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RESEARCH ARTICLE

An \mathcal{H}_{∞} -Norm-Based Approach for Operating Point Selection and LPV Model Identification from Local Experiments

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Abstract

When the identification of linear parameter-varying (LPV) models from local experiments is considered, the question of the necessary number of local operating points as well as the problem of the efficient interpolation of the locally-estimated linear time-invariant models arise. These challenging problems are tackled herein by using the H_{α} -norm. First, thanks to the nu-gap metric, an heuristic technique is introduced to optimize the number as well as the position of the local operating points (along a given trajectory of the scheduling variables) with respect to the information brought by the local models. Having access to a reliable set of local models, the second step of the procedure, i.e., the parameter estimation step, consists of the optimization a second H_x-norm-based cost function measuring the fit between the local information (represented by the locally-estimated LTI models) and the local behavior of a parameterized global LPV model. A special attention is given to parameterized LPV models satisfying a fully-parametrized or a physically-structured linear fractional representation.

Keywords

linear parameter-varying model \cdot linear fractional representation \cdot H_{x} -norm \cdot non-smooth optimization \cdot nu-gap metric

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1 Introduction

Nowadays, the extensive demand for reliable models of nonlinear and/or time-varying systems requires the development of special model structures in which the non-linear behavior is broken down into several local models [16]. Among all the multi-model structures available in the literature, a particular attention has been paid to the linear parameter-varying (LPV) models during the last two decades (see [28, Chapter 1] for a historical review of LPV modeling and identification). This interest can be mainly explained by the following reasons. First, an LPV model can be seen as a combination of local models with parameters evolving as a function of measurable variables (called the scheduling variables) which can be related to the different operating points of the system. By this way, the model structure is close to the standard linear time-invariant (LTI) one but with a structural flexibility able to cope with time-varying, even non-linear behaviors. Second, the development of LPV models is linked to control engineering, where a controller must be designed in order to guarantee a suitable closed-loop performance for a given plant in different operating conditions. A well-known example of controller design technique using this "divide and conquer" basic idea is the gain scheduling approach [25]. A wide body of LPV controller design techniques is now available for this problem, which can be solved reliably, provided that a suitable model in a parameter-dependent form has been derived.

LPV model identification methods based on I/O data [7] can be divided into two sub-classes generally called the global approach and the local approach, respectively. On the one hand, the global approach assumes that one global experiment can be performed during which the control inputs, as well as the scheduling variables, are both excited (see among others *e.g.*, [15, 5, 32, 14, 31] and the references therein). By this way, all the non-linearities of the system are excited at the same time by passing through a large number of operating points. On the other hand, local methods are based on a multi-step procedure where

 local experiments are performed in which the operating points (corresponding to fixed values of the scheduling variables) are held constant and the control inputs are (persistently) excited,

- local LTI models are estimated by using the locally gathered input/output (I/O) measurements,
- an interpolation phase is performed in order to determine a final global parameter-dependent model,

Such a multi-step approach is a lot closer to the standard procedure used for non-linear system identification or the one dedicated to gain scheduling. Such a viewpoint has been considered, e.g., in [17, 30, 19, 38, 12, 8]. Both classes of methods have advantages and drawbacks. From a practical point of view, the global approach can suffer from the difficulty to satisfy the "rich" excitation of the control inputs and the scheduling variables simultaneously. It is obvious that such an experimental procedure may not be reasonable for specific applications, mainly for safety reasons. On the contrary, applying small variations around particular operating points, as considered by the local approach, is more conceivable in many practical cases. However, when state-space forms are concerned (as considered in the sequel), the interpolation step involved in the local approach requires that the local LTI state-space models are written in a coherent basis [29], basis which, on top of that, must be related to the global LPV state-space structure. In order to address this coherent basis issue, a new approach, sharing ideas from D. Petersson's work [21], is suggested hereafter. As explained first in [22, 23], then in [36, 35], the standard interpolation step involved in the local approach can be efficiently discarded by optimizing one global cost-function which fits the local behavior of the sought global LPV model (with coherent local and global state basis) to the available local information available in the local LTI models estimated during the second step of the local approach. However, contrary to the developments available in [21, 24], we do not consider an H_2 -normbased optimization solution hereafter but an H_{α} -norm-based one. The main reason why we focus on this specific norm is linked to the fact that the H_{α} -norm is a direct and efficient tool for adapting reliable and convergent non-smooth optimization algorithms [1, 2, 3] (suggested initially for H_{∞} -synthesis) for parameterized model identification. Notice also that the optimization techniques introduced hereafter do not involve the bounded real lemma [6] and thereby lead to moderate-size optimization routines even when huge data-sets are handled. Interesting from a consistency point of view [3], the iterative H_{∞} -norm-based optimization algorithm used hereafter may be very sensitive to the number of operating points involved during the minimization. Considering too many data-sets can be indeed computationally cumbersome. Unfortunately, according to the Authors' knowledge, apart from a first attempt in [13], the problem of the optimal selection of the operating points is often eluded when local methods are applied. Indeed, when the final LPV model is determined based on local experiments, the operating points are usually assumed to be given in a well-chosen and well-placed manner along a given trajectory of the scheduling variables. In this article, inspired by the developments suggested in [26], an extension of the approach presented in [35] is developed with the feature to give access to an ad-hoc number of local operating points on-line. More precisely, a new combination of two H_{∞} -norm-based methods is suggested first to determine a sufficient number of local LTI models (in order to ensure that the final global LPV model captures the whole dynamics of the system to identify), second to estimate the unknown parameters of the sought parameterized LPV model by matching the local LTI models to the frozen [28] global LPV model. The first problem, *i.e.*, the working points selection, is tackled by resorting to the nu-gap metric [33]. As shown in [34], the nu-gap metric can be considered as a non-linearity measure. This latter feature can be seen as an important asset when LPV modeling and identification problems are tackled. An optimal experimental design technique is proposed in [13] for input/output representations while an operating point reduction approach based also on the nu-gap metric is suggested in [26]. In this latter article, however, a user-defined set of local input/output representations has to be available before the application of the method. Furthermore, a simple weighted interpolation between the selected local linear models is only suggested to obtain the final LPV one. As mentioned above, after such an interpolation step, the resulting LPV model may not be consistent. On the contrary, our contribution avoids the interpolation step by considering a global cost function comparing the global LPV model and the available local information. As shown hereafter, such a viewpoint leads to an efficient optimization procedure which connects the theoretical soundness of the H_{m} -norm with a quite simple implemention. Notice also that the method presented in this paper does not require any initial set of local models but only assumes the availability of two estimated fully-parametrized model as a starting point. Then, the estimation of the other models is performed step-bystep along the trajectory of the scheduling variables by optimizing the step size between each working point.

The paper is organized as follows. In Section 2, the problem statement and some important definitions are given. Section 3 is dedicated to the working points selection step. The problem of the estimation of the unknown parameters of the LPV model is postponed to Section 4. In Section 5, the performance of the developed method is presented through a simulation example. Section 6 concludes this article.

