

# A Distributed Voltage Stability Monitoring Method Based on Local Measurements and Line Parameters

Huaichang Ge<sup>1</sup>, Zhijun E<sup>2</sup>, Qinglai Guo<sup>1\*</sup>, Chenghu Gong<sup>2</sup>,  
Hongbin Sun<sup>1</sup>, Bin Wang<sup>1</sup>

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## Abstract

*This paper proposes a distributed real-time voltage stability monitoring method based on local measurements and line parameters. Local PV sensitivity (PVS) is adopted as the indicator of voltage stability. Compared with traditional methods, the proposed method only requires local voltage magnitude, active power flow measurements and the parameters of transmission lines connected to local node. The proposed method will check the operation status of each transmission line first and thus formulate the calculation expression of the voltage stability index. Then we will calculate the PVS accordingly. And we have analyzed the uncertainty of PVS led by the measurement errors. Simulation studies using IEEE 39 nodes system will be introduced and used to demonstrate the effectiveness of the proposed method.*

## Keywords

*local voltage stability monitoring, PV sensitivity, measurement uncertainty*

## 1 Introduction

Voltage stability is one of the most severe stability issues of large-scale power systems, which has induced several well-known blackouts such as July 2, 1996 in western American power grid, August 14th 2003 in Northeastern American power grid and so on. With respect to time scale, voltage stability can be categorized into long-term voltage stability and short-term voltage stability [1, 2]. In this paper, we focus on the problem of long term voltage stability. Long-term voltage stability concerns about the steady state operation of power system, which involves the capability of transferring enough energy to the load side. If the load demand is beyond the upper limit of transmitted energy, long-term voltage instability will happen. With the rapid development of power systems, the electric distance between generators and loads becomes larger and the problem is extremely serious at the receiving end. So voltage instability detection and control is a serious problem to ensure the reliability of power system.

However, direct monitoring the voltage magnitude is not an effective way to detect the voltage instability. So the researchers have been motivated to study on the effective monitoring method of voltage instability. There are a lot of researches on the evaluation of voltage stability margin, the closeness from current operation point to the collapse point. Many of the voltage stability monitoring methods are based on power flow equations, such as voltage stability index based on sensitivity analysis [3, 4], continuous power flow method [5, 6], and minimum singular value of Jacobian Matrix [7, 8] and so on. Those approaches are based on the model of the entire power system. The topological information and real-time measurements are required for these methods. Besides, they may come up with several numerical difficulties. The methods have to be completed in the control center.

On the other hand, some of the researches are based on the Thevenin equivalent model. Such approaches are based on Thevenin equivalent network behind a load bus [9, 10]. The Thevenin equivalent network is estimated based on real time measurements. According to the impedance match principle, the transmitted power reaches its maximum when absolute value of

<sup>1</sup> Department of Electrical Engineering, State Key Laboratory of Power Systems, Tsinghua University, Beijing 100084, China

<sup>2</sup> State Grid Tianjin Electric Power Corporation, Tianjin 300010, China

\* Corresponding author, e-mail: [guoqinglai@tsinghua.edu.cn](mailto:guoqinglai@tsinghua.edu.cn)

the load side equivalent impedance is equal with that of the network side equivalent impedance. This method doesn't need the model of power system, and only the phasor measurements are required for the parameter estimation. So it could be distributed in the substation level. However, due to its simplicity, the accuracy of the algorithms can't be guaranteed. Many researches and implements have been done to improve the methods. [11]

As is introduced above, the operation pattern of voltage stability monitoring can be categorized as centralized and distributed. The control action against voltage instability of power system is under voltage load shedding (UVLS) scheme [12, 13]. It is completed in the substation level. With respect to the centralized voltage stability monitoring methods, the results don't have direct guiding significance to the operation of UVLS. UVLS still operates according to local voltage measurements. In order to combine the functionality of voltage stability monitoring and control, we focus on the distributed voltage stability monitoring methods and overcome the problem of inaccuracy caused by simplicity of system model.

In this paper, we will introduce a distributed PV sensitivity calculation method for voltage stability monitoring. As is introduced above, traditional PV sensitivity calculation must be conducted in the control center due to the need of system model. The proposed distributed PV sensitivity calculation method is based on local measurements and line parameters of the transmission lines connected to the monitored node. The conduction of the proposed method could be distributed in substations. And it doesn't rely on the simplification of the topology of power system compared with Thevenin equivalent model. The rest of the paper is organized as follows. Section 2 introduces the algorithm of the proposed distributed voltage stability monitoring method and the uncertainty analysis of the proposed method. The simulation work is presented in Section 3 and the conclusions are drawn in Section 4.

