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RESEARCH ARTICLE

Continuous PWM Strategies of Multi-Phase Inverter-Fed AC Drives

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Abstract

Different continuous PWM strategies of multi-phase inverter-fed ac motors are investigated from the point of view of ac motor harmonic losses. It is shown that for phase number over seven - in comparison with the space vector or with the harmonic injection methods - the natural or regular sinusoidal PWM of multi-phase system gives the best solution both from the point of view of the simplicity of realization and the value of harmonic losses. However, the stator harmonic current losses are slightly higher while the rotor harmonic current losses considerably lower than those of the three-phase system. The theoretically derived expressions for stator and rotor loss (for infinite value of switching frequency (MATHCAD program) are checked with resulting of direct computation for finite value of switching frequency (MATLAB program) and by simulation.

Keywords

Multi-phase system · AC motor drives · PWM · harmonic losses

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1 Introduction

The structure of multi-phase inverter-fed ac drive is presented in Fig. 1. In this case the m phase number of motor and inverter is higher than 3. The multi-phase inverter-motor system has better fault tolerance, lower motor torque ripples, smaller power rating of converter semiconductors and lower phase current for a given voltage rating [1–4] .

These advantages may be very useful in some areas of industry. Therefore a wider application of multi-phase system is expected.

The paper is the improvement and a continuation of [5], which deals with the investigation of stator and rotor different harmonic losses as well as by computing the stator currents for phase numbers from 3 to 9. Three PWM methods are investigated: sinusoidal PWM, space vector PWM and harmonic injection PWM.

The present paper expands these investigations up to phase number 13 (but virtually to infinitively large number of phases), presents the theoretically derived expressions for stator and rotor loss components and further on deals with stator and rotor harmonic current formation. This paper and [5] don't use the space vector (Park-vector) interpretation.

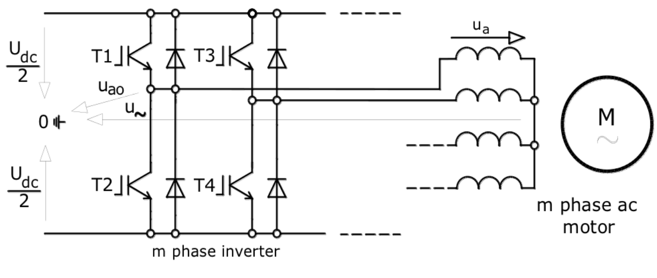


Fig. 1. Structure of multi-phase inverter-fed ac drive

Several papers were published dedicated to very similar problems of the multiphase inverter-fed drives [6–10]. These papers also concentrate on estimation of motor losses and on currents of these drives by the determination of HDF (harmonic distortion factors. These papers use space vectors approach which leads to quite different method of computation.

In [6] the 5-phase system is analyzed for three different PWM methods. The final theoretical results are completely the same as in [5] and they are supported by experimental measurements. The [7] uses polygon approach which requires the determination of different line voltages and fluxes of multi-phase system similarly to [5]. The final HDF value is the same as in [6]. The discussion method mainly for delta connected motors can be applied. The [8, 9] investigates 5-phase system and compares the application of two large and two middle voltage vectors per switching period with the use of four large vectors for this. However these PWM solutions are not better than a carrier based PWM.

For 3-phase system the space vector investigation gives very sensible advantages but for the multi-phase system this investigation seems more difficult. Really, the number of possible phase-end connections to dc bars are equal to 2^m. Thus the number of possible space vectors e.g. $m = 7$ for are 128. Two from them are zero vectors while the other ones create $126/14 = 9$ different vector systems. Each system consists of 14 vectors with equal amplitudes and phase shift $\pi/7$ between two neighboring vectors. For 9-phase the number of possible vectors are already 512. At the same time for 3-phase system the space vectors and motor torque ripples can be measured and observed on oscilloscopes while for multi-phase systems it is at least today impossible.

Therefore probably it is better to investigate the phase and line values (voltages, current, flux etc.). In latter case the basic point will be the investigation of $u_{a0}(t)$ phase end voltage function for different modulation techniques.

2 State of the Art

The motor harmonic losses can be characterized by the generalized loss-factor that for the pure inductive load is given as follows [11]:

$$G_f = f_t^2\, \Delta\Psi^2, \tag{1}$$

where

$$\Delta\Psi^2 = \sum_{\nu>1}^{\infty} \Psi_\nu^2, \quad \Psi_\nu = U_\nu/\nu$$

and U_ν, Ψ_ν are the harmonic voltage and the harmonic flux of the order ν, $\Delta\Psi$ is the RMS value of the harmonic flux, f_t is the switching frequency of one transistor (GTO) which for continuous PWM equals to carrier (sampling) frequency. All the values in (1) re in p.u. system. In the above equations it is assumed that the motor rated voltage amplitude is equal to the maximal inverter voltage $U_{1\,\max} = 2U_{dc}/\pi$, where U_{dc} is the inverter input dc voltage and the rated frequency f_r corresponds $U_{1\,\max}$ voltage.

The computed general loss-factor can be converted to harmonic distortion factor by multiplication by a constant

$$\mathrm{HDF} = G_f 160/\pi^4.$$

Between the generalized loss-factor and HDF the difference is only in the normalization factor: for HDF the normalization factor is a fictive flux $U_{dc}/(8 f_c)$ while in (1) the flux is related to the rated one. In latter case the current computation doesn't require a multiplication by fictive flux.

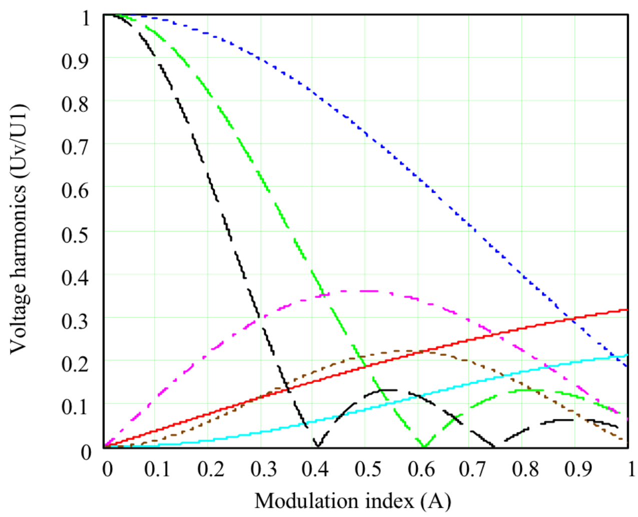


Fig. 2. The most important voltage harmonics of multi-phase inverters, ($f_c/f_1 \pm 2$: red; $2f_c/f_1 \pm 1$: blue; $2f_c/f_1 \pm 3$: lightblue; $3f_c/f_1 \pm 2$: purple; $4f_c/f_1 \pm 1$: green; $4f_c/f_1 \pm 3$: brown; $5f_c/f_1 \pm 1$: black)

For an odd number of phases the modulation index $A = U_1/(U_{dc}/2)$ at the beginning of overmodulation region is given by

$$A_m = 1/\cos(\pi/(2m)) \tag{2}$$

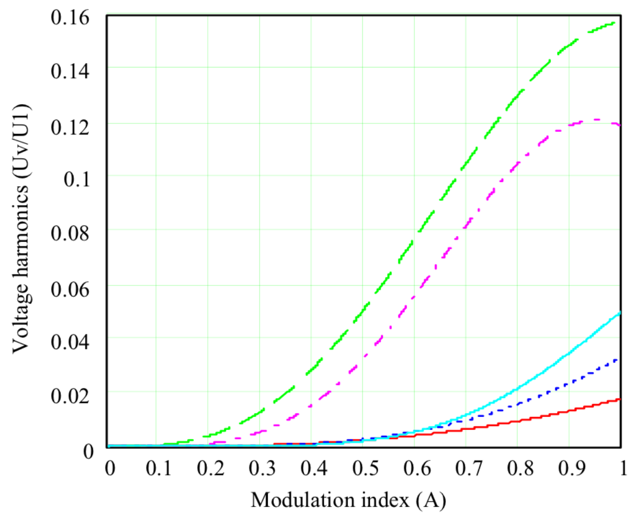


Fig. 3. Voltage harmonics which can be zero sequence harmonics, ($f_c/f_1 \pm 4$: red; $2f_c/f_1 \pm 5$: blue; $3f_c/f_1 \pm 4$: green; $4f_c/f_1 \pm 5$: purple; $4f_c/f_1 \pm 7$: lightblue)

For any even number of phases the overmodulation region starts from $U_1 = U_{dc}/2 = \pi U_{1\,\max}/4 = 0.785 U_{1\,\max}$ where $A = 1$. Therefore the span of overmodulation region is considerably bigger than for the three-phase system [12].

