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Robust Aerodynamic Parameter Estimation of Unmanned Aircraft Based on Two-step Identification

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Abstract

This paper presents the estimation of stability and control derivatives of an unmanned aircraft. The aerodynamics are described using regressors composed of velocity, angular rates, flow angles and control surface deflections. The flight data is generated from numerical simulation of postulated equations of motion describing the aerodynamics model. Least squares based on the equation error method is used to estimate the parameters representing the different force and moment aerodynamic coefficients. Statistical analysis is done on the estimates to determine the accuracy and adequacy of the estimates to describe the aerodynamic model. A dynamic simulation based on the identified aerodynamic model is used to improve the parameter estimates through regression of the errors between the flight data and the model response. The aircraft under consideration is a scaled Yak-54 fixed wing unmanned aerial vehicle.

Keywords

unmanned aircraft, parameter estimation, least squares, equation error, heteroscedasticity

1 Introduction

System identification seeks to determine a model inclusive of the parameters, from test data; the outcome heavily depends on the models considered [1]. This means that for system identification, the model describing the physical process may not be unique. If the structure of the model is known a priori or it is pre-defined, the process of system identification results to parameter estimation; determination of the numerical values of parameters in the model [2]. The objective of parameter estimation in aircrafts is to obtain a reasonably accurate estimate of the aerodynamic model to adequately represent the dynamics of the system. An estimated system is not a fact but rather an attempt to closely approximate the original system either in "form" and/or behavior.

Several methods have previously been used in parameter estimation in aircrafts with great successes. Most of these methods often seek to minimize the squared difference of the observations and predicted values. The output error method [1] based on the maximum likelihood principle was used in [3–5] together with Nelder-Mead and Levenberg-Marquadt algorithms to extract aerodynamic parameters of a jet aircraft. In [3], the uncertainties of parameter estimates were quantified using Cramer-Rao bounds. Least squares estimation based on the equation error method was used in [6–8]. Segmentation of flight data was carried out in [6], then a step-by-step procedure using least squares was used to estimate aerodynamic coefficients from the different segments. Other techniques including filter error method [9], neural networks [10–12], and filtering [13–15] have also been used for aircraft system and parameter identification.

During system identification, two sets of test data are required; one data set for identification, and a complimentary data set for model validation. Several sources of data are available for system identification such as flight data [13, 16], computational fluid dynamics data [17, 18], and wind tunnel tests [19]. Many at times, either the data is not readily available, or data collection would involve a significant cost. Moreover, for real flight data, it is difficult to find that which encompasses specific flight profiles necessary for system identification. Synthetic flight test data from online repositories [20, 21], simulated flight data [10, 22] and surrogate models [23, 24] provide good alternatives.

Physical aircraft models derived from aerodynamics are usually available, however they cannot be used due to their complex nonlinear behaviour. Simplified representations

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of physical models can be developed so as to achieve specific tasks such as control synthesis, motion planning, and parameter estimation [25]. Parameter estimation based on test data extracted from surrogate models [26] and experimental simulation [27] result in heteroscedastic models [28] in which the variance of the residual terms in a regression model varies widely. This is attributed to aerodynamic structure inaccuracies, approximations, model simplifications and inconsistency in simulations. Presence of heteroscedasticity increases the likelihood that the estimates are further from the correct value, and similarly the variances of the estimates [27].

The use of equation error method for system identification is not a novel concept nor its application for parameter estimation in unmanned aircraft. However, this work moves a step further to correct the effect of heteroscedasticity on the parameter estimate variances, and effects an improvement in the parameter estimates using a two-step identification procedure. It is assumed that the reader is familiar with the fundamental concepts in statistics and rigid body kinematics and dynamics [1, 2].

The rest of the paper is organized as follows: Section 2 introduces briefly the concepts of least squares parameter estimation, and statistical analysis of the parameter estimates. The formulation of aircraft aerodynamics model to fit the precepts of the equation error identification method is discussed in Section 3. Numerical results and analysis are presented in Section 4, and the conclusions thereafter in Section 5.

