

# Variable Recursive Least Square Algorithm for Lithium-ion Battery Equivalent Circuit Model Parameters Identification

Mouncef El Marghichi<sup>1\*</sup>, Azedine Loulijat<sup>1</sup>, Issam El Hantati<sup>2</sup>

<sup>1</sup> Faculty of Sciences and Technology, Hassan First University, 26002 Settati, P.O.B. 577, Morocco

<sup>2</sup> Laboratory of Mechanics Production and Industrial Engineering (LMPGI), High School of Technology (ESTC), Hassan II University of Casablanca, Route d'ElJadida Km 7, 8012 Casablanca, Morocco

\* Corresponding author, e-mail: [m.elmarghichi@uhp.ac.ma](mailto:m.elmarghichi@uhp.ac.ma)

Received: 13 October 2022, Accepted: 02 March 2023, Published online: 28 April 2023

## Abstract

For SOC (state of charge) assessment techniques based on electrical circuit models, the parameters of the model are strongly biased by: battery aging, temperature, causing some errors in the estimation of the SOC. One approach to solve this problem is to update the model parameters constantly. We suggest a new algorithm VRLS (variable recursive least squares) to update the parameters of a 2-resistor-capacitor (RC) network and to estimate the output battery voltage. VRLS is compared to the recursive least squares (RLS) and the adaptive forgetting factor recursive least squares (AFFRLS) algorithms. For algorithm assessment, we utilized real experimental data conducted on the Samsung 18650-20R lithium-ion cell. The tests indicate that compared to RLS and AFFRLS methods, VRLS recorded a low distribution in the high error range, in addition to small predictive performance indicators (RMSE, MAE, and MAPE) in all tests, which implies that VRLS has a good parameter identification ability.

## Keywords

recursive least squares (RLS), variable recursive least squares (VRLS), adaptive forgetting factor recursive least squares (AFFRLS), battery

## 1 Introduction

The lithium-ion battery (LIB) has emerged as a prominent energy storing device for EVs (electric vehicles) due to its long durability. The LIBs are supervised by a BMS (battery management system) in these applications. The involved responsibilities of the BMS include state of charge and health (SOC and SOH) estimation, battery cell balancing, etc. The global effectiveness of these functionalities is strongly related to the correctness of the model developed [1].

There are three groups of battery models: Electrochemical models, Data-based models, and Equivalent circuit models (ECMs) [2]. Electrochemical models show high potential to capture battery dynamics. But, because of their computational complexity, they are not suitable for online applications [3]. One of the key issues with data-driven models is that they use exhaustive tests to capture large sets of battery data in order to learn from the data to represent the battery's behavior in various regimes [4].

ECMs' (Equivalent circuit models) robustness and simplicity have made them a viable choice for BMS applications [5]. The ECM of a battery contains resistors, capacitors, and a source voltage to represent the cell's behavior,

ECMs typically contain  $n$  RC resistor capacitor networks. The second order ECM has been shown to provide an accurate simulation of the cell's dynamical behavior [6, 7].

Using high order battery models with over two RC networks causes computational overhead while not significantly improving the accuracy of the battery model [8]. Furthermore, along with the battery model, a precise identifier of battery model parameters is a major contributor to enhance the accuracy of the model and the estimation of SOC and SOH. We can categorize the techniques used for ECM parameter identification into two types: offline and online identification.

Electrochemical impedance spectroscopy [9, 10] and hybrid pulse power characterization [11] are amongst the most popular offline methods. In these methods, the identified parameters are constants. Tests have shown that battery parameter values vary with SOC, temperature, and charge levels. Considering fixed values for these parameters results in an imprecise ECM. Hence, an online parameter estimator is needed for the battery model to enhance BMS performance [12].

In this context, various studies have been conducted, here we are going to focus on the most recent ones. In [13], Thevenin model parameters are identified from voltage feedback data collected from a constant current discharge battery. From this voltage response, a sensitiveness test is utilized to assess the parameter identifiability. To tackle the identification, a non-linear least squares optimization problem is formulated, bounds have been set to reduce the search space. The identification problem is solved by a confidence region method. In [14] an exponential regression algorithm (ERA) based online parameter estimator is proposed for the identification of the parameters of the ECM. In addition, the authors have also proposed an adaptive sliding mode observer based on the proposed adaptive ECM for SOC estimation. Wang et al. [15] suggested in an online adaptive prediction algorithm for a fractional equivalent circuit model on the basis of the fractional order theory of computation and the indirect Lyapunov method. Mouncef et al. [16] and Elmarghichi et al. [17] adapted the sunflower optimization algorithm (SFO) to derive the parameters of a first order RC battery model, and to predict battery terminal voltage. In [18], a particle filter is employed to continuously identify the battery model parameters in real time taking into account the battery states. At the same time, a cubature Kalman is employed to predict the SOC. The partial least squares regression with the moving window structure was implemented on the second-order ECM in [19] to derive a set of linear piecewise battery models. The estimation of the SOC was made simpler by this method as a linear Kalman filter (KF) can be applied to the linear model developed. In [20], an ECM that is temperature dependent was constructed and embedded in the ensemble membership technique for state of charge estimation. The major characteristic of this approach is that it addresses the effects of measurement noise in the estimation process.

