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Enhancing Decision-making in Uncertain Domains through Optimized Fuzzy Logic Systems

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Abstract

Fuzzy logic helps manage human-like reasoning in system control, mainly when traditional analysis does not work due to complex control processes. Despite its usefulness, fuzzy logic faces challenges in decision-making, especially in complex business situations and when combined with expert systems. It struggles with uncertainty and relies on various beliefs and assumptions, which is limiting compared to other methods for handling uncertainty. However, fuzzy logic can improve traditional control systems by adding a layer of intelligence. This study adapts mathematical functions like the straight-line point-slope equation, the absolute value function, and the Gaussian equation to develop accurate and flexible membership functions for fuzzy logic systems. By analyzing 10,000 tasks of different sizes, we found our methods significantly more precise than traditional approaches, especially in determining degrees of membership for uncertain and complex environments. Our MATLAB research shows the potential of using varied membership functions to enhance fuzzy logic systems' accuracy and flexibility.

Keywords

fuzzy logic, fuzzy decision making, fuzzy control systems, membership function

1 Introduction

Fuzzy logic has become a cornerstone of intelligent control systems, seamlessly integrating with advanced methodologies such as neural networks and genetic algorithms. It is widely applied to interpret, analyze, and resolve the inherent ambiguities associated with complex human-centric needs and challenges. Its unique ability to handle imprecise and uncertain data through fuzzy sets and rules positions it as a powerful tool for decision-making in dynamic and intricate systems. The core processes of fuzzy logic—fuzzification, inference (driven by IF-THEN rules and an extensive knowledge base), and defuzzification—facilitate the conversion of vague inputs into precise, actionable outputs, ensuring effective and reliable system performance.

This capability supports the suitability of robust control and decision-making across various applications. Integrating fuzzy logic with adaptive systems enhances its flexibility and optimization capabilities, making it indispensable in robotics, industrial automation, and artificial intelligence (AI) domains. These fields frequently encounter inaccuracies from sensor data or other unpredictable inputs, whereas fuzzy logic systems demonstrate exceptional efficiency and reliability. The Mamdani fuzzy inference system (FLS) is widely favored among the many fuzzy logic approaches for its straightforward structure and interpretability. In electric drive systems, fuzzy logic has been employed to develop an adaptive proportional-integral (PI) speed controller for vector control of induction motors (IM) [1]. This controller uses an Adaptive Neuro-Fuzzy Inference System (ANFIS) to optimize control gains, ensuring resilience against parametric variations. Validation through MATLAB-Simulink simulations demonstrated its robust performance and suitability for enhancing electric drive reliability. In agriculture, fuzzy logic has addressed environmental uncertainty.

For instance, a wheeled robot with a microcontroller was developed for autonomous pesticide spraying, achieving high decision-making accuracy in weed identification despite challenging environmental conditions [2]. Hydraulic systems have also benefited from fuzzy logic. Researchers proposed a discrete-time switching controller strategy for pumping stations, integrating fuzzy-PD or

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fuzzy-PID controllers with PI controllers. A fuzzy supervisor facilitates controller switching, ensuring robustness, stability, and asymptotic error correction [3]. In high-performance electric motor applications, integrating Model Reference Adaptive Systems (MRAS) with fuzzy logic has significantly improved rotor speed and resistance estimation in induction motors. The study "High-Performance Control of IM using MRAS-Fuzzy Logic Observer" highlights this advanced control strategy's effectiveness in high-demand environments [4]. Further advancements include a method for simultaneously estimating rotor resistance and speed using two independent adaptive observers alongside a streamlined algorithm for optimal controller gains [5].

The adaptability of fuzzy logic extends to managing ambiguity and vagueness, which occur when boundaries and alternatives are unclear. By employing fuzzy numbers and membership functions, fuzzy logic offers a structured approach to handling uncertainty, surpassing traditional Boolean logic [6, 7]. This flexibility allows fuzzy logic systems to adapt to tasks such as navigation, object handling, and decision-making in uncertain environments, enabling human-like control in artificial intelligence (AI) systems [8, 9].

Classical information theory reduces uncertainty by increasing information; however, fuzzy logic uses membership functions to quantify degrees of association between inputs and sets within a universal discourse. These functions form the backbone of fuzzy logic systems, linking input values to degrees of membership and enabling approximate reasoning in complex scenarios [10–12]. Optimization algorithms enhance fuzzy logic by refining membership functions and improving actuator precision and control, especially in autonomous systems [13].

The development of fuzzy logic systems hinges on constructing fuzzy partitions and defining the shape and number of membership functions (MFs). These MFs are essential as they quantify the degree to which a specific input belongs to a fuzzy set. Expert knowledge is pivotal in this process, guiding the selection and parameterization of appropriate MFs. Optimizing these systems minimizes reliance on subjective trial-and-error approaches, thereby enhancing the accuracy of input/output mappings [14]. Membership functions are fundamental to representing the degree of membership for each variable, serving as critical inputs for the inference rules that drive system functionality [15].

This study introduces a mathematical model to categorize crisp inputs, representing a universe of discourse, into groups corresponding to fuzzy logic membership function sets. The model determines the membership degree for each input, quantifying its association across relevant membership functions. Optimization algorithms are incorporated to refine membership degrees for triangular, trapezoidal, and Gaussian membership functions to improve classification accuracy and precision. The model was implemented using MATLAB and tested on a dataset categorizing user tasks in a cloud computing environment. Comparative analysis with the Mamdani FLS revealed higher accuracy and precision in determining membership degrees, particularly in scenarios characterized by ambiguity and uncertainty. This advancement demonstrates significant potential for improved decision-making in complex systems like cloud computing.

This work contributes a mathematical model that enhances fuzzy logic systems by refining membership functions through optimization techniques. The proposed model surpasses conventional adaptability and decision-making precision systems, addressing gaps in current methodologies, particularly for task categorization in cloud computing environments.

The remainder of this paper is structured as follows:

- Section 1 introduces the integration of fuzzy logic and its value in various fields.
- Section 2 provides a literature review on fuzzy set theory and its applications in uncertainty management.
- Section 3 explores the background and fundamentals of fuzzy logic, focusing on the Mamdani system and membership function criteria.
- Section 4 outlines the methodology and application of the proposed mathematical model.
- Section 5 presents experimental results from 10,000 user tasks, accompanied by figures and comparative tables illustrating the model's effectiveness.
- Section 6 concludes the study, summarizing insights and contributions that enhance its impact.

2 Literature review

Fuzzy logic systems have become influential in decision-making, particularly in uncertain contexts. They offer flexibility and approximate reasoning; however, the literature points to challenges such as the complexity of fuzzy rule formulations and computational inefficiencies. These challenges underscore the need for further optimization to enhance the applicability and effectiveness of fuzzy logic across various fields.

In his seminal work on fuzzy sets, Zadeh [16] defined a fuzzy set as "a class of objects with a continuum of grades of membership", where a membership function assigns each object a grade ranging from zero to one. This work extends traditional notions such as inclusion, union, intersection, and complement to fuzzy sets, establishing various properties within this context. Notably, Zadeh [16] also proved a separation theorem for convex fuzzy sets without requiring the sets to be disjoint. Building on this foundation, researchers expanded fuzzy set theory by exploring its theoretical underpinnings and practical applications in managing uncertainty and imprecision across various domains [17].

However, these approaches often overlook the computational inefficiencies that arise when applying fuzzy logic in real-world decision-making scenarios. Recent advancements have attempted to address these inefficiencies. For instance, researchers have proposed a novel approach to healthcare decision-making that integrates fuzzy logic with machine learning [18]. This hybrid model aims to improve diagnostic accuracy and resource utilization, particularly when dealing with incomplete and uncertain data, thus addressing traditional inefficiencies.