2 Parameter estimation and statistical analysis

2.1 Equation-error method

Denote $y(k) = x(k)\theta$ as the linear hypothesis about the parameter dependence. The equation-error method based on the least squares, LS is simple yet powerful, and sufficiently captures the underlying dependencies in the noisy observation data [1]. Consider Eq. (1):

$$z(k) = x(k)\theta + \varepsilon(k) \tag{1}$$

where $k = 1,\ldots,N$ and $\theta = (1,\theta_2,\ldots,\theta_n)^T$ represents the unknown parameters; z represents the observation, x represents the independent variable, also referred to as regressors and ε is the uncorrelated error representing lumped up noise in the observations. Parameter θ is assumed constant over all N data samples, and that the independent variable x is assumed to be error-free. For the N data points, Eq. (1) becomes $Z = X\theta + E$, where Z, E, and X are $N \times 1$, $N \times 1$ and $N \times n$ matrices respectively. The errors in the N data points become $E = Z - X\theta$. The unknown parameters θ are obtained by minimizing the sum of the squares of the errors in a least squares sense. Let $J(\theta)$ be a cost function and define:

$$J(\theta) = \frac{1}{2}\sum_{k-1}^{N} \varepsilon^2(k) = \frac{1}{2}[Z - X\theta]^T [Z - X\theta]. \tag{2}$$

Taking the gradient of the cost function Eq. (2) with respect to θ, results to

$$\frac{\partial J(\theta)}{\partial \theta} = -Z^T X + \theta^T (X^T X). \tag{3}$$

Equating Eq. (3) to zero, and making the unknown parameters the subject results to

$$\hat{\theta} = (X^T X)^{-1} X^T Z, \tag{4}$$

where $\hat{\theta}$ is the least squares parameter estimate of the true value θ that minimizes the error in response between the identified model and the measured response. If $(X^TX)^{-1}$ exists, then $\hat{\theta}$ is unique, if it doesn't exist, then $\hat{\theta}$ will have multiple candidate solutions [1]. The condition number of (X^TX) can provide a quick check on whether $(X^TX)^{-1}$ exists, or the severity of the ill-conditioning [2]. To achieve reliable parameter estimates a large data set is vital.

2.2 Statistical analysis of parameter estimates

The statistical analysis of aerodynamic parameter estimates provides a means of judging the sufficiency of the identified model parameters to replicate the system adequately. The analysis involves calculation of confidence intervals, parameter variances, cross-covariance and the fit error. Confidence intervals can be expressed as the Cramer-Rao bounds which are given by the standard deviations of parameter estimates. (X^TX) is the Fisher information matrix, the inverse of which gives the estimation error covariance matrix $P \equiv (X^T X)^{-1} = [P_{ij}]$, $i,j = 1,2,\ldots,n$. The covariance matrix of the parameter estimates $\hat{\theta}$ is given as

$$Cov(\theta) = \sigma^2 (X^T X)^{-1}. \tag{5}$$

The Cramer-Rao bounds are calculated by $\sigma_j = \sqrt{\sigma^2 P_{jj}}$ where P_{jj} are the main diagonal elements of P. Cramer–Rao bounds indicate the theoretically maximum achievable accuracy of the estimates in statistical terms. Parameter estimates are statistically accurate if the standard deviations and correlation coefficients are small. The error variance σ^2 in Eq. (5) is usually not known a priori and thus an unbiased estimate for σ^2 is determined from the residuals using Eq. (6).

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{N}\left[z(i)-\hat{y}(i)\right]^2}{N-n} \equiv s^2 \tag{6}$$

The fit error is given by $\sqrt{s^2}$ [2]. Residuals are expected to be random, independently distributed and reasonably normal since they are elements of variation not captured in the fitted model. Any deviations from these assumptions mean that the residuals contain a pattern/trend that is not accounted for in the model. Identification of the trend and adding terms to the original model to improve the regression may lead to a better model.

2.3 Model validation of aircraft parameter estimates

Was a model formulated such that unique parameter values can be found? Do the estimated parameters have physical realistic values? Does the identified model sufficiently represent the system dynamics? After parameters of a model have been estimated, it is important to determine the correctness, adequacy, and sufficiency of that model.

Investigation of an identified model for correctness should be through the plausibility and polarity of the estimates from a theoretical background. Adequacy of parameter estimates can be checked through the relative magnitudes of parameters contributing to a given aerodynamic force or moment. Equation (7) gives the coefficient of determination R^2 [2] which represents the proportion of the variation in the measured output that is explainable by an identified model. It can be used to check the sufficiency of an identified model to represent the system dynamic. $R^2 \in [0,1]$, where a value of 1 represents a perfect fit to the data.

$$R^2 = \frac{\hat{\theta}^T X^T z - N\bar{z}^T\bar{z}}{z^T z - N\bar{z}^T\bar{z}} \tag{7}$$

External validation of an identified model involves a comparison of the behaviour of variables within the simulation model with the corresponding measured quantities in a real/nominal system. Proof-of-match (POM) is one such method, and the general rule is to use complementary test data [1].