It is well known that the voltage control region of sinusoidal PWM is limited by $A \leq 1$ while in case of space vector PWM or that with m order harmonic injection it can be widen to (2). However (2) for $m = 5$ gives $A_m = 1.051$ and for $m = 7$ gives

$A_m = 1.026$ therefore the sinusoidal modulation gives virtually the same voltage control region as other PWMs when $m \geq 7$. It should be noted that for odd phase number the discontinuous PWM also extends the linear control region up to (2) however for $m \geq 7$ there are no sensible advantages in comparison with the sinusoidal PWM [10, 13].

3 Voltage Harmonics of Multi-Phase Inverters

For carrier-based natural or regular sinusoidal PWM the order of the harmonics can be given as follows [14]:

$$\nu = \pm K \frac{f_c}{f_1} \pm n > 0 \tag{3}$$

where K and n are positive integers, f_c is the frequency of triangular carrier wave and f_1 is the frequency of sinusoidal reference wave (motor fundamental frequency).

With (3) the motor voltage harmonics of natural sinusoidal PWM (related to $U_1 = AU_{dc}/2$ fundamental one) are expressed as follows [14, 15]:

$$\frac{U_\nu}{U_1} = \frac{2}{\pi AK} J_n\left(KA\frac{\pi}{2}\right)\left[(-1)^k - (-1)^n\right],\ \varphi_\nu = n\varphi \tag{4}$$

where $J_n\left(KA\frac{\pi}{2}\right)$ is the first-kind Bessel function of n order, φ is the phase angle between sinusoidal reference and triangular carrier and φ_ν is the phase angle of ν order harmonic. Very similar expression is valid for regular PWM but for $f_{c_1}/f_1 > 20$ the harmonic amplitudes of a sinusoidal and a regular PWMs virtually are the same. Using (4) the most important voltage harmonics of multi-phase inverters ($m > 3$) are presented in Fig. 2. These harmonics exist for any phase number higher than 3. In Fig. 3. those voltage harmonics are given which can have sensible value but become zero sequence harmonics if $n = m$. Really, e.g. for four-phase system the harmonics of order $f_c/f_1 \pm 4$, $3f_c/f_1 \pm 4$ etc. according to (4) have between neighboring phases the phase shift $n(2\pi/4) = 4(2\pi/4) = 2\pi$. Hence these harmonics are zero sequence harmonics. Naturally the harmonics of order m, $3m$, $5m$ ... and f_c/f_1, $3f_c/f_1$, $5f_c/f_1$ ... in any case are zero sequence harmonics.

From Figs. 2-3 it can be concluded that with very good approximation for $m > 3$ and especially for $m > 5$ the multi-phase system consists of the same order of important voltage harmonics with equal amplitudes. Therefore for a given carrier frequency the generalized loss-factor only in small degree depends on the phase number. The 3-phase system produces lower value of harmonic losses since in this case $2f_c/f_1 \pm 3$ and $4f_c/f_1 \pm 3$ order harmonics (Fig. 2) are zero sequence harmonics. This is also valid e.g. for 6-phase (or 9-phase) inverter when it is controlled as two (or three) 3-phase ones and the motor star points of two (or three) 3-phase system are isolated [16]. It should be noted that this paper deals only with those even or odd phase numbers which have only one star point.

4 The Generalized Loss-Factor

The natural or regular continuous PWM can be realized by three different PWM methods:

1 sinusoidal PWM

2 space vector PWM and

3 PWM with injection of m order harmonic.

4.1 Sinusoidal Modulation

The generalized loss-factor determination can be carried out by two different methods. In the first one the determination of phase harmonic flux-time function is needed. It requires the star point voltage computation for the cancellation of zero sequence harmonics or use space vector approach. In this case the switching points of all the phases must be computed. The second and possibly simpler method is the computation of line harmonic flux-time function since the line voltage does not contain the zero sequence harmonics.

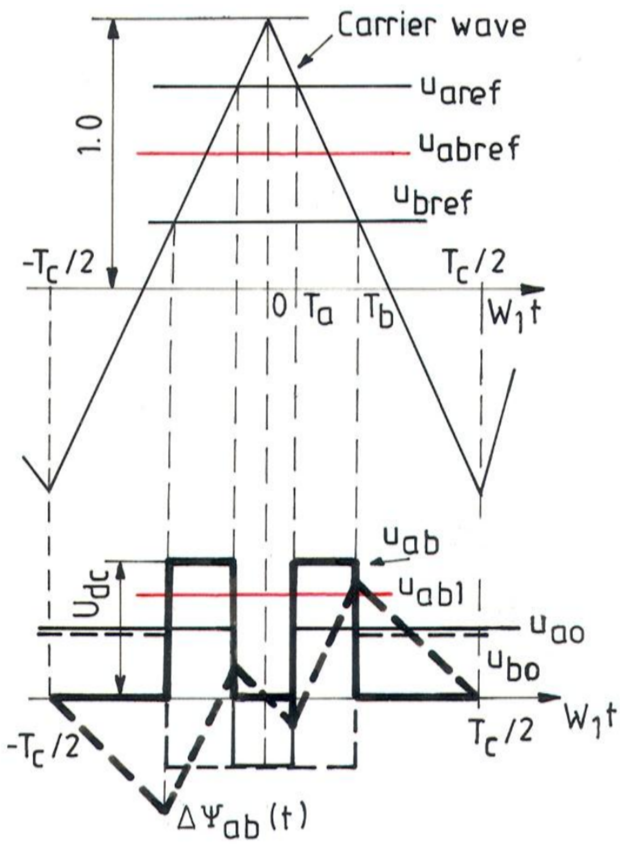


Fig. 4. Modulation process for continuous PWM technique

The line voltage expressions are:

$$U_{abref} = U_{aref} - U_{bref};$$
$$U_{ab} = U_{a0} - U_{b0}$$

while the fundamental line voltage is equal

$$U_{ab1} = U_{a1} - U_{b1}$$

In Fig. 4 the reference voltages for natural sinusoidal modulation are:

$$\begin{aligned} u_{aref} &= A\cos W_1 t \\ u_{bref} &= A\cos(W_1 t - 2\pi/m) \end{aligned} \tag{5}$$

The line voltage $u_a - u_b$ equals to U_{dc} or to zero while u_{a0} and u_{b0} are $U_{dc}/2$ or $-U_{dc}/2$. For a carrier period the square of RMS value of the line harmonic flux $\Delta\Psi_{ab}^2$ is determined according to Fig. 4 where the switching (intersection) times T_a and T_b are

$$T_a = \frac{T_c}{4}[1 - A\cos(W_1 t)]$$

$$T_b = \frac{T_c}{4}[1 - A\cos(W_1 t - 2\pi/m)]$$

After that the square of RMS value of the line harmonic flux on a fundamental period can be computed [11]. Nevertheless, when m>3 several different line voltage amplitudes exist. Really, e.g. the fundamental line voltage between a and b phases and as well between a and c ones respectively are:

$$\begin{aligned} u_{ab1} &= U\cos W_1 t - U\cos(W_1 t - 2\pi/m) \\ &= -2U\sin(\pi/m)\sin(W_1 t - \pi/m), \\ u_{ac1} &= -2U\sin(2\pi/m)\sin(W_1 t - 2\pi/m) \end{aligned} \tag{6}$$

Thus in the first case the amplitude is $2U\sin(\pi/m)$ while in the second one it is $2U\sin(2\pi/m)$.