However, it has faced criticism for relying on subjective inputs, which can introduce biases and affect the consistency of outcomes [19]. Moreover, researchers have highlighted limitations in the fuzzy linguistic approach, particularly regarding information loss during fusion processes. They propose a 2-tuple model to enhance precision and extend aggregation operators [20], although its complexity continues to pose challenges for practitioners, making implementation cumbersome [21].

Further research has discussed adaptive fuzzy systems, which show promise but frequently experience stability issues [22], leading to inconsistent decision-making in dynamic environments [23]. The Mamdani fuzzy inference model, while foundational, is often critiqued for its limited robustness under varying conditions [24]. Although recent studies have sought to enhance this model's applicability, challenges persist in managing time-sensitive decisions effectively [25].

Additionally, the researchers provided extensive insights into fuzzy systems but focused primarily on theoretical aspects [26], which hinders practical application and adoption by industry practitioners [27]. Dong et al. [28] explored fuzzy risk assessment in engineering, yet their approach does not adequately address the interactions among risk factors, potentially oversimplifying complex decision-making contexts [29]. In the context of business applications, researchers reviewed fuzzy decision-making [30], underscoring the pressing need for improved methodologies to handle severe uncertainties, particularly when data is sparse or incomplete [31]. Lastly, the integration of fuzzy logic with genetic algorithms has been explored [32]. However, this approach often struggles with computational efficiency and convergence issues, complicating its practical use in real-time decision-making scenarios [33].

In summary, the literature underscores significant gaps in the application of fuzzy logic systems within uncertain domains, highlighting the need for optimized methodologies to enhance robustness, efficiency, and applicability in decision-making processes. This study aims to address these critical gaps by focusing on accurately determining the degree of membership of input elements and their association with the most appropriate membership functions. The proposed mathematical model seeks to improve fuzzy logic systems' capacity to handle uncertainty and make accurate decisions by refining the process of selecting the best membership function and aligning it with closely related decisions.

3 Background and conceptions

3.1 Fuzzy logic system

Fuzzy logic is a form of many-valued logic that deals with approximate rather than fixed and exact reasoning. Unlike traditional binary logic, which operates with true or false values, fuzzy logic allows for a range of values between 0 and 1, which makes it particularly useful for handling the concept of partial truth. This approach is often referred to as "computing with words" because it can model the way humans think and reason with imprecise information [34, 35]. Fig. 1 depicts the architecture of a fuzzy logic system.

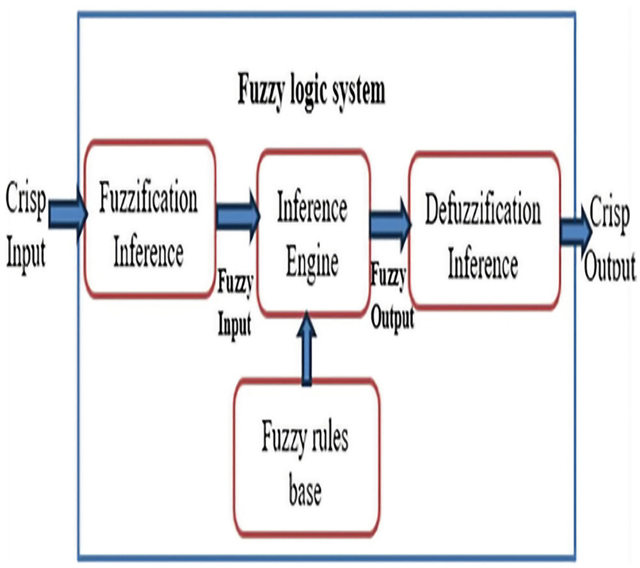

Fig. 1 Architecture of a fuzzy logic system

3.1.1 Key components of fuzzy logic systems

Crisp input

In fuzzy logic, a crisp set refers to a set in which each element has a membership value that is strictly either 0 or 1, signifying complete exclusion or inclusion. This differs from fuzzy sets, where membership values can vary continuously between 0 and 1, enabling partial membership.

In a crisp set, individuals are categorized into two distinct groups: members, who belong unequivocally to the set, and non-members, who are definitively excluded. Crisp sets adhere to classical binary logic, emphasizing a clear and absolute boundary for set membership. The indicator function for a crisp set, *A*, where elements in the set are assigned a value of 1 and those outside the set are assigned a value of 0, can be expressed as:

$$\mu_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}. \quad (1)$$

Fuzzification inference

Fuzzification inference is a process that converts input data into fuzzy sets, which are subsequently used to generate outputs based on a predefined set of rules, typically expressed in the "IF…THEN" format. This process plays a vital role in fuzzy inference systems, facilitating the transformation of uncertain or imprecise information into structured, actionable outcomes for decision-making [36].

Inference engine

An inference engine is a critical component of an expert system, employing logical rules to derive information or make decisions based on a knowledge base. It maps fuzzified inputs (obtained through the fuzzification process) to the rule base, generating fuzzified outputs for the applicable rules. The fuzzy inference engine follows a structured process comprising several key steps. Initially, it performs rule matching by identifying relevant rules from the knowledge base and comparing the input data to the conditions specified in each rule. Once the relevant rules are identified, the engine evaluates the degree of truth for each rule, determining the extent to which the input satisfies the conditions. Subsequently, it aggregates the conclusions derived from the matched rules by combining their outputs to generate a coherent decision or conclusion. This process is iterative, with the engine continuously applying rules and updating the knowledge base until a solution is achieved or no further rules apply. This systematic approach enables the fuzzy inference engine to handle complex and dynamic scenarios effectively. Inference engines are widely used in artificial intelligence applications, including diagnostic systems, recommendation systems, and other decision-making tasks [37].

Fuzzy rule base

A fuzzy rule base is a set of fuzzy rules that describe the relationship between input variables and output results in a fuzzy logic system. These rules, often derived from linguistic expressions, characterize the dynamic behavior of the system. Each rule consists of an antecedent (the "IF" part) and a consequent (the "THEN" part) based on the knowledge and expertise of a domain expert. Fuzzy rules generally follow the format:

$$\text{if} \rightarrow \text{antecedent}(\text{s}) \ \text{then consequent}(\text{s}).$$

Enabling the system to infer outputs under various input conditions. These rules are crucial for managing uncertainty and imprecision in control algorithms within systems [38, 39].

Defuzzification

Defuzzification is the final step in a fuzzy system and is responsible for converting the fuzzy output generated by the inference engine into a precise numerical value. This process translates the fuzzy set produced during inference into a specific, actionable numerical value suitable for decision-making or control applications. Standard defuzzification techniques, such as the Centre of Gravity (COG) method illustrated in Eq. (2), derive a crisp result by calculating a representative value from the combined fuzzy sets generated by multiple rules. This step ensures the system's outputs are interpretable and practical for real-world implementation [40].

$$Z = \sum_{i=1}^{n} (\mu_i \beta_i) \Big/ \sum_{i=1}^{n} \mu_i \quad (2)$$

Where:

- Z is the crisp output (defuzzified value),
- μ_i is the membership degree of the fuzzy set for the i^{th} rule,
- β_i is the representative value (often the centroid) of the output fuzzy set for the i^{th} rule,
- n is the total number of rules in the system.

3.2 Membership functions and their criteria

The membership function is a core concept in fuzzy logic. It quantifies the degree of belonging of a given input to a fuzzy set. Mapping inputs to values from 0 to 1 provides a nuanced representation of uncertainty and partial truth, enabling more flexible and accurate modelling than traditional binary logic. The function adheres to specific constraints and has a range of [0, 1]. For every $x \in X$, $\mu_A(x)$ must be unique [41]. In this study, we have selected three widely used membership functions recognized as essential in fuzzy logic systems: triangular, trapezoidal, and Gaussian.

