 4 can be considered as the real counterpart of the first Hoeffding inequality (3), because in the case of identical upper bounds (peak rates), i.e. $p_j = 1, \forall j$, we get back this inequality. Also, in this case the optimal s^* from equation (16) will be $s^* = \log \frac{C(n-M)}{M(n-C)}$, which is the same as the optimal s obtained in the derivation of the first Hoeffding bound (3).

According to the results in [10] it is also true that the upper estimation in Theorem 4 is the *best possible one* with the given assumptions, namely, the Chernoff bound is bounded above using only the number of flows, the peak rates of the flows and the aggregate mean rate as the available information on the traffic.

3.3. Closed Form Approximations

Although the numerical evaluation of the upper bound in (9) can be straightforward, often a closed form expression can be more useful and expressive, even if it is not optimal with respect to the transformation parameter s. The problem with the result in (9) is that although the optimal parameter s^* can be numerically computed, the optimization may be a time consuming process and can hardly be accomplished in real time.

In the following, we present two ways of how to find suboptimal but closed form formulae for s which can be substituted into (9) obtaining closed form expressions for the improved bound.

One method is as follows:
First, we take the first three terms in the Taylor series expansion of the fraction $p_j/(e^{sp_j}-1)$ with respect to s at $s=0$. Next putting them into the logarithm in (15), then taking the first two terms of the Taylor series expansion of the received formula in the same way, we get

$$sM + \left(\frac{1}{8}P_2 - \frac{1}{2N}\left(M - \frac{1}{2}P\right)^2\right)s^2 - sC,$$

where $M = \sum_{j=1}^{J}\sum_{i=1}^{n_j} m_{ji}$, $P = \sum_{j=1}^{J} n_j p_j$, $P_2 = \sum_{j=1}^{J} n_j p_j^2$.

The expression above should be minimized in order to obtain a sub-optimal value for s. Thus we get

$$\tilde{s}_1^* = \frac{C-M}{\frac{1}{4}P_2 - \frac{1}{N}(M - \frac{1}{2}P)^2}.$$

Now we can express an upper bound for the tail distribution, which does not contain the transformation parameter s. Hence, substituting $\tilde{s}_1^*$ into (9), we finally obtain

$$P(S \geq C) \leq T(\tilde{s}_1^*, C, M, \mathbf{n}, \mathbf{p}) =$$

$$\left(\frac{M + \sum_{j=1}^{J} \frac{n_j p_j}{e^{\frac{C-M}{K}p_j}-1}}{N}\right)^N e^{-\frac{(C-M)C}{K}} \prod_{j=1}^{J}\left(\frac{e^{\frac{C-M}{K}p_j}-1}{p_j}\right)^{n_j}, \tag{17}$$

where $K = \frac{1}{4}P_2 - \frac{1}{N}\left(M - \frac{1}{2}P\right)^2$.

Our second suggestion for a suboptimal s has the same form as the optimal parameter (5) obtained at the first Hoeffding inequality. Substituting each peak rate with the arithmetic mean of the peak rate of all the sources, that is $p = p_j = \frac{1}{N}\sum_{j=1}^{J} n_j p_j, \forall j$, we obtain the optimization parameter:

$$s^* = \frac{1}{p}\log\frac{C\,(Np-M)}{M\,(Np-C)}. \tag{18}$$

Taking into account that (2) is true for $\forall s$ we may apply (18) for (9) achieving a new closed form bound, the performance of which, however, should be investigated carefully. Apparently, we may use other types of means as well, e.g. if

$$p = \sqrt{\frac{1}{N}\sum_{j=1}^{J} n_j p_j^2}\,, \forall j,$$

we get

$$\tilde{s}_2^* = \sqrt{\frac{N}{P_2}}\log\frac{C\,(P-M)}{M\,(P-C)}, \tag{19}$$

or in general

$$\tilde{s}_{2k}^* = \sqrt[k]{\frac{N}{\sum_{j=1}^{J} n_j p_j^k}} \log \frac{C\,(P-M)}{M\,(P-C)}, \tag{20}$$

each of which would equally be applicable, however, through extensive numerical investigation the use of $\tilde{s}_2^*$ seems to show the best properties. This choice still satisfies the requirement that $\tilde{s}_2^*$ should remain positive since neither M nor C can exceed the sum of the peak rates. Another important property of $\tilde{s}_2^*$ is, that as opposed to $\tilde{s}_1^*$, it becomes the original optimal value of s if all the peak rates are equal to each other. From this we can expect that if the deviation of the peak rates is small, this latter solution gives better results than (17).