3 Unmanned aircraft aerodynamic model formulation

Let $(u,v,w)^T$ and $(p,r,r)^T$ represent the velocity and angular velocity components respectively in the body frame's center of gravity. The force and moment equations of a UAV in the body frame are given by Eqs. (8), (9) [2].

$$\left.\begin{aligned} F_x &= m(\dot{u}+qw-rv) \\ F_y &= m(\dot{v}+ru-pw) \\ F_z &= m(\dot{w}+pv-qu) \end{aligned}\right\} \tag{8}$$

$$\left.\begin{aligned} M_x &= \dot{p}I_x - \dot{r}I_{xz} + qr(I_z - I_y) - qpI_{xz} \\ M_y &= \dot{q}I_y + pr(I_x - I_z) + (p^2 - r^2)I_{xz} \\ M_z &= \dot{r}I_z - \dot{p}I_{xz} + pq(I_y - I_x) + qrI_{xz} \end{aligned}\right\} \tag{9}$$

$F_{x,y,z}$ and $M_{x,y,z}$ are composed of contributions from aerodynamics, thrust and gravity as expressed in Eqs. (10), (11)

$$\begin{aligned} F_{x,y,z} &= F_{\text{aero}} + F_g + F_T \\ &= \bar{q}S\begin{bmatrix} C_x \\ C_y \\ C_z \end{bmatrix} + \begin{bmatrix} -mg\sin\theta \\ mg\sin\phi\cos\theta \\ mg\cos\phi\cos\theta \end{bmatrix} + \begin{bmatrix} T \\ 0 \\ 0 \end{bmatrix} \end{aligned} \tag{10}$$

$$M_{x,y,z} = M_{\text{aero}} = \bar{q}S\begin{bmatrix} bC_l \\ \bar{c}C_m \\ bC_n \end{bmatrix} \tag{11}$$

where $\bar{q} = \rho V^2/2$ is the dynamic pressure. The other variables $\rho, b, \bar{c}, S, I_*$ have their usual meaning according to aircraft literature [2, 19]. The effect of a rotating mass in the propulsion system has been neglected. The force and moment equations can be written as in Eqs. (12), (13) after substituting Eqs. (10), (11) into Eqs. (8), (9), and making the relevant rearrangements [2, 3].

$$\begin{aligned} m\dot{u} &= (rv - qw) + \bar{q}SC_x - mg\sin\theta + T \\ m\dot{v} &= (pw - ru) + \bar{q}SC_y + mg\sin\phi\cos\theta \\ m\dot{w} &= (qu - pv) + \bar{q}SC_z + mg\cos\phi\cos\theta \end{aligned} \tag{12}$$

$$\begin{aligned} \dot{p}I_x - \dot{r}I_{xz} &= qr(I_y - I_z) + qpI_{xz} + \bar{q}SbC_l \\ \dot{q}I_y &= pr(I_z - I_x) + (r^2 - p^2)I_{xz} + \bar{q}S\bar{c}C_m \\ \dot{r}I_z - \dot{p}I_{xz} &= pq(I_x - I_y) - qrI_{xz} + \bar{q}SbC_n \end{aligned} \tag{13}$$

It is often appropriate to express force equations in terms of airspeed V, angle of attack α, and side slip angle β, rather than u,v,w. The force equations, Eq. (12) are transformed to the wind axes using an appropriate rotation matrix and the resultant force equations become Eq. (14).

$$\begin{aligned} \dot{V} &= \frac{\bar{q}S}{m}C_{D_W} + \frac{T}{m}C_\alpha C_\beta - gW_1 \\ \dot{\beta} &= \frac{\bar{q}S}{mV}C_{Y_W} + \Gamma_1 - \frac{T}{mV}C_\alpha S_\beta + \frac{gW_2}{V} \\ \dot{\alpha} &= -\frac{\bar{q}S}{mC_\beta}C_L + q - \Gamma_2 - \frac{T}{mVC_\beta}S_\alpha + \frac{gW_3}{VC_\beta} \end{aligned} \tag{14}$$