3.2.1 Fuzzy triangular membership function

Fuzzy triangular membership function can be represented by the parameters $\{a, b, c\}$ as in Eq. (3).

$$\mu_F(x;a,b,c) = \begin{cases} 0; & x \le 0 \\ \frac{x-a}{b-a}; & a < x \le b \\ \frac{c-x}{c-b}; & b < x < c \\ 0; & x \ge c \end{cases} \tag{3}$$

3.2.2 Fuzzy trapezoidal membership function

Fuzzy trapezoidal MF is defined by the parameters $\{a, b, c, d\}$ as in Eq. (4).

$$\mu_F = \begin{cases} 0; & x \le a \\ \frac{x-a}{b-a}; & a < x < b \\ 1; & b \le x \le c \\ \frac{d-x}{d-c}; & c < x < d \\ 0; & x \ge d \end{cases} \tag{4}$$

3.2.3 Fuzzy Gaussian membership function

A fuzzy Gaussian membership function uses the Gaussian distribution to measure membership levels within a fuzzy set. It creates bell-shaped curves that manage uncertainty and vagueness. The function provides a continuous range of membership values between 0 and 1.

The general formula for a fuzzy Gaussian membership function is:

$$\mu_A(x) = e^{-\left(\frac{x-c}{\sigma}\right)^2} . \tag{5}$$

4 Methodology

The max-min compositional Mamdani fuzzy logic inference method employs a classification approach that integrates IF-THEN conditions with AND (fuzzy t-norm) and OR (fuzzy s-norm) operators to categorize and filter input values based on their compatibility with specific functions. In the max-min compositional Mamdani method the t-norm selects the minimum degree of membership among comparable values, while the s-norm selects the maximum degree of membership. In this framework, every value within the universe of discourse is associated with a distinct degree of membership function, irrespective of its membership in other functions. This attribute empowers our proposed method to gauge the membership level of a value across all relevant membership functions within the problem-solving model.

Our method provides adaptability and utility, rendering it a valuable tool for scientists and researchers when confronted with decision-making in ambiguous situations, necessitating precise and comprehensive insights. It facilitates the assessment of a value's impact on the environment in connection with the decision-making process. We have drawn upon mathematical principles embodied by Eqs. (6)–(13) and principles.

4.1 Mathematical formulation of proposed triangular and trapezoidal membership functions

The general equation for a straight line is expressed as in Eq. (6).

$$y = mx + c \tag{6}$$

Here, m represents the slope of the line, and c stands for the y-intercept. This is the most used equation form for a straight line in geometry. However, the straight-line equation can be presented in various forms, including point-slope. The equation of a straight line with a slope m that passes through a specific point (x_1, y_1) is derived using the point-slope form, which is expressed as in Eq. (7).

$$y - y_1 = m(x - x_1) \tag{7}$$

Where (x, y) denotes an arbitrary point on the line.

The absolute value parent function is represented as:

$$f(x) = |x| . \tag{8}$$

It is defined as:

$$f(x) = \begin{cases} x, & \text{if} \quad x > 0 \\ 0, & \text{if} \quad x = 0 \\ -x, & \text{if} \quad x < 0 \end{cases} . \tag{9}$$

The stretching or compressing of the absolute value function $y = |x|$ is defined by the function $y = \alpha|x|$ where α is a constant. The graph opens if $\alpha > 0$ and opens down when $\alpha < 0$. In a more general context, Eq. (10) for an absolute value function takes the form:

$$y = \alpha |x - h| + k \quad (10)$$

$$\alpha = \frac{y2 - y1}{x2 - x1}. \quad (11)$$

Here, h signifies the horizontal translation, and k represents the vertical translation [33].

4.2 Mathematical formulation of proposed Gaussian membership function

The Gaussian random variable is the most utilized and highly significant when investigating random variables. A Gaussian random variable is characterized by a probability density function (PDF) that can be expressed in a general form.

$$fX^{(x)} = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-m)^2}{2\sigma^2}\right) \quad (12)$$

$$\sigma = \sqrt{\frac{\sum (x_i - \bar{\bar{x}})^2}{n-1}} \quad (13)$$

The PDF of the Gaussian random variable has two parameters, m and σ, which have the interpretation of the mean and standard deviation (σ), respectively. The parameter σ^2 is referred to as the variance [42, 43].

4.3 Partitioning the inputs and determining membership degrees within uncertain domains

The proposed method provides a systematic approach for partitioning inputs into uncertain domains, enabling more precise and efficient determination of membership levels for various functions. This approach incorporates three distinct algorithms derived from the mathematical formulations used in this study. Algorithm 1 enhances the construction of accurate triangular membership functions, Algorithm 2 refines the formation of trapezoidal membership functions, and Algorithm 3 optimizes the generation of Gaussian membership functions.

This method's core is a robust mathematical model that simplifies the computation of membership degrees, resulting in significantly improved processing speed compared to traditional fuzzy inference systems. Conventional systems often rely on extensive rule bases and complex interdependencies, which can lead to considerable computational overhead. In contrast, the proposed algorithms directly calculate membership values, reducing complexity and streamlining the process. One key advantage of this method is its systematic categorization of input values according to specific membership functions. By effectively addressing issues related to ambiguity and uncertainty, this approach ensures a more accurate determination of membership degrees and supports enhanced decision-making outcomes.

Algorithm 1 Input partitioning and membership classification as similar work as triangular MF

Input:
V: Set of input values representing the universal discourse variables.
n: Total number of parameter values (PV) for which the degree of membership is to be calculated.
Output:
A collection of triangular Membership Functions (MF) and their corresponding degrees for each input value V.
Procedure:
1. Initialization:
$\text{Max}(V_i) \leftarrow \max(V_i)$
// Calculate the maximum value of sets V in the universe discourse.
2. Parameter value calculation:
$PV_1 \leftarrow (\text{Max}(V_i)/n)$
// Determine the first parameter value.
$PV_n \leftarrow n \times PV_1$
// Compute the last parameter value.
3. Iterate over each input value V_i in the set of parameter values:
for each $V_i \in V$:
Case 1: *if* $V_i \geq 0$ *and* $V_i \leq PV_1$
$MF_1 \leftarrow (-V_i/PV_2) + 1$
Output $\leftarrow (MF_1, \text{Degree}(V_i))$
// Compute Membership Function 1.
Output $\leftarrow (MF_2, MF_3, \ldots, MF_{m-1}, \text{Degree}(V_i))$
// Determining the degree of element in the remaining membership functions domain.
Case 2: if $V_i \geq PV_1$ *and* $V_i \leq PV_2$
$MF_1 \leftarrow (-V_i/PV_2) + 1$
Output $\leftarrow (MF_1, \text{Degree}(V_i))$
// Compute the degree of element affiliated with both domains MF_1 and subsequent it, as MF_2.
$\alpha \leftarrow (V_i - PV_2)$
// Define the alpha variable.
$MF_2 \leftarrow (-1/(PV_2 - PV_1)) \times (|\alpha| + 1)$
// Compute the degree of element affiliated with both domains MF_2 and previous it, as MF_1.
Output $\leftarrow (MF_3, MF_4, \ldots, MF_{m-1}, \text{Degree}(V_i))$
//Determining the element's degree of membership across the remaining membership functions.
Case 3: *if* $V_i \geq PV_{n-1}$ *and* $V_i \leq PV_n$
$MF_m \leftarrow (1/(PV_n - PV_{n-1})) \times (V_i - PV_{n-1})$
Output $\leftarrow (MF_m, \text{Degree}(V_i))$
// Calculate Membership Function m.
Output $\leftarrow (MF_1, MF_2, \ldots, MF_{m-1}, \text{Degree}(V_i))$
// Determining the element's degree of membership across the remaining membership functions.
4. End of Algorithm 1

