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Approximation and Analysis of Single Band FIR Pass Integrator Centered around Mid-band Frequencies with Degree *k* = 1, 2, 3...

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Abstract

In this paper, a modified Finite Impulse Response based linear Pass integrator for centered frequency, ranging between 0.1 *π* to 0.9 *π* has been realized. Both the cases have been considered i.e. for what values the phase response is of use and where the phase response has zero value. An iterative formula has been used to calculate the weights depending upon the Transfer Functions, and applying differentiation method. A flat output approximation for the desired frequency *ω* has been applied for which the results overlap with the ideal integrator. Performance comparison of the proposed integrator has been done with the previous one and relative percentage errors have been observed for both cases implemented. Graphical analysis has also been carried out for frequency responses having degree greater than one (i.e. *k* = 2, 3, 4) for both cases of proposed integrator and compared with the ideal integrator's response.

Keywords

digital differentiator, digital integrator, Finite Impulse Response, Infinite Impulse Response

1 Introduction

Integrators are widely used in many fields related to SONAR, average speed calculations, image processing, navigations, and many others where digital signal processing is being utilized [1–3]. Digitization of integrators has increased the dependability and accuracy of these calculations. Digital integrators constitute important members for operations required in numerous subsystems of the signal processing domain. The dynamic behavior of vibrating mechanical structures, for example is analyzed by first measuring the acceleration (using accelerometer) and then deriving displacement with the help of a digital integrator. Another example is the use of digital integrators in the field of medical for quantifying the activity of muscle where electromyograph signals can be integrated to quantify the activity of the muscle. Hence it leads us to work in this direction for reducing error, time and complexity of the existing techniques.

A range of traditional algorithms such as McClellan et al. are available to design Finite Impulse Response (FIR) integrators which may appear to be complex in terms of calculations and time consuming [4, 5]. Al-Alaoui [6–9] made use of fractional delay for efficient control of magnitude and phase for both integrators and differentiators, focusing mainly on Infinite Impulse Response (IIR) types. Papamarkos and Chamzas [10] proposed an alternative method of calculating the unknown weights of the transfer function from the ideal magnitude responses of the integrator by taking the absolute difference between the ideal response $H(\omega)$, obtaining $H_n(\omega)$ and minimizing the obtained difference. Dam et al. [11] has designed first order FIR integrator consisting of FIR filter as successor with a sampling rate of higher numbers. The employed method includes least square error criterion. This is in the frequency domain and uses oversampling to improve the performance of the filter of the FIR integrator. Also, involving primarily the derivation of transfer function using trapezoidal integration rule and differential equation of low order formulas another form of integrator was implemented with polyphase decomposition by Tseng and Lee [12] and for integrators with fractional delay by Tseng [13].

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Ngo [14] proposed a wideband filter for low order which is highly accurate for the Nyquist frequency range.

Kansal and Upadhya [15] developed a linear phase integrator and differentiator with Genetic Algorithm for fourth order integrator. Jain et al. [16] implemented integrators with higher orders where magnitude was improved using pole-zero and constant optimization method. Many authors [17–20] gave techniques for IIR integrators with techniques like backward integrator, low orders approximation avoiding fractional delays, group delay compensation based on Taylor series, presenting compact formula related to the length of the filter using Lagrange interpolators, etc. Mathematical formula to calculate weights for the specified filter design, related to order [21–24] lead to the implementation of the digital integrators popularly.

The frequency response for an ideal differentiator is given as follows in Eq. (1):

$$D(\omega) = j\omega, -\pi \le \omega \le \pi. \tag{1}$$

The above-mentioned frequency response is typically realized using IIR filters. As these filters have poles at *dc*, the input signal is required to be multiplied by the frequency response. It is imperative for the integrators to be designed keeping in mind the order that makes them useful for applications. Our approximations for a designed integrator are based on the cognitive recognition of frequency. The alterations are made within the limits of the response range of an ideal integrator. The frequency response of the ideal integrator is given by:

$$H(\omega) = \frac{1}{D(\omega)}, -\pi \le \omega \le \pi. \tag{2}$$

By using Z transform, the transfer function in Eq. (2) can be split into the function of sine and cosine. Constant phase of $\pi/2$ radians at all frequencies is represented by the factor 'j' in Eq. (3). Therefore, where a compulsion constant phase of $\pi/2$ radians exists in addition to magnitude response requirement, we can approximate the frequency response by using the Z transform as:

$$H_n(\omega) = \sum_{i=1}^{n} a_t^{(n)} \sin(t\omega), \tag{3}$$

for $n = \dfrac{N-1}{2}$ and $t = 1,2\ldots n$.