The large amount of research published for parameter identification indicates that this is still an issue that needs to be addressed (refer to Table 1 for a summary of the most common battery model parameter estimation techniques). On one side, higher order battery models are required to obtain proper transient and static behavior in battery voltage prediction. However, the improved models increase the complexity and require more powerful and expensive CPUs. Thus, there is still a need to develop algorithms that can efficiently and correctly address the nonlinearity of the battery at a rational computational cost. For this purpose, we suggest a new algorithm, the VRLS (variable recursive

**Table 1** Summary of the common method used to estimate battery parameters

Method	Merits	Disadvantages
Electrochemical models	High potential to capture battery dynamics	Not suitable for online applications due to computational complexity
Data-based models	Capture large sets of battery data in various regimes	Require exhaustive tests
Equivalent circuit models	Robustness and simplicity	High order models cause computational overhead and may not improve accuracy significantly
Offline identification	Electrochemical impedance spectroscopy, hybrid pulse power characterization	Identified parameters are constants and not accurate for varying battery conditions
Online identification	Various studies conducted	Need to develop efficient algorithms that address battery nonlinearity at a rational computational cost

least square) to update battery model parameters and to estimate the terminal voltage, we also propose a new expression to adapt the forgetting factor. VRLS is contrasted against the Adaptive Forgetting Factor Recursive Least Squares (AFFRLS) and the Recursive Least Squares (RLS) applied to a second order ECM model. The main contributions of this paper include:

- We propose the VRLS algorithm to estimate the parameters of a second order battery model and to estimate the output terminal voltage.
- Real data collected from the CALCE research group was used to evaluate the algorithm performance. Results show that VRLS has high accuracy and reliability.
- Comparison of the suggested approach with two robust recursive methods (AFFRLS, RLS). Assessment tests showed high accuracy of VRLS with low errors in all scenarios in comparison with AFFRLS and RLS algorithms.

The reminder of the document is structured as follows: in Section 2, we describe the battery model used in this study and formulate the required equations. Section 3, contains the steps for the algorithm implementation. Section 4 shows the implementation setup for the SAMSUNG cell. In Section 5 we give a detailed discussion of the results. In Section 6 we draw the conclusion.

## 2 Modeling of the lithium-ion battery

The model of the cell employed is represented in Fig. 1. In this design, an  $R_0$  resistor is included to emulate the momentary voltage drop, two parallel  $R$ - $C$  elements to depict the transitory regime ( $R_1, C_1$ , and  $R_2, C_2$ ). The open circuit voltage (OCV) Eq. (1) is defined with an adjustment of NERNST's equation with three coefficients  $K_2, K_1$  and  $K_0$ . The hysteresis effect is denoted by  $s$ - $M$ , where  $s$  is linked to the sign of the current, and  $M$  is a ratio term.  $V_{out}$  is the terminal voltage.

The open circuit voltage  $V_{ocv}$  is defined as (adjustment of NERNST's equation [21]):

$$V_{ocv}(k) = K_2 \ln(1-z(k)) + K_1(k) \ln(z(k)) + K_0(k). \quad (1)$$

$z$  is the state of charge defined as:

$$z(k+1) = z(k) - \frac{\eta I(k) \Delta t}{Cn}. \quad (2)$$

$Q$  is the battery nominal capacity,  $\eta$  stands for the coulombic efficiency assumed to be 0.98 while charging and 1 while discharging.  $I(k)$  is the battery current flowing out and in,  $\Delta t$  is the interval of sampling time.

VRLS is used to recursively estimate the battery model parameters ( $K_2, K_1, K_0, M, R_2, R_1, R_0, C_1, C_2$ ) and the terminal voltage  $V_{out}$ :

$$V_{out}(k) = K_2 \ln(1-z(k)) + K_1(k) \ln(z(k)) + I(k)R_0(k) + K_0(k) + s(k)M(k) - A_1(k)U_1(k-1) - B_1(k)I(k-1) - A_2(k)U_2(k-1) - B_2(k)I(k-1). \quad (3)$$

$V_{out}(k)$  is the output voltage in time  $k$ , with:

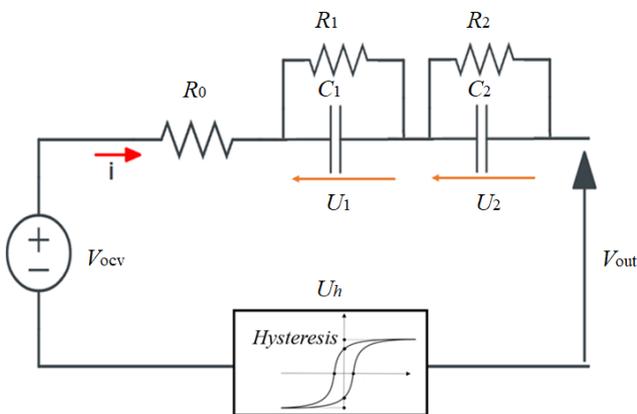


Fig. 1 The second order battery model with hysteresis

$$A_1(k) = \exp\left(\frac{-\Delta t}{R_1(k)C_1(k)}\right), \quad (4)$$

$$B_1(k) = R_1(k) \left[ 1 - \exp\left(\frac{-\Delta t}{R_1(k)C_1(k)}\right) \right], \quad (5)$$

$$A_2(k) = \exp\left(\frac{-\Delta t}{R_2(k)C_2(k)}\right), \quad (6)$$

$$B_2(k) = R_2(k) \left[ 1 - \exp\left(\frac{-\Delta t}{R_2(k)C_2(k)}\right) \right]. \quad (7)$$