2 Problem formulation and definitions

In this article, the identification of LPV models described by the following state-space representation is addressed

$$\gamma \mathbf{x}(t) = \mathbf{A}(\mathbf{p}(t), \Theta) \mathbf{x}(t) + \mathbf{B}(\mathbf{p}(t), \Theta) \mathbf{u}(t)$$
(1a)

$$\mathbf{y}(t) = \mathbf{C}(\mathbf{p}(t), \Theta)\mathbf{x}(t) + \mathbf{D}(\mathbf{p}(t), \Theta)\mathbf{u}(t)$$
(1b)



Fig. 1. Linear fractional representation as partial feedback interconnection of M and Δ_p .

where $\mathbf{u}(t) \in \mathbb{R}^{n_u}$ is the input signal vector, $\mathbf{y}(t) \in \mathbb{R}^{n_y}$ is the output signal vector, $\mathbf{x}(t) \in \mathbb{R}^{n_x}$ is the state vector and $t \in \mathbb{R}$ or \mathbb{Z} . Herein, γ stands for the forward shift operator when discrete-time systems are considered or for the differential operator when continuous-time systems are handled. The vector $\Theta \in \mathbb{R}^{n_\Theta}$ contains the unknown parameters to identify. In the LPV framework, the system matrices (**A**, **B**, **C**, **D**) are rational functions of measurable time-varying signals gathered into the vector $\mathbf{p}(t) \in \mathbb{P} \subseteq \mathbb{R}^{n_p}$ and called the scheduling variables, *i.e.*,

 $\mathbf{A}: \mathbb{P} \to \mathbb{R}^{n_x \times n_x} \qquad \qquad \mathbf{B}: \mathbb{P} \to \mathbb{R}^{n_x \times n_u}$ (2a)

$$\mathbf{C}: \mathbb{P} \to \mathbb{R}^{n_y \times n_x} \qquad \qquad \mathbf{D}: \mathbb{P} \to \mathbb{R}^{n_y \times n_u}, \qquad (2b)$$

where

$$\mathbb{P} = \{ \mathbf{p} \in \mathbb{R}^{n_p} \mid \underline{p}_i \le p_i \le \overline{p}_i, \forall i \in \{1, \cdots, n_p\} \},$$
(3)

is the so-called scheduling "space" [27] which is a compact set. It is furthermore assumed that the system matrices satisfy a static dependence on \mathbf{p} [28], *i.e.*, they do not depend on the time-shifted versions ($\mathbf{p}(k+1)$, $\mathbf{p}(k+2)$, ...) or the time-derivatives ($\dot{\mathbf{p}}(t)$, $\ddot{\mathbf{p}}(t)$,...) of the scheduling variables when discrete or continuous-time models are considered, respectively. Such an assumption (static dependence) can be justified by noticing that no dynamical dependency can be observed from local experiments. Thus, only static dependent LPV models can be handled when a local procedure is considered.

In this paper, a specific attention is paid to state-space matrices satisfying a rational dependence on $\mathbf{p}(t)$. Such a parameter dependence is indeed more general than the affine dependence usually encountered in system identification. It is also often used in the robust control literature [39]. More precisely, we focus on LPV models which can be written into the so-called linear fractional representation (LFR) [39]. The block-diagram of the linear fractional representation can be seen in Fig. 1. The Δ_n matrix is $\mathbf{p}(t)$ -dependent and defined as

$$\Delta_p = \operatorname{diag}(p_1(t)\mathbf{I}_{r_1}, \cdots, p_{n_p}(t)\mathbf{I}_{r_{n_p}}).$$
(4)

The signals $\mathbf{w}(t) \in \mathbb{R}^{n_w}$ and $\mathbf{z}(t) \in \mathbb{R}^{n_z}$ are inner auxiliary signals with $r = \sum_{k=1}^{n_p} r_k$ linked by the equation $\mathbf{w}(t) = \Delta_p \mathbf{z}(t)$. **M** is an LTI system with the following state-space representation

$$\begin{bmatrix} \dot{\mathbf{x}}(t) \\ \mathbf{z}(t) \\ \mathbf{y}(t) \end{bmatrix} = \begin{bmatrix} A_0(\Theta) & B_w(\Theta) & B_0(\Theta) \\ C_z(\Theta) & D_{zw}(\Theta) & D_{zu}(\Theta) \\ C_0(\Theta) & D_{yw}(\Theta) & D_0(\Theta) \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{w}(t) \\ \mathbf{u}(t) \end{bmatrix}$$
(5)

where $(A_0(\Theta) \quad B_w(\Theta) \cdots \quad D_0(\Theta))$ are time-invariant matrices with appropriate dimensions.

Thanks to this linear fractional representation, the statespace form described by Eq. (1) can then be obtained from the following state matrices [39]

$$\mathbf{A}(\mathbf{p}(t),\Theta) = A_0(\Theta) + B_w(\Theta)\Delta_p (\mathbf{I}_{n_z} - D_{zw}(\Theta)\Delta_p)^{-1}C_z(\Theta)$$
 (6a)

$$\mathbf{B}(\mathbf{p}(t),\Theta) = B_0(\Theta) + B_w(\Theta)\Delta_p (\mathbf{I}_{n_z} - D_{zw}(\Theta)\Delta_p)^{-1} D_{zu}(\Theta)$$
 (6b)

$$\mathbf{C}(\mathbf{p}(t),\Theta) = C_0(\Theta) + D_{yw}(\Theta)\Delta_p (\mathbf{I}_{n_z} - D_{zw}(\Theta)\Delta_p)^{-1}C_z(\Theta) \qquad (6c)$$

$$\mathbf{D}(\mathbf{p}(t),\Theta) = D_0(\Theta) + D_{yw}(\Theta)\Delta_p (\mathbf{I}_{n_z} - D_{zw}(\Theta)\Delta_p)^{-1} D_{zu}(\Theta)$$
 (6d)

provided that the inverse $(\mathbf{I}_{n_z} - D_{zw}\Delta_p)^{-1}$ exists [39, Chapter 10] for all the possible trajectories of $\mathbf{p}(t)$. Notice that the elements of the system matrices in Eq. (6) (*i*) are polynomial or rational functions of the scheduling variable vector when $D_{zw} \neq 0$, (*ii*) are affine functions when $D_{zw} = 0$. The LFR of the LPV model can also be compactly written as

$$\mathcal{G}(\Delta_p, \Theta) = \mathcal{F}(\mathbf{M}(\Theta), \Delta_p), \tag{7}$$

where $\mathbf{M}(\mathbf{\Theta})$ represents the LTI system and \mathcal{F} stands for the upper linear fraction transformation as presented in Fig. 1 (see [39, Chapter 10] for details).

As explained in Section 1, such an LPV model can be identified from local or global experiments. Herein, a local approach is considered. First, in Section 3, a specific attention is paid to the reliable selection of N_{op} different local operating points. According to the Authors' knowledge, apart from the contributions [13, 26], this problem is often overlooked in the literature. In this paper, it is shown how these N_{op} operating points can be extracted on-line from local information without reducing the quality of the final LPV model. Based on this local information, the identification of the global LPV model is then studied in Section 4.