Subscripts α, β are inputs to trigonometric functions, that is $C_\alpha = \cos(\alpha)$, $C_\beta = \cos(\beta)$, $S_\alpha = \sin(\alpha)$, $S_\beta = \sin(\beta)$. $C_{D_W} = C_D \cos\beta - C_Y \sin\beta$, $C_{Y_W} = C_Y \cos\beta + C_D \sin\beta$, $\Gamma_1 = pS_\alpha - rC_\alpha$, $\Gamma_2 = T_\beta(pC_\alpha + rS_\alpha)$ and gW_1, gW_2, gW_3 are the gravity acceleration components in the wind axis frame. Equations (13), (14) are the equations of motion, which form the dynamic model. They are developed with the assumption that the aircraft is a rigid body and thrust acts along the x-axis through the aircraft center of gravity.

For a conventional aircraft at low Mach number M < 1, the functional form of the nondimensional force and moment coefficients is expressed as

$$C_* = C_*\left(\alpha, \beta, \hat{p}, \hat{q}, \hat{r}, \hat{\dot{\alpha}}, \hat{\dot{\beta}}, \delta_s\right) \tag{15}$$

where $* \Rightarrow D, Y, L, l, m, n$ and δ_s represent aircraft controls of aileron, elevator, rudder and thrust command. For quasi steady flow, the functional form is reduced to $C_* = C_*\left(\alpha, \beta, \hat{p}, \hat{q}, \hat{r}, \delta_s\right)$ [2]. The independent variables of the dynamic model Eqs. (13), (14) are expressed using the quasi-steady functional form to become as in Eq. (16) [3, 6].

$$\begin{aligned}
C_D &= C_{D0} + C_{D\alpha}\alpha \\
C_Y &= C_{Y0} + C_{Y\beta}\beta + C_{Yp}\hat{p} + C_{Yr}\hat{r} + C_{Y\delta_a}\delta_a + C_{Y\delta_r}\delta_r \\
C_L &= C_{L0} + C_{L\alpha}\alpha + C_{Lq}\hat{q} + C_{L\delta_e}\delta_e \\
C_l &= C_{l0} + C_{l\beta}\beta + C_{lp}\hat{p} + C_{lr}\hat{r} + C_{l\delta_a}\delta_a + C_{l\delta_r}\delta_r \\
C_m &= C_{m0} + C_{m\alpha}\alpha + C_{mq}\hat{q} + C_{m\delta_e}\delta_e \\
C_n &= C_{n0} + C_{n\beta}\beta + C_{np}\hat{p} + C_{nr}\hat{r} + C_{n\delta_a}\delta_a + C_{n\delta_r}\delta_r
\end{aligned} \tag{16}$$

The structure of the identification model culminates to the solution of θ_j, j = 1,…,n, which represent the aerodynamic coefficients in Eq. (16). The parameters θ_j can be estimated together using a combined maneuver exciting all the modes [2], although a possibility exists where force and moment parameters can be estimated independently. The model dynamics after reorganization of Eqs. (13), (14) result to Eq. (17).

$$\begin{aligned}
C_l &= \left(\dot{p}I_x + qr\left(I_z - I_y\right) - \left(qp + \dot{r}\right)I_{xz}\right) / \bar{q}Sb \\
C_m &= \left(\dot{q}I_y + pr\left(I_x - I_z\right) + \left(p^2 - r^2\right)I_{xz}\right) / \bar{q}S\bar{c} \\
C_n &= \left(\dot{r}I_z + pq\left(I_y - I_x\right) + \left(qr - \dot{p}I_{xz}\right)I_{xz}\right) / \bar{q}Sb \\
C_{D_W} &= \left(m\dot{V} - TC_\alpha C_\beta - mgW_1\right) / \bar{q}S \\
C_{Y_W} &= \left(mV\left[\dot{\beta} + \Gamma_1\right] + TC_\alpha S_\beta - gW_2\right) / \bar{q}S \\
C_L &= \left(mVC_\beta S_\alpha\left[-\dot{\alpha} + q - \Gamma_2\right] - TS_\alpha + mgW_3\right) / \bar{q}S
\end{aligned} \tag{17}$$

The right side of Eq. (17) form the observations and the right side of Eq. (16) form the regressors.

4 Identification results

The goal of the identification experiments was to determine the nonlinear dynamical system parameters using signal processing in order to validate their convergence properties.