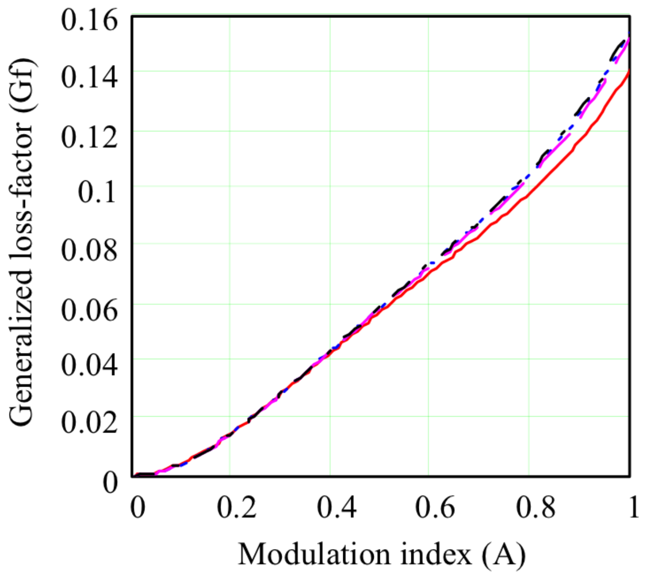


Fig. 5. Generalized loss-factor of multi-phase inverters, sinusoidal PWM, (m = 3: red, m = 5: blue, m = 7: green, m = 9: black)

On other side for $m > 3$ the line voltage amplitudes of certain harmonic orders are not proportional to fundamental ones, e.g. if $\nu = f_c/f_1 + 2$ than according (3) and (4) $n = 2$ and line voltage is:

$$\begin{aligned} u_{ab2} &= u_{a2} - u_{b2} \\ &= U_2\cos(2\pi f_c t + 2\pi f_1 t) - U_2\cos(2\pi f_c t + 2\pi f_1 t - n2\pi/m) \\ &= -2U_2\sin(2\pi/m)\sin(2\pi f_c t + 2\pi f_1 t - 2\pi/m) \end{aligned} \tag{7}$$

Therefore this voltage harmonics will produce by $\sin(2\pi/m)/\sin(\pi/m)$ higher harmonic line current than one is in phase current. These phenomena require the computation of all line fluxes and generalized loss-factors which belong to the different line voltage amplitudes. This means that in case e.g. $m = 4$ or $m = 5$ the $\Delta\Psi_{ab}$ and $\Delta\Psi_{ac}$ while for $m = 8$ and $m = 9$ the $\Delta\Psi_{ab}, \Delta\Psi_{ac}, \Delta\Psi_{ad}$ and $\Delta\Psi_{ae}$ must be computed. After that the generalized loss-factor is determined as it is shown in Appendix.

For sinusoidal PWM the computation results are presented in Fig. 5 where the generalized loss-factor is drawn for $m = 3, 4, 5$ and 9 phase number while the derived equation are listed in Table 1. For any phase number the generalized loss-factor expression is as follows:

$$G_f = \frac{\pi^4 A^2}{192}[1 - yA + (0.75 + z)A^2] \tag{8}$$

where y depends only on phase number while z depends on phase number and the method of simulation.

As it was concluded from voltage spectra investigation the generalized loss-factor scarcely depends on the number of phases and for $m > 7$ virtually depends only on modulation index. Really, e.g. for $A = 1$ and number of phase 3, 4, 5, 7 and 9 it is respectively 0.1420, 0.1527, 0.1552, 0.1564 and 0.1566 (Table 2).

From Table 1 and (8) it is seen that the generalized loss-factor in case of sinusoidal modulation depends on number of phase due to the change of the middle term in the bracket. The coefficient $y = 1.4418$ for $m = 7$ and $y = 1.4413$ for $m = 9$, thus there is virtually no difference in loss-factor values for $m \geq 7$.

4.2 Space Vector Modulation

For odd phase number the space vector modulation is realized similarly to the 3-phase system. Every π/m the reference waves in (7) are modified in so manner that the maximum of each reference wave must be at $\pm\pi/(2m)$ distance from the fundamental component maximum of the reference wave (Fig. 6). This is realized when the reference waves are modified by an average value of the biggest and the smallest values of reference waves, e.g. in $-\pi/m \leq t \leq 0$ by

$$0.5A\left[\cos(t) + \cos\left(t - \frac{m-1}{m}\pi\right)\right] = A\cos\left(t - \frac{m-1}{2m}\pi\right)\sin\frac{\pi}{2m}$$

or by $\sin(\pi/(2m))(-1)^{(m+1)/2}$ part of those phase reference wave whose absolute value is the lowest one in this time interval. Thus e.g. for $m = 5$ in $-\pi \leq t \leq 0$ (Fig. 6) the reference of a phase will be:

$$u_{aref} = A\left[\cos(t) + \sin\left(\frac{\pi}{10}\right)(-1)^3\cos\left(t - \frac{2\pi}{5}\right)\right] \tag{9}$$

The derived generalized loss-factor expressions are presented in Fig. 7 and Table 1. It is seen that in the expression of the general loss-factor only the last z term in brackets depends on the modulation method. It is interesting that for three-phase system

Tab. 1. Generalized Loss-Factor of Multi-Phase System, Sinusoidal PWM, Space Vector PWM and m Order Harmonic Injection PWM

Phase number	$G_f = \frac{\pi^4 A^2}{192}[(1 - yA + (0.75 + z)A^2]$		
	Sin. PWM (z=0), SPV, m harm. injection	SPV	m harmonic injection
m	y,	z	z
3	$8/(\pi\sqrt{3}) = 1.470$	−0.0902	$1.5A_3^2 - 0.75A_3$; $(-8.33 \cdot 10^{-2})$
4	$(4\sqrt{2} + 8)/(3\pi) = 1.449$	–	–
5	$2\sqrt{2 + 2/\sqrt{5}}/(3\pi) = 1.444$	0.0092	$1.5A_9^2$; $(5.53 \cdot 10^{-3})$
6	$4(5 + 3\sqrt{3})/(9\pi) = 1.4425$	–	–
7	1.4418	0.0027	$1.5A_7^2$; $(1.52 \cdot 10^{-3})$
8	1.4414	–	–
9	1.4413	0.0009	$1.5A_9^2$; $(5.58 \cdot 10^{-4})$
10	1.4412	–	–
11	1.4411	0.0004	$1.5A_{11}^2$; $(2.51 \cdot 10^{-4})$
12	1.44110	–	–
13	1.44107	0.0002	$1.5A_{13}^2$; $(1.29 \cdot 10^{-4})$
∞	1.4409		

Tab. 2. Generalized Loss-Factor for $A = 1$, Space Vector PWM and m Order Harmonic Injection PWM

Phase	Generalized Loss-Factor, G_f		
Number m	Sinusoidal PWM	Space vector PWM	m harmonic injection PWM
3	0.1420	0.0962	0.0997
4	0.1527	–	–
5	0.1552	0.1599	0.1581
6	0.1560	–	–
7	0.1564	0.1577	0.1571
8	0.1565	–	–
9	0.1566	0.1571	0.1569
10	0.1567	–	–
11	0.1567	0.1569	0.1568
12	0.1567	–	–
13	0.1567	0.1568	0.1568
∞	0.1568	0.1568	0.1568