3 Operating point selection algorithm 3.1 Nu-gap metric: a short reminder

In order to describe the working point selection technique, let us first introduce the measure of model fit, the present contribution is based on. The nu-gap metric [34], as a distance measurement between two LTI models, is defined in [33] as

$$v_g = \delta(\mathfrak{G}_1(s), \mathfrak{G}_2(s)) \tag{8a}$$

$$= \| (\mathbf{I} + \mathfrak{G}_{2}(s)\overline{\mathfrak{G}}_{2}(s))^{-\frac{1}{2}} (\mathfrak{G}_{2}(s) - \mathfrak{G}_{1}(s)) (\mathbf{I} + \mathfrak{G}_{1}(s)\overline{\mathfrak{G}}_{1}(s))^{-\frac{1}{2}} \|_{\infty},$$
(8b)

where $\mathfrak{G}_i(s)$, $i \in \{1,2\}$, stands for the transfer functions of each LTI model while $\overline{\mathfrak{G}}_i$, $i \in \{1,2\}$, and $|| \bullet ||_{\infty}$ stand for the complex conjugate and the H_{∞} -norm of \bullet , respectively. This measure is bounded between 0 and 1 [33]. In addition, it can be shown that if v_g is close to 0, then the two considered models have similar

dynamic behavior. Contrarily, when v_g is close to 1, then the two models behave differently [33, 34]. So, the nu-gap metric can be considered as a normalized H_{∞} -norm of the difference between two LTI models. Thanks to its advantageous properties, this tool can efficiently be used during the operating point selection procedure before the final LPV model estimation step.

3.2 Selection algorithm

First, it is important to notice that the following selection algorithm can be used with any identification method based on local experiments. In the next section, this technique will be associated with a specific H_{α} -norm-based optimization algorithm but such an LPV model identification technique can be replaced by any other efficient interpolation technique usually developed in the local approach framework. For this selection procedure, it is assumed that the maximal $(p_{max} = \overline{\mathbf{p}})$ and minimal $(p_{min} = \mathbf{p})$ possible values of the scheduling variables are known (see Eq. (3) for the definition of \mathbf{p} and \mathbf{p} , respectively). By assuming that this (weak) assumption is satisfied, the natural questions which arise when dealing with a local approach are the following: how many working points do you need to get a consistent final LPV model? How can you select the good working points for the local experiments? How do you measure the information brought by a working point? Indeed, roughly speaking, by assuming that an initial operating point is available, the next one should be (i) far enough from the first one to decrease the complexity of the LPV model identification step, (ii) close enough to the first one to ensure that the global behavior of the system is well-captured. These important questions can guide the user in developing an heuristic technique for selecting the working points in an efficient way. They more precisely show that a distance between two consecutive local models must be introduced to decide if a local model must be kept because it gives access to new and reliable information about the global behavior of the system to identify. In this paper, it has been chosen to use the nu-gap metric to measure such a distance because of its ability to quantify (between 0 and 1) the behavioral similarities or differences between to LTI models. Once this frequency fit measurement is introduced, according to prior knowledge or specific practical constraints satisfied by the system to identify, the user must choose a specific range of admissible values for this match measurement v_a (related to a user-defined threshold called in the following), range which must picture the confidence the user has in the capability of the selected local models to capture the global behavior of the system. Then, depending on whether the current value of v_{σ} is in or out of the user-defined range of admissible values, the distance between the local models must be updated (increased or decreased).

This scenario being set, it is now necessary to translate it into an iterative algorithm. Hereafter, the actual operating point and the step size are denoted by p_{act} and p_{step} , respectively. The

following procedure can be performed in order to determine the best¹ operating points and to identify the local black-box models from the local information available at these points. In the following, the algorithm is presented by using only one scheduling variable. Notice that the extension of the algorithm to several scheduling variables can be carried out straightforwardly.

- Estimate a black-box LTI state-space model 𝔅_i, i = 1, in the initial operating point (p_{act} = p_{min}) by using any dedicated identification method [16, 10, 9, 18]. Define the initial p_{step} as p_{step} ∈ [0.01, 0.05]p_{max} and set the operating point index as i = i + 1. Compute the position of the next working point (p_i = p_{act} + p_{step}).
- 2. Perform the following steps until it holds true that $p_{act} \leq p_{max}$.
 - Estimate a black-box LTI state-space model (\mathfrak{G}_i) in the actual operating point $(p_{act} = p_i)$ by using any dedicated identification method [16, 10, 9, 18].
 - Compute the nu-gap metric by using the definition given in Eq. (8a), as

$$\sigma = \delta(\mathfrak{G}_{i-1}, \mathfrak{G}_i), \tag{9}$$

- If $\underline{\beta} < \sigma < \overline{\beta}$, then the actual model (\mathfrak{G}_i) is kept, i = i + 1 and, where the user-defined threshold $\underline{\beta}$ and $\overline{\beta}$ are chosen to be equal to $(\beta 0.1)$ and $(\beta + 0.1)$, respectively.
- Else if $\sigma > \overline{\beta}$, then the actual model (\mathfrak{G}_i) is discarded, $p_{step} = p_{step} - \Delta p_{step}$ and $p_i = p_{act} - p_{step}$.
- $p_{step} = p_{step} \Delta p_{step} \text{ and } p_i = p_{act} p_{step}.$ • Else if $\sigma < \underline{\beta}$, then the actual model (\mathfrak{G}_i) is discarded, $p_{step} = p_{step} + \Delta p_{step}$ and $p_i = p_{act} + p_{step}.$

The resulting set contains the kept models, the dimension of which is denoted by N_{op} in the following.

By construction, such an algorithm is heuristic. A statistical investigation of the effect of the parameters β and the effect of noisy data during local model identification is tackled in Section 5 by resorting to simulations. The theoretical analysis of this selection algorithm is referred to a future work. However, in order to guide the user, we can herein give some important hints in order to select the hyper-parameters of the technique efficiently. First, the initial value of p_{step} should be chosen quite small w.r.t. p_{max} , e.g., $p_{step} \in [0.01, 0.05] p_{max}$. This choice can be justified by the fact that the step size should be initially small enough to be able to capture the non-linearities of the system. Then the algorithm will determine (increase or decrease) at each iteration a new value of p_{step} if it is necessary. Second, as far as β is concerned, as can be seen from the results obtained in Section 5, the admissible choice of this hyper-parameter depends on the noise involved in the local I/O data sequences

¹ In this context, by best selection, it is meant that the final set of working points is the best in terms of the user-defined β which is a maximal allowable distance, in the nu-gap metric sense [33], between each local model.

and on the considered framework². Finally, Δp_{step} is chosen to be equal to $0.25p_{step}$ during the simulation example in Section 5.

Notice that the operating point determination procedure can be started from the maximal value (p_{max}) and from any other intermediate value of the scheduling variables as well by (only) slightly modifying the afore-described algorithm. When the algorithm can not find any dynamical changes en route to p_{max} , two working points will be selected anyway, the ones corresponding to p_{min} and p_{max} .

Notice finally that this selection algorithm can be easily implemented by using the gapmetric function of the Robust Control Toolbox available in MATLAB. As mentioned above, the proposed algorithm can be applied with any identification technique based on local experiments (such methods can be, among others, [20, 19, 35]). In this article, the procedure is embedded into the H_{∞} -norm-based identification method developed first in [35].