4.1 Assumptions

To desist deviating from the objective of the study, the following assumptions were made:

1. Flight data was obtained during simulation experiments under closed loop control, and it was available and sufficient for parameter estimation.
2. High precision state estimation solution based on low cost IMU, MEMS, and GPS was available, for example [29]. However, the differences between the real and the estimated states were negligible at this level of experiments.
3. In order to validate the model parameters, besides their convergence properties, knowledge of their numerical values was available during the experiments.

4.2 Simulation flight data

The physical properties of a Yak-54 scaled unmanned aircraft are given in Table 1. Equations (13), (14) were used to carry out numerical simulations in Matlab using the model and structure adapted from [30]. The simulation was designed using an appropriate maneuver such that the control deflections were not too large to yield impractical data nor too small to not excite the dynamic modes.

The continuous time histories of state and control variables shown in Figs. 1, 2 were realized.

The angular accelerations $\dot{p}, \dot{q}, \dot{r}$ in Eq. (17) were obtained through numerical differentiation of the time histories of the angular velocity quantities in Fig. 2.

4.3 Initial parameter estimates

The estimates of aerodynamic coefficients were computed according to the method discussed in Section 2 and were

Table 1 Aircraft properties [30]

Property	Value
m	12.755 kg
I_x	1.3059 kg m^2
I_y	3.9208 kg m^2
I_z	5.1597 kg m^2
I_{xz}	0.0500 kg m^2
b	2.4079 m
$\bar{c}$	0.4420 m

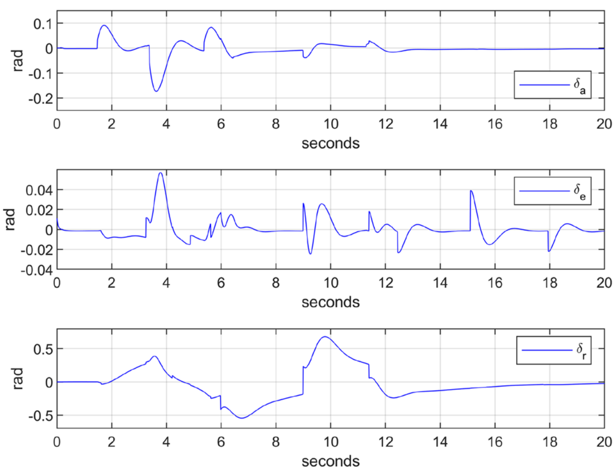

Fig. 1 Control deflections time histories used for identification

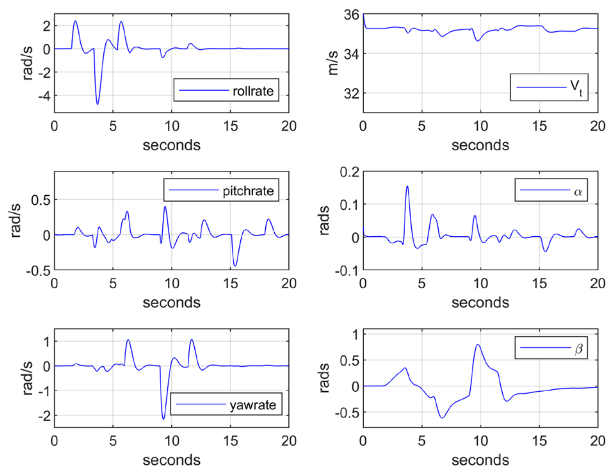

Fig. 2 State variable time histories used for identification

compared to two other techniques; feasible weighted least squares, and bagged residual.

Residual bagging [31] is not a system identification method. It is a bootstrapping approach whose objective is to "naturally break the heteroscedasticity of the data and impose homoscedasticity in the bootstrapped samples" and thus increase the confidence intervals of parameter estimates. This technique creates multiple regression models by resampling the residuals with replacement, regresses the samples with addition to the observations, and takes the average to form a new set of parameter estimates [32]. The variability of the new parameter estimates is derived solely from the residuals. For this work, 1000 residual bootstrap samples were created, and the means of the parameter predictions were taken.