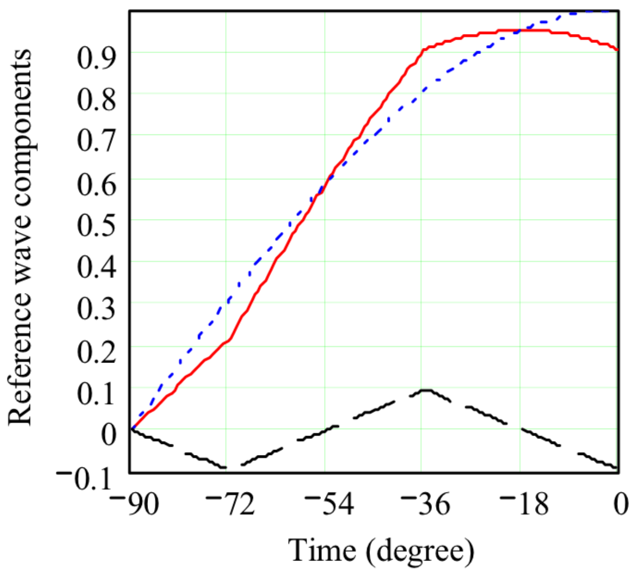


Fig. 6. Reference wave and its components, m=5, SPV PWM, (reference wave: red line; its fundamental: blue line; added wave: black line)

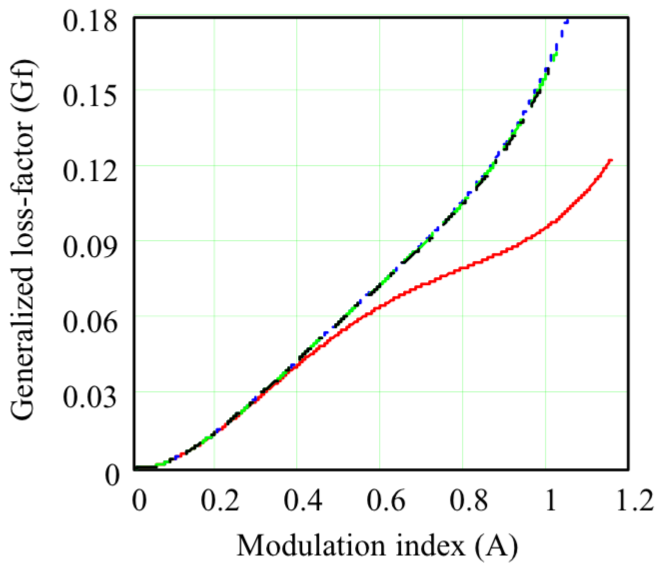


Fig. 7. Generalized loss-factor of multi-phase inverters, space vector PWM, (m = 3: red, m = 5: blue, m = 7: green, m = 9: black)

the generalized loss-factor is considerably lower than in case of the sinusoidal PWM while for multi-phase system the space vector modulation produces slightly higher loss-factor (Tables 1 and 2) than the sinusoidal PWM since z> 0 in (8). It is explained mainly by low value of losses from harmonic currents $m \pm 2$ and $m \pm 4$ order in case of 3-phase space vector modulation (and third order harmonic injection). For $m \geq 3$ however these losses in case of sinusoidal and other two PWMs are practically the same. Especially for $m \geq 7$ the voltage spectra of vector modulation (and third order harmonic injection) are the same as in case of sinusoidal modulation. Therefore (4) and Figs 2- 3 are also valid for multi-phase system.

The loss-factor of space vector modulation decreases with enlarging the phase number over $m = 5$. It is clear that with increasing phase number the difference between these two modulation techniques tends to zero.

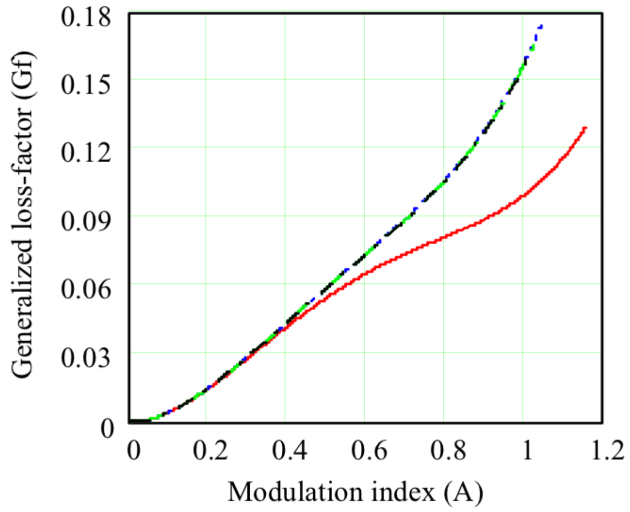


Fig. 9. Generalized loss-factor of multi-phase inverters, m order harmonic injection, (m = 3: red, m = 5: blue, m = 7: green, m = 9: black)

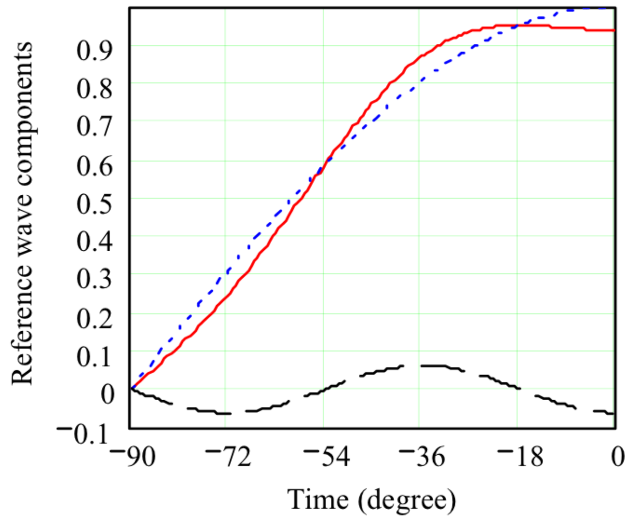


Fig. 8. Reference wave and its components, m=5, m order harmonic injection PWM, (reference wave: red line; its fundamental: blue line; added wave: black line)

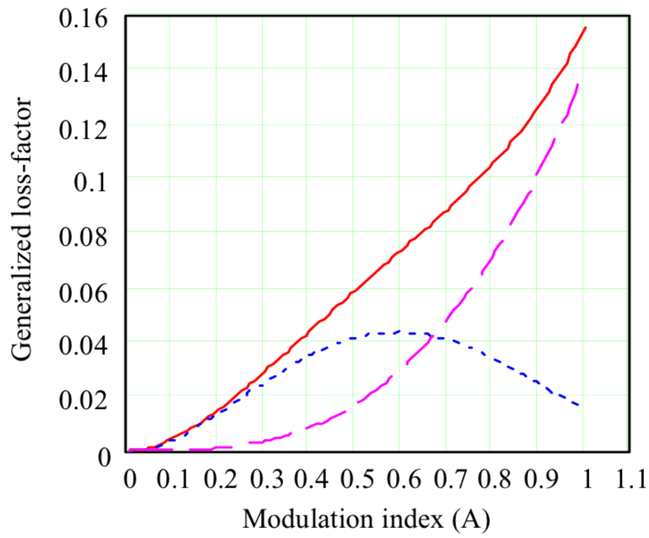


Fig. 10. Generalized loss-factor (red solid line) its components (G_1: blue line, G_2: purple line, m = 5

4.3 The *m* Order Harmonic Injection

In this case the reference waves of (5) are modified by injection of m order harmonic with amplitude AA_m. In case of *a* phase the following expression is valid:

$$u_{aref} = A\cos(W_1 t) - AA_m \cos(mW_1 t) \qquad (10)$$

The reference wave is presented in Fig. 8 together with its fundamental and added waves.

The generalized loss-factor computation is carried out similarly to the previous computations. The results are shown in Fig. 9 and Tables 1- 2. In Table 1 z values for the widest linear region where $A_m = \sin(\pi/(2m))/m$ are also shown (last column, in brackets). It is seen that there is no sensible difference between space vector and this PWM: the loss-factor values are virtually the same and only slightly decrease with the increasing phase number approaching to the values of sinusoidal PWM. From Table 1 it is seen that z> 0 therefore the harmonic injection method does not decrease the motor losses below the losses of sinusoidal PWM.