4 H_{∞} -norm-based identification method from local experiments

As explained in the former section, the working point selection algorithm described in this paper yields N_{op} local LTI models $\mathfrak{G}_i(s)$, $i \in \{1, \dots, N_{op}\}$. The problem which remains to solve is how this local information can be efficiently combined in order to get, in the final step, an accurate LPV model. In this paper, LPV models satisfying a linear fractional representation are handled. It is now well-known [29, 20, 28] that, in a framework using LPV state-space representations, this interpolation step must be performed with caution if the user does not want to face (dynamical) similarity transformation issues. Herein, this problem is bypassed by resorting to a specific cost function which focuses on the preservation of the input-output behavior of the model. More precisely, having access to reliable local models $\mathfrak{G}_i(s)$, $i \in \{1, \dots, N_{op}\}$, as well as an LPV model structure $\mathcal{G}(s, \Delta_p, \Theta)$ effective, inter alia, for frozen values of the scheduling variables \mathbf{p}_{i} , $i \in \{1, \dots, N_{op}\}$, the idea used herein consists in estimating the unknown parameters of the LPV model by optimizing the following global cost function

$$\min_{\Theta} \sum_{i} \| \mathfrak{G}_{i}(s) - \mathfrak{G}(s, \Delta_{p_{i}}, \Theta) \|_{0}^{2}$$
(10)

where \diamond stands for any system norm while

$$\mathcal{G}(s, \Delta_{p_i}, \Theta) = \mathbf{D}(\mathbf{p}_i, \Theta) + \mathbf{C}(\mathbf{p}_i, \Theta)(s\mathbf{I} - \mathbf{A}(\mathbf{p}_i, \Theta))^{-1}\mathbf{B}(\mathbf{p}_i, \Theta).$$
(11)

This technique echos back the method developed in [21] but, hereafter, a specific attention is paid to the H_{∞} -norm. More precisely, we consider the following -norm-based cost-function

$$\min_{\Theta} \max_{i \in \{1, \cdots, N_{op}\}} \| \mathfrak{G}_{i}(s) - \mathcal{G}(s, \Delta_{p_{i}}, \Theta) \|_{\infty} .$$
(12)

The use of the H_{∞} -norm can first be justified by noticing that, by definition, the H_{∞} -norm is the maximal singular values of the complex gain matrix over all the frequencies. Thus, thanks to the norm property, if the cost function in Eq. (12) is small enough (*i.e.*, a lot smaller than 1), then the distance between the actual system and the identified one is small as well in the considered local working points. In other words, if the maximal value of the H_{μ} -norm found in Eq. (12) is small enough, the LPV model can be considered as a decent approximation of all the local LTI state-space models for all the considered fixed values of the scheduling variables. Notice that the local models perfectly captures the true local behavior. Second, the H_{∞} -norm is used herein as an efficient tool for adapting reliable and convergent non-smooth optimization algorithms [1, 2, 3] (suggested initially for H_{μ} -synthesis) for parameterized model identification. The proximity control algorithm developed in [3] is indeed particularly adapted for the optimization of (non-smooth) structurally-constrained functions. The structured model identification framework considered in this paper explains as well the reasons why we direct our attention to this kind of techniques.

In order to be able to separately manage the involved local information, a block diagonal reformulation of the cost-function defined by Eq. (12) is proposed herein. By doing so, a structured H_{∞} synthesis problem is obtained (see [35] for more details). Let us more precisely introduce the block-diagonal term $\overline{G}(s, \mathbf{\bar{p}}, \Theta)$ having the form

$$\breve{G}(s, \mathbf{\bar{p}}, \Theta) = \text{blockdiag}\Big(G_1(s, \Theta), \cdots, G_{N_{op}}(s, \Theta)\Big), \quad (13)$$

where $\mathbf{\overline{p}} = [p_1, \cdots, p_{N_{op}}]^\top$ is a vector containing the scheduling variables. As a result of the block diagonal structure, it can be written that

$$\|\breve{G}(s, \mathbf{\bar{p}}, \Theta)\|_{\infty} = \max_{i} \|G_{i}(s, \Theta)\|_{\infty} .$$
⁽¹⁴⁾

By replacing the above obtained structure into Eq. (12), we obtain the reformulated cost function as

$$\min_{\Theta} \| \tilde{G}(s, \mathbf{p}, \Theta) \|_{\infty} .$$
 (15)

In order to minimize the H_{∞} -norm of $\tilde{G}(s, \mathbf{p}, \Theta)$, it is necessary to compute the value of this function efficiently. Clearly,

$$\left\| \breve{G}(s, \mathbf{\bar{p}}, \Theta) \right\|_{\infty}^{2} = \max_{\omega \in [0, \infty]} \lambda_{1}(\breve{G}^{H}(j\omega, \mathbf{\bar{p}}, \Theta) \breve{G}(j\omega, \mathbf{\bar{p}}, \Theta)).$$

Thus, solving Eq. (15) is equivalent to the following optimization problem

$$\min_{\Theta} \max_{\omega \in [0,\infty]} \lambda_1(\vec{G}^H(j\omega, \mathbf{\bar{p}}, \Theta)\vec{G}(j\omega, \mathbf{\bar{p}}, \Theta))$$
(16)

where $\lambda_1(\bullet)$ is the maximum eigenvalue function which is a convex but a non-smooth function [2]. Notice that the computation of the objective function, defined by Eq. (16), is a hard task because, by construction, the global optimum of a non-smooth and non-convex function over $[0, \infty]$ must be estimated. Fortunately, when $\mathcal{G}(\Delta_{p,}, \Theta)$, $i \in \{1, \dots, N_{op}\}$, has a state-space

² By framework, it is meant that during the local LTI model estimation step, a black or a gray-box LPV model is sought to be estimated.



Fig. 2. System used for demonstration.

representation (which is the case herein), this step can be performed efficiently by using a bisection algorithm based on a hamiltonian calculus [3].

Several efficient methods to solve the optimization problem defined by Eq. (16) can be found in [1, 3]. In this present work, the hinstruct function is applied to minimize the cost-function defined by Eq. (12). This optimization tool developed by Apkarian *et al.* is available in the Robust Control Toolbox of MATLAB. A detailed description of the optimization algorithm is available in [1]

In the next section, the performance of the proposed technique is studied with the help of a simulation example.

5 Simulation example

5.1 System description

In order to illustrate the performance of the identification method developed in this paper, a translational two-massspring system is used (see Fig. 2). The masses of the two vehicles are denoted by m_1 and m_2 , respectively. On this system, the linear speed v_1 of the first vehicle is assumed to be actuated with a force F and effected by $-f_1v_1$. Similarly, the velocity v_2 of the second vehicle suffers from a dissipative force $-f_2v_2$. The two vehicles are linked with a flexible transmission of strength k and friction ratio f. By denoting δ the distance between the two vehicles, the force produced on the first vehicle is $-k\delta - f(v_1 - v_2)$ and the opposite on the second vehicle. The driving vehicle is equipped with a linear speed controller: F = $K(u - v_1)$, where u is the velocity reference and K is the control gain. The velocity of the first vehicle is considered as the output $(y = v_1)$. The second vehicle is loaded with a mass $m_c(t)$, which is considered to be time-varying and measurable at any time. By considering $x = [m_1v_1 (m_2 + m_c)v_2 \delta]$ as the state vector, an LPV model structure similar to the one described in Eq. (1) can be obtained with $\mathbf{p}(t) = m_{a}(t)$ and where

$$\begin{bmatrix} \mathbf{A}(\mathbf{p}(t)) & \mathbf{B}(\mathbf{p}(t)) \\ \mathbf{C}(\mathbf{p}(t)) & \mathbf{D}(\mathbf{p}(t)) \end{bmatrix} = \begin{bmatrix} \frac{-f - f_1}{m_1} & \frac{f}{m_2 + m_c(t)} & -k & K \\ \frac{f}{m_1} & \frac{-f - f_2}{m_2 + m_c(t)} & k & 0 \\ \frac{1}{m_1} & \frac{-1}{m_2 + m_c(t)} & 0 & 0 \\ \frac{1}{m_1} & \frac{1}{m_2 + m_c(t)} & 0 & 0 \end{bmatrix}.$$