4.3.1 A detailed application overview of Algorithm 1

In fuzzy logic-based systems, membership functions play a crucial role in determining the degree to which an input

Algorithm 2 Input partitioning and membership classification as similar work as trapezoidal MF

Input:
V: Set of input values representing the universal discourse variables.
n: Total number of parameter values (PV) for which the degree of membership is to be calculated.
Output:
A collection of trapezoidal Membership Functions (MF) and their corresponding degrees for each input value V.
Procedure:
1. Initialization:
$\text{Max}(V_i) \leftarrow \max(V_i)$
// Calculate the maximum value from the sets V.
2. Parameter value calculation:
$PV_1 \leftarrow (\text{Max}(V_i)/n)$
// Determine the first parameter value.
$PV_n \leftarrow n \times PV_1$
// Compute the last parameter value.
3. Iterate over each input value V_i in the set of parameter values:
for each $V_i \in V$:
Case 1: *if* $V_i \geq 0$ *and* $V_i \leq PV_1$
Degree $(V_i) \leftarrow 1$
Output $\leftarrow (MF_1, \text{Degree}\ (V_i))$
// Compute Membership Function 1.
Output $\leftarrow (MF_2, MF_3, \ldots, MF_{m-1}, \text{Degree}\ (V_i))$
// Determining the element's degree of membership across the remaining membership functions.
Case 2: *if* $V_i \geq PV_1$ *and* $V_i \leq PV_2$
$MF_1 \leftarrow ((-V_i/PV_2) - PV_1) + 1$
Output $\leftarrow (MF_1, \text{Degree}\ (V_i))$
// Compute the degree of element affiliated with both domains MF_1 and subsequent it, as MF_2.
$\alpha \leftarrow (V_i - PV_2)$
// Define the alpha variable.
$MF_2 \leftarrow ((-1/(PV_2 - PV_1)) \times (abs(\alpha))) + 1$
Output $\leftarrow (MF_2, \text{Degree}\ (V_i))$
// Compute the degree of element affiliated with both domains MF_2 and previous it, as MF_1.
Output $\leftarrow (MF_3, MF_4, \ldots, MF_{m-1}, \text{Degree}\ (V_i))$
// Determining the element's degree of membership across the remaining membership functions.
Case 3: *if* $V_i \geq PV_{n-1}$ and $V_i \leq PV_n$
Degree $(V_i) \leftarrow 1$
Output $\leftarrow (MF_m, \text{Degree}\ (V_i))$
// Calculate Membership Function m.
Output $\leftarrow (MF_1, MF_2, \ldots, MF_{m-1}, \text{Degree}\ (V_i))$
// Determining the element's degree of membership across the remaining membership functions.
4. End of Algorithm 2

value belongs to a particular fuzzy set. Algorithm 1 provides a structured approach to partitioning input values and classifying them into triangular Membership Functions (MF), ensuring an efficient representation of fuzzy variables.

Algorithm 1 takes a set of input values, representing the universal discourse variables, and determines the corresponding membership functions based on the total number of parameter values (PV). It systematically computes the maximum input value, derives the required parameter values, and iterates through each input to classify them accordingly. This process enables accurate membership function allocation, facilitating enhanced decision-making within fuzzy systems. The overview of Algorithm 1 is shown in the following listing:

Algorithm 3 Input partitioning and membership classification as similar work as Gaussian MF

Input:
V: Set of input values representing the universal discourse variables.
n: Total number of parameter values (PV) for which the degree of membership is to be calculated.
Output:
A collection of Gaussian Membership Functions (MF) and their corresponding degrees for each input value V.
Procedure:
1. Initialization:
$\text{Max}(V_i) \leftarrow \max(V_i)$
// Calculate the maximum value from the sets V.
$\sigma \leftarrow 16339$
//Define standard deviation of the Gaussian MF.
2. Parameter value calculation:
$PV_1 \leftarrow 0$
$PV_2 \leftarrow \text{MAX}(V_i)/2$
$PV_n \leftarrow \text{MAX}(V_i)$
MF_1 center $\leftarrow PV_1$
MF_2 center $\leftarrow PV_2$
MF_m center $\leftarrow PV_n$
3. Iterate over each input value V_i in the set of parameter values:
for each $V_i \in V$:
Case 1: *if* $V_i \geq 0$ *and* $V_i \leq PV_n$
$MF_1 \leftarrow \text{EXP}\ (-((V_i - PV_1)^2)/(2 \times \sigma^2))$
Output $\leftarrow (MF_1, \text{Degree}\ (V_i))$
// Compute Membership Function 1.
Output $\leftarrow (MF_2, MF_3, \ldots, MF_{m-1}, \text{Degree}\ (V_i))$
// Determining the element's degree of membership across the remaining membership functions.
Case 2: $MF_2 \leftarrow \text{EXP}\ (-((V_i - PV_2)^2)/(2 \times \sigma^2))$
Output $\leftarrow (MF_2, \text{Degree}\ (V_i))$
// Compute Membership Function 2.
Output $\leftarrow (MF_3, MF_4, \ldots, MF_{m-1}, \text{Degree}\ (V_i))$
// Determining the element's degree of membership across the remaining membership functions.
Case 3: $MF_m \leftarrow \text{EXP}\ (-((V_i - PV_m)^2)/(2 \times \sigma^2))$
Output $\leftarrow (MF_m, \text{Degree}\ (V_i))$
// Compute Membership Function m.
Output $\leftarrow (MF_1, MF_2, MF_3, \ldots, MF_{m-1}, \text{Degree}\ (V_i))$
// Determining the element's degree of membership across the remaining membership functions.
4.End of Algorithm 3

- Maximum value: 67,170
- Point1 = Maximum value/4
- Point2 = 2 × Point1
- Point3 = 3 × Point1
- Point4 = 4 × Point1
- μ_{small}: [0 0 point2]
- μ_{medium}: [point1 point2 point3]
- μ_{big}: [point2 point4 point4].
- When 0 ≤ value ≤ point1
  Consider input value is 165.

Calculate small membership function:

- μ_{smal} (165) = (−value/point2) + 1
- μ_{small} (165) = (−165/33585) + 1
- μ_{small} (165) = −0.00491 + 1
- μ_{small} (165) = 0.995087092.

The "μ_{medium} (165)*" remains* 0 *since the input value falls within the* 0 *to Point*1 *range.*

The "μ_{big} (165)*" remains* 0 *since the input value falls within the* 0 *to Point*1 *range.*

- When point1 ≤ value ≤ point2 then:
  Consider input value is 20892
  Calculate small membership function:
  - μ_{small} (20892) = (−value/point2) + 1
  - μ_{small} (20892) = −0.6218 + 1
  - μ_{small} (20892) = 0.377936579.

Calculate α:

- α = value − point2
- α = 20892 − 33585
- α = −12693.

Calculate medium membership function:

- μ_{medium} (20892) = (−1/point2 − point1) × |α| + 1
- μ_{medium} (20892) = (−1/33585 − 16792.5) × |12693| + 1
- μ_{medium} (20892) = (−1/16792.5) × 12693 + 1
- μ_{medium} (20892) = −0.7560 + 1
- μ_{medium} (20892) = 0.244126842.