The feasible weighted least squares [33], FWLS method seek to correct heteroscedasticity by accounting for prediction uncertainty in the parameter estimates through the use of a heteroscedasticity consistent covariance matrix [34, 35]. The procedure is first to estimate $\hat{\theta}$ using ordinary linear least squares Eq. (4). Calculate the residuals ε_i and their expectation $E\left(\varepsilon_i^2\right) = \sigma^2 \cdot f\left(z_i'\theta\right)$, by noting that $z_i \equiv x_i\theta + c_i\varepsilon_i$, where $c_i^2 = f\left(z_i'\theta\right)$ is the heteroscedasticity function. Thereafter, adjust the observation and predictor matrices as $z_i^* = z_i \cdot \left[f\left(z_i'\theta\right)\right]^{-0.5}$, $x_i^* = x_i \cdot \left[f\left(z_i'\theta\right)\right]^{-0.5}$, then carry out feasible weighted least squares using x_i^*, z_i^* and $\hat{\Lambda} = diag\left[f\left(z_i'\theta\right)\right]$ into Eq. (18) for N data points.

$$\hat{\theta}_{\text{FWLS}} = \left(X^T \hat{\Lambda}^{-1} X\right)^{-1} X^T \hat{\Lambda}^{-1} Z. \tag{18}$$

Table 2 and Table 3 show a comparison of the identified parameters using the three methods in columns 3, 4, and 5. Column 2 shows the nominal values of the parameters.

The estimates based on LS and FWLS have correct polarity for all the estimates; bagged residual has an exception only on $C_{n\delta_a}$ where the polarity is incorrect. The LS and FWLS parameter estimates, closely resemble each other with the exception of the lift force coefficients in FWLS, where the magnitudes of estimates are theoretically not plausible. The other parameter estimates are reasonably good estimates in relation to the nominal values. Other than $C_{n\delta_a}$, the LS estimates lie within a close neighborhood of the perturbed estimates based on residual bagging.

The variance of measurement errors computed using Eq. (5) showed non-constant variances, and covariances, an indication of presence of heteroscedasticity. The standard errors computed would give inaccurate values. Therefore, standard deviations in columns 6 of Table 2 and Table 3 were computed from a heteroscedasticity-consistent covariance matrix estimator $\left(X^T X\right)^{-1}\left(X^T \hat{\Lambda} X\right)\left(X^T X\right)^{-1}$, with $\hat{\Lambda} = diag\left[\left(z_i - x_i'\hat{\theta}_{\text{LS}}\right)^2\right]$ for the least squares parameter estimates. Aslam et al. [36] note that for a high degree of cross correlation, ordinary least squares always performs better than FWLS. The least squares parameter estimates were thus selected as the initial estimates.

4.4 Parameter estimates improvement and analysis

A numerical simulation was carried out using the initial model parameters estimates. The errors between the flight data (c.f. Figs. 2, 3) and the initial estimated model response were used to form a compound observation variable on addition to the initial observation variable. The new observation variable was regressed using the initial regressors variables. The moments' parameter estimates did not record any changes during this process, implying optimality of the estimates. However, there were some adjustments in the force parameter estimates. Table 4 shows the initial, $\hat{\theta}_{\text{LS}}$ and the updated, $\hat{\theta}_{\text{LS}}^*$ force parameter

Table 2 Parameter estimates for the moment coefficients

Parameter	θ	$\hat{\theta}_{LS}$	$\hat{\theta}_{FWLS}$	Bagged residual	$s(\hat{\theta}_{LS})$
$C_{l\beta}$	−0.0255	−0.0194	−0.0185	−0.0213	0.0011
C_{lp}	−0.3817	−0.2967	−0.2995	−0.3121	0.0137
C_{lr}	0.0504	0.0421	0.0424	0.0499	0.0028
$C_{l\delta_a}$	0.3490	0.2762	0.2785	0.2917	0.0127
$C_{l\delta_r}$	0.0154	0.0112	0.0102	0.0130	0.0008
C_{m0}	−0.0018	−0.0017	−0.0017	−0.0021	0.0000
$C_{m\alpha}$	0.3701	0.3495	0.3498	0.3506	0.0063
C_{mq}	−8.5026	−8.1499	−8.1546	−8.1402	0.1575
$C_{m\delta_e}$	−0.8778	−0.8382	−0.8389	−0.8406	0.0143
$C_{n\beta}$	0.0954	0.0901	0.0898	0.0887	0.0041
C_{np}	−0.0156	−0.0277	−0.0276	−0.0468	0.0056
C_{nr}	−0.1161	−0.0957	−0.0939	−0.0912	0.0061
$C_{n\delta_a}$	−0.0088	−0.0031	−0.0029	0.0148*	0.0035
$C_{n\delta_r}$	−0.0996	−0.0938	−0.0935	−0.0928	0.0046