5 The Generalized Loss-Factor Component

According to (2) and as well assuming a sinusoidal flux distribution in the air gap only the current harmonics with

$$n = \pm k_1 m \pm 1, \; (k_1 = 0, 1, 2, 3, \ldots) \qquad (11)$$

are able to produce the counter rotor currents and torque. Among these the $Kf_c/f_1 \pm 1$ harmonic orders ($k_1 = 0$) are independent of phase number while when $k_1 \neq 0$ the $Kf_c/f_1 \pm k_1 m \pm 1$ harmonic orders depend on phase number. Thus, e.g. for $m = 5$ and $k_1 = 1$ harmonics $Kf_c/f_1 \pm 4$ and $Kf_c/f_1 \pm 6$, while for $m = 7$ harmonics $Kf_c/f_1 \pm 6$ and $Kf_c/f_1 \pm 8$ produce rotor currents and torque. These harmonics produce the first component of generalized loss-factor (G_1) thus determine the generalized loss-factor of the rotor.

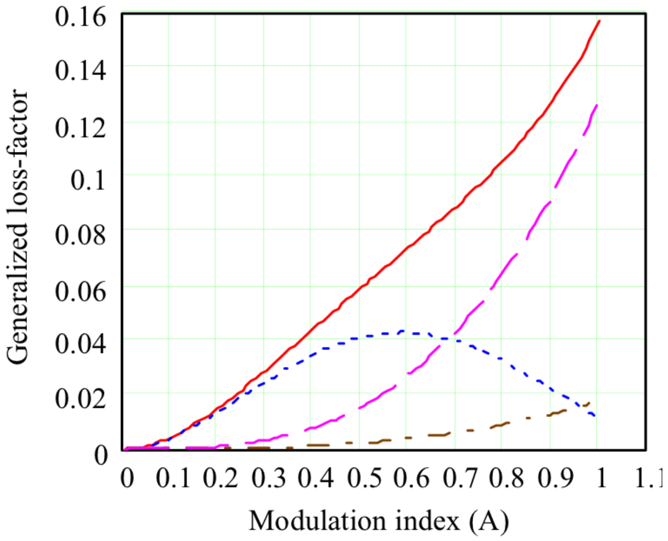


Fig. 11. Generalized loss-factor (red line) its components (G_1: blue line, G_2: purple line, G_3: brown line), $m = 7$

The other current harmonics give zero value of the resultant MMF and contain further components of the loss-factor presented in Figs. 10-13 for five, seven, nine and eleven phase numbers. Number of these components is one for $m = 4$ and 5 and two for $m = 6$ and 7 etc. They are restricted only by L_σ stator leakage inductance and stator resistance. The stator leakage inductance is lower than the L' transient one therefore many stator current harmonics will be higher than in case of 3-phase system. On the other side the RMS value of the rotor harmonic current will be considerably lower. The L' and L_σ are computed from induction motor equivalent circuit [8].

The square of RMS. value of the stator harmonic current is determined from (1), since e.g. $\Delta\Psi_1^2 = (\Delta i\, L')^2$ and $\Delta\Psi_2^2 = (\Delta i\, L_\sigma)^2$:

$$\Delta i^2 = \frac{G_1}{(f_c L')^2} + \frac{G_2}{(f_c L_\sigma)^2} + \frac{G_3}{(f_c L_\sigma)^2} + \ldots \qquad (12)$$

For a rough approximation the RMS value of stator and rotor harmonic currents is computed with $L_\sigma =\simeq L' = 0.10 =$ const neglecting the influence of resistance on harmonic current but taking into account the strong skin effect in the rotor by which the rotor leakage reactance tends to zero. These approximations give only qualitative information of performances but considerably simplify the computations.

For continuous PWM the carrier frequency is equal to the switching frequency of one transistor. When the rated frequency is 50Hz and $f_t = 4127$ Hz the switching frequency in p.u. will be $f_c = 4127/50 = 82.54$. With these data, e .g. for $m = 5$ and $A = 1$ the loss-factor $G_f = 0.1552$ produces

$$\Delta i^2 = \frac{0.1552}{(82.54 \cdot 0.1)^2} = 0.0023$$

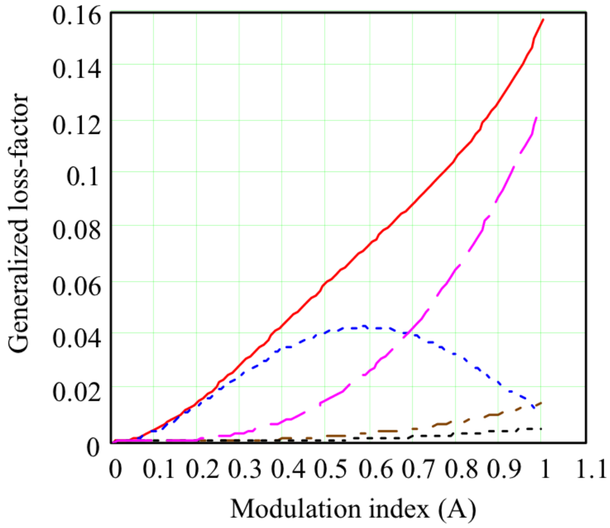


Fig. 12. Generalized loss-factor (red solid line) its components (G_1: blue line, G_2: purple line, G_3: brown line, G_4: black line) $m = 9$

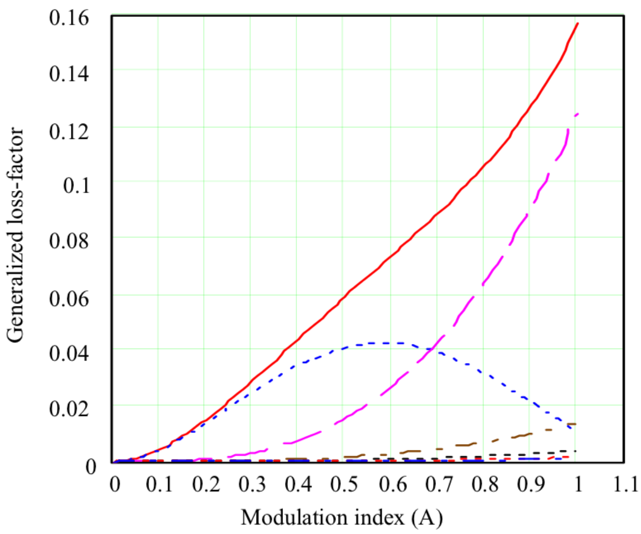


Fig. 13. Generalized loss-factor (red solid line) its components (G_1: blue line, G_2: purple line, G_3: brown line, G_4: black line, G_5: orange line), m=11

Tab. 3. The Generalized Loss-Factor Components for $A = 1$, (G_1 : sinusoidal PWM)

m	G_1	G_2	G_3	G_4	G_5	G_6
4	0.0266	0.1260	–	–	–	–
5	0.0152	0.1400	–	–	–	–
6	0.0124	0.1295	0.0141	–	–	–
7	0.0115	0.1270	0.0179	–	–	–
8	0.0111	0.1261	0.0155	0.0038	–	–
9	0.0109	0.1258	0.0147	0.0054	–	–
10	0.0107	0.1256	0.0138	0.0045	0.0015	–
11	0.0107	0.1256	0.0142	0.0038	0.0021	–
12	0.0106	0.1254	0.0141	0.0040	0.0018	0.0007
13	0.0106	0.1254	0.0140	0.0040	0.0017	0.0011

Thus the stator harmonic losses are equal to 0.23% of stator rated coil losses when the skin effect in stator can be neglected. The computation of all components of the generalized loss-factor is given in Appendix. According to this e.g. for $m = 5$ the rotor loss-factor will be only $G_1 = 0.0152$ thus $\Delta i_r^2 = 0.022\%$. The rotor skin effect may increase the rotor harmonic losses considerably e.g. up to 0.35-0.40% of the rotor rated coil losses.