In the next section we compare the new bounds derived from (9), (17) and (19) to each other and to the second Hoeffding bound (7), numerically.

4. Comparison with Numerical Examples

The comparison is made between the negative exponents of the bounds derived from (9), (17) and (19). This means that the behavior of the difference functions

$$\mathrm{Diff}_{o,s1}(C) = -\log T(s^*, C, M, \mathbf{n}, \mathbf{p}) + \log T(\tilde{s}_1^*, C, M, \mathbf{n}, \mathbf{p}), \tag{21}$$

$$\mathrm{Diff}_{o,s2}(C) = -\log T(s^*, C, M, \mathbf{n}, \mathbf{p}) + \log T(\tilde{s}_2^*, C, M, \mathbf{n}, \mathbf{p}), \tag{22}$$

$$\mathrm{Diff}_{s1,s2}(C) = -\log T(\tilde{s}_1^*, C, M, \mathbf{n}, \mathbf{p}) + \log T(\tilde{s}_2^*, C, M, \mathbf{n}, \mathbf{p}), \tag{23}$$

and

$$\mathrm{Diff}_{s1,ho2}(C) = -\log T(\tilde{s}_1^*, C, M, \mathbf{n}, \mathbf{p}) - \frac{2(C-M)^2}{\sum_{j=1}^{J} n_j p_j^2} \tag{24}$$

are to be analyzed, where

$$\log T(s, C, M, \mathbf{n}, \mathbf{p}) = N \log\left(\frac{M + \sum_{j=1}^{J} \frac{n_j p_j}{e^{sp_j} - 1}}{N}\right) + \\ + \sum_{j=1}^{J} n_j \log \frac{e^{sp_j} - 1}{p_j} - sC. \tag{25}$$

From our large number of numerical examples hereafter we show four different cases to represent the results. *Table 1* shows the most important parameters of the different traffic situations, which are the number of service classes J, the vector of the number of the sources in the service classes $\mathbf{n}$, the vector of the peak rates $\mathbf{p}$,

Table 1 The parameter sets of different traffic situations

	a.	b.	c.	d.
J	2	5	10	10
n	(10,1)	(10,8,6,5,6)	(4,5,4,5,4,5,4,5,4,5)	(15,12,12,5,6,6,4,10,3,6)
p	(1,10)	(1,2,4,8,10)	(1,2,3,4,5,6,7,8,9,10)	(1,2,3,4,5,10,11,20,24,25)
P	20	150	195	651
M	8	80	60	100
D_P	0.13	0.023	0.014	0.013
M/P	0.4	0.53	0.31	0.15

the sum of the peak rates P, the aggregate mean arrival rate M, and a normalized deviation like parameter D_P for comparison purposes defined as:

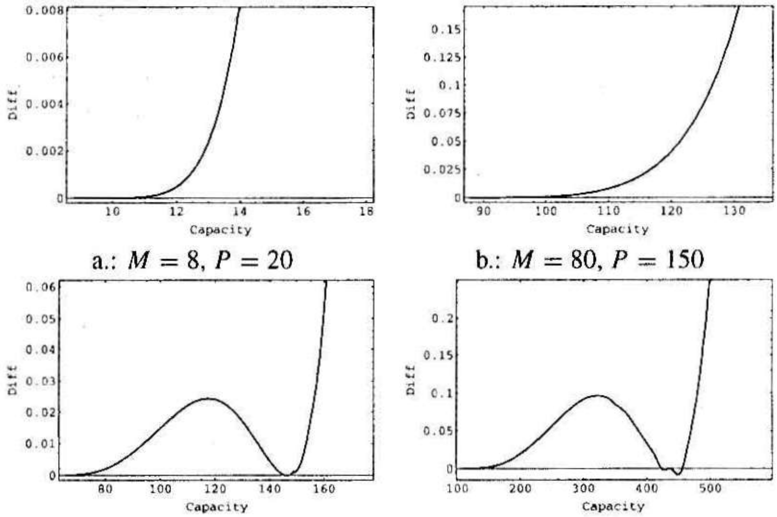
$$D_P = \frac{1}{P}\sqrt{\frac{1}{N}\sum_{j=1}^{J} n_j\left(p_j - \frac{1}{N}P\right)^2}.$$

The mean to peak ratio M/P is also noted which may give an outline about the total traffic activity in the different cases.