The "μ_{big} (20892)*" remains* 0 *since the input value falls within the Point*1 *to Point*2 *range.*

4.3.2 A detailed application overview of Algorithm 2

In fuzzy logic systems, accurately determining the degree of membership of input values is crucial for effective decision-making. The trapezoidal Membership Function (MF) is widely used due to its ability to represent uncertainty and gradual transitions between membership categories. Algorithm 2 provides a structured approach similar to work trapezoidal for partitioning input values and classifying their membership using a mathematical approach. By leveraging its affiliated condition criterion, Algorithm 2 ensures smooth transitions and flexible representation of fuzzy sets, making it highly suitable for applications requiring precise input classification. The overview of Algorithm 2 is shown in the following listing:

- Maximum value: 67,170
- Point1 = Maximum value/5
- Point2 = 2 × Point1
- Point3 = 3 × Point1
- Point4 = 4 × Point1
- Point5 = 5 × point1
- μ_{small}: [0 0 point1 point2]
- μ_{medium}: [point1 point2 point3 point4]
- μ_{big}: [point3 point4 point4 point5].
- When 0 ≤ value ≤ point1 then:
  μ_{small} (value) = 1.
  The "μ_{medium} (value)*" remains* 0 *since the input value falls within the* 0 *to Point*1 *range.*
  The "μ_{big}(value)*" remains* 0 *since the input value falls within the* 0 *to Point*1 *range.*

- When point1 ≤ value ≤ point2
  Consider input value is 17132.
  Calculate small membership function degree:
  - μ_{small} (value) = (−value/point2) + 1
  - μ_{small} (17132) = (−17132/33585) + 1
  - μ_{small} (17132) = −0.6376 + 1
  - μ_{small} (17132) = 0.362364151.

Calculate α:

- α = value − point2
- α = 17132 − 26868
- α = −9736.

Calculate medium membership function degree:

- μ_{medium} (17132) = (−1/point2 − point1) × |α| + 1
- μ_{medium} (17132) =(−1/26868 − 13434) × |−9736| + 1
- μ_{medium} (17132) = (−1/13434) × 9736 + 1
- μ_{medium} (17132) = −0.7248 + 1
- μ_{medium} (17132) = 0.275271699.

The "μ_{big} (17132)*" remains* 0 *since the input value falls within the Point*1 *to Point*2 *range.*

4.3.3 A detailed application overview of Algorithm 3

In fuzzy logic systems, accurately defining membership degrees is crucial for robust decision-making. The Gaussian Membership Function (MF) is widely used due to its smooth, continuous representation of uncertainty, offering gradual transitions and minimizing sharp classification boundaries. Algorithm 3 systematically partitions input values, assigns Gaussian membership degrees, and optimally distributes MF centers while defining the standard deviation (σ). By ensuring precise classification, this approach enhances fuzzy systems' adaptability. It is particularly useful for uncertainty modeling applications, including cloud resource optimization, SLA evaluation, and real-time decision-making, where accurate and smooth membership transitions are essential for performance and reliability. The overview of Algorithm 3 is shown in the following listing:

- Maximum value: 67,170
- Point1 = 0
- Point2 = Maximum value/2
- Point4 = Maximum value
- Standard deviation $\sigma = 16339$
- Small center = c_{small} = point1
- μ_{small}: [σ point1]
- Medium center = c_{medium} = point2
- μ_{medium}: [σ point2]
- Big center = c_{big} = point4
- μ_{big}: [σ point4]

Consider input value is 11381.

Calculate small membership function degree:

μ_{small} (11381) = Exp(−(11381 − 0)^2/2 × (16339)^2).

Calculate the squared difference:

(11381 − 0)^2 = 129564361.

Compute:

2 × σ^2 = 2 × (16339)^2 = 533906642.

Divide and apply the exponent:

- μ_{small} (11381) = Exp(−129564361/533906642)
- μ_{small} (11381) = Exp(−0.2426)
- μ_{small} (11381) = 0.784590058.

Calculate medium membership function degree:

μ_{medium} (11381) = Exp(−(11381 − 33585)^2/2 × (16339)^2).

Calculate the squared difference:

(11381 − 33585)^2 = 494383296.

Divide and apply the exponent:

- μ_{medium} (11381) = Exp (−494383296/533906642)
- μ_{medium} = Exp(−0.9263)
- μ_{medium} = 0.397173449.

Calculate big membership function degree:

μ_{big} (11381) = Exp(−(11381 − 67170)^2/2 × (16339)^2).

Calculate the squared difference:

(11381 − 67170)^2 = 3104115681.

Divide and apply the exponent:

- μ_{big} (11381) = Exp(−3104115681/533906642)
- μ_{big} (11381) = Exp(−5.8146)
- μ_{big} (11381) = 0.002940142.

5 Experimental and results

Our proposed method has been applied to a dataset comprising over 10,000 user tasks of varying sizes, which was extracted from the Parallel Workloads Archive [44]. This archive is a comprehensive repository that contains detailed logs of job-level usage data from large-scale parallel supercomputers, clusters, and grids.

The logs encompass crucial information about the size of user tasks, which can vary significantly depending on the specific workload and system specifications. Given that each user base requests the cloud environment to perform its tasks, the data size is measured per request. For further specifics regarding user task sizes, you can explore the raw workload logs and models available at the website of the Parallel Workloads Archive [44]. These task sizes are generally random and unstructured, encompassing "small", "medium", and "big". The recorded data consists of task sizes measured in bytes, ranging from a minimum of 0 to a maximum of 67170 bytes. This wide range reflects the diverse nature of user activities. The data were obtained directly from the database in their original form without preprocessing. Fig. 2 depicts the database titles selected for the work.

The task column data, specifically identified and prepared for analytical purposes, was systematically extracted from the database to serve as the foundation for the subsequent experimentation; Fig. 3 explains the tasks before classifying.

The extracted data underwent processing and search operations using the MATLAB® (R2018b) software [45]. This program was selected due to its robust computational capabilities, enabling precise mathematical analysis, data manipulation, and visualization. The processing steps included data filtering and targeted analysis to derive meaningful insights and ensure the integrity of the results.

Task	Number of Allocated processors	Average CPU time used	Request Time	User ID	Executable Number

Fig. 2 Database addresses

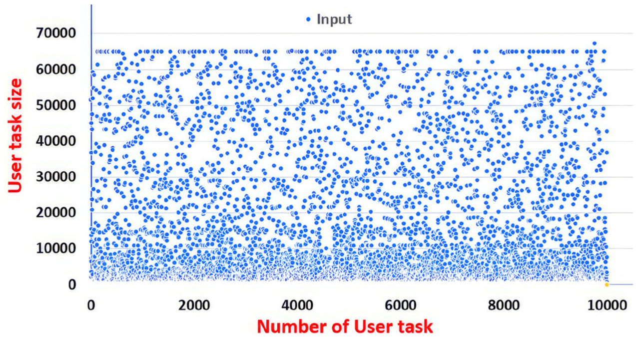

Fig. 3 User task before classify

5.1 Determine the degree of membership as the triangular membership function

In this context, tasks are classified by size using the proposed method, as outlined in Section 4. To demonstrate this, we determine the degree of membership through the triangular membership function by applying Algorithm 1 to values within the universal discourse. The implementation results are systematically illustrated to demonstrate the classification processes based on fuzzy logic principles.

Fig. 4 presents a classified single triangular membership function, showcasing the initial classification structure with a single membership function type for clarity and precision. Fig. 5 extends this analysis by depicting the classification of all nested membership functions, emphasizing the hierarchical arrangement and interactions between multiple membership functions within the system.

In contrast, Fig. 6 demonstrates the classification of the membership function achieved through the application of the Mamdani fuzzy logic system, which integrates fuzzy rules and inference mechanisms to produce comprehensive and interpretable classification results. Figs. 4–12 collectively highlight the progressive refinement of membership function classification, illustrating the effectiveness of fuzzy logic systems in managing uncertainty and delivering accurate outcomes.