And when the constant phase is not the constraint any more, the response of the integrator is approximated by:

$$F_n(\omega) = \sum_{i=1}^{n} b_i^{(n)} \cos(t\omega), \tag{4}$$

for $n = \dfrac{N-1}{2}$ and $t = 1, 2\ldots n$.

Where, coefficients a_t and b_t are filter coefficients, 'N' denotes the length of the filter and 'n' is the order of the filter. Generally, taking an even number of N requires a practical consideration of fractional delay, which is not suitable for many applications. Computing weights using exact mathematical formulas involved in this design of integrators will give us more stable FIR integrator with less error involved for the Nyquist Range.

In this brief, the approximations of $H(\omega)$ and $F(\omega)$ have been modified to has the maximum flatness points, in terms of ω, where ω is in the range 0 to π. In Section 2, calculations of weights for $F_n(\omega)$, which are dependent on the transfer function have been done. This results in the equation for the transfer function of FIR integrator of degree, $k \ge 1$. In Section 3, the magnitude response of the designed FIR integrators has been plotted with respect to the normalized frequency. Analysis has also been done on the response of integrators for various degrees, $k > 1$. In Section 4, performance comparison of the proposed FIR integrator has been done with the already existing integrators in terms of magnitude response and absolute values of relative error.

2 Proposed single band FIR pass integrator

2.1 Calculation of dependent variable and weights for $H_n(\omega)$

The magnitude response characteristics of the ideal integrator in Eq. (2) is for degree 1. As the degree is increased, the magnitude response characteristics will tend to go deeper. Hence, it will be a challenge to adjust the weights in order to prevent the diversion. Since the related characteristics for $H_n(\omega)$ requires fractional delays, we will be focusing mainly on $F_n(\omega)$ for different value of ω. Odd order integrator is preferred over even order integrator as the later requires half sample delay. The generic equation for the calculation of weights for n, from previous values n–1 and n–2 weights is given by Eq. (5):

$$H_{n-2}^{(v)}(\omega)\Big|_{\omega=\omega_0} = H^{(v)}(\omega)\Big|_{\omega=\omega_0,} = (-1)^v\, v! \left(\frac{1}{\omega_0}\right)^{v+1}, \tag{5}$$

where n = 2, 3, 4, 5...

Here, ω_0 is the desired frequency at which the weights are being calculated. 'v^{th}' time derivation is done for calculation of weights. 'v' varies from 1 to n–3. The values of a_t^n have been calculated and then the weights for the next higher order approximation $H_{m+1}(\omega)$ have been computed through the use of $H_{m-1}(\omega)$ and $H_m(\omega)$.

Let value of be approximated by Eq. (6) as:

$$H_n(\omega) = \bar{A}_n H_n(\omega) + \left[\bar{B}_n H_n(\omega) - \bar{C}_n H_{n-1}(\omega)\right] \left(\cos\omega - \cos\omega_0\right), \tag{6}$$

where $\bar{A}_n, \bar{B}_n$ and $\bar{C}_n$ are functions of n which we shall compute. After calculation and determination of the multipliers $\bar{A}_n, \bar{B}_n$ and $\bar{C}_n$, we obtain the values $\bar{A}_n = 1$ and $\bar{B}_n = \bar{C}_n$.

Taking n^{th} derivative of Eq. (6), also putting $\omega = \omega_0$ and after simplification as done in, we obtain finally at:

$$\bar{B}_n = \left[\frac{-1}{(n-1)\sin\omega_0}\right] \frac{(-1)^{n-1}\, n-1!\left(\dfrac{1}{\omega_0}\right)^n - H_{n-1}^{(n-1)}(\omega_0)}{(-1)^{n-2}(n-2)!\left(\dfrac{1}{\omega_0}\right)^{n-1} - H_{n-2}^{(n-2)}(\omega_0)}, \tag{7}$$

where n = 2, 3, 4, 5…

Now, the above relation shows that ω_0 is related to $\bar{B}_n$. The desired weights i.e. $a_t^{(n)}$ can be determined from $a_t^{(n-1)}$ and $a_t^{(n-2)}$ if there is prior knowledge of $a_i^{(n-1)}$ and $a_i^{(n-2)}$. For the estimation of $a_i^{(n)}$, manual calculation of the previous two weights $a_t^{(n-2)}$ and $a_t^{(n-1)}$ is required. This concludes that computation of all weights up to n–2 has to be done. Since computation of $a_t^{(n-2)}$ and $a_t^{(n-1)}$ has been done manually and the value of n cannot be negative, so $\bar{B}_n$ will be calculated from n = 2.