Fig. 1, *Fig. 2* and *Fig. 3* show the functions $\mathrm{Diff}_{o,s1}$, $\mathrm{Diff}_{o,s2}$ and $\mathrm{Diff}_{s1,s2}$ with respect to the capacity C. To better understand the meaning of the curves let us take, for example, the third plot of the first comparison in *Fig. 1*. At $C = 120$, it shows that the difference of the negative exponents of the probability counted with the optimal method and that of counted with (17) is about 0.024, which means that (17) gives $e^{0.024}(\approx 1.024)$ times larger (i.e. worse) upper bound than the optimal technique does. The difference between the optimal and the other two methods in *Fig. 1* and *Fig. 2* gives us a general overview about the accuracy of using the suboptimal $\tilde{s}_1^*$ and $\tilde{s}_2^*$ instead of s^*. Note that in *Fig. 2* in most of the cases the difference $\mathrm{Diff}_{o,s2}$ remains under 0.5, which appears to be a negligible error, considering that these bounds have around the middle of the interval $[M, P]$ a value of 0.01 with a slope of about -10 as capacity increases, resulting in a waste of less than 0.1% of the total capacity.

Investigating the relation between the two suboptimal methods we can first draw the conclusion that the one using $\tilde{s}_2^*$ gets worse as D_P increases (as we would expect), while the other one shows little dependence on it. However, considering the above mentioned facts, this often small deviation of about 0.5 has little significance. On the other hand, the effect of the change of the value M/P has considerable impact. While the formula using $\tilde{s}_2^*$ becomes more precise as M/P decreases, the one with $\tilde{s}_1^*$ shows strong degradation. Note that this tendency remains overshadowed due to the high deviations ($D_P \approx 0.02$) with respect to the number of sources in our discussed traffic situations.

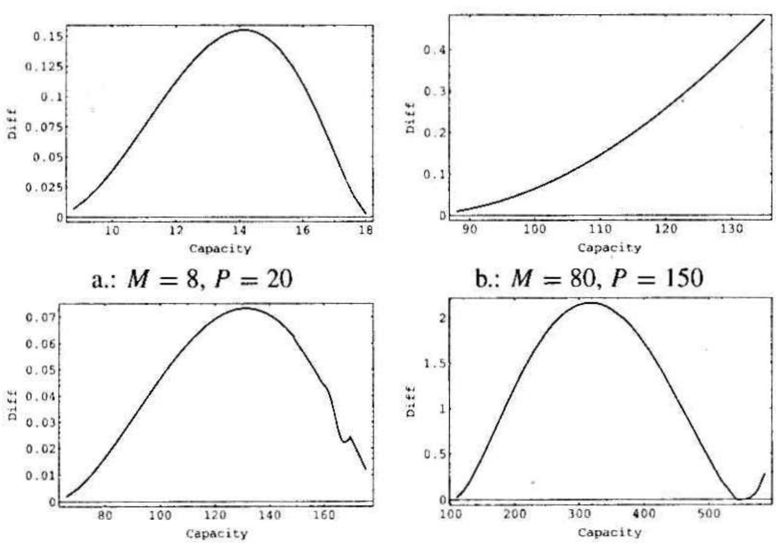
Finally, we present comparisons between our first closed-form formula using (17) and the second Hoeffding inequality (7) via the difference function $\mathrm{Diff}_{s1,ho2}$. *Fig. 4* illustrates strict increase in the difference with increasing capacity C. It means