Table 3 Parameter estimates of the forces

Parameter	θ	$\hat{\theta}_{LS}$	$\hat{\theta}_{FWLS}$	Bagged residual	$s(\hat{\theta}_{LS})$
C_{D0}	0.0526	0.0484	0.0484	0.0487	0.0004
$C_{D\alpha}$	−0.0863	−0.2083	−0.2083	−0.2080	0.0197
$C_{Y\beta}$	−0.3462	−0.3533	−0.3523	−0.3537	0.0138
C_{Yp}	0.0073	0.0130	0.0064	0.0132	0.0639
C_{Yr}	0.2372	0.3325	0.3359	0.3339	0.0448
$C_{Y\delta_r}$	0.1928	0.2002	0.1992	0.2007	0.0169
C_{L0}	0.1470	0.0256	0.3178	0.0260	0.0008
$C_{L\alpha}$	4.5363	4.0874	0.0211	4.0871	0.0655
C_{Lq}	5.1515	8.9726	0.0005	8.9588	1.4689
$C_{L\delta_e}$	0.3762	0.6458	0.0046	0.6465	0.0913

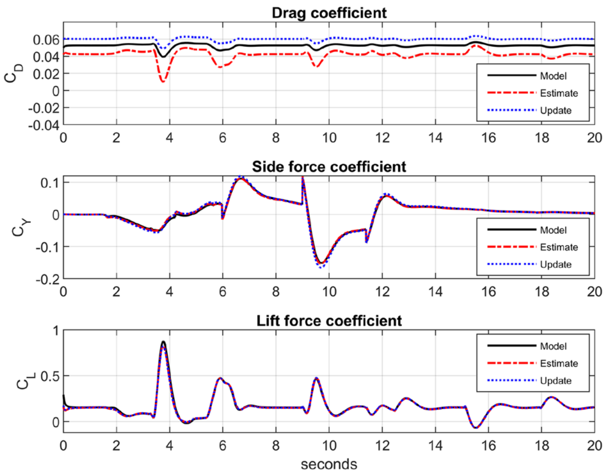

Fig. 3 Drag, side force and lift force coefficients

Table 4 Updated force parameter estimates

Parameter	θ	$\hat{\theta}_{LS}$	$\hat{\theta}^*_{LS}$
C_{D0}	0.0526	0.0484	0.0620
$C_{D\alpha}$	−0.0863	−0.2083	−0.0741
$C_{Y\beta}$	−0.3462	−0.3533	−0.3589
C_{Yp}	0.0073	0.0130	0.0244
C_{Yr}	0.2372	0.3325	0.3137
$C_{Y\delta_r}$	0.1928	0.2002	0.1897
C_{L0}	0.1470	0.0256	0.0399
$C_{L\alpha}$	4.5363	4.0874	4.1394
C_{Lq}	5.1515	8.9726	8.5000
$C_{L\delta_e}$	0.3762	0.6458	0.6132
R^2 %	–	78.90	83.28

estimates respectively. Most of the force parameter estimates realized positive improvements with only exceptions from C_{D0}, $C_{y\beta}$ and C_{yp}. It is noted that a significant improvement in C_{Lq} could not be achieved as deviation of the estimate from the nominal is still big. However, since we considered only quasi steady flow variables, chances are that some other non-quasi steady flow variable(s) could have been lumped therein.

The coefficient of determination of the force parameters improved from 78.90% to 83.28%, a change of 4.38%. A comparison of drag, side force and lift force coefficients as formulated in Eq. (16) and the parameter estimates in Table 4 are shown in Fig. 3. The side force and lift coefficients of the updated model closely predict the nominal model and the initial estimate. For the drag coefficient, the updated estimates significantly improve the profile and adequacy of predicting the nominal model, albeit with a little shift.

The drag coefficient in this identification formulation comprised of a bias, C_{D0} and an α dependent coefficient $C_{D\alpha}$. The steady state error in the drag coefficient is attributable to the differences in magnitude of the bias terms, which creates a steady state shift. The rolling, pitching and yawing moments coefficients shown in Fig. 4 are from to the initial parameter estimates cf. Table 2 column 3, since there was no parameter estimate improvement realized on these coefficients.

A POM validation was carried out using a different flight regime other than that used for identification. Figs. 5, 6 show the comparison of the angular rates, velocity, flow angles, and control deflections between the nominal and the identified model.