Further, performing mathematical manipulations and using general trigonometric formulas, it can be found that:

$$a_i^{(n)} = a_i^{(n-1)} + \frac{\bar{B}_m}{2} \begin{bmatrix} \left(a_{i-1}^{(n-1)} - 2\cos\omega_0 a_i^{(n-1)} + a_{i+1}^{(n-1)}\right) \\ -\left(a_i^{(n-2)} - 2\cos\omega_0 a_i^{(n-2)} + a_{i+1}^{(n-2)}\right) \end{bmatrix}, \tag{8}$$

for $t = 1, 2, \ldots, n+1$.

2.2 Calculations of weights for $F_n(\omega)$

The generic equation for the $F_n(\omega)$ is given by:

$$F_n(\omega) = \sum_{t=1}^{n} b_t^{(n)} \cos i\,\omega,\ n = \frac{N-1}{2}. \tag{9}$$

Same as $H_n(\omega)$, weights of the filter have been represented by $b_t^{(n)}$ in Eq. (9). As discussed above, the odd value of N requires delays.

Again, proceeding similarly as for that of $H_n(\omega)$, and deriving at the desired frequency:

$$F_n^{(v)}(\omega)\Big|_{\omega=\omega_0} = \bar{F}^{(v)}(\omega)\Big|_{\omega=\omega_0} = \frac{d^v}{d\omega^v}\left(\frac{1}{\omega}\right)\Bigg|_{\omega=\omega_0}, \tag{10}$$

for $v = 0, 1, \ldots, n$.

Now, calculating the value of first two weights $b_t^{(n-1)}$ and $b_t^{(n-2)}$ manually, the value of $b_t^{(n)}$ and higher weights can be calculated. $F_n(\omega)$ can be made flat at the desired frequencies using the above relation and by calculating $F_{n-1}(\omega)$ and $F_{n-2}(\omega)$. Thus $F_n(\omega)$ is given by:

$$F_n(\omega) = \bar{A}_{n-1} F_{n-1}(\omega) + \left[\bar{B}_{n-1} F_{n-1}(\omega) - \bar{C}_{n-1} F_{n-2}(\omega)\right]\left(\cos\omega - \cos\omega_0\right). \tag{11}$$

Calculating the value of coefficients, $\bar{A}_{n-1}, \bar{B}_{n-1}$ and $\bar{C}_{n-1}$ to satisfy Eq. (13):

$$F_{n-1}^{(v)}(\omega)\Big|_{\omega=\omega_0} = \bar{F}^{(v)}(\omega)\Big|_{\omega=\omega_0},\ = (-1)^v v!\left(\frac{1}{\omega_0}\right)^{v+1}, \tag{12}$$

for $v = 0, 1, 2, \ldots, n-2$.

It will result in $\bar{A}_{n-1} = 1$ and $\bar{B}_{n-1} = \bar{C}_{n-1}$. Now, for the n^{th} derivative at the desired frequency ω_0 will result in Eq. (13):

$$\bar{B}_n = \left[\frac{-1}{(n-1)\sin\omega_0}\right] \frac{(-1)^{n+1}(n+1)!\left(\dfrac{1}{\omega_0}\right)^{n+2} - F_n^{(n+1)}(\omega_0)}{(-1)^n\, n!\left(\dfrac{1}{\omega_0}\right)^{n+1} - F_{n-1}^{(n)}(\omega_0)}, \tag{13}$$

for n = 2, 3, 4.

$\bar{B}_n$ will have positive real finite values for the desired band i.e. $0 < \omega < \pi$, but will not work for 0 or π, which are the starting and end points of the desired band. The weights of higher degree will be calculated by putting the value of $\bar{B}_n$ in $F_n(\omega)$. After applying mathematical logics and general trigonometric relations, we get:

$$\bar{b}_t^{(n)} = \bar{b}_t^{(n-1)} + \frac{\bar{B}_{n-1}}{2} \begin{bmatrix} \left(\bar{b}_{t-1}^{(n-1)} - 2\cos\omega_0 \bar{b}_t^{(n-1)} + \bar{b}_{t+1}^{(n-1)}\right) \\ -\left(\bar{b}_{t-1}^{(n-2)} - 2\cos\omega_0 \bar{b}_t^{(n-2)} + \bar{b}_{t+1}^{(n-1)}\right) \end{bmatrix}, \tag{14}$$

for t = 2, 3, … m + 1, $t \neq 1$,

$$\bar{b}_t^{(n)} = \bar{b}_t^{(n-1)} + \frac{\bar{B}_{n-1}}{2} \begin{bmatrix} \left(2\bar{b}_0^{(n-1)} - 2\cos\omega_0 \bar{b}_t^{(n-1)} + \bar{b}_2^{(n-1)}\right) \\ -\left(2\bar{b}_0^{(n-2)} - 2\cos\omega_0 \bar{b}_1^{(n-2)} + \bar{b}_2^{(n-2)}\right) \end{bmatrix}, \tag{15}$$

for t = 1.