a.: $M = 8$, $P = 20$ b.: $M = 80$, $P = 150$

c. $M = 60$, $P = 195$ d. $M = 100$, $P = 651$

Fig. 1 The difference function $\mathrm{Diff}_{o,s1}(C)$



a.: $M = 8$, $P = 20$ b.: $M = 80$, $P = 150$

c. $M = 60$, $P = 195$ d. $M = 100$, $P = 651$

Fig. 2 The difference function $\mathrm{Diff}_{o,s2}(C)$

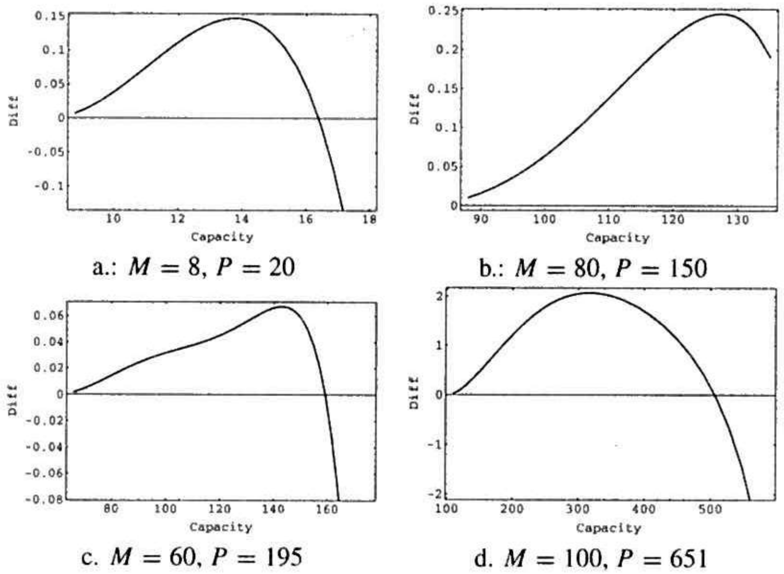


Fig. 3 The difference function $\mathrm{Diff}_{s1,s2}(C)$

that our first closed-form upper bound seems to be always better (which is also experienced in large numbers of numerical examples) than the second Hoeffding inequality.

To sum up the results of our comparisons it can be stated that among the approximation techniques presented in this paper, the method using (17) seems to be the most appropriate one, for practical cases.

5. Conclusions

In this paper we have developed a new upper bound for tail distribution of aggregate traffic using the well-known Chernoff bounding technique. This bound uses the aggregate mean arrival rate, the number of sources and the individual peak rates. This estimation can be considered as a generalized version of the Hoeffding bound assuming identical and normalized peak rates of traffic sources, because that can be obtained as a special case. This new upper bound is the best possible one assuming the condition that we apply the Chernoff bounding method using the given information (aggregate mean rate, peak rates, number of random variables) on the random variables. Due to the fact that our newly developed bound cannot be expressed in closed form, we have also presented weaker but closed form upper bounds for the tail distribution of aggregate traffic. Several numerical examples have also shown that one of the closed-form bounds is the most appropriate one for practical use.

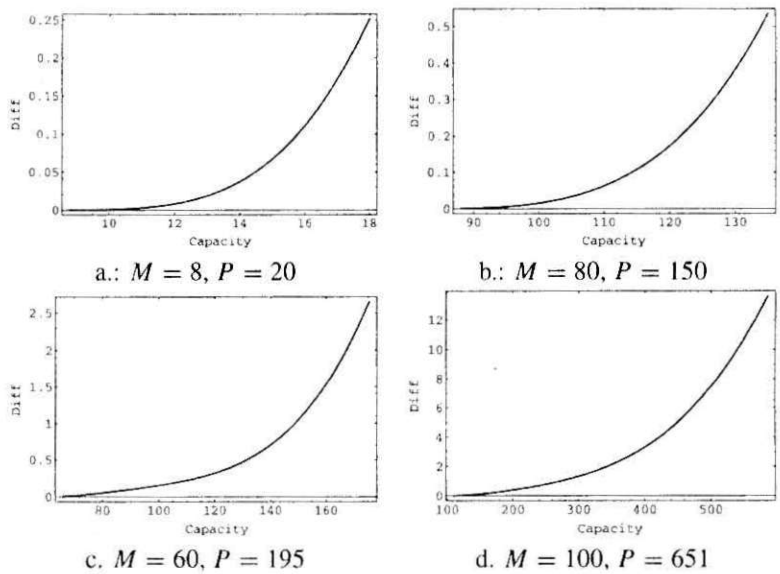


Fig. 4 The difference function $\mathrm{Diff}_{s1,ho2}(C)$

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