$U_1(k-1)$  and  $U_2(k-1)$  represent respectively the prior voltage drop in  $R_1$  and  $R_2$ :

$$U_1(k-1) = V_{ocv}(k-1) + s(k-1)M(k-1) - I(k-1)R_0(k-1) - U_2(k-1) - V(k), \quad (8)$$

$$U_2(k-1) = V_{ocv}(k-1) + s(k-1)M(k-1) - I(k-1)R_0(k-1) - U_1(k-1) - V(k). \quad (9)$$

$s(k)$  is defined as the sign of the current:

$$s(k) = \begin{cases} 1 & \text{if } I(k) > \alpha \\ -1 & \text{if } I(k) < -\alpha \\ s(k-1) & \text{if } |I(k)| \leq \alpha \end{cases}, \quad (10)$$

where  $\alpha$  is a constant small number and  $k$  is the time index.

## 3 VRLS implementation

The parameters ( $K_2, K_1, K_0, M, R_0, R_2, R_1, C_1$ ) are saved in  $\theta(k)$ :

$$\theta(k) = \begin{bmatrix} K_0(k), K_1(k), K_2(k), M(k), \\ R_0(k), A_1(k), B_1(k), A_2(k), B_2(k) \end{bmatrix}^T, \quad (11)$$

with:

$$\varphi(k) = \begin{bmatrix} 1, \ln(z(k)), \ln(1-z(k)), \\ s(k), -I(k), -U_1(k-1), \\ -I(k-1), -U_2(k-1), -I(k-1) \end{bmatrix}, \quad (12)$$

$$G(K) = \frac{P(k-1)\varphi(k)}{\lambda + \varphi^T(k)P(k-1)\varphi(k)}, \quad (13)$$

$$P(K) = \frac{P(k-1) - G(k)\varphi^T(k)P(k-1)}{\lambda}. \quad (14)$$

$\mathbf{G}(\mathbf{K})$ ,  $\mathbf{P}(\mathbf{K})$  are respectively the error and gain covariance matrices. The forgetting factor  $\lambda$  is used to assign weights to new and old data. The model parameters (in  $\theta(\mathbf{k})$ ) are upgraded using the expression below:

$$\theta(\mathbf{k}) = \theta(\mathbf{k}-1) + \mathbf{G}(\mathbf{k}) [V_{out}(k) - \varphi^T(k)\theta(\mathbf{k}-1)]. \quad (15)$$

VRLS uses the above equations to update the battery model parameters in  $\theta(\mathbf{k})$  based on the prior values  $\theta(\mathbf{k}-1)$ , the error covariance and the gain matrices  $\mathbf{P}(\mathbf{K})$  and  $\mathbf{G}(\mathbf{K})$ . Fig. 2 displays the chart of the VRLS approach applied to extract the parameters.

VRLS employs an adjustable forgetting factor  $\lambda$  to update the battery parameters.  $\lambda$  allocates weights to new and old data (generally lies ranging from 0.95 to 1 [21, 22]).

The forgetting factor should vary with the error of the identification parameter adaptively, particularly when the error is very high, in order for the online identification to have a more rapid convergence speed and in order to decrease the identification error [22]. Here, we suggest a new equation to compute the forgetting factor given by:

$$\lambda(k) = \min(\lambda_{min} + (1 - \lambda_{min}) \times \exp(\beta(k)), 1), \quad (16)$$

$$\beta(k) = V_{est}(k) - V_{out}(k). \quad (17)$$

$\lambda_{min}$  is the smallest value of  $\lambda$  which is fixed at 0.98 to achieve a better trade-off between precision and rapidness [22].  $\beta(k)$  is the error between the real measured and the output voltage estimated at time  $k$ .

From Eq. (16), it is obvious that the lower the value of the error  $\beta(k)$ , the lower the forgetting factor, conversely when the error is high. The value of the forgetting factor lies between 0.98 and 1. In this way, the forgetting factor does change with the identification error.

The second-order RC model Fig. 1 is employed as a model wherein parameters to be extracted are hold in the vector  $\theta(\mathbf{k})$  Eq. (11). VRLS is applied in a recursive manner to compute  $\theta(\mathbf{k})$  vector with an adaptive weight factor  $\lambda$  Eq. (16).

The presented approach is demonstrated in Fig. 2. It comprises two steps: data pre-processing and an estimation step.

In step 1: corrupted data are eliminated through cleaning (cleansing): data cleaning is extremely critical. It is used to clean the data to prepare it for the next step. Here, we attempt to delete all data that are potentially corrupted, incomplete, improper or duplicate data; then, the data extracted are evenly spaced.

In step 2: the VRLS algorithm takes over to estimate the model parameters and the output voltage. First, we provide