On the one hand, the linear fractional representation given in Eq. (5) can be obtained with $\mathbf{\Delta}_p = \mathbf{p}(t) = m_c(t)$ and

N

$$\mathbf{A}(\Theta) = \begin{bmatrix} A_0(\Theta) & B_w(\Theta) & B_0(\Theta) \\ C_z(\Theta) & D_{zw}(\Theta) & D_{zu}(\Theta) \\ C_0(\Theta) & D_{yw}(\Theta) & D_0(\Theta) \end{bmatrix} \\ = \begin{bmatrix} \frac{-f - f_1}{m_1} & \frac{f}{m_2} & -k & \frac{-f}{m_2} & k \\ \frac{f}{m_1} & \frac{-f - f_2}{m_2} & k & \frac{f + f_2}{m_2} & 0 \\ \frac{1}{m_1} & \frac{-1}{m_2} & 0 & \frac{-f}{m_2} & 0 \\ \frac{1}{m_1} & \frac{1}{m_2} & 0 & \frac{-1}{m_2} & 0 \\ \frac{1}{m_1} & 0 & 0 & 0 & 0 \end{bmatrix},$$
(17)

where $\Theta = [m_1, m_2, K, k, f, f_2]^{\top}$ denotes the vector of the physical parameters of the system to identify. On the other hand, when $\mathbf{M}(\Theta)$ is considered to be a fully-parametrized matrix, *i.e.*,

$$\mathbf{M}(\Theta) = \begin{bmatrix} A_{0}(\Theta) & B_{w}(\Theta) & B_{0}(\Theta) \\ C_{z}(\Theta) & D_{zw}(\Theta) & D_{zu}(\Theta) \\ C_{0}(\Theta) & D_{yw}(\Theta) & D_{0}(\Theta) \end{bmatrix}$$

$$= \begin{bmatrix} \vartheta_{1} & \vartheta_{2} & \vartheta_{3} & \vartheta_{4} & \vartheta_{5} \\ \vartheta_{6} & \vartheta_{7} & \vartheta_{8} & \vartheta_{9} & \vartheta_{10} \\ \vartheta_{11} & \vartheta_{12} & \vartheta_{13} & \vartheta_{14} & \vartheta_{15} \\ \vartheta_{16} & \vartheta_{17} & \vartheta_{18} & \vartheta_{19} & \vartheta_{20} \\ \vartheta_{21} & \vartheta_{22} & \vartheta_{23} & \vartheta_{24} & \vartheta_{25} \end{bmatrix},$$
(18)

the LFR given in Eq. (5) can be calculated with $\mathbf{\Delta}_{p} = \mathbf{p}(t) = m_{c}(t)$. The parameters to identify are $\boldsymbol{\Theta} = [\boldsymbol{\vartheta}_{1}, \cdots, \boldsymbol{\vartheta}_{25}]^{\mathsf{T}}$.

Remark 5.1 At this step, it is important to discuss the identifiability issue related to the LPV model parameterization. When fully-parameterized LPV models are concerned, it is obvious that, as in the linear time-invariant framework, the parameter vector Θ cannot be estimated uniquely. Considered initially as a drawback, this feature can be used efficiently by the optimization algorithm introduced to estimate Θ . Indeed, in a certain way, the H_{∞} -norm-based algorithm can use this extra degree of freedom to fit, as well as possible, the sought parameter vector Θ to the available local information, by passing thus the coherent basis constraint. In the gray-box case, i.e., when the model structure is physically-parameterized (see Eq. (17)), getting a unique parameter vector is essential because the user wants to estimate the real physical parameters, e.g., $[m_1, m_2, K, k, f, f_2]^{\dagger}$, accurately. Thus, in this framework, using an identifiable model structure is compulsory. For our simulation example, we will assume that the following two definitions are satisfied

Definition 5.1 A model structure χ is a differentiable map from a parameter set $\Phi \subset \mathbb{R}^{n_{\theta}}$ to a set of models

$$\chi: \Phi \subset \mathbb{R}^{n_{\theta}} \to \mathbb{R}^{n_{y} \times n_{u}}$$
$$\Theta \quad \mapsto \quad \mathcal{F}(\mathbf{M}(\Theta), \Delta).$$
(19)

Definition 5.2 A model structure χ is locally identifiable at $\Theta^* \in \Phi$ if a neighborhood $v(\Theta^*)$ exists such that

$$\overline{\Theta} \in v(\Theta^*) \quad and \quad \chi(\overline{\Theta}) = \chi(\Theta^*) \Longrightarrow \overline{\Theta} = \Theta^*.$$
(20)

Notice that, for the simulation example considered herein, these definitions are satisfied at least for frozen values of the scheduling variable. A further analysis of the identifiability and wellposedness of gray-box LFRs in the local LPV model identification framework is devoted to a future work. Notice however that the good estimation results obtained in Paragraph 5.3.2 lead us to conclude that this assumption should be satisfied by our model parameterization.

5.2 Working point selection

The first step of the method used in this paper consists in selecting N_{an} local operating points by applying the algorithm described in Section 3. As explained previously, this selection algorithm contains an identification step yielding local fullyparametrized LTI state-space models from which the nu-gap metric test can be applied for selecting the more informative operating points. In this paper, it has been chosen to use the Simplified Refined Instrumental Variable method for Continuous-time systems (SRIVC) [10, 11] to get these local black-box models. This choice can be justified first by noticing that the SRIVC method, which can be considered as one of the most efficient identification techniques for the direct estimation of continuous-time LTI models, is known to yield consistent estimates when the model belongs to the system class [10] and second because a convenient implementation is available in the CONTSID Toolbox for identifying MISO models, a constraint satisfied by the simulated system studied in the current section. Notice also that the implementation available in the CONTSID Toolbox has the advantage of not requiring any design parameters to be specified. Of course, users familiar with other techniques can use their favorite algorithms to estimate the initial state-space representations $(\mathfrak{A}_i, \mathfrak{B}_i, \mathfrak{C}_i), i \in \{1, \dots, N_{on}\}$, as long as these fully-parameterized models mimic the system behavior accurately. As explained in Section 3, the working point selection algorithm depends on hyper-parameters the user must tune correctly in order to ensure that the selected local models are enough (in quantity and quality) to get a reliable final LPV model. In this paper, we first focus on the influence of $p_{step_{uu}}$ as well as the direction of the search, *i.e.*, if starting from p_{min} or p_{max} influences the final result. The study of the impact of β is postponed to the next sub-section because this parameter is highly linked to the quality of the LPV model obtained from the optimization of the H_{∞} -norm-based criterion (15). Thus, herein,

the following Monte-Carlo simulation is carried out. Starting first from p_{min} , then from p_{max} , for 9 successive values of β in the range [0.1 : 0.9] with a step of 0.1, 20 randomly selected values of $p_{step_{min}}$ are generated so that it can vary in the range [1% 50%] p_{max} . For each run, $\Delta p_{step} = 0.25 p_{step}$ while the local I/O data-sets are obtained by exciting the real system with a PRBS composed of 600 samples designed so that the whole local dynamics of the system is visited, then modified by adding up, on the noise-free output, a zero mean white Gaussian noise satisfying an SNRequal³ to 30 *dB*. First, in Fig. 3 and 4, respectively, we plot, for each value of $\beta \in [0.1:0.9]$, the median (cross), the minimum (lower limit) and the maximum (upper limit) number of working points obtained with the nu-gap-metric-based technique starting from p_{min} and from p_{max} , respectively. These bar-plots show that

- increasing the value of β leads to a decrease of N_{ap} ,
- the algorithm is not too sensitive to the search direction, *i.e.*, starting from p_{min} or p_{max} does not very influence the number of selected working points,
- apart from specific cases (for instance $\beta = 0.1$, starting point = p_{min}), the range of values satisfied by N_{op} is small.