## 2 Distributed voltage stability monitoring method and uncertainty analysis

In this section, we will introduce a distributed voltage stability monitoring method. The proposed method contains two parts: the calculation method of PV sensitivity and the uncertainty analysis of the proposed method.

### 2.1 PV Sensitivity Calculation Method

In this paper, PV sensitivity is adopted as the indicator of voltage stability condition. PV sensitivity is the sensitivity of local active power consumption with respect to local voltage measurement. The voltage magnitude will decrease along with the increase of power consumption. At the normal operation point, the absolute value of PV sensitivity is a negative number which implies that the increase of power consumption will cause some voltage decline. However, when the power system operates close to the voltage collapse point, the absolute value

of PV sensitivity will be very close to zero which means that slight increase of power consumption will cause large voltage decline. So real time monitoring of the value of PV sensitivity is able to judge the voltage stability status. Traditional PV sensitivity calculation is based on the measurements and topology of the entire power system at the control center, which can't be distributed. In this part, we will introduce a distributed PV sensitivity (PVS) calculation method.

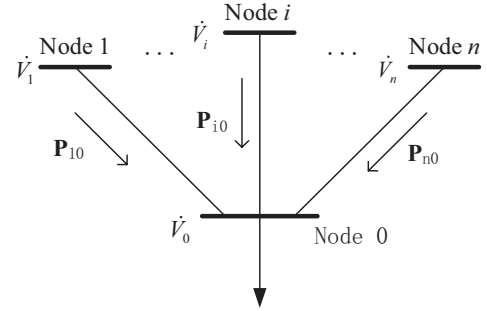


Fig. 1 Simplified power system model

The Thevenin equivalent model is only composed of an equivalent electromotive force and an equivalent impedance. It is the simplification of the actual power system, so it will lead to the inaccuracy of the calculation result. In this paper, the monitored node is modeled according to its actual operation status.

Figure 1 shows a diagram of system model with the monitored node 0. The local monitored node 0 is connected with  $n$  nodes by transmission lines. The voltage phasor of node  $i$  ( $i = 0, 1, 2, \dots, n$ ) is expressed as  $\dot{V}_i$  and the current phasor in transmission line  $i-0$  ( $i = 0, 1, 2, \dots, n$ ) is expressed as  $\dot{I}_{i0}$ . The admittance of each transmission line  $i-0$  is expressed as Equation (1).

$$y_{i0} = g_{i0} + jb_{i0}, \quad y_{i0} = -Y_{i0}, \quad Y_{i0} = G_{i0} + jB_{i0} \quad (1)$$

Where  $Y_{i0}$  is the corresponding element of the admittance matrix. The apparent power transmitted from node  $i$  to node 0,  $S_{i0}$ , can be obtained according to Equation (2).

$$S_{i0} = \dot{V}_0 \cdot \hat{I}_{i0} = P_{i0} + jQ_{i0} \quad (2)$$

Where  $\hat{I}_{i0}$  is the conjugation of  $\dot{I}_{i0}$ ,  $P_{i0}$  is the active power transmitted from node  $i$  to node 0 and  $Q_{i0}$  is the reactive power transmitted from node  $i$  to node 0. The following equations could be derived.

$$\dot{I}_{i0} = y_{i0} \cdot (\dot{V}_i - \dot{V}_0) \quad (3)$$

$$\begin{aligned} P_{i0} - jQ_{i0} &= \dot{V}_0 \cdot (g_{i0} + jb_{i0}) \cdot (\dot{V}_i - \dot{V}_0) \\ &= \dot{V}_0 \cdot (g_{i0}\dot{V}_i - g_{i0}\dot{V}_0 + jb_{i0}\dot{V}_i - jb_{i0}\dot{V}_0) \end{aligned} \quad (4)$$

Assuming that the reference node is node 0, so the phasors can be expressed as follows.

$$\begin{aligned} \dot{V}_0 &= V_0 \angle 0 \\ \dot{V}_i &= V_i \angle \theta_i \end{aligned} \quad (5)$$

Where  $V_i$  and  $\theta_i$  are the magnitude and angle of voltage phasor  $\dot{V}_i$ . The expression of  $\mathbf{P}_{i0}$  and  $\mathbf{Q}_{i0}$  is shown in the following equations.

$$\mathbf{P}_{i0} = V_i V_0 (g_{i0} \cos \theta_i - b_{i0} \sin \theta_i) - g_{i0} V_0^2 \quad (6)$$

$$\mathbf{Q}_{i0} = -V_i V_0 (g_{i0} \sin \theta_i + b_{i0} \cos \theta_i) + b_{i0} V_0^2 \quad (7)$$