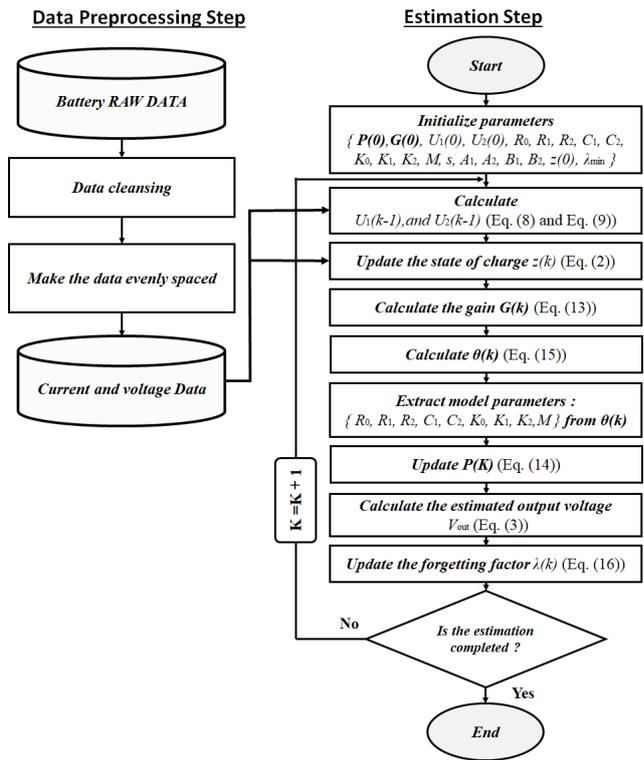


Fig. 2 The proposed VRLS method

the starting values of: the vector  $\theta(0)$ , the error and gain covariance matrices ( $\mathbf{P}(0)$ ,  $\mathbf{G}(0)$ ), and the forgetting factor  $\lambda$ . For each iteration, VRLS computes the past voltage drops  $U_1(k-1)$  and  $U_2(k-1)$  using Eq. (8) and Eq. (9) respectively, then readjusts the state of charge  $z(k)$  using Eq. (2). The gain matrix  $\mathbf{G}(k)$  is evaluated in accordance with Eq. (13) before calculating  $\theta(k)$  vector.

The parameters ( $K_2, K_1, K_0, M, R_2, R_1, R_0, C_1, C_2$ ) can be then extracted from the vector  $\theta(k)$ . Afterwards, the algorithm updates the covariance matrix  $\mathbf{P}(k)$  Eq. (14) using the  $\mathbf{G}(k)$  matrix and the forgetting factor  $\lambda$ . Lastly, the voltage output is evaluated using Eq. (3), and the forgetting factor is readjusted considering the current error  $\beta(k)$  Eq. (17). The same process is reiterated until the estimation is completed.

#### 4 Simulation setup

In this section, we are going to compare the VRLS Fig. 2 with RLS (recursive least square) and AFFRLS (adaptive forgetting factor recursive least squares) algorithms presented respectively in [21] and [23], we applied the algorithms to the model Fig. 1 to estimate the battery model parameters and the terminal voltage.

The performance of the algorithms is verified using experimental data given by the CALCE Research Group performed on the Samsung INR 18650-20R [24–26] cell

in two EV dynamic profiles: The Beijing dynamic stress test (BJDST), and the supplemental federal test procedure (US06) [24–26]. The configuration of the experiment is shown in [24–26].

The Samsung 18650-20R dataset [24–26] is a widely-used benchmark dataset for evaluating the performance of battery estimation algorithms and models. It is important to evaluate the performance of these algorithms under different driving conditions, as this can have a significant impact on battery life and safety.

The BJDST and US06 dynamic profiles are two common driving cycles that are used for this purpose. The BJDST cycle is a drive cycle that was developed by the Beijing Joint Driving Simulation Team, while the US06 cycle is a driving cycle developed by the US Environmental Protection Agency [24–26]. These cycles have been designed to simulate real-world driving conditions and are commonly used for testing and evaluation of battery systems.

The current and voltage data for the Samsung 18650-20R battery cycled under the BJDST and US06 profiles [24–26] are shown in Fig. 3. It is evident from Fig. 3

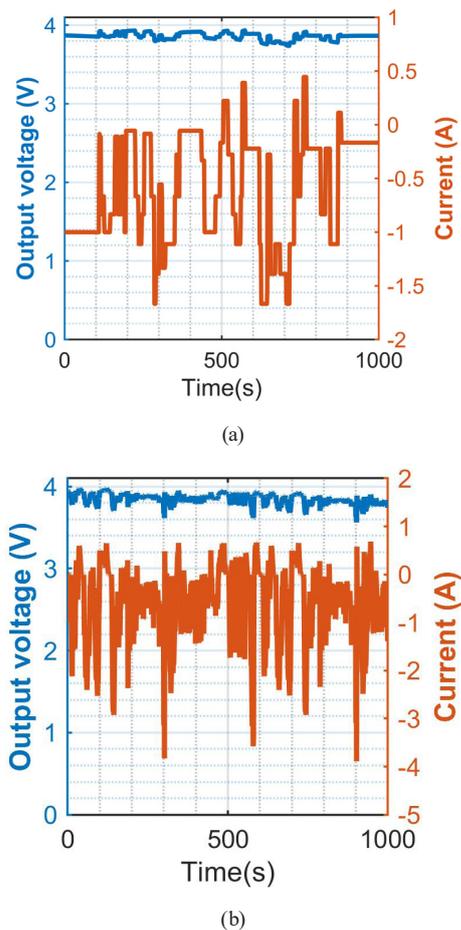


Fig. 3 Current for: (a) BJDST, (b) US06 profiles

that the current in these profiles oscillates vigorously, which can make it challenging to accurately estimate the battery parameters. Therefore, the correctness of the algorithms for parameter identification needs to be carefully checked under such conditions.

Battery estimation algorithms typically use a combination of electrical and thermal models to predict the behavior of the battery. These models rely on accurate estimates of the battery parameters, such as capacity, internal resistance, and open-circuit voltage. The accuracy of these estimates is critical for ensuring the safe and reliable operation of the battery.