Also, one important assumption to mention here is that $\bar{b}_t^{(n)} = 0$ for $t < 0$ or for $n > t$. Hence, in order to find the $F_1(\omega)$ and $F_2(\omega)$, the first step is to find the weights $\bar{b}_0^{(1)}$, $\bar{b}_1^{(1)}$ for $F_1(\omega)$ and $\bar{b}_0^{(2)}$, $\bar{b}_1^{(2)}$, $\bar{b}_2^{(2)}$ for $F_2(\omega)$ and then will be computing higher order weights progressively.

An important point to be mentioned here is that the above solution is for degree, $k = 1$. Equation for order $k = 1$ is given by modifying Eq. (3) as:

$$H_1(\omega,1) = \sum_{i=1}^{1} c_{t,1}^{(1)} \sin(t\omega). \tag{16}$$

In Eq. (16), the argument of H is 1, denoting the degree. For degree 2, the equation for ideal integrator is given by:

$$H(\omega,2) = [H(\omega)][H(\omega)] = \frac{1}{(j\omega)^2}. \tag{17}$$

Now replacing '$1/\omega$' with its optimal approximation $H_n(\omega,1)$ we arrive at the optimal approximation for $1/(j\omega)k$ i.e. the approximation for integrators of k^{th} degree. We finally obtain:

$$H_n(\omega,k) = \begin{cases} \sum_{i=1}^{n} c_{i,k}^{(n)} \sin i\omega, & r \text{ odd} \\ \sum_{i=1}^{n} c_{i,k}^{(n)} \cos i\omega, & r \text{ even} \end{cases}, \tag{18}$$

where

$$c_{i,k}^{(n)} = \begin{cases} \dfrac{(-1)^{\frac{k-1}{2}} i^{k-1} c_i^{(n)}}{(k-1)!}, & r \text{ odd} \\ \dfrac{(-1)^{\frac{k}{2}} i^{k-1} c_i^{(n)}}{(k-1)!}, & r \text{ even} \end{cases}. \tag{19}$$

3 Results and performance

In Fig. 1, the magnitude response of the proposed FIR integrator, $F_n(\omega)$ is plotted against the normalized frequency. As the FIR integrator is designed for different frequencies, we have chosen the frequencies that satisfies Eq. (3) and Eq. (4). This results in the response shown in Fig. 1, for ω_0 =0.33 π. Different values of n i.e. 10, 12, 15, 16, 20, and 31 are taken and plotted along with characteristics of ideal integrator. It has been marked in the graph that proposed integrator's magnitude response is overlapping on and around the desired ω_0 for ideal integrator, centerd around ω_0 =0.33 π. The band explodes near the edges and exhibits characteristic different from its ideal nature. The degree of the designed FIR integrator is $k = 1$.

In Fig. 2, the centered frequency, ω_0 is taken as 0.4 π, and the order n is taken as n = 10, 12, 15, 16, 20 and 31.

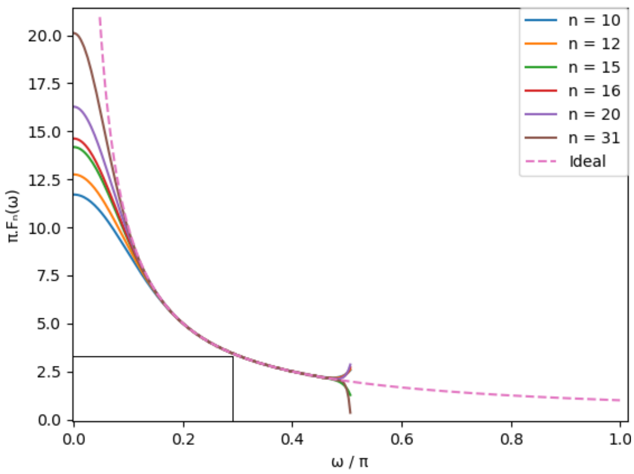

Fig. 1 Magnitude response of designed FIR integrator $F_n(\omega)$, for ω_0 = 0.33 π and different n, in comparison to ideal integrator