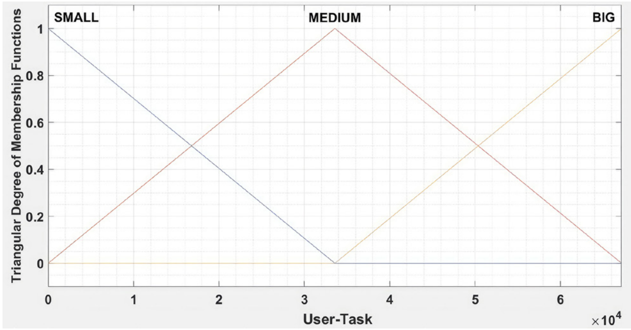

Fig. 6 Mamdani triangular MF

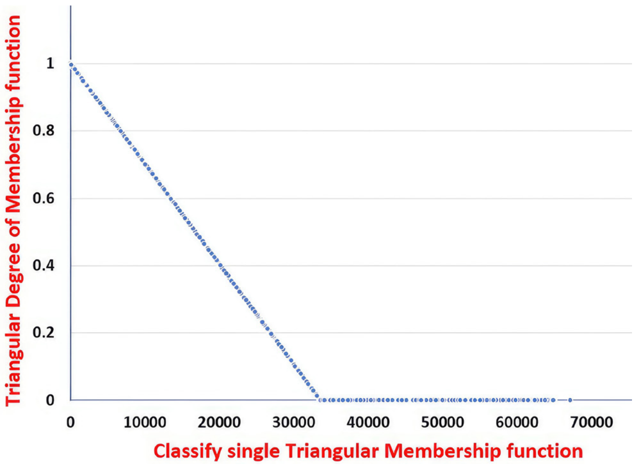

Fig. 4 Classify single triangular MF

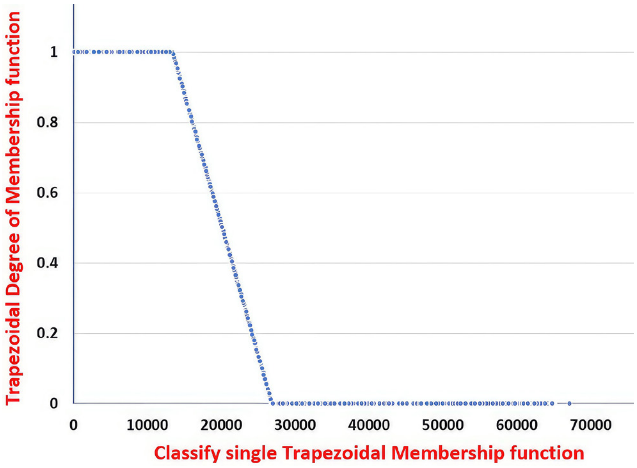

Fig. 7 Classify single trapezoidal MF

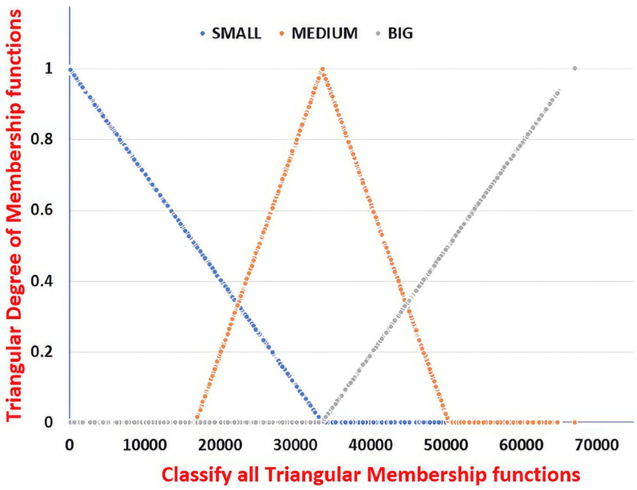

Fig. 5 Classify all triangular MF

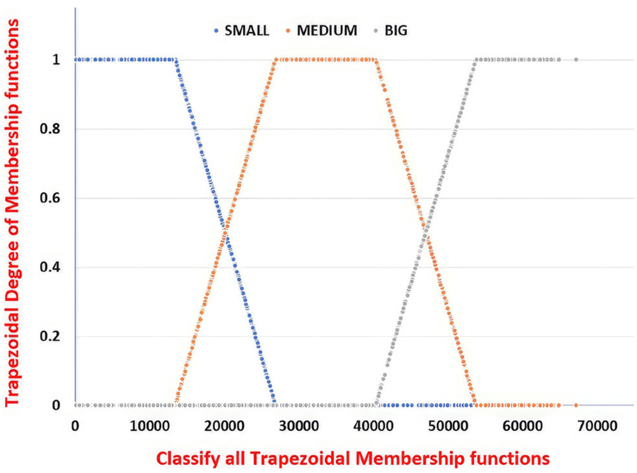

Fig. 8 Classify all trapezoidal MF

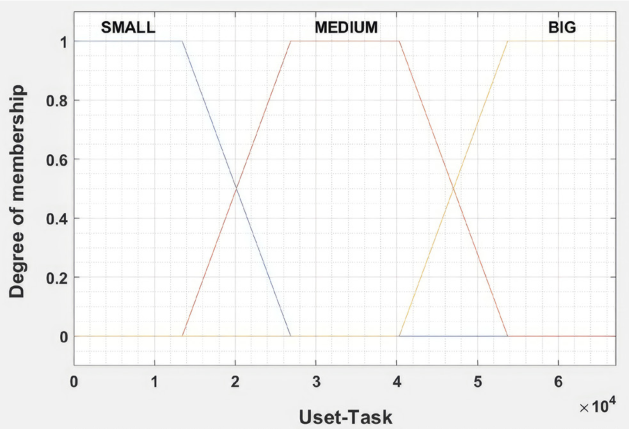

Fig. 9 Mamdani trapezoidal MF

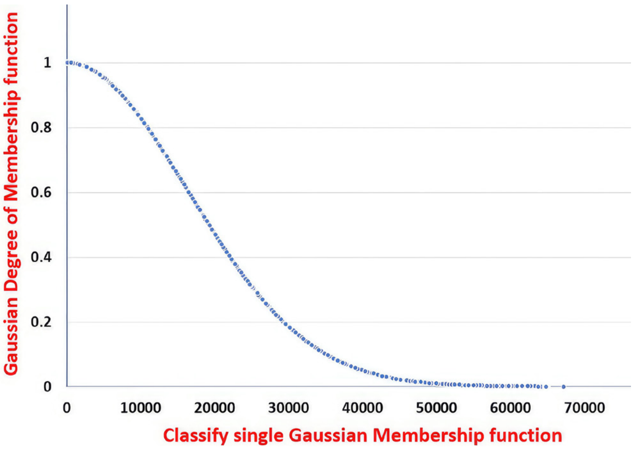

Fig. 10 Classify single Gaussian MF

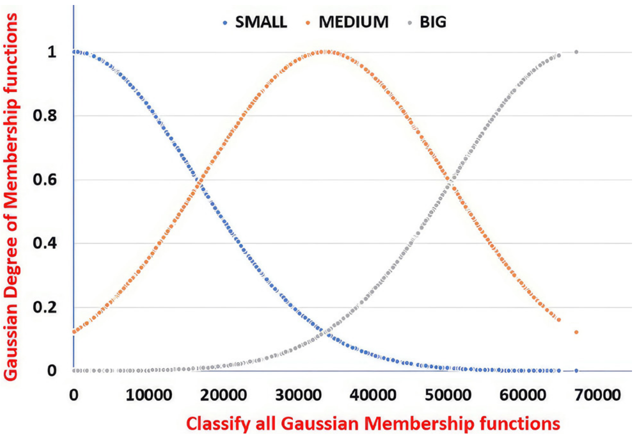

Fig. 11 Classify all Gaussian MF

5.2 Determine the degree of membership as the trapezoidal membership function

In this context, tasks are classified based on their size using the proposed method, as detailed in Section 4. To achieve this classification, we determine the degree of membership by utilizing a trapezoidal membership function, which is applied by implementing Algorithm 2 to values within the defined universal discourse. This approach ensures