To compare the performance of VRLS, RLS, and AFFRLS algorithms in estimating battery parameters and output voltage, all three algorithms were initialized with the same values. These initial values included the battery parameter vector  $\theta(0)$ , error and gain matrices  $P(0)$  and  $G(0)$ , initial voltage drops across resistors  $R_2$  and  $R_1$  ( $U_2(0)$  and  $U_1(0)$ ), and the forgetting factor  $\lambda$ .

The forgetting factor is an important parameter in adaptive filtering algorithms as it determines the influence of past observations on the estimation process. A higher value of  $\lambda$  indicates a higher level of adaptability to changing conditions, while a lower value of  $\lambda$  gives more weight to older observations. In this study, the forgetting factor for RLS was set to 0.9996, as in [21], while for AFFRLS, it was set as in [23]. For VRLS, the forgetting factor was initialized as specified in Section 3 of the paper. Once the algorithms were initialized, they were applied to estimate the battery parameters and output voltage using the current and voltage measures provided in the Samsung 18650-20R [24–26] dataset for the appropriate test cycle. The algorithms were run for multiple cycles to evaluate their performance over time and under varying conditions. The use of the same initialization values for all three algorithms ensures a fair comparison of their performance in estimating battery parameters and output voltage. The inclusion of the forgetting factor further enhances the adaptability of the algorithms to changing conditions and improves the accuracy of the estimates.

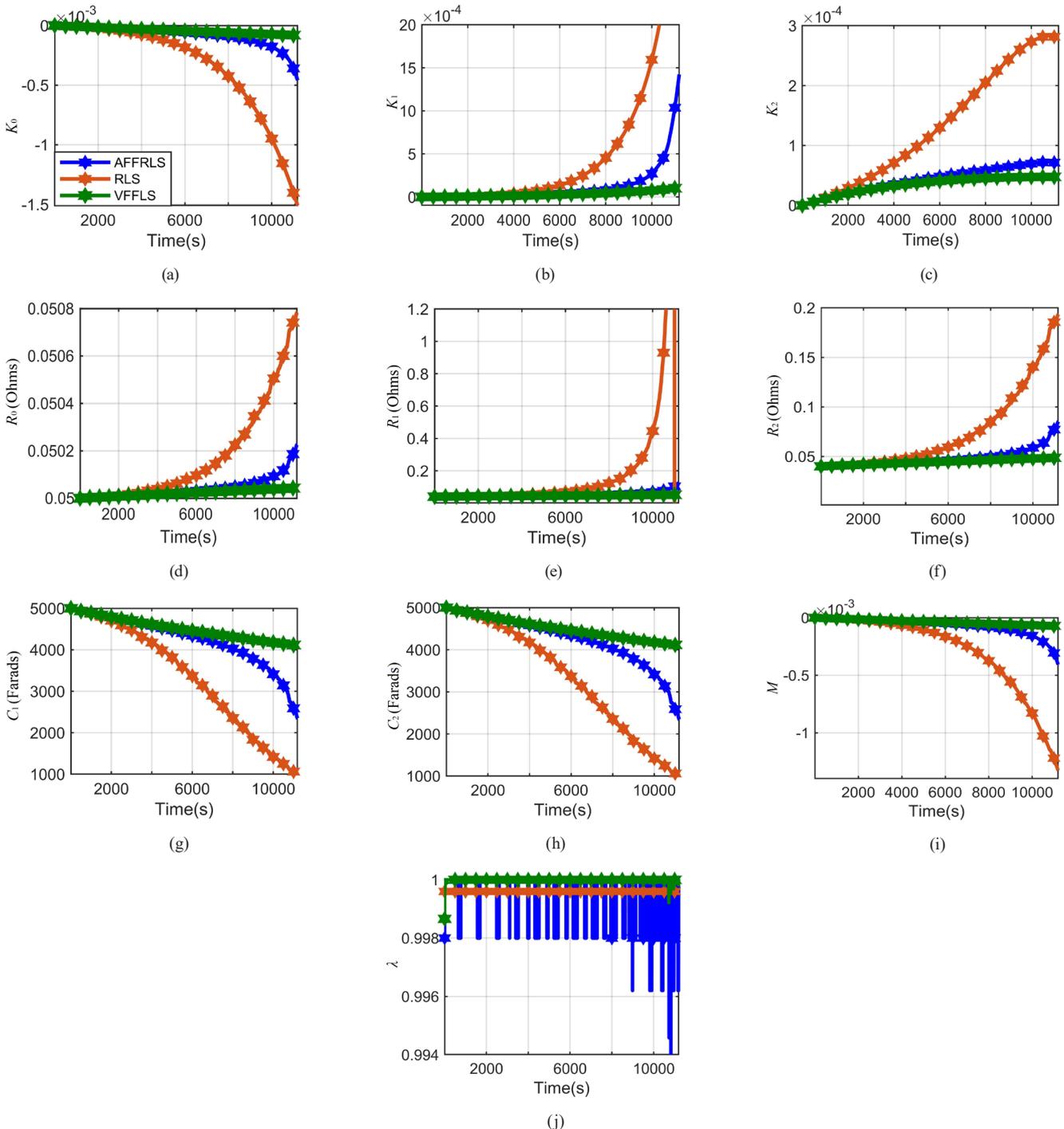
Overall, the methodology employed in this study provides a robust and comprehensive approach to evaluate the performance of battery estimation algorithms in real-world conditions. The use of a benchmark dataset and standard driving cycles, along with careful initialization and parameter selection, enables the comparison of different algorithms and the identification of their strengths and limitations.