Notice that, for each value of β , the median values are reached at least 12 times (out of the 20 Monte-Carlo simulations). These good conclusions can be reinforced by looking at the distribution of the values of the selecting scheduling variables for $\beta \in [0.1:0.9]$ and $i \in \{1, \dots, N_{op}\}$, respectively. For the sake of conciseness, results obtained with $\beta = 0.5$ are only shown in this paper. Notice however that similar conclusions can be drawn for the other values of β . Figure 5 and Figure 6 depict the box-plot for $\beta = 0.5$ starting from p_{min} and p_{max} , respectively. It is clear from these curves that the selected working points remain almost the same for the each value of $i \in \{1, \dots, N_{op}\}$.

Based on the results presented above, it can be concluded that the working point selection algorithm is not very sensitive to the initial working point (p_{min} or p_{max}) as well as the userdefined value of $p_{step_{min}}$ when mild noisy conditions are encountered. Notice however that, according to the Authors' experience, this conclusion must be moderated when low SNR are handled. Large ranges of scheduling variables can indeed come out when the SNR is set to be equal to 10 dB or less because of the sensitivity of the nu-gap-metric to model misfit. This issue is still under investigation and future publications will be dedicated to the developed solutions.

5.3 LPV model estimation

The second step of the procedure aims at estimating the final LPV model through the minimization of the cost-function defined by Eq. (12) from a set of local models. The hinstruct MATLAB function is used herein to minimize the

³ The chosen noisy conditions are mild in order to avoid misleading conclusions which could be related to the noise effect and not to the hyper-parameter choice.



Fig. 3. Distribution of the number of the selected working points (N_{op}) w.r.t. β . Starting point: p_{min} .



Fig. 5. Distribution of the value of the scheduling variable (p_i) in the *ith* working point w.r.t. $i \in \{1, \dots, N_{oo}\}$ when $\beta = 0.5$. Starting point: p_{min} .

involved cost-function. Notice that other algorithms such as HIFOO [4] or the proximity control method recently described in [37] should be used instead. The main advantage of hinstruct is its availability in MATLAB. For this LPV model identification step, two LPV model structures (a black-box and a gray box structure, respectively) are handled, as explained in Sub-Section 5.1. As far as the estimation procedure is concerned, the following scenario is used. For $\beta = 0.3$, 0.6 and 0.9, respectively, for each selected working point chosen as by the mean value of the scheduling variable p_{i} , $i \in \{1, \dots, N_{op}\}^4$ (shown by the red lines in Figure 5 and 6, respectively), 40 noise-free local I/O data-sets are first generated, then disturbed by a zero-mean white Gaussian noise satisfying an SNR = 20dB. The system is, more precisely, excited by applying PRBS signals having a sampling-time equals to 0.1s. By using again the SRIVC algorithm, black-box models are estimated locally from these noisy data-sets. For each working point, the best



Fig. 4. Distribution of the number of the selected working points (N_{op}) w.r.t. β . Starting point: p_{max} .



Fig. 6. Distribution of the value of the scheduling variable (p_i) in the *ith* working point w.r.t. $i \in \{1, \dots, N_{op}\}$ when $\beta = 0.5$. Starting point: p_{max} .

local model, the worst one and the median one are then kept for the last step of the procedure, *i.e.*, the H_{∞} -norm-based procedure. By best, worst and median, we mean the model giving the best, worst and median I/O fit defined as

BTF =
$$100 \times \max\left(1 - \frac{\|y - \hat{y}\|}{\|y - \operatorname{mean}(y)\|_{2}}, 0\right)$$
 (21)

where y stands for the output of the real system while \hat{y} denotes the model output. Notice that this fit evaluation is performed with noise-free validation data-sets. Table 1 contains the obtained best, worst and median averaged⁵ BFT for each value of β . The procedure presented above is performed by using the sets of working points obtained by starting the selection algorithm from p_{min} and from p_{max} as well. The figures gathered in Table 1 show that the SRIVC algorithm leads to accurate local LTI models quite often because, for each value of β , the median BFT values are very close to the best ones.

⁴ The number of working points are $N_{op}^{(0.3)} = 10$, $N_{op}^{(0.6)} = 3$, $N_{op}^{(0.9)} = 2$, respectively.

⁵ Notice that for each β , different numbers of BFT values are averaged according to the number of working points $(N_{op}^{(0.3)} = 10, N_{op}^{(0.6)} = 3, N_{op}^{(0.9)} = 2)$.

 Tab. 1. Best, median and worst I/O BFT measurements of the estimated black-box LTI models.

	$p_{\min} \rightarrow p_{\max}$			$p_{max} \rightarrow p_{min}$			
β	0.3	0.6	0.9	0.3	0.6	0.9	
Best BFT [%]	90.07	90.07	90.08	90.06	90.05	90.07	
Median BFT [%]	90.02	90.03	90.03	90.03	90.03	90.02	
Worst BFT [%]	63.65	55.61	89.91	54.98	59.84	64.97	

As explained previously, the behavioral interpolation suggested in this paper involves the minimization of an H_{∞} -norm. Because of the non-convexity of such a cost function, initialization issues can occur. Herein, in order to test the robustness of our technique w.r.t. the initialization, a Monte-Carlo simulation of dimension 10 is performed. In the black-box framework, the 25 sought parameters are initialized randomly by picking up uniformly distributed values $(r_j^{(i)}, i \in \{1, \dots, 25\}, j \in \{1, \dots, 10\})$ in the range [0, 1]. In the gray-box one, the parameters are initialized by using the following expression

$$\Theta_{i_{init}} = \Theta_{i_{real}} (1 + 4(r_j^{(i)} - 0.5)), \tag{22}$$

where, again, $r_j^{(i)}$, $i \in \{1, \dots, 6\}$, $j \in \{1, \dots, 10\}$, denotes a uniform random number in the range [0, 1] while $\Theta_{i_{real}}$, $i \in \{1, \dots, 6\}$, stands for the real values of the parameters, respectively. Notice that, for this simulation example, it is assumed that we know *a priori* that m_1 and m_2 are equal. Such an assumption can be made according to the available prior information about the system to identify.