The identified model predicts the data well with exception of some steady state error in the angle of attack, α. The reason for this can be attributable to static stability whereby, by changing the elevator position, an aircraft settles to a specific "equilibrium" angle of attack. Simply put, the angle of attack follows the elevator position [37], and this can be seen by the steady state error in the control deflection δ_e in Fig. 6. Since the experiments were carried out assuming a steady flight, it is usual that the response of the angle of attack should oscillate above zero.

Identification of dynamic models from experimental data has often been driven by the creation of approximate models that mimic the behavior of the simulation model as closely as possible, and the ability to use the identified model as a basis for control design. The identified model tried to compensate the response of the angle of attack; no positive lift can be generated in a steady flight by a symmetric wing at zero or negative angle of attack [37].

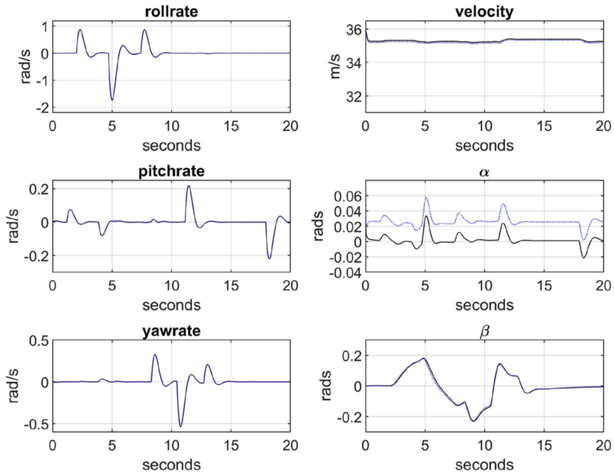

Fig. 5 State variables (blue-Estimate)

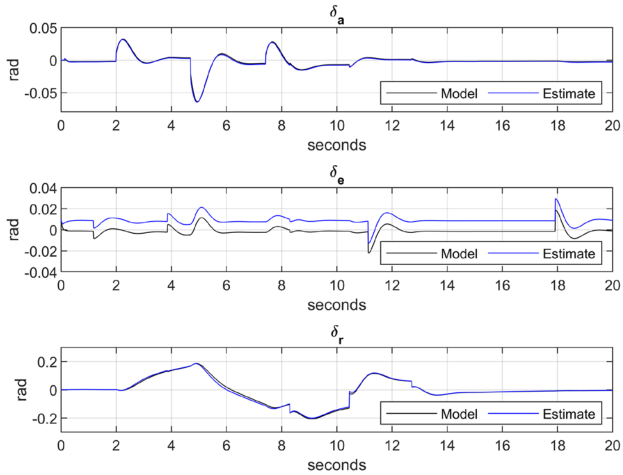

Fig. 6 Control variables

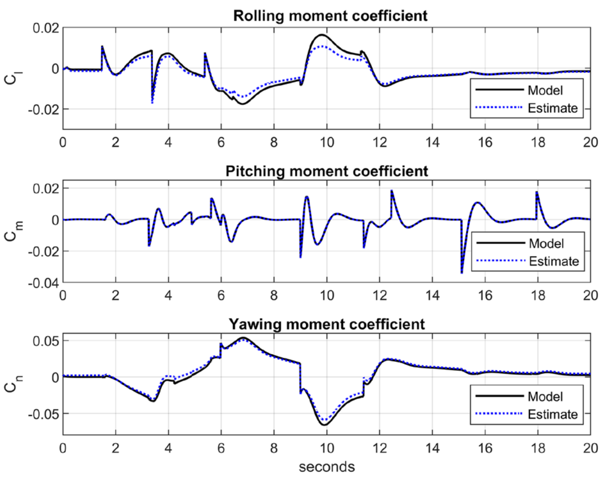

Fig. 4 Rolling, pitching, and yawing moments coefficients

5 Conclusion

The identification of aerodynamic parameters based on a two-step procedure has been presented, by regressing the errors between flight test data and the identified model response. The observation variables were adjusted by accounting for the differences in response. The estimated parameter error bounds obtained through normal calculation would be too optimistic in the presence of heteroscedasticity, therefore giving wrong inferences. A correction was done to remedy this. The identified model was able predict well the response of the nominal model. In future, we aim at investigating the convergence of the force coefficient bias terms.

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