It is worth mentioning that for a given number of phases only the first component of the generalized loss-factor depends on PWM method. The next ones (the number of these equations increases with the number of phases) are valid for any type of modulation (sinusoidal, space vector, the m order harmonic injection or – disregarding $(m-1)^2/m^2$ multiplier – different types of discontinuous PWM) [7, 13]. This result is a bit unexpected since the voltage harmonic amplitudes of different PWM methods are not the same.

The computation of general loss-factor components is given in the Appendix. The magnitudes of these components for $A = 1$ are given in Table 3. Except of G_1 (which in Table 3 is given for a sinusoidal PWM) these components are proportional to A^3 therefore they can easily be computed for any A.

The above computation is valid when the motor rated voltage is equal to $2U_{dc}/\pi$. In general case in p.u. system the flux must be divided by the square of the rated flux. When the rated voltage is $U_{rated} = k \cdot U_{dc}$ the rated flux is $k \cdot U_{dc}/W_{1rat}$ therefore according to (8) in p.u.:

$$G_f = \frac{A^2\pi^2}{48(k^\star)^2}[1 - yA + (0.75 + z)A^2]$$

It should be noted that the for $f_c/f_1 \to \infty$ derived by MATCAD program generalized loss-factor values were checked by direct computation of reference and triangular wave intersection points with follow determination of voltage harmonics and generalized loss-factors (MATLAB program). For all investigated PWM strategies the deviation of two computation methods was inside 1% when $f_c/f_1 \geq 75$. The results are equally valid for natural and regular PWMs, as well for general loss-factor components.

6 Simulation Results

The stator current is determined by simulation of the system for $A = 1$ and for all three modulation strategies. The no-load fundamental current was taken as $i_1 = 0.5\sin(W_1 t)$ p.u. (current is related to the amplitude of the rated current). This fundamental current was summed up with harmonic components computed from voltage harmonics with reactance $L_\sigma = L' = 0.1$. The results for $m = 3, 5$ and $m = 7$ are drawn in Fig. 14, where the time is given in radian ($t' = W_1 t$) and for all cases $f_c/f_1 = 105$ was selected. The upper row in Fig. 14 shows the phase current in case of sinusoidal PWM while the next row belongs to the space vector PWM. In last row of Fig. 14 the phase currents are drawn for m harmonic injection PWM. The value of the loss-factor and its first component are everywhere given.

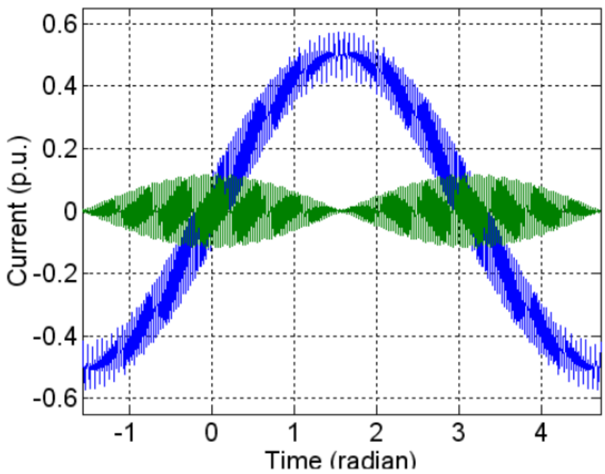
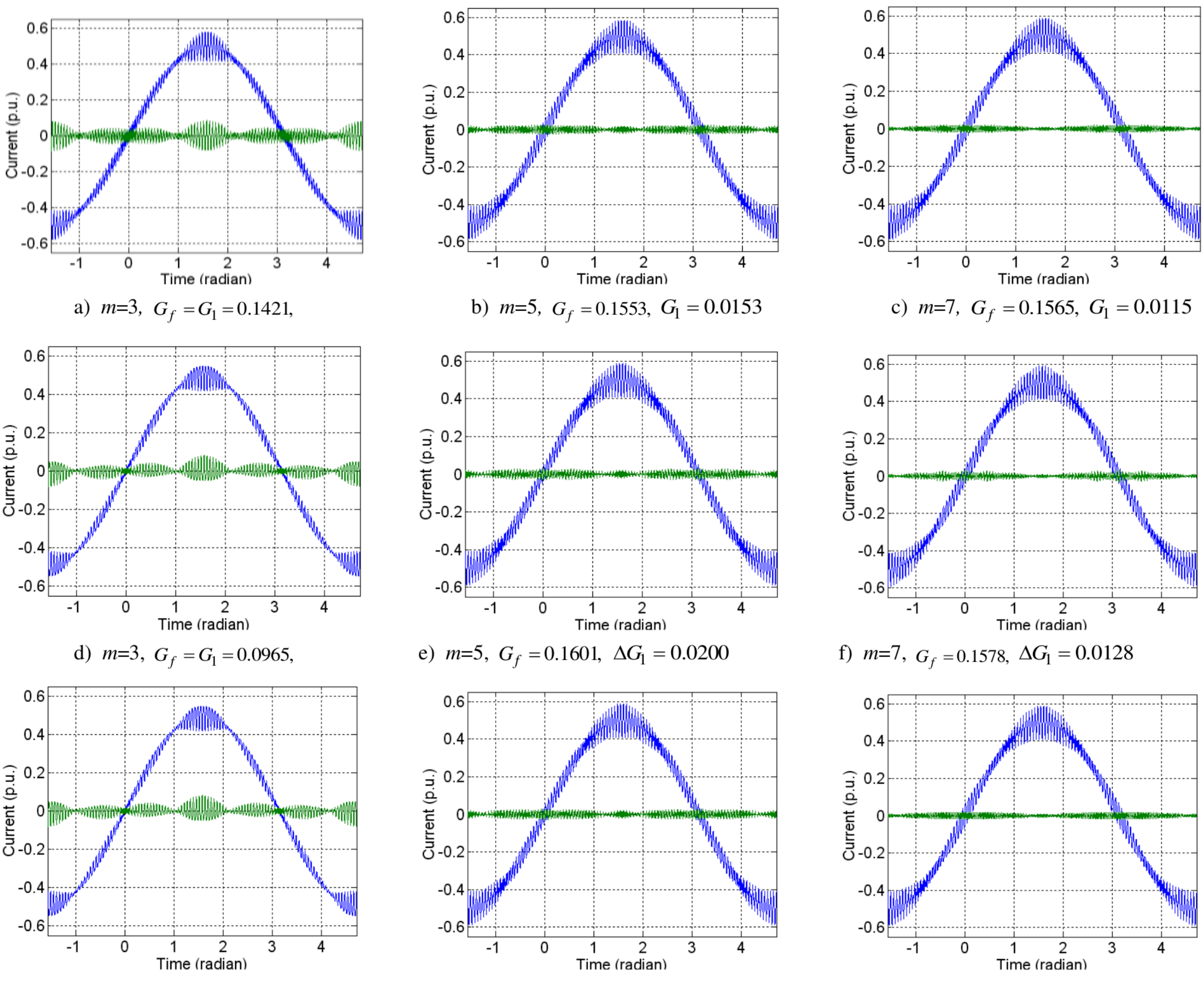


Fig. 15. Stator (blue line) and rotor (green line) currents no-load condition, $A = 0.2$, $m = 7$, $G_f = 0.0150$; $G_1 = 0.0139$

It is seen that motor harmonic losses of multi-phase systems really scarcely depend on number of phases. The space vector PWM produces considerable decrease of these losses only for 3-phase case while in other cases there is really slight increase of harmonic losses. The comparison of the generalized loss-factor value and its first component in Figs. 14 and in Table 1 shows that the deviation is lower than 0.3% when $f_c/f_1 = 105$. For 50Hz rated frequency the case of $A = 1$ responds to $f_1 = 39.3$



a) m=3, $G_f = G_1 = 0.1421$,

b) m=5, $G_f = 0.1553$, $G_1 = 0.0153$

c) m=7, $G_f = 0.1565$, $G_1 = 0.0115$

d) m=3, $G_f = G_1 = 0.0965$,

e) m=5, $G_f = 0.1601$, $\Delta G_1 = 0.0200$

f) m=7, $G_f = 0.1578$, $\Delta G_1 = 0.0128$

g) m=3, $G_f = G_1 = 0.0998$,

h) m=5, $G_f = 0.1582$, $G_1 = 0.0182$

i) m=7, $G_f = 0.1573$, $G_1 = 0.0123$

Fig. 14. Motor stator (blue line) and rotor (green line) currents, no-load condition, A = 1; a), b) and c): sinusoidal PWM; d), e) and f) : space vector PWM; g), h) and i) : m-harmonic injection.