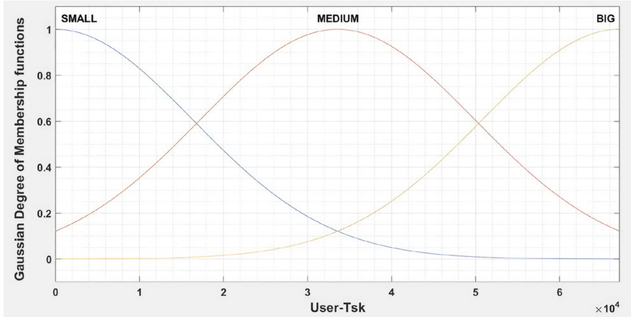

Fig. 12 Mamdani Gaussian MF

a systematic and accurate classification of tasks according to their size. Figs. 7, 8, and 9 illustrate the results of this implementation. Specifically, Figs. 7 and 8 present the outcomes of the task classification using the proposed algorithm, highlighting its ability to assign membership values effectively. In contrast, Fig. 9 depicts the corresponding Mamdani system membership functions, showcasing the fuzzy inference process and its integration into the classification framework. This detailed presentation highlights the proposed method and algorithms' role in accurately determining membership degrees, enabling precise and meaningful task classification within the system.

5.3 Determine the degree of membership as the Gaussian membership function

In this context, tasks are classified based on their size using the proposed method, as outlined in Section 4. To demonstrate the effectiveness of this approach, the degree of membership is determined using the Gaussian membership function by implementing Algorithm 3 on values within the defined universal discourse.

The Gaussian membership function, chosen for its smooth and continuous nature, ensures precise membership value assignment, facilitating accurate classification of task sizes. The results of this implementation are presented as follows. Fig. 10 illustrates the classification using a single Gaussian membership function, providing a clear and focused representation of membership values for task sizes.

Fig. 11 expands on this by presenting the classification of all Gaussian membership functions simultaneously, showcasing the system's ability to handle multiple overlapping membership functions effectively. In contrast, Fig. 12 depicts the classification results using the Mamdani fuzzy system membership functions, highlighting the integration of fuzzy inference rules with membership functions to produce comprehensive, interpretable, and consistent outcomes. These results collectively validate the robustness and flexibility of the proposed method, demonstrating the precision of Gaussian membership functions and the effectiveness in managing uncertainty and enhancing task size classification.

5.4 Analysis and selection of sample data for results

To evaluate the performance of the proposed method in comparison to the classical approach, we conducted a detailed analysis using ten representative input samples aligned with key points within the universe of discourse. This approach assessed the algorithm's ability to determine the membership degree for each input, whether it falls within a single membership function or between nested functions. Membership degrees were calculated for all applicable functions, while functions outside the input's range in the universe of discourse were assigned a degree of zero. The results were compared to those obtained using the classical Mamdani fuzzy logic system, which served as a benchmark.

Table 1 presents the outcomes of the proposed method, while Table 2 summarizes the results from the Traditional Method, providing a direct comparison of their performance. To further validate the proposed method, a large-scale analysis was conducted on 10,000 inputs representing diverse task sizes. The results of this evaluation are depicted in Figs. 13–18, illustrating the application of various membership functions and their integration with Mamdani's fuzzy logic system. Fig. 13 demonstrates the application of Algorithm 1 with the triangular membership function, showcasing its precision and effectiveness in task size classification. Fig. 14 extends this by integrating the triangular membership function with Mamdani's fuzzy logic system, highlighting the enhanced classification results achieved through fuzzy inference rules. Similarly, Fig. 15 illustrates the application of Algorithm 2 with the trapezoidal membership function, emphasizing its ability to manage overlapping task ranges effectively. Fig. 16 complements this by applying Mamdani's fuzzy logic system to the trapezoidal membership function, producing refined and interpretable outcomes.

Moving to Algorithm 3, Fig. 17 depicts the implementation of the Gaussian membership function, highlighting its smooth and continuous classification capabilities. Finally, Fig. 18 presents the results of integrating Mamdani's fuzzy logic system with the Gaussian membership function, illustrating its effectiveness in combining fuzzy inference with the Gaussian approach for comprehensive and accurate classification. These results collectively demonstrate the robustness and versatility of the proposed method compared to the classical Mamdani approach. The proposed method showcases superior accuracy and adaptability, particularly in managing uncertainty and achieving precise task size classification across triangular, trapezoidal, and Gaussian membership functions. This comprehensive evaluation highlights the significant advancements introduced by the proposed method in fuzzy logic-based classification systems.

Table 1 Results of the proposed method applied to selected samples

Samples of degree of triangular membership function			
value	small	medium	big
0	1	0	0
16823	0.499091856	0.001816287	0
17129	0.489980646	0.020038708	0
17361	0.4830728	0.033854399	0
17579	0.476581807	0.046836385	0
25978	0.226499926	0.547000149	0
26931	0.198124163	0.603751675	0
28842	0.141223761	0.717552479	0
31475	0.062825666	0.874348668	0
33565	0.000595504	0.998808992	0
Samples of degree of trapezoidal membership function			
value	small	medium	big
20162	0.499181182	0.500818818	0
21582	0.393479232	0.606520768	0
23875	0.222792914	0.777207086	0
25331	0.114411195	0.885588805	0
26846	0.001637636	0.998362364	0
46120	0	0.566919756	0.433080244
45451	0	0.616718773	0.383281227
44329	0	0.700238202	0.299761798
42852	0	0.810183117	0.189816883
40336	0	0.997469108	0.002530892
Samples of degree of Gaussian membership function			
value	small	medium	big
0	1	0.120934543	0.000213895
1	0.999999998	0.120949757	0.000213949
10090	0.826402652	0.355634634	0.002238294
32026	0.146469985	0.995458374	0.098946015
49791	0.009627715	0.611475933	0.567984183
54045	0.004209592	0.456574063	0.724241188
61138	0.000911417	0.241274197	0.934125619
64852	0.000379417	0.160259114	0.989987311
65069	0.000359903	0.156223736	0.991766863
67170	0.000213895	0.120934543	1

Table 2 Results of the traditional method applied to selected samples

value	small	medium	big
Samples of degree of triangular membership function			
0	1	0	0
16823	0.499076941, 400667	0.001846117, 1986660315	0
17129	0.489965459, 74273464	0.020069080, 51453073	0
17361	0.483057408, 28966176	0.033885183, 420676514	0
17579	0.476566222, 01048116	0.046867555, 979037634	0
25978	0.226476893, 75893282	0.547046212, 4821344	0
26931	0.198100285, 8504	0.603799428, 2991901	0
28842	0.141198189, 61410197	0.717603620, 7717961	0
31475	0.062797760, 83849452	0.87440447, 83230109	0
33565	0.000565745, 5931395903	0.998868508, 8137208	0
Samples of degree of trapezoidal membership function			
20162	0.499181182, 07533124	0.500818817, 9246688	0
21582	0.393479231, 7999107	0.606520768, 2000894	0
23875	0.222792913, 50305197	0.777207086, 496948	0