5.3.1 Black-box LPV model

Having access to $3 \times N_{op}$ local LTI models for each value of $\beta \in [0.3:0.3:0.9]$, we have now to interpolate these "worst", "best" and "median" models to get final LPV black-box models. This problem is solved by minimizing the cost function (12) as explained in Section 4. Using this technique for the current simulation example leads to 3×10 black-box LPV models for each value of $\beta \in [0.3:0.3:0.9]$. Now, the question of LPV model validation arises. Herein, this validation step is performed by comparing the time evolution of the model output with the output of the real system. During this validation step, the scheduling variable is continuously excited in the range [0, 0.8] by using a chirp signal with a frequency varying linearly with respect to time in [0-15] rad/s. Notice that there is no noise during this LPV model validation. Again, an I/O fit measurement, similar to the one defined in Eq. (21), is used to quantify the capabilities of the model to mimic the system behavior. Table 2 contains the mean (based on 10 LPV blackbox models) of the obtained BFT measurements.

The figures gathered in Table 2 show that, except when the worst local models are used for the H_{∞} -norm-based optimization, the procedure dedicated to the LPV black-box model identification (*i*) leads to LPV models able, in average, to mimic the behavior of the real system efficiently even when few local

	$p_{min} \rightarrow p_{max}$			$p_{max} \rightarrow p_{min}$			
β	0.3	0.6	0.9	0.3	0.6	0.9	
Best BFT [%]	96.6	91.5	91.2	96.8	90.7	90.8	
Median BFT [%]	96.3	90.2	90.3	96.2	90.1	90.01	
Worst BFT [%]	48.9	53.2	89.1	49.2	56.2	61.4	

models are involved, (*ii*) is not sensitive to the initialization while the 25 sought parameters are initialized in the range [0, 1].

5.3.2 Gray-box LPV model

In the gray-box framework, our goal is to focus on the capabilities of the identification algorithm to estimate the physical parameters of the real system accurately. Indeed, in this context, the structure of the LPV model is fixed a priori. By using the same local models as the ones considered in the blackbox case, LPV physically-structured models are estimated by resorting, again, to the cost function (12). Table 3 gathers the mean of the estimated physical parameters (as well as the real parameter values) by considering the "best" and "median" sets of local models while, in Table 4, I/O fit measurements (see Eq (21)) satisfied by the estimated LPV models (by following the black-box LPV model I/O validation procedure considered previously) can be seen. Notice that the worst set of local models is discarded in this study because this set of local models leads to very bad physical parameter estimations (of magnitude, even sign, far from or in contradiction with the physics of the real system) which can be discriminated easily thanks to the available prior information on the sought physical parameters.

According to the obtained results, it can be concluded that the physical parameters of the system under study are wellestimated based on both the "best" and "median" sets of local models. As far as the time-domain fit measurements defined by Eq (21) are concerned, it can be seen in Table 4 that the estimated gray-box LPV models are able to capture the behavior of the system even when little local information is involved.

Notice that in the gray-box framework, local minima have arisen resulting in 1, maximum 2 (out of 10) erroneous sets of estimated parameters. Again, these local minima problem can be discarded thanks to the available prior knowledge about the magnitude and/or sign of the real parameters. The resulting estimated parameters are indeed a lot out of the range of the real parameter values.

6 Conclusion

In this article, a nu-gap metric-based [33] working point selection algorithm associated with an H_{∞} -norm-based identification method (initially developed in [35]) is introduced for LPV model identification. As shown in the former sections, the proposed technique is able to determine, on-line, a set of the local black-box LTI state-space models along the trajectory of

Tab. 3. Estimated physical parameters based on the best and median sets of local models.

		$p_{min} \rightarrow p_{max}$					$p_{max} \rightarrow p_{min}$				
Parameter	<i>m</i> _{1,2}	k	Κ	f_2	f	<i>m</i> _{1,2}	k	Κ	f_2	f	
Real Value	0.2	10	4	0.04	0.1	0.2	10	4	0.04	0.1	
β	0.3										
Best	0.19	9.96	4.00	0.038	0.1	0.19	10.02	4.01	0.04	0.098	
Median	0.19	9.89	3.98	0.037	0.11	0.2	9.9	4.04	0.039	0.102	
β 0.6											
Best	0.2	10.01	3.93	0.04	0.09	0.19	9.79	3.97	0.039	0.009	
Median	0.19	9.98	3.98	0.039	0.09	0.19	9.88	3.99	0.04	0.101	
β	0.9										
Best	0.21	9.75	4.09	0.039	0.098	0.2	9.83	4.02	0.038	0.098	
Median	0.2	9.86	3.93	0.039	0.10	0.199	9.88	3.92	0.04	0.102	

Tab. 4. I/O BFT measurements of the estimated gray-box LPV models.

	$p_{\min} \rightarrow p_{\max}$			$p_{max} \rightarrow p_{min}$			
β	0.3	0.6	0.9	0.3	0.6	0.9	
Best BFT [%]	97.4	96.8	96.1	97.2	96.9	96.2	
Median BFT [%]	97.4	96.9	96.4	97.1	96.1	96.1	

the scheduling variables, then yields an accurate global LPV model from local experiments by resorting to a behavioral

interpolation procedure, discarding thus the coherent basis problem encountered in the local approach. Contrary to the standard solutions which focus on affine LPV model structures, linear fractional physically-structured and fully-parametrized LPV representations are considered. As illustrated by a simulation example study, this contribution can be seen as a promising solution to the working point selection problem when LPV model identification from local experiments is considered.

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References

- Apkarian P., Noll D., Nonsmooth H_∞ synthesis. IEEE Transactions on Automatic Control, 51 (1), pp. 71–86, (2006). DOI: <u>10.1109/TAC.2005.860290</u>
- Apkarian P., Noll D., Prot O., A trust region spectral bundle method for non-convex eigenvalue optimization. SIAM Journal on Optimization, 19 (1), pp. 281–306, (2008).
 DOI: <u>10.1137/060665191</u>
- 3 Apkarian P., Noll D., Prot O., A proximity control algorithm to minimize nonsmooth and nonconvex semi-infinite maximum eigenvalue functions. Journal of Convex Analysis, 16 (3), pp. 641–666, (2009).
- 4 Arzelier D., Deaconu G., Gumussoy S., Henrion D., *H2 for H1-FOO*. in 'Proceedings of the International Conference on Control and Optimization with Industrial Applications, Ankara, Turkey', August 2011.
- Bamieh B., Giarré L., Identification of linear parameter varying models. International Journal of Robust and Nonlinear Control, 12 (9), pp. 841–853, (2002).
 DOI: <u>10.1002/rnc.706</u>
- 6 Boyd S., El Ghaoui L., Feron E., Balakrishnan V., *Linear matrix inequalities in system and control theory*. Society for Industrial and Applied Mathematics, (1994).