Hz i.e. the carrier frequency of the simulation is as follows:

$$f_c = 105 \cdot 39.3 = 4127 \text{ Hz}$$

At the same time from Fig. 14 it is clearly visible that the rotor currents for number of phases more than three are considerably lower. This is valid for modulation index $0.3 \le A \le A_{max}$. For $A < 0.2$ the generalized loss-factors G_f and G_1 have about the same values as it is seen on Fig. 15.

In Fig. 16 the phase current is given for $A = 1$ and the even number of phases: m=4, 6 and 8. In this case $f_c/f_1 = 96$ ($f_c =$ 3773Hz) and natural sinusoidal PWM is selected. It should be noted that in case of even m the harmonics $Kf_c/f_1 \pm n$ with even n don't produce rotor currents and torque. In case of m=6 e.g. only harmonics with $n = \pm 1$, $n = \pm 5$ and $n = \pm 7$ etc. are able to produce rotor currents and torque while in case of $m = 8$ only harmonics with $n = \pm 1$, $n = 7$ and $n = 9$ etc.. Therefore the rotor loss-factor e.g. for $m = 6$ will be only $G_1 = 0.0125$ and the rotor current harmonics produce

$$\Delta i_r^2 = G_1/(3773/50)^2/(L')^2 = 2.2 \cdot 10^{-4}.$$

Since the ratio of $I_{\text{rated}}/I_{\text{r rated}} \simeq 0.9$ therefore the rotor current harmonics increase the rotor losses by $0.022/(0.81) = 0.027\%$ from rated rotor losses. The skin effect increases the rotor resistance by 8-12 times thus the rotor harmonic losses will be about 0.30%.

In all cases the motor harmonic losses virtually do not depend on the method of modulation. This is supported by Fig. 17 where the motor voltage spectra are given for $A = 0.5$ and $A = 1.0$ in case 7-phase and $f_c/f_1 = 21$. It is seen that the important voltage harmonics have the same amplitudes in case of sinusoidal and SPV PWMs.

7 Conclusions

Three different natural and regular PWM methods of multi-phase inverter-fed ac motors are investigated from point of view of harmonic losses of ac motor. These methods are: a) sinusoidal PWM, b) space vector PWM and c) PWM with injection of m order harmonic.

For the infinitively high carrier frequency theoretically derived expressions of the generalized loss-factor and its components are given. It is shown that only the first component of the loss-factor depends on the method of modulation while the other ones are independent of that and depend only on the phase number. For the phase number over three the natural or regular sinusoidal PWM of multi-phase system produces the best solution from point of view of the simplicity of realization and the value of harmonic losses. However, these stator losses are slightly higher while the rotor harmonic losses of multi-phase machine are considerably lower as compared to 3-phase machines. For number of phases $m > 3$ the motor voltage spectra of the space vector or the m order harmonic injection PWMs are virtually the same as in the case of sinusoidal PWM.

The load of inverter-fed induction motor is usually limited by rotor overheating. Therefore the decrease of rotor current harmonics of multi-phase motors is an important advantage of multi-phase drives over the three phase drives.

The theoretical results are checked with the results of direct computation of generalized loss-factors for finite value of switching frequency (MATLAB program) and with those obtained by simulation. The difference between theoretically computed generalized loss-factor and its components and the results of direct computations was inside 1% when the carrier frequency is 75 times higher than the fundamental one.

Appendix

The generalized loss-factor computation is shown in an example of seven-phase system. In this case there are three fundamental line voltages with different amplitudes (6) :

$$\begin{aligned} u_{ab1} &= -2U \sin(\pi/7)\sin(W_1 t - \pi/7), \\ u_{ac1} &= -2U \sin(2\pi/7)\sin(W_1 t - 2\pi/7), \\ u_{ad1} &= -2U \sin(3\pi/7)\sin(W_1 t - 3\pi/7) \end{aligned} \tag{13}$$

The possible harmonic orders are $14k \pm 1, 14k \pm 3$ and $14k \pm 5$ ($k = 1, 2, 3, \ldots$). From (13) it is seen that line voltage can be $2\sin(\pi/7), 2\sin(2\pi/7)$ or $2\sin(3\pi/7)$ higher than phase voltage. According to (4) those harmonics which have order of $14k \pm 1$ change in the same rate as fundamental ones while e.g. the line harmonic amplitudes of $1k \pm 3$ order change with $2\sin(3\pi/7), 2\sin(6\pi/7)$ or $2\sin(9\pi/7)$ rate respectively. Therefore, in (14) each square of phase fluxes must be multiplied by the square of ratio the each line voltages to phase ones. Denoting the square of phase harmonic flux components from $14k \pm 1, 14k \pm 5$ and $14k \pm 3$ harmonics by $\Delta\Psi_{\mathrm{I}}^2, \Delta\Psi_{\mathrm{II}}^2$ and $\Delta\Psi_{\mathrm{III}}^2$ respectively, the next expressions are valid:

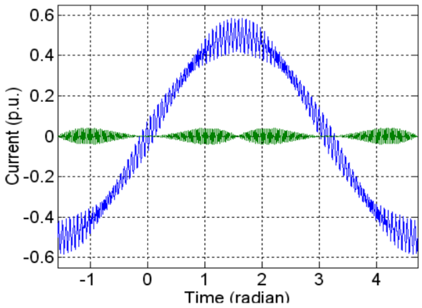
$$\begin{aligned} \Delta\Psi_{ab}^2 &= 4\sin^2\left(\frac{\pi}{7}\right)\Delta\Psi_{\mathrm{I}}^2 + 4\sin^2\left(\frac{2\pi}{7}\right)\Delta\Psi_{\mathrm{II}}^2 + 4\sin^2\left(\frac{3\pi}{7}\right)\Delta\Psi_{\mathrm{III}}^2 \\ \Delta\Psi_{ac}^2 &= 4\sin^2\left(\frac{2\pi}{7}\right)\Delta\Psi_{\mathrm{I}}^2 + 4\sin^2\left(\frac{4\pi}{7}\right)\Delta\Psi_{\mathrm{II}}^2 + 4\sin^2\left(\frac{6\pi}{7}\right)\Delta\Psi_{\mathrm{III}}^2 \\ \Delta\Psi_{ad}^2 &= 4\sin^2\left(\frac{3\pi}{7}\right)\Delta\Psi_{\mathrm{I}}^2 + 4\sin^2\left(\frac{6\pi}{7}\right)\Delta\Psi_{\mathrm{III}}^2 + 4\sin^2\left(\frac{9\pi}{7}\right)\Delta\Psi_{\mathrm{III}}^2 \end{aligned} \tag{14}$$

Computation according to Fig. 4 for sinusoidal PWM gives:

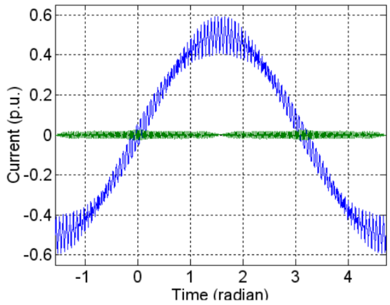
$$\begin{aligned} \Psi_{ab}^2 &= \frac{U_{dc}^2}{\pi^4 192}(1 - 0.7366A + 0.75A^2) \\ \Psi_{ac}^2 &= \frac{U_{dc}^2}{\pi^4 192}(1 - 1.3773A + 0.75A^2) \\ \Psi_{ad}^2 &= \frac{U_{dc}^2}{\pi^4 192}(1 - 1.6551A + 0.75A^2) \end{aligned} \tag{15}$$