In the same way, we can get the expression of local active power consumption  $\mathbf{P}_0$  and reactive power consumption  $\mathbf{Q}_0$ :

$$\mathbf{P}_0 = V_0 \sum_{i=1,2,\dots,n} V_i (g_{i0} \cos \theta_i - b_{i0} \sin \theta_i) - V_0^2 G_{0-0} \quad (8)$$

$$\mathbf{Q}_0 = V_0 \sum_{i=1,2,\dots,n} V_i (g_{i0} \sin \theta_i + b_{i0} \cos \theta_i) - V_0^2 B_{0-0} \quad (9)$$

The sensitivity of  $\mathbf{P}_0$  or  $\mathbf{Q}_0$  with respect to the voltage magnitude  $V_0$  can be expressed as the following equations.

$$\frac{\partial \mathbf{P}_0}{\partial V_0} = \sum_{i=1,2,\dots,n} V_i \cdot (g_{i0} \cos \theta_i - b_{i0} \sin \theta_i) - 2V_0 G_{0-0} \quad (10)$$

$$\frac{\partial \mathbf{Q}_0}{\partial V_0} = \sum_{i=1,2,\dots,n} V_i \cdot (g_{i0} \sin \theta_i + b_{i0} \cos \theta_i) - 2V_0 B_{0-0} \quad (11)$$

In Equation (10) and (11), PVS could be calculated with the awareness of line parameters and the measurements of different nodes. To the local nodes, the voltage magnitude and angle of other nodes isn't accessible. This is the limitation of traditional PVS calculation method.

However, at the substation level, not only the voltage magnitude is accessible, the power flow in the transmission lines can also be measured. According to Equation (6) and (7), the expressions of power flow in transmission lines also contain the voltage phasor information and transmission line parameters. PVS could be expressed by the power flow measurements in transmission lines. The following equations could be derived from Equation (6) and (7).

$$V_i \cdot (g_{i0} \cos \theta_i - b_{i0} \sin \theta_i) = \frac{\mathbf{P}_{i0}}{V_0} + g_{i0} V_0 \quad (12)$$

$$V_i \cdot (g_{i0} \sin \theta_i + b_{i0} \cos \theta_i) = -\frac{\mathbf{Q}_{i0}}{V_0} + b_{i0} V_0 \quad (13)$$

Substitute Equation (12) and (13) into Equation (10) and (11) respectively, it could be derived that:

$$\frac{\partial \mathbf{P}_0}{\partial V_0} = \sum_{i=1,2,\dots,n} \left( \frac{\mathbf{P}_{i0}}{V_0} + g_{i0} V_0 \right) - 2V_0 G_{00} \quad (14)$$

$$\frac{\partial \mathbf{Q}_0}{\partial V_0} = \sum_{i=1,2,\dots,n} \left( -\frac{\mathbf{Q}_{i0}}{V_0} + b_{i0} V_0 \right) - 2V_0 B_{00} \quad (15)$$

According to Equation (14), only the local measurements and line parameters are needed for the calculation of PV sensitivity. Besides, the calculation process is very concise and it will not

come up with numerical problems, which makes the distributed real time application of the proposed method practical.

In the real-time operation of power system, the voltage stability problem may supervene with topological change of power system. If the transmission line connected to the monitored node is out of work, the transmitted active power through this transmission line is zero. However we couldn't directly substitute 0 into Equation (14) and assume that the topology doesn't change at all. The calculation expression of the proposed PVS index should be reformulated. The corresponding transmission line should be rejected. Assuming that the tripped line is n-0, if we don't correct the expression, the value of PVS could be expressed as the following equation.

$$\begin{aligned} \frac{\partial \mathbf{P}_0}{\partial V_0} &= \sum_{i=1,2,\dots,n-1} \left( \frac{\mathbf{P}_{i0}}{V_0} + g_{i0} V_0 \right) + (0 + g_{n0} V_0) - 2V_0 G_{00} \\ &= \sum_{i=1,2,\dots,n-1} \left( \frac{\mathbf{P}_{i0}}{V_0} + g_{i0} V_0 \right) - (2V_0 G'_{00} + g_{n0} V_0) \end{aligned} \quad (16)$$

Where  $G'_{00}$  is the corresponding element of Y matrix after the topological change. As is shown in Equation (16), there will be an error term,  $g_{n0} V$ , in the equation. The reason for the appearance of the error term is that if the transmission line is at work and there is no power flow in it, the voltage phasor of both ends of the transmission line are the same. Apparently, such a condition could not be satisfied when the transmission line is actually out of work. So the error term is due to the violence of the implicit equal-voltage condition.

So we should check the operation status of each transmission line and formulate the calculation expression of the proposed index before we calculate the value of the proposed index. If the power flow in some transmission line is zero, we assume that the transmission line is out of work. The rationality of the above assumption is that in the actual operation of power systems the equal