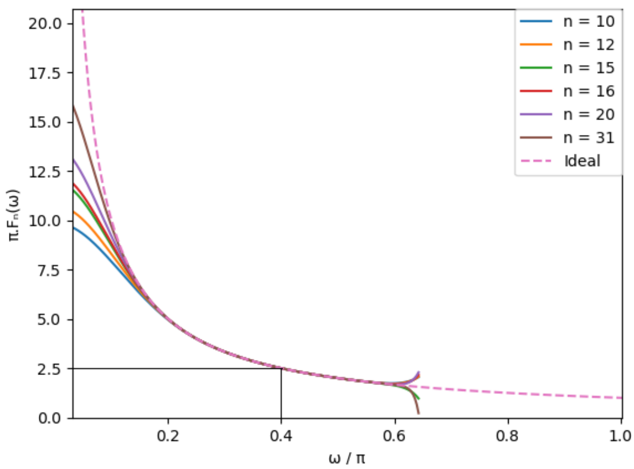

Fig. 2 Magnitude response of designed FIR integrator $F_n(\omega)$, for ω_0 = 0.4 π and different n, in comparison to ideal integrator

Graph here clearly reveals that proposed integrator is overlapping with the ideal integrator response at the specified frequency. In Fig. 3, ω_0 = 0.5 π, taken as centered frequency, and the magnitude response are overlapping for the desired frequency with the ideal integrator frequency response. For Fig. 4, centered frequency, ω_0 = 0.6 π is taken for designing of integrator. In Fig. 5, the magnitude response of $H_n(\omega)$ is plotted against the normalized frequency.

The graphs in Fig. 1, Fig. 2, Fig. 3 and Fig. 4, follow the curve of cosine. The graph in Fig. 5 follows the sine curve, thus becoming negative between the normalized frequency range of 0 and π.

Fig. 6 represents the integrator of degree, k = 2 as explained by Eq. (18) The centerd frequency, ω_0, of interest is taken as 0.5 π. Hence giving the flat response for the desired frequency ω_0 different values of n = 0, 12, 15, 16, 20 and 31 is taken. However, as we gradually increase the degree

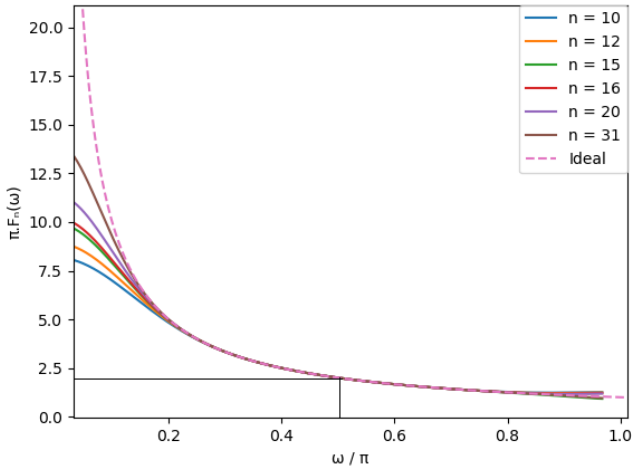

Fig. 3 Magnitude response of designed FIR integrator $F_n(\omega)$, for $\omega_0 = 0.5\,\pi$ and different n, in comparison with ideal integrator

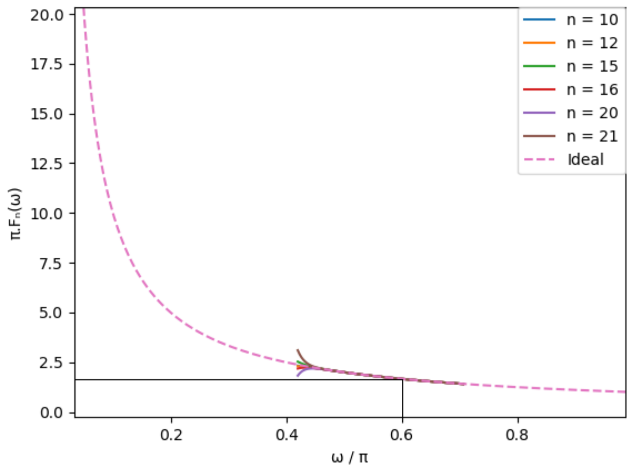

Fig. 4 Magnitude response of designed FIR integrator $F_n(\omega)$, for $\omega_0 = 0.6\,\pi$ and different n, in comparison with ideal integrator