From (14) and (15) the square of harmonic fluxes are as follows:

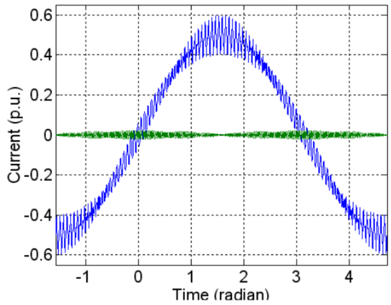
$$\begin{aligned} \Delta\Psi_{\mathrm{I}}^2 &= -0.1341\Delta\Psi_{ab}^2 + 0.1554\Delta\Psi_{ac}^2 + 0.9787\Delta\Psi_{ad}^2 \\ \Delta\Psi_{\mathrm{II}}^2 &= 0.1938\Delta\Psi_{ab}^2 - 0.4356\Delta\Psi_{ac}^2 + 0.2417\Delta\Psi_{ad}^2 \\ \Delta\Psi_{\mathrm{III}}^2 &= 0.0479\Delta\Psi_{ab}^2 + 0.6294\Delta\Psi_{ac}^2 + 0.6722\Delta\Psi_{ad}^2 \end{aligned} \tag{16}$$



a) m=4, $G_f = 0.1529$, $G_1 = 0.0266$

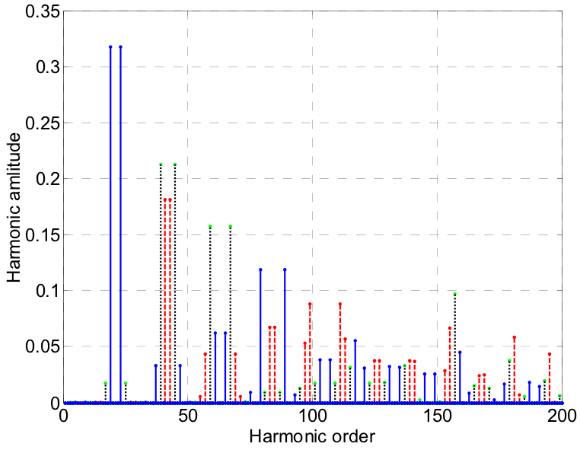


b) m=6, $G_f = 0.1562$, $G_1 = 0.0125$

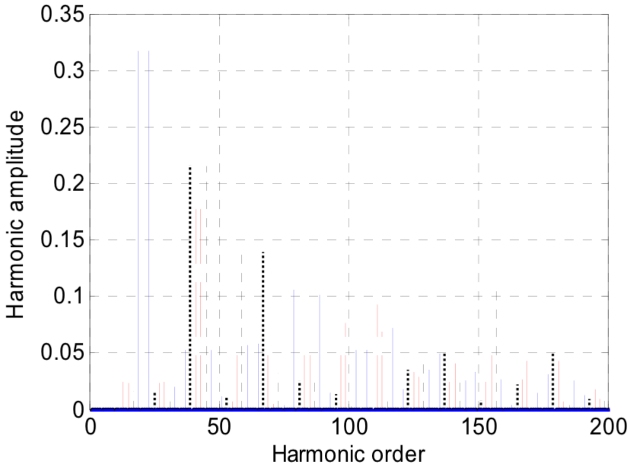


c) m=8, $G_f = 0.1567$, $G_1 = 0.0111$

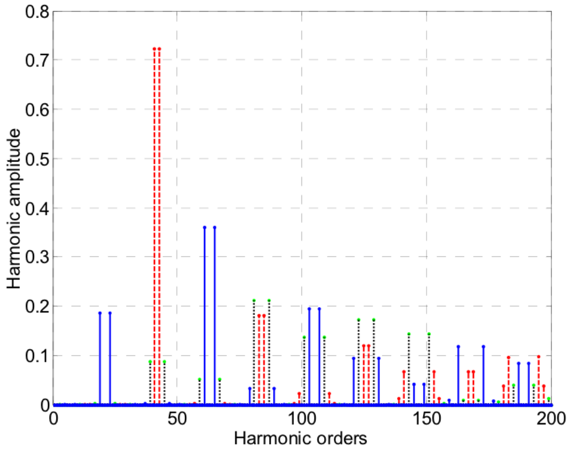
Fig. 16. Motor phase current (blue line) and rotor current (green line), no-load condition, sinusoidal PWM, A=1



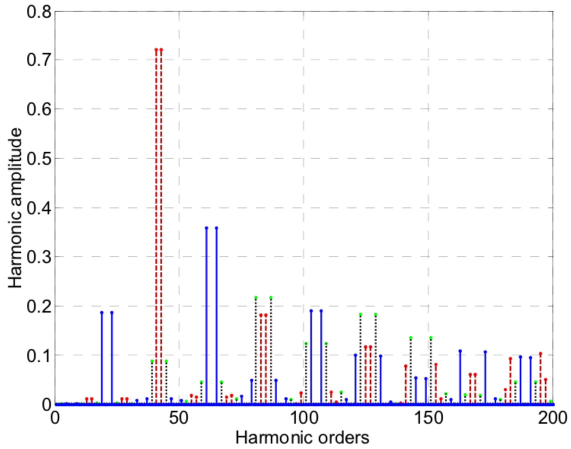
a) A=1, sinusoidal PWM



c) A=1, space vector PWM



b) A=0.5, sinusoidal PWM



d) A=0.5, space vector PWM

Fig. 17. Voltage spectra for $A = 0.5$ and $A = 1.0$, sinusoidal and space vector PWMs, $f_c/f_1 = 105$, $m = 7$, (1. group: blue; 2. group: red; 3. group harmonics: black)

and

$$\Delta\Psi^2 = \Delta\Psi_{\mathrm{I}}^2 + \Delta\Psi_{\mathrm{II}}^2 + \Delta\Psi_{\mathrm{III}}^2$$

Naturally the line harmonic fluxes have to computed according to the PWM methods applied.

In p.u. system (16) and (17) must be divided by the square of the rated phase flux:

$$\Delta\Psi_r^2 = \frac{4}{\pi^2} U_{dc}^2 \frac{1}{4\pi^2 f_r^2}$$

In case of sinusoidal modulation the results are:

$$G_f = \pi^4 A^2 (1 - 1.4418A + 0.75A^2)/192;$$

$$G_1 = \pi^4 A^2 (1 - 1.727A + 0.75A^2)/192;$$

$$G_2 = \pi^4 A^3 0.25/192;$$

$$G_3 = \pi^4 A^3 0.00353/192.$$

For a given number of phases the first two equations depend on the PWM method while the last ones (the number of those equations increases with the increase of phase number) are valid for any type of modulation (sinusoidal, space vector, the m harmonic injection or different types of discontinuous PWM) [13]. It should be noted that harmonic losses from these components – in case of PWM with constant number of pulses on fundamental period – are proportional to the modulation index.

A simpler expression [13] is obtained for $\Delta\Psi^2$ after summation of (14) equations since:

$$4\sin^2(\pi/7) + 4\sin^2(2\pi/7) + 4\sin^2(3\pi/7) = 7 \qquad (17)$$

and therefore

$$\Delta\Psi^2 = \frac{\Delta\Psi_{ab}^2 + \Delta\Psi_{ac}^2 + \Delta\Psi_{ad}^2}{7}.$$

The above expression is valid for any odd phase number:

$$\Delta\Psi^2 = \frac{1}{m} \sum_{i=1}^{(m-1)/2} \Psi_{ai}^2$$

where the summation expands on all different line fluxes. For even phase number similar expressions are

$$4\sin^2(\pi/m) + 4\sin^2(2\pi/m) + 4\sin^2(3\pi/m) + \ldots = m + 2$$

and

$$\Delta\Psi^2 = \frac{1}{m+2} \sum_{i=1}^{m/2} \Psi_{ai}^2$$

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