### 5 Results and discussion

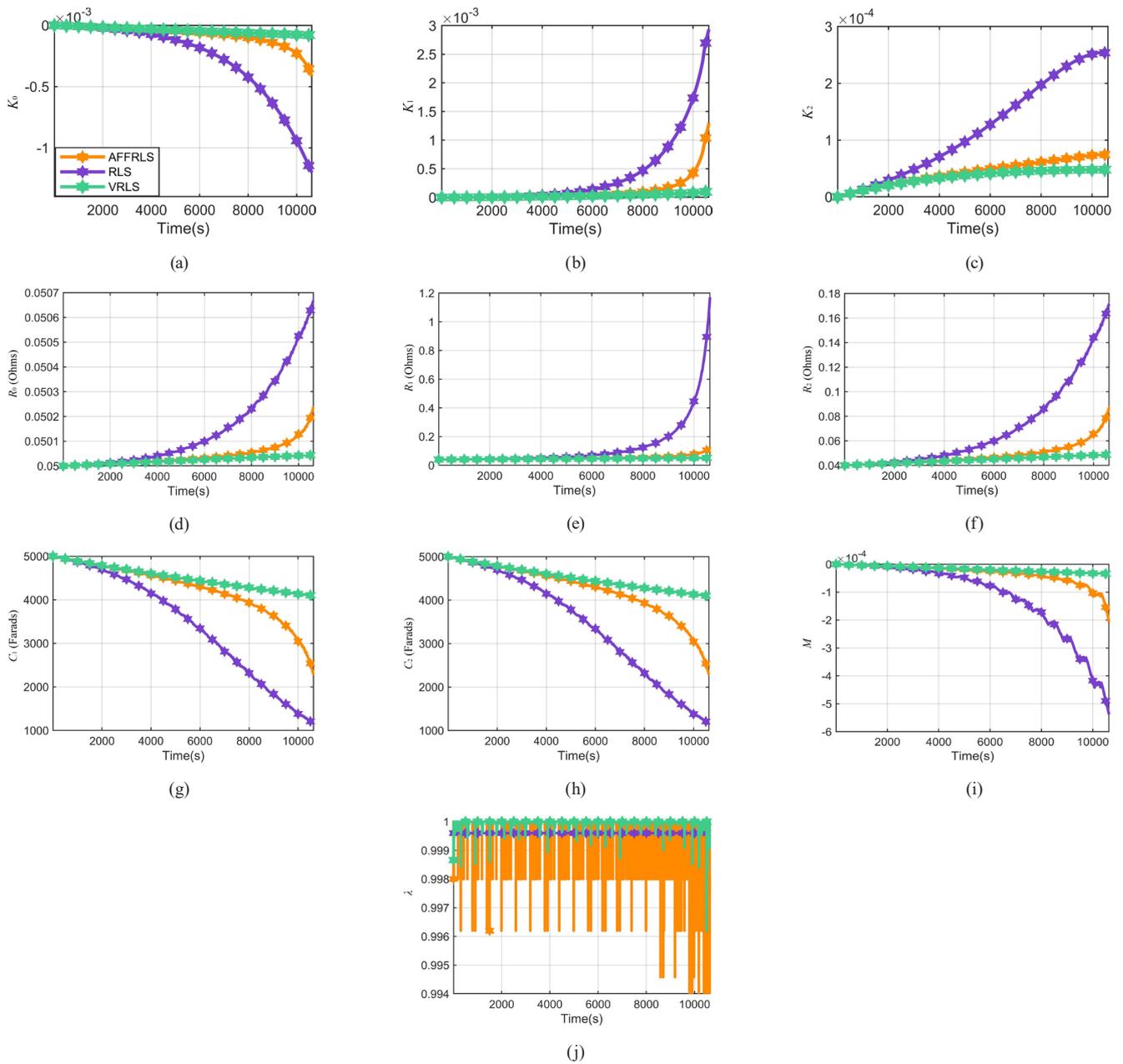
Figs. 4 and 5 demonstrate the findings of the identification of the parameters using the three algorithms. It is evident from Figs. 4 and 5 that the parameters extracted by VRLS and AFFRLS are more stable, whereas the parameters identified by RLS change gradually with current variation. This can be attributed to the fact that both VRLS and AFFRLS allow the forgetting factor to change

dynamically to reduce the error between the measured and estimated terminal voltage. In contrast, RLS has a steady forgetting factor, and to minimize the error, it varies the parameters in the vector  $\theta(k)$ , leading to more fluctuations and occasional peaks in the battery parameters collected by this method.