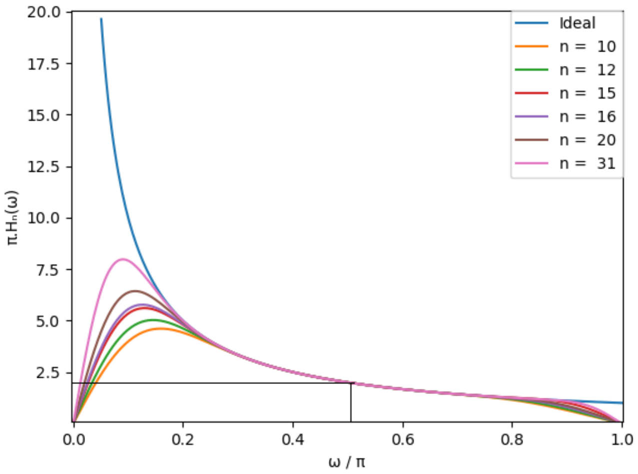

Fig. 5 Magnitude response of designed FIR integrator $H_n(\omega)$, for $\omega_0 = 0.5\,\pi$ different n, in comparison with ideal integrator

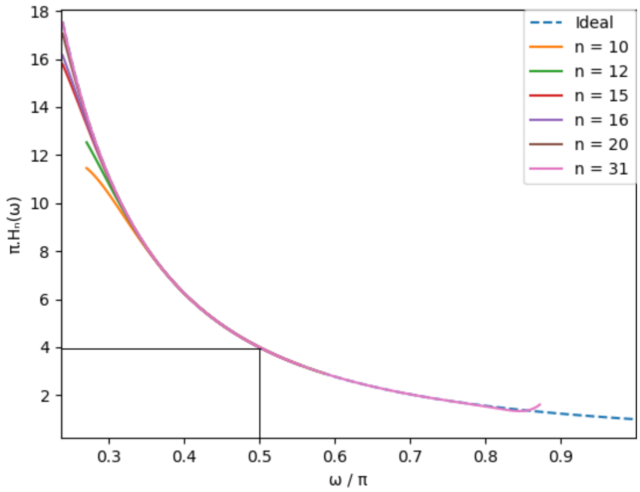

Fig. 6 Magnitude response of designed FIR integrator $H_n(\omega)$ with degree $k = 2$, for $\omega_0 = 0.5\,\pi$ and different n, in comparison with ideal integrator

of desired filter, for $k = 3$, as seen in Fig. 7, the response of the designed filter diverges away from the ideal integrator response as shown in Fig. 6 having degree $k = 2$. Fig. 8 represents the integrator with degree, $k = 4$, and as explained, with the increase in the degree, k, the amplitude of the integrator's magnitude response increases.

In order to find the relative percentage of error corresponding to the approximation of both $H_n(\omega)$ and $F_n(\omega)$ will be used within the following formula (Eq. 20):

$$\text{R.P.E} = \frac{H_n(\omega) - H(\omega)}{H(\omega)}, \quad (20)$$

where $H_n(\omega)$ represents the approximated integrator function and $H_n(\omega)$ represents the ideal integrator response. The same formula is used for the calculation of relative

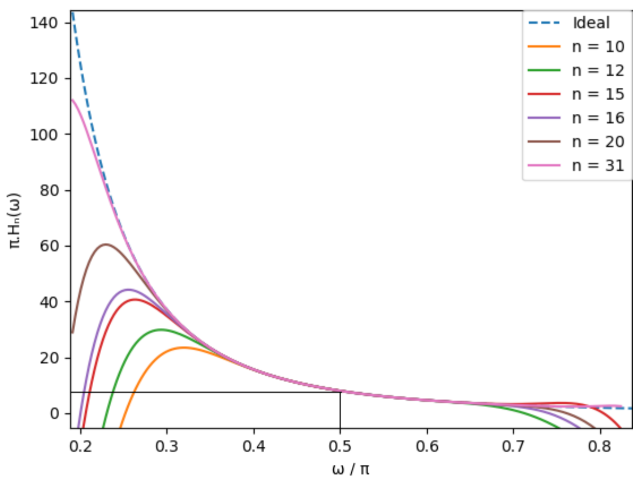

Fig. 7 Magnitude response of designed FIR integrator $H_n(\omega)$ with degree $k = 3$, for $\omega_0 = 0.5\,\pi$ and different n, in comparison with ideal integrator