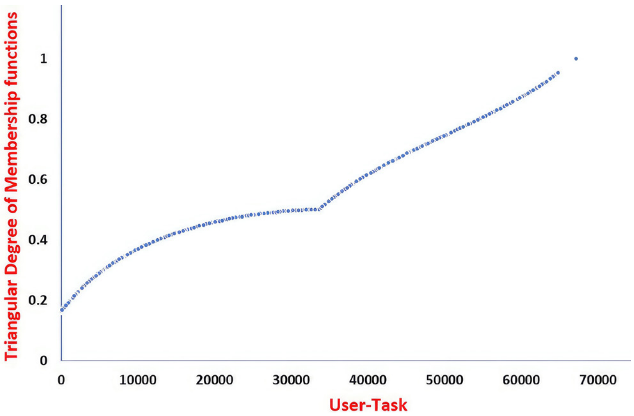

Fig. 13 Result of Algorithm 1

Table 2 Results of the traditional method applied to selected samples (continued)

value	small	medium	big
Samples of degree of trapezoidal membership function			
25331	0.114411195, 47417002	0.885588804, 52583	0
26846	0.001637635, 849337502	0.998362364, 1506625	0
46120	0	0.783443757, 9096255	0.216556242, 09037444
45451	0	0.808345120, 2263083	0.19165487, 977369167
44329	0	0.850107943, 1251396	0.149892056, 87486043
42852	0	0.905084493, 4117472	0.094915506, 5882528
40336	0	0.998734459, 9121566	0.001265540, 0878433707
Samples of degree of Gaussian membership function			
0	1	0.122	0.0002
1	1	0.122	0.0002
10090	0.8418	0.7201	0.0053
32026	0.2931	0.996	0.1097
49791	0.0304	0.5364	0.7211
54045	0.0124	0.2917	0.8431
61138	0.0028	0.1097	0.9959
64852	0.0011	0.0566	0.9881
65069	0.0010	0.0532	0.9926
67170	0.0002	0.1218	1

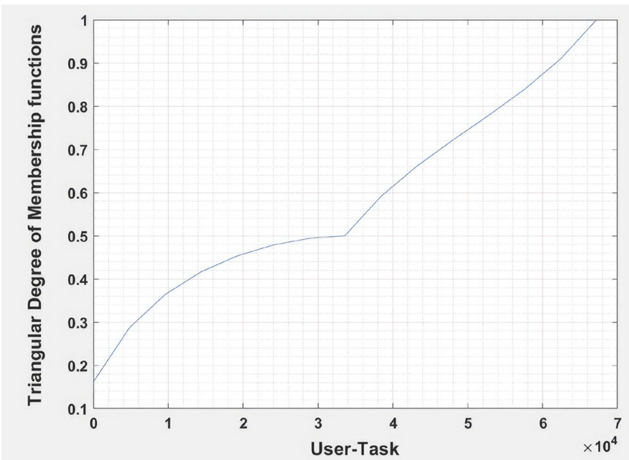

Fig. 14 Mamdani's performance versus Algorithm 1

6 Conclusions

This paper uses a coherent mathematical model to present a new speculative execution method for estimating membership degrees within different membership functions defined in a universal context. This method generates triangular, trapezoidal, and Gaussian membership functions, each consisting of three membership levels: small, medium, and large. Our approach employs three algorithms to classify and organize ambiguous input data into structured categories, each associated with a specific membership function to address particular problem contexts.

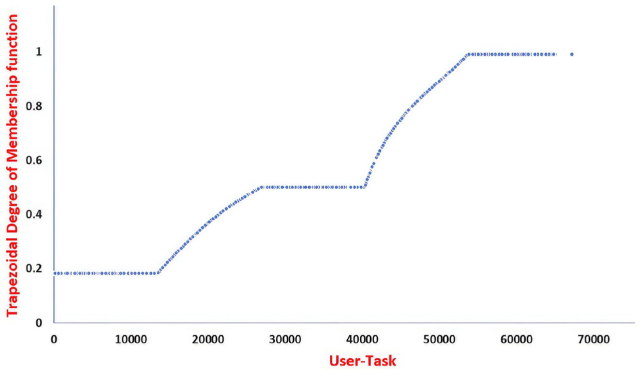

Fig. 15 Result of Algorithm 2

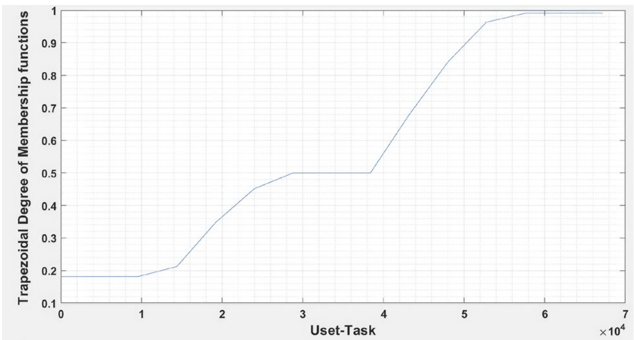

Fig. 16 Mamdani's performance versus Algorithm 2

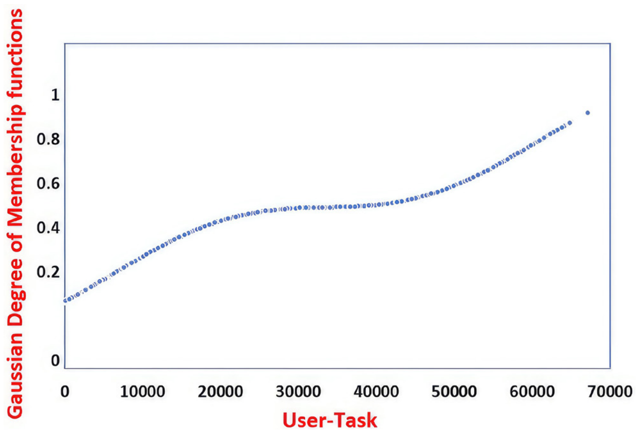

Fig. 17 Result of Algorithm 3

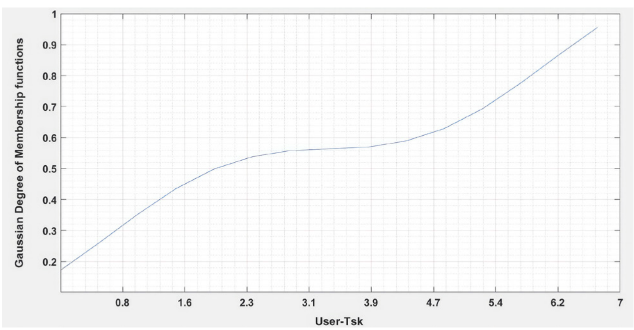

Fig. 18 Mamdani's performance versus Algorithm 3

The performance of this model is about the same as the Mamdani fuzzy inference system when it comes to showing the membership degree of each element across different membership functions. In decision-making contexts, we apply the linear point-slope equation and the absolute value function to determine the membership degree of a given value within either a single membership function or two nested membership functions, depending on their shared universal context.

The Gaussian membership function is also used with the Gaussian random variable equation to sort and organize the membership degrees for each input value in the universal discourse. To evaluate the effectiveness of our approach, we conducted experiments using a variety of task sizes from the Parallel Workloads Archive [44], a comprehensive repository of job-level data. The experiments focused on two primary objectives:

1. Designing membership functions with symmetric shapes but using asymmetric techniques, compared to the Mamdani fuzzy inference system, across the three predominant types of membership functions: triangular, trapezoidal, and Gaussian.
2. Enhancing precision in organizing and classifying membership degrees for each input value.

Our experiments yielded the following insights:

1. Our method made it much more accurate to get membership degrees in the universal context. It gave results between 0 and 1 for both unique and nested membership functions that were linked to a given input value.
2. Mathematical models employing flexible equations demonstrated substantial potential for improving membership degree accuracy. Compared to previous constrained approaches, the fuzzy logic decision-making system exhibited higher precision in determining membership degrees. These findings highlight the importance of precise membership degree proportions, validating the proposed approach. Future studies may explore scalability, real-time applications, advanced optimization algorithms, dynamic membership function models (e.g., sigmoid, polynomial), and the Fuzzy Logic Toolbox's membership functions, extending the methodology to domains like healthcare, finance, and smart cities.

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