- 7 Casella F., Lovera M., LPV/LFT modelling and identification: overview, synergies and a case study. in 'Proceedings of the IEEE International Conference on Computer-Aided Control Systems, 2008. CACSD 2008. San Antonio, Texas, USA', pp. 852–857 (2008). DOI: 10.1109/CACSD.2008.4627358
- 8 De Caigny J., Pintelon R., Camino J., Swevers J., Interporlated modeling of LPV systems based on observability and controllability. in 'Proceedings of the IFAC Symposium on System Identification, Brussels, Belgium', July 2012.
- Garnier H., Gilson M., Bastogne T., Mensler M., The CONTSID toolbox: a Software Support for Data-based Continuous-time Modelling. in 'Identification of Continuous-time Models from Sampled Data,' Springer Verlag, London, UK, pp. 249–290, 2008.
 DOI: 10.1007/978-1-84800-161-9_9
- Garnier H., Gilson M., Young P. C., Huselstein E., An optimal IV technique for identifying continuous-time transfer function model of multiple input systems. Control Engineering Practice, 15 (4), pp. 471–486, (2007).
 DOI: 10.1016/j.conengprac.2006.09.004
- 11 Garnier H., Wang L., Identification of Continuous-time Models from Sampled Data. Springer Verlag, London, UK, (2008).

- Huang J., Ji G., Zhu Y., van den Bosch P., Identification of multimodel LPV models with two scheduling variables. Journal of Process Control, 22 (7), pp. 1198–1208, (2012).
 DOI: <u>10.1016/j.jprocont.2012.05.006</u>
- 13 Khalate A. A., Bombois X., Tóth R., Babuska R., Optimal experimental design for LPV identification using a local approach. in 'Proceedings of the 15 th IFAC Symposium on System Identification. Saint-Malo, France', 1, pp. 162–167 (2009).
- 14 Laurain V., Tóth R., Gilson M., Garnier H., Direct identification of continuous-time LPV input/output models. IET Control Theory and Applications, 5 (7), pp. 878–888, (2011). DOI: 10.1049/iet-cta.2010.0218
- Lee L. H., Poolla K., Identification of linear parameter-varying systems using non linear programming. Journal of Dynamic Systems, Measurements, and Control, 121 (1), pp. 71–78 (1999).
 DOI: 10.1115/1.2802444
- 16 Ljung L., *System identification: Theory for the user*. Prentice Hall, Upper Saddle River, N. J., USA 2nd edition, (1999).
- 17 Lovera M., Mercère G., Identification for gain scheduling: a balanced subspace approach. in 'Proceedings of the American Control Conference', New York, USA, July 2007. DOI: 10.1109/ACC.2007.4282899
- 18 Mercère G., Regression techniques for subspace-based black-box state-space system identification: an overview. Technical report, Poitiers University, Laboratoire d'Automatique et d'Informatique pour les Systemes, France (2013).

URL: http://dblp.uni-trier.de/db/journals/corr/corr1305. html#abs-1305-7121

- 19 Mercère G., Laroche E., Lovera M., Identification of a flexible robot manipulator using a linear parameter-varying descriptor statespace structure. in 'Proceedings of the 50th IEEE Conference on Decision and Control and European Control Conference, Orlando, Florida, USA, pp. 818–823, (2011). DOI: <u>10.1109/CDC.2011.6160683</u>
- Mercère G., Lovera M., Convergence analysis of instrumental variable recursive subspace identification algorithms. Automatica, 43 (8), pp. 1377–1386, (2007).

DOI: 10.1016/j.automatica.2007.01.014

- 21 Petersson D., A nonlinear optimization approach to H2-optimal modeling and control. PhD thesis, Linköping University, Linköping, Sweden, 2013.
- 22 Petersson D., Löfberg J., Optimization based LPV-approximation of multimodel systems. in 'Proceedings of the European Control Conference', Budapest, Hungary, August 2009.
- 23 Petersson D., Löfberg J., Identification of LPV State-Space Models Using H2 Minimisation. in 'Optimization Based Clearance of Flight Control Laws. (Varga A., Hansson A., Puyou G. eds.)' pp. 111–128, 2012.

DOI: <u>10.1007/978-3-642-22627-4_6</u>

- Petersson D., Löfberg J., Optimisation-based modelling of LPV systems using an H2-objective. International Journal of Control, 87 (8), pp. 1536–1548, (2014).
 DOI: <u>10.1080/00207179.2013.878477</u>
- 25 Shamma J., *Linearization and gain-scheduling*. in 'The control handbook' (ed.: Levine W. S.) CRC Press, pp. 388–396, (1996).

- 26 Shen H., Zhao J., Xu Z., Benchmark operating points determining method of operating trajectory LPV model based on gap metric. in 'Proceedings of the 31st Chinese Control Conference. Hefei, China', pp. 1972–1977, (2012).
- 27 Tanelli M., Ardagna D., Lovera M., On- and off-line model identification for power management of Web service systems. in 'Proceedings of the 47th IEEE Conference on Decision and Control, Cancun, Mexico', pp. 4497–4502 (2008).
 DOI: 10.1109/CDC.2008.4738945
- 28 Tóth R., Identification and Modeling of Linear Parameter-Varying Systems. Series Lecture Notes in Control and Information Sciences, 403, Springer Verlag, Berlin Heidelberg, (2010).
 DOI: 10.1007/978-3-642-13812-6
- 29 Tóth R., Felici F., Heuberger P., van den Hof P., *Discrete-time LPV I/O and state-space representations. differences of behavior and pitfalls of interpolation.* in 'Proceedings of the European Control Conference, Kos, Greece', pp. 5418-5425, (2007).
- Tóth R., Heuberger P. S. C., Van den Hof P. M. J., Asymptotically optimal orthonormal basis functions for LPV system identification. Automatica, 45 (6), pp. 1359–1370, (2009).
 DOI: <u>10.1016/j.automatica.2009.01.010</u>
- 31 Tóth R., Laurain V., Zheng W., Poolla K., Model Structure Learning: A Support Vector Machine Approach for LPV Linear-Regression Models. in 'Proceedings of the Conference on Decision and Control and European Control Conference', Orlando, Florida, USA, December 2011.
- van Wingerden J. W., Verhaegen M., Subspace identification of Bilinear and LPV systems for open- and closed-loop data. Automatica, 45 (2), pp. 372–381, (2009).
 DOI: <u>10.1016/j.automatica.2008.08.015</u>
- Vinnicombe G., Frequency domain uncertainty and the graph topology. IEEE Transactions on Automatic Control, 38 (9), pp. 1371–1383, (1993).

DOI: <u>10.1109/9.237648</u>

- 34 Vinnicombe G., Uncertainty and feedback-H_α, loop-shaping and the v-gap metric. Imperial College Press, (2001).
- **35** Vizer D., Mercère G., Prot O., Laroche E., Lovera M., *Linear fractional LPV model identification from local experiments: an* $H\infty$ -*based optimization technique.* in 'Proceedings of the IEEE 52nd Annual Conference on Decision and Control, Florence, Italy', pp. 4559–4564, (2013).

DOI: <u>10.1109/CDC.2013.6760594</u>

- 36 Vizer D., Mercère G., Prot O., Ramos J., *A local approach framework for black-box and gray-box LPV system identification*. in 'Proceedings of the European Control Conference', Zurich, Switzerland, July 2013.
- 37 Vizer D., Mercère G., Prot O., Laroche E., H∞-norm-based optimization for the identification of gray-box linear time-invariant statespace representations. Submitted to IEEE Transactions on Control Systems Technology.
- 38 Zhao Y., Huang B., Su H., Chu J., Prediction error method for identification of LPV models. Journal of Process Control, 22 (1), pp. 180–193, 2012.

DOI: 10.1016/j.jprocont.2011.09.004

39 Zhou K., Doyle J. C., Glover K., *Robust and optimal control*. Prentice Hall, Upper Saddle River, N.J., USA (1996).