Figs. 6 and 7 show the comparison between the estimated voltage for the three methods and the real terminal



**Fig. 4** Parameter identification results (BJDST): (a)  $K_0$ , (b)  $K_1$ , (c)  $K_2$ , (d)  $R_0$ , (e)  $R_1$ , (f)  $R_2$ , (g)  $C_1$ , (h)  $C_2$ , (i)  $M$ , (j)  $\lambda$



**Fig. 5** Parameter identification results (US06): (a)  $K_0$ , (b)  $K_1$ , (c)  $K_2$ , (d)  $R_0$ , (e)  $R_1$ , (f)  $R_2$ , (g)  $C_1$ , (h)  $C_2$ , (i)  $M$ , (j)  $\lambda$

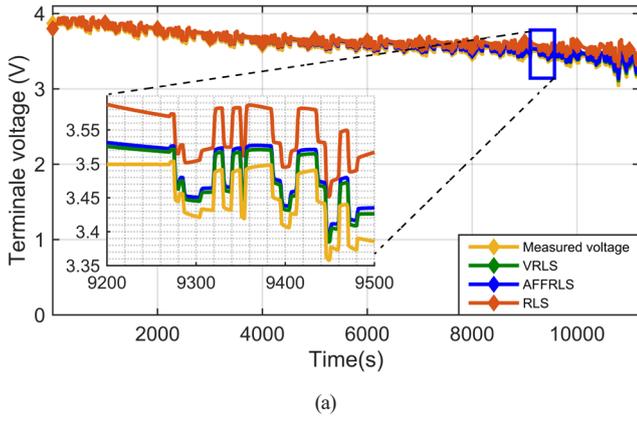
voltage (measured value recorded in the dataset). The estimation error of the output voltage is also displayed in Figs. 6 and 7. The voltage estimated by the algorithms is close to the actual measured voltage, with VRLS showing a slightly lower accuracy.

Figs. 8 and 9 provide a more detailed analysis of the absolute relative error distribution for each algorithm. Figs. 8 and 9 reveal that VRLS is more accurate in terms of parameter identification than AFFRLS and RLS methods. The absolute relative error distribution of AFFRLS has a higher proportion for small errors ( $< 1\%$ ) than VRLS and RLS. However, the distribution decreases for high intervals

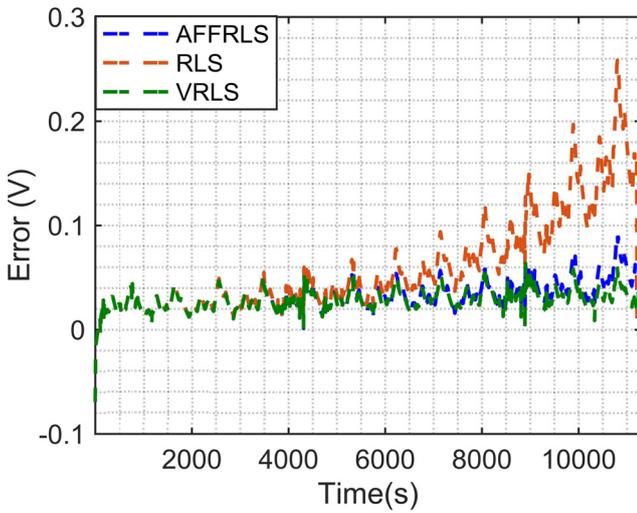
(greater than 2%), up to 6% less compared to the other methods. This means that VRLS is superior in terms of accuracy in identifying the lithium battery model parameters.

To further demonstrate the efficiency of VRLS, we calculated three predictive performance indicators that reveal the validity of the algorithms: mean absolute percentage error (MAPE), mean absolute error (MAE), and root mean square error (RMSE):

$$\text{MAPE}(\%) = \frac{100}{n} \sum_{i=1}^n \frac{|V_{est}(i) - V_{true}(i)|}{V_{true}(i)}, \quad (18)$$



(a)



(b)

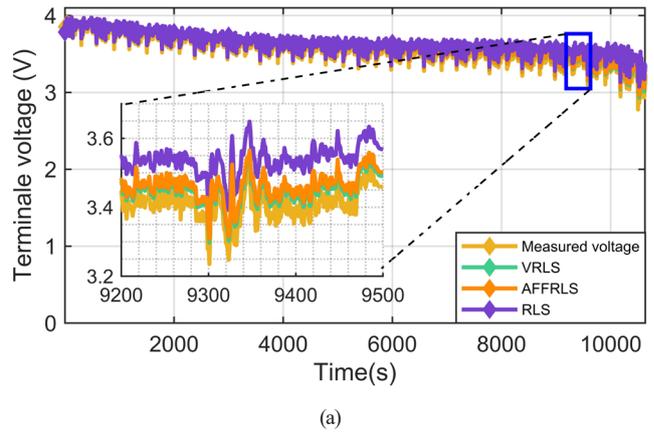
Fig. 6 BJDST profile results: (a) true measured voltage vs estimated output voltage, (b) error of estimation by the algorithms

$$MAE = \frac{1}{n} \sum_{i=1}^n |V_{est}(i) - V_{true}(i)|, \quad (19)$$

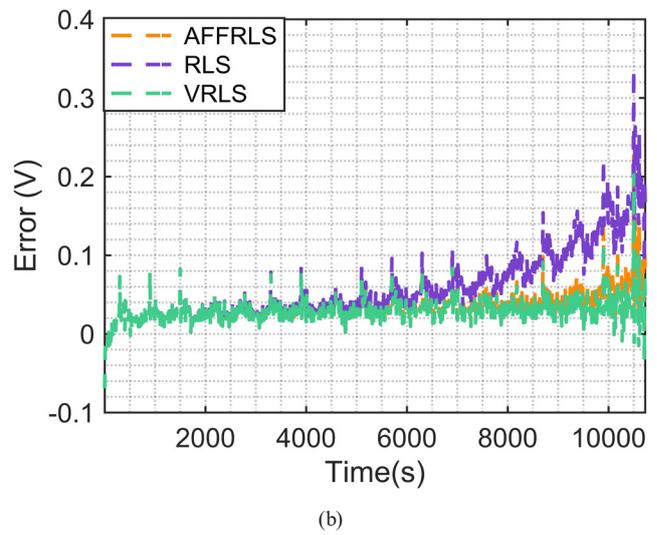
$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (V_{est}(i) - V_{true}(i))^2}, \quad (20)$$

where  $n$  denotes the cycle number,  $V_{true}$  is the measured output voltage, and  $V_{est}$  is the estimated value.