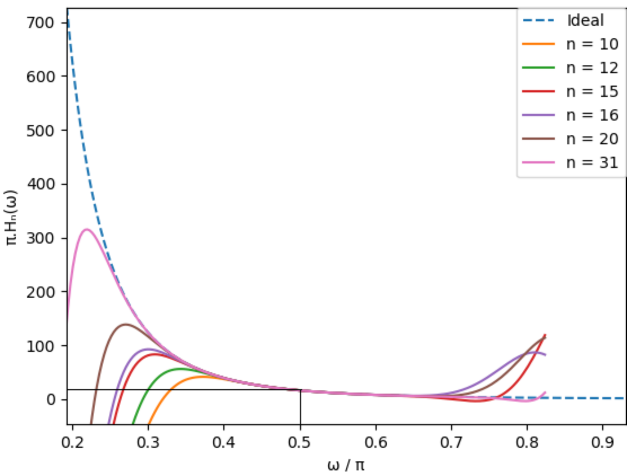

Fig. 8 Magnitude response of designed FIR integrator $H_n(\omega)$ with degree $k = 4$, for $\omega_0 = 0.5\ \pi$ and different n, in comparison with ideal integrator

percentage error for $F_n(\omega)$ also. However, to calculate the relative error, the range around the flatness point ω_0 is taken at equal intervals. Different values of n have been taken for calculating the relative error for both $H_n(\omega)$ and $F_n(\omega)$ the same value of n for both approximations has been taken.

Table 1 depicts the relative error percentage for and is calculated using the relation in Eq. (20). Relative error is calculated for different values of n, where $n = 10$, 20 and 30 and for center frequency being 0.33 π, 0.4 π and 0.5 π. As the value of n is increased, the relative error increases proportionally and is highest for $n = 30$, being 28.631 % for $\omega = 0.33\ \pi$. For $\omega = 0.4\ \pi$, the maximum error observed is 22.215 %, which is again for $n = 30$. However, if we look at the relative error, it can be seen that the range of error is quiet less and is gradually decreasing as the operating frequency of the designed FIR integrator is increased towards the end of the band i.e. at π. The range of error for same value of ω, and two different value of n, for $n = 10$ and 30, the latter has slightly high percentage range of relative error in the interval [15.707, 17.689] (%).

Table 2 delineate relative error percentage of $F_n(\omega)$, varying the value of n, $n = 10$, 20 and 30. The value of frequency ω is taken same as what was taken for $H_n(\omega)$. This is done to aid the examination of relative error between $H_n(\omega)$ and $F_n(\omega)$. In contrary to $H_n(\omega)$, the pattern observed in $F_n(\omega)$ is of ascending error. As the value of frequency ω

Table 1 $H_n(\omega)$ Relative error for different values of ω and n

n	Centered frequency		
	0.33 π	0.4 π	0.5 π
10	22 %	21 %	15.707 %
20	27 %	22 %	16 %
30	28.631 %	22.215 %	17.689 %

Table 2 $F_n(\omega)$ Relative error for different values of ω and n

n	Centered frequency		
	0.33 π	0.4 π	0.5 π
10	1.098 %	3 %	4.140 %
20	1.819 %	3.109 %	4.158 %
30	2.861 %	4.132 %	5.632 %

is increased, the percentage relative error is increased. As, from $n = 10$ to $n = 30$ for $\omega = 0.33\ \pi$ the range of error increases from 1.098 % to 2.861 % and from 4.140 % to 5.632 % for $\omega = 0.5\ \pi$. It can be noted that as the degree of the integrator is increased the error will also increase.

4 Performance comparison

Fig. 9 compares the magnitude response of the designed FIR integrator with previous existing integrators plotted against normalized frequency. The proposed integrator represents the magnitude of the designed integrator which is in orange. It can be seen that for the Nyquist range, the magnitude response of the proposed integrator overlaps quite well with the response of the ideal integrator. Reference [6] represents the magnitude response of the Al-Alaoui first-order integrator, depicted by a green continuous line. Reference [9] represents Al-Alaoui second-order integrator via the red line. The integrator designed by Papamarkos and Chamzas is represented by reference [10]. As it is evident from the graph, amongst all, the response of the integrator proposed integrator offers maximum overlaps with the ideal integrator response.

Fig. 10 offers a comparison of the relative error for the designed FIR integrator with previous existing integrators. In terms of relative error, it can be clearly seen that the relative error of the designed integrator is better than remaining integrators. In Fig. 10 the relative error

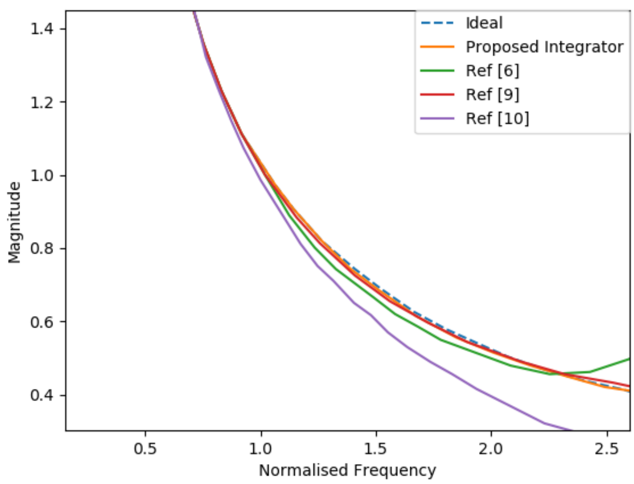

Fig. 9 Magnitude comparison of proposed integrator with existing integrators

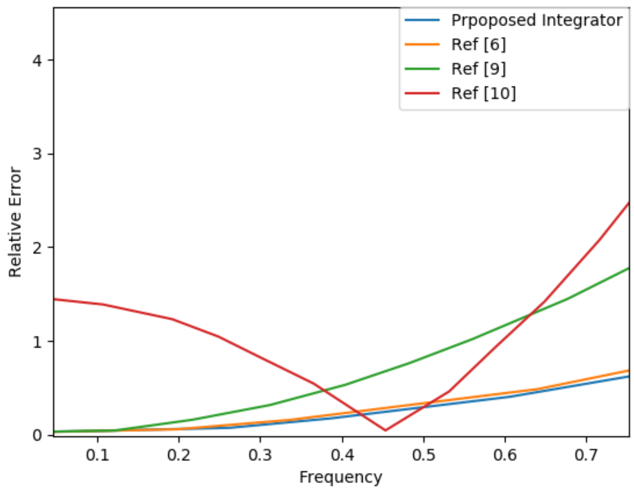

Fig. 10 Relative error comparison of proposed integrator with existing integrators

for the proposed integrator is marginally lower than reference [6] for the range $0 < \omega < \pi$, performing better than reference [6] for the Nyquist range. Also, in comparison to reference [9] proposed integrator performs better for the Nyquist range. When compared to reference [10], the proposed integrator performs better for the range of $\omega > 0.5\ \pi$ and $\omega < 0.4\ \pi$. Relative error for the Nyquist range of frequencies is improved marginally, approximately 2 % as compared to reference [9]. As compared to previous existing integrators the relative error has been successfully reduced by more than 20 % as the maximum value for reference [10] goes to 2.5 %. However, in comparison reference [6] relative error value is reduced by 16 % as after the Nyquist range, the relative error increases sharply. The main factors for this good performance of the proposed integrator are low order and reduced complexity.

Such kind of single band FIR pass integrator approximated above has advantages that they can be used in real time applications including oil expedition, wireless data transmission, or through telephonic channels, seismology, etc. where single operating frequency is existing. The above approximated integrators are highly suitable for applications requiring fast calculations like for freezing in an IC, storing the weights vectors, and providing various output taps to simulate approximation of various lower order filters. Another practical implementation of these integrators can be in situations where the equiripple (Chebyshev type) approximations are generally preferred. These designs are accomplished by optimization techniques using iterations which are time-consuming and non-analytical. In such applications, our approximation will reduce the complexity by giving the exact mathematical formula that will help in designing the integrator required for equiripple passband of frequencies. The disadvantage of the proposed integrator although, is that in case of multiple frequencies, several FIR integrators will be required to operate simultaneously. The system in such cases becomes more complex in terms of calculations and hardware. Another scope of improvement is that since after the Nyquist range there is a rise in relative error because of the approximation done using sine and cosine functions, hence this divergence from the ideal integrator response should be optimized further and is the area to be worked upon. Authors are working to develop novel FIR integrator for working on multiband pass frequencies at a time.

5 Conclusion

FIR based linear pass integrator has been proposed, operating in the frequency range of $0.1\ \pi$ to $0.9\ \pi$. Dependent cases on phase are considered and weights have been derived using the iterative formula depending upon the Transfer functions and . Respective graphs for both transfer functions have been plotted depicting the overlapping of the designed FIR integrator with the ideal integrator at the desired frequency, ω. For desired order of the filter, n, the relative percentage errors have been calculated and found in the range of interval [15.707, 28.631] % and [1.098, 5.632] % for and respectively and the same is used for performance comparison with the existing integrators. Also, FIR based linear pass integrator having degree, $k = 2$, 3 and 4 have been studied and plotted graphically. Such integrators are useful for real-time applications like digital signal processing systems, in navigations (RADAR, SONAR) etc.

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