

# Efficient Method for Computing Capacitance Changes between Electrodes due to Foreign Objects in Capacitive WPT Systems

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## Abstract

In this paper, a foreign object detection method based on an integral formula is derived for capacitive wireless power transfer systems. The formula expresses the capacitance changes of the system and can predict even very small changes with high accuracy and computational efficiency, as the change is directly evaluated from the electric field in the volume of the foreign object. First, the derivation of the integral formula is presented, then a simple configuration is investigated using the finite element method.

## Keywords

foreign object detection, wireless power transfer (WPT), finite element method (FEM)

## 1 Introduction

With Capacitive Power Transfer (CPT) emerging as an alternative to Inductive Power Transfer (IPT) in electrical vehicle (EV) charging [1], there is an urgent need to solve various problems. Among them foreign object detection (FOD) is a major one. The CPT system must be able to detect foreign objects in the operational area (i.e., between the charging plates). Dielectric objects in this area will detune the resonators decreasing the transfer efficiency and will be heated due to dielectric losses. The losses in the dielectric objects are also a fire hazard, as the dissipated power could cause the objects to catch fire and cause damage to the charging station [2].

In the past, several methods have been presented for FOD in inductively coupled systems [3–6], but given that CPT systems are not widespread yet, only a few methods have been presented for FOD in CPT systems [7–8]. These methods include measuring the voltages and currents at the resonator inputs [8], trying to determine the capacitance change between the charging plates. These changes can be very small, especially with low dielectric constant objects, so a numerically robust and accurate method is needed. In this paper, a numerical simulation method is proposed which aims at calculating this very small change of the capacitance due to foreign objects. This method can be a useful tool in the design and analysis of CPT systems.

First an integral formula for the capacitance changes based on Green's second identity and the theorem of reciprocity in electrostatics is derived. A similar method has been presented for the two-port characteristics in IPT systems in [6]. In [9] one can also find an efficient method of computing field distortions due to foreign objects, called the perturbation method. Utilizing the perturbation method and the derived formulas one can compute the changes in the capacitance coefficients of an arbitrary electrode system due to the presence of a foreign object with high computational efficiency. A typical structure of a CPT system is shown in Fig. 1.

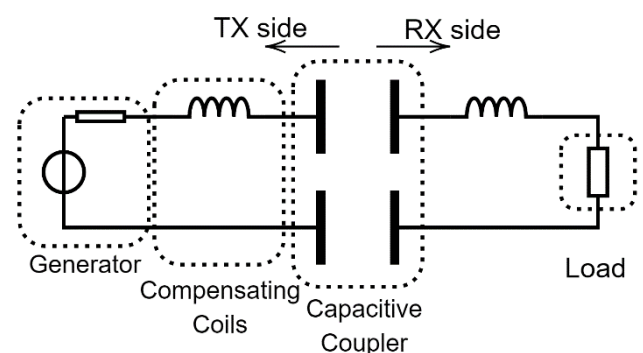


Fig. 1 Typical CPT system realization. The operating frequency is determined by the coupler and the compensating coils

The transmitter and receiver are realized by metal plates. For simplicity, a 4-plate resonator system is studied to investigate the accuracy of the method using the finite element method (FEM). The capacitance changes are evaluated while a foreign object (a dielectric sphere) is placed between the charging plates.

## 2 Capacitance change from reciprocity

### 2.1 Configuration

Let us consider a simple 4-plate CPT system shown in Fig. 2. The subscripts denote the two cases of the configurations (i.e., with and without the foreign object present). In case A, denoted by subscript  $a$ , the foreign object is not present, the volume in  $\Omega_j$  is filled with air. In case B, denoted with subscript  $b$ , the foreign object is present in volume  $\Omega_f$ .

### 2.2 Computation of capacitance changes

The Maxwell's equations for the two configurations are:

$$\nabla \times \mathbf{E}_a = 0, \quad \nabla \cdot \mathbf{D}_a = \rho_a, \quad (1)$$

$$\nabla \times \mathbf{E}_b = 0, \quad \nabla \cdot \mathbf{D}_b = \rho_b. \quad (2)$$

As a starting point, we will look at Green's second identity, where  $\phi_a$  and  $\phi_b$  are the electric scalar potential fields we get from Eq. (1) and Eq. (2),  $\Omega$  is an arbitrary closed volume and  $\partial\Omega$  it's surface:

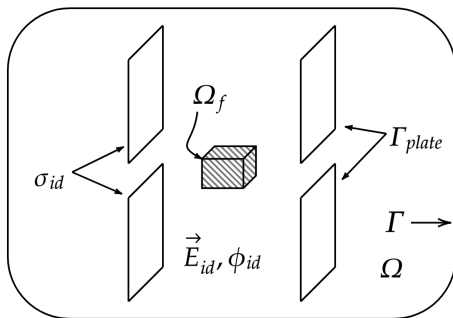
$$\int_{\Omega} (\phi_a \nabla^2 \phi_b - \phi_b \nabla^2 \phi_a) d\Omega = \int_{\partial\Omega} (\phi_a \partial_n \phi_b - \phi_b \partial_n \phi_a) ds. \quad (3)$$

We can get a more descriptive form if we use the following formulas:

$$\nabla^2 \phi = -\rho / \varepsilon \text{ in } \Omega, \quad (4)$$

$$\partial_n \phi = -\sigma / \varepsilon \text{ on conducting surfaces in } \partial\Omega, \quad (5)$$

where  $\rho$  is the volume charge and  $\sigma$  is the surface charge. Substituting these into Eq. (3) we get the following formula:



**Fig. 2** Two configurations for the derivation of the formulas. Configuration A: without the foreign object present in volume  $\Omega_f$ ,  $id = a$ . Configuration B: foreign object present in volume  $\Omega_f$ ,  $id = b$ .

$$\int_{\Omega} \frac{\phi_b \rho_a}{\varepsilon} - \frac{\phi_a \rho_b}{\varepsilon} d\Omega = \int_{\partial\Omega} \frac{\phi_b \sigma_a}{\varepsilon} - \frac{\phi_a \sigma_b}{\varepsilon} ds \quad (6)$$

On the right-hand side, the integral is only non-zero on the electrode surfaces because there is no other surface in the configuration with non-zero surface charge. The left-hand side is only non-zero in the volume of the foreign object  $\Omega_f$ , as we assume that the foreign object has a bound charge  $\rho_b$ , and because in case A the object is not present  $\rho_a$  is zero. Thus, we can write:

$$\sum_{i=1}^4 \oint_{\Gamma_i} \sigma_a \phi_b d\Gamma = \sum_{i=1}^4 \oint_{\Gamma_i} \sigma_b \phi_a d\Gamma + \int_{\Omega_f} \rho_b \phi_a d\Omega, \quad (7)$$

where  $\Gamma_i$  is  $i$ -th plate's surface. Equation 7 can be simplified knowing that the electrode surfaces are equipotential surfaces, so the electrode potentials can be brought outside the integration. What remains inside is the surface charge of the electrode, in which case the integral is by definition equal to the electrode charge:

$$\phi_{b,i} \oint_{\Gamma_i} \sigma_{a,i} d\Gamma = \phi_{a,i} Q_{a,i}, \quad (8)$$

$$\sum_{i=1}^4 \phi_{b,i} Q_{a,i} = \sum_{i=1}^4 \phi_{a,i} Q_{b,i} + \int_{\Omega_f} \rho_b \phi_a d\Omega, \quad (9)$$

where the first subscript labels the case and the second the electrode. The electrode charges can be expressed using the capacitance coefficients  $c_{ij}$  in both cases:

$$Q_{a,i} = \sum_{j=1}^4 c_{ij} \phi_{a,j}, \quad (10)$$

$$Q_{b,i} = \sum_{j=1}^4 (c_{ij} + \Delta c_{ij}) \phi_{b,j}. \quad (11)$$

Here  $\Delta c_{ij}$  is the change of the corresponding capacitance coefficient due to the presence of the foreign object. Substituting Eqs. (10) and (11) into Eq. (9) we get the following:

$$\begin{aligned} & \sum_{j=1}^4 \phi_{b,j} \sum_{j=1}^4 c_{ij} \phi_{a,j} \\ &= \sum_{j=1}^4 \phi_{a,j} \sum_{j=1}^4 (c_{ij} + \Delta c_{ij}) \phi_{b,j} + \int_{\Omega_f} \rho_b \phi_a d\Omega. \end{aligned} \quad (12)$$

Subtracting the double sum on the right side from the left side we will be left with only the coefficient changes inside the summation:

$$\sum_{i=1}^4 \sum_{j=1}^4 (-\Delta c_{ij} \phi_{a,i} \phi_{b,j}) = \int_{\Omega_f} \rho_b \phi_a d\Omega. \quad (13)$$

The reciprocity of the electrodes assures that  $\Delta c_{ij} = \Delta c_{ji}$ . The bound volume charge of the foreign object can be expressed with the polarization vector  $\mathbf{P}_b$  as

$$\rho_b = -(\nabla \cdot \mathbf{P}_b), \quad (14)$$

$$\int_{\Omega_f} \rho_b \phi_a d\Omega = \int_{\Omega_f} -(\nabla \cdot \mathbf{P}_b) \phi_a d\Omega \equiv \int_{\Omega_f} \mathbf{P}_b \cdot \nabla \phi_a d\Omega. \quad (15)$$

The polarization vector can be expressed using the electric field and the dielectric constant  $\epsilon_r$  of the foreign object:

$$\mathbf{P}_b = \mathbf{D}_b - \epsilon_0 \mathbf{E}_b = \epsilon_0 (\epsilon_r - 1) \mathbf{E}_b. \quad (16)$$

Also, the gradient of the electric potential is equal to the negative electric field:

$$\nabla \phi_a = -\mathbf{E}_a. \quad (17)$$

Substituting Eq. (16) and Eq. (17) into Eq. (15) and then Eq. (11) we get the following:

$$\sum_{i=1}^4 \sum_{j=1}^4 \Delta c_{ij} \phi_{a,i} \phi_{b,j} = \int_{\Omega_f} \epsilon_0 (\epsilon_r - 1) \mathbf{E}_a \mathbf{E}_b d\Omega. \quad (18)$$

The individual coefficient changes can be obtained using linearly independent combinations of electrode potentials. As an example, if we are looking for the change in the capacitance coefficient between the first and second electrode we can use the following potential vectors as excitations:

$$\begin{aligned} \phi_a &= [1 \ 0 \ 0 \ 0], \ \phi_b = [0 \ 1 \ 0 \ 0] \text{ or} \\ \phi_a &= [0 \ 1 \ 0 \ 0], \ \phi_b = [1 \ 0 \ 0 \ 0], \end{aligned} \quad (19)$$

where the  $i$ -th element of  $\phi_a$  and  $\phi_b$  are the corresponding electrode potentials. As we can see, the individual coefficient can be obtained by evaluating the integral shown in Eq. (18) with the excitation shown above:

$$\Delta c_{12} = \frac{-1}{\phi_{b,1} \phi_{a,2} + \phi_{a,1} \phi_{b,2}} \int_{\Omega_f} \epsilon_0 (\epsilon_r - 1) \mathbf{E}_a \mathbf{E}_b d\Omega. \quad (20)$$

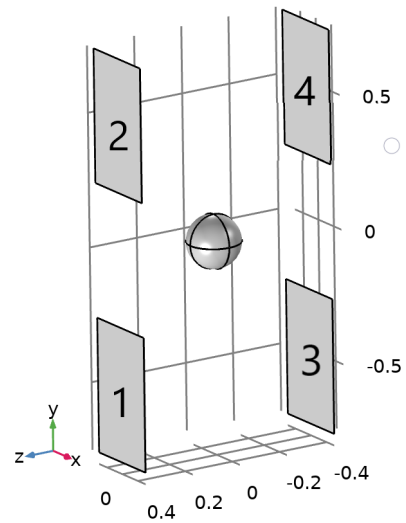
The main advantage of this method comes from the fact, that said integral only needs to be evaluated in the volume  $\Omega_f$ . Not only we get a more robust way to compute the capacitance changes, but we can greatly increase the accuracy of the computations without a significant increase in the required computational capacity.

### 3 A numerical example

We investigate a symmetrical 4-plate electrode system, where the electrodes are thin aluminum plates. The electrode system is disturbed by a foreign object, represented by a dielectric sphere. The parameters of the geometry are shown in Table 1, and configuration in Fig. 3. The problem is studied using the FEM software COMSOL Multiphysics.

**Table 1** Parameters of the electrode system used in the simulations

Name	Value
Plate side $a$	30 cm
Plate side $b$	50 cm
Gap between plates	50 cm
Distance between capacitors	80 cm
Plate thickness	1 mm
Sphere radius	10 cm
Dielectric constant of foreign object	6



**Fig. 3** The configuration used in the numerical simulations. The numbers represent the electrode indexes

For this study, the change of the capacitance coefficients of electrode 1 are calculated using both the perturbation method and the traditional method (i.e., by taking the difference of the two capacitances we get from the two simulations). The position of the foreign object is also varied along the  $y$ -axis with the  $y = 0$  point being the midpoint, as shown in Fig. 3. The results of the simulations are shown in Fig. 4. In this case the change of  $c_{12}$  shows the largest difference between the two methods. Also, interesting to note, that the change of the largest capacitance value, which is  $c_{13}$  in this case, shows the least deviation between the two curves. The drawback of the traditional method can be observed by looking at the values in Table 2, which shows that the capacitance coefficients that we need to subtract from each other are very close to each other.

In order to validate the method, we take the difference of the capacitance changes  $\Delta c_{13}$  we get from the two methods while changing the dielectric constant of the sphere. The position of the sphere is fixed at  $y = -0.5$  m (between plates 1 and 3). We expect that the difference will decrease

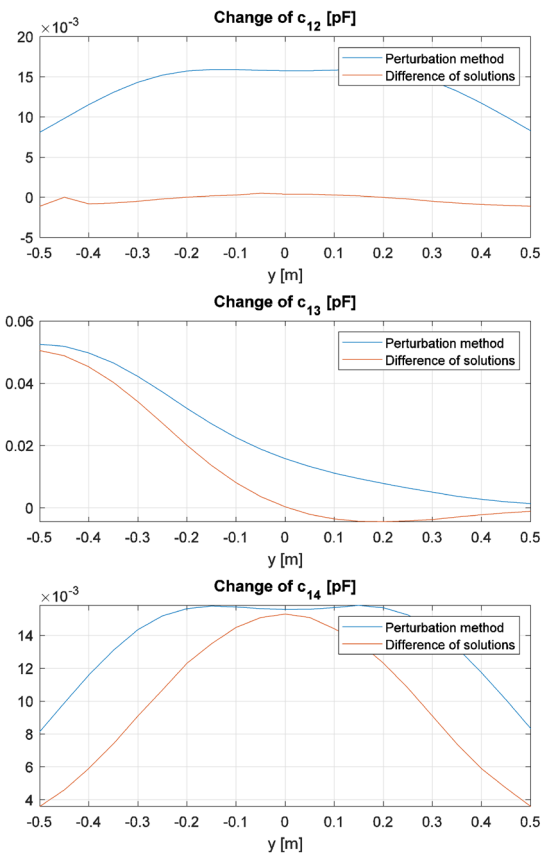


Fig. 4 The capacitance changes as a function of the foreign objects position between the plates

as the field perturbation increases. The result can be seen in Fig. 5, which confirms our expectations.

#### 4 Conclusion

Based on the reciprocity of electrostatics an integral formula giving the change of the capacitances between electrodes when a foreign object is introduced was derived

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Table 2 The actual capacitance values of  $c_{12}$  in pF in various sphere positions

Sphere position [m]	Without Foreign Object	With Foreign Object	Difference
-0.5	4.6982	4.6971	-0.0011
-0.25	4.6963	4.6961	-0.0002
0	4.7025	4.7029	0.0004
0.25	4.7006	4.7004	-0.0002
0.5	4.6977	4.6966	-0.0011

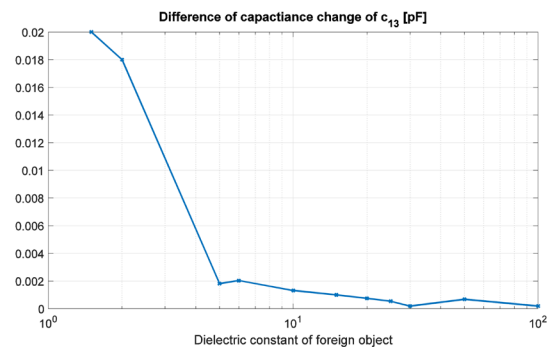


Fig. 5 Difference of from the two methods as a function of the dielectric constant of the sphere. As the field perturbation increases the difference between the two methods decreases

and evaluated. The main advantage of this formula is that very small changes can be obtained with high accuracy, as the method evaluates the change directly from the electric field in the volume of the foreign object. A 3D FEM model has been used for the calculations, and it was found that the proposed method gives a more accurate result over the traditional one when the perturbation of the electric field due to the presence of the foreign object is small.

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