Table 2 provides a summary of the predictive performance of all the algorithms in terms of RMSE, MAE, and MAPE. These are commonly used metrics to evaluate the accuracy of prediction models. The results show that VRLS performs better than RLS and AFFRLS in all three metrics. The lower values of RMSE, MAE, and MAPE for VRLS indicate that the algorithm has better predictive accuracy and a lower degree of prediction error. This is an important finding since accurate prediction of battery performance is crucial for optimizing battery usage and extending its lifetime.



(a)



(b)

Fig. 7 US06 profile results: (a) True measured voltage vs estimated output voltage, (b) error of estimation by the algorithms

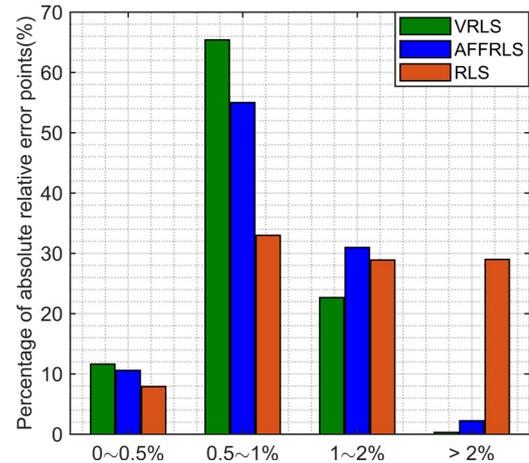


Fig. 8 Percent of absolute relative error points of the three algorithms for the BJDST profile

Table 3 summarizes the minimum, maximum, and mean error values for the two battery cells. The results reveal that VRLS has the lowest maximum and mean errors compared to RLS and AFFRLS. The maximum error represents the worst-case scenario in terms of prediction accuracy, while

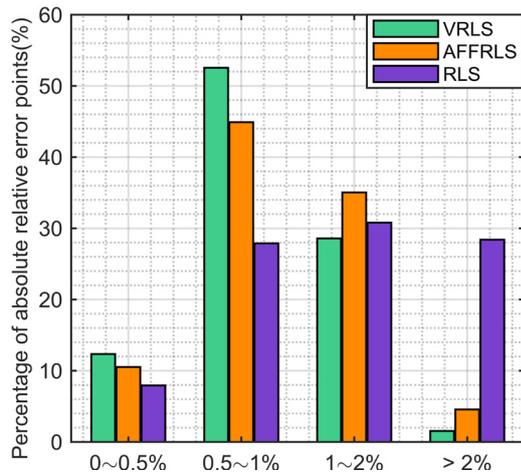


Fig. 9 Percent of absolute relative error points of the three algorithms for the US06 profile

Table 2 Predictive performance indicators

Profiles	Methods	RMSE (mV)	MAE (mV)	MAE (%)
BJDST	VRLS	30.61	28.39	0.79
	AFFRLS	34.64	31.62	0.88
	RLS	59.31	49.35	1.39
US06	VRLS	32.87	30.53	0.85
	AFFRLS	35.88	33.08	0.92
	RLS	63.02	52.85	1.49

Table 3 Max, min and mean error

Profiles	Methods	Max (mV)	Min (mV)	Mean (mV)
BJDST	VRLS	105.04	0.026	28.84
	AFFRLS	330	0.02	68.79
	RLS	330.6	0.026	96.98
US06	VRLS	124	0.007	30.53
	AFFRLS	141.42	0.007	33.08
	RLS	232.11	0.008	52.85

the mean error gives an overall idea of the accuracy of the algorithm. The highest mean error was recorded by the

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RLS method, which indicates that this method is less accurate than the other two methods. This finding is consistent with the results presented in Table 2.

Overall, the results suggest that VRLS outperforms RLS and AFFRLS in terms of prediction accuracy and estimation precision. This is an important contribution to the field of battery modeling since accurate battery modeling is crucial for the development of efficient battery management systems. The findings of this study can help researchers and practitioners to choose the most appropriate algorithm for battery parameter identification and prediction, depending on their specific needs and applications.

## 6 Conclusion

In this work, a second order Thevenin model is used, the OCV is described with an adjustment of Nernst equation, and the hysteresis effect is denoted with a zero-correction term.

The parameters of the equivalent model are depicted by the proposed VRLS method and compared to RLS and AFFRLS methods. The output voltage estimated by all methods is compared with the measured voltage (saved in the dataset).

The accuracy of the algorithms was verified using experimental data for the lithium-ion batteries: SMASUNG INR 18650 [24–26] cycled in two dynamic profiles.

We provided the data to the algorithms and compared the voltage estimated by the three algorithms. We have shown that the distribution of the relative absolute error of VRLS is small for errors greater than 2% less than AFFRLS and RLS. This means that VRLS is superior in terms of accuracy. In view of this work. The suggested algorithms can be adjusted and coupled with approached like Kalman filters, sliding mode observers, sunflower optimization algorithm (SFO),  $H^\infty$  filter or particle filtering (PF) to evaluate the state of health or charge of a lithium-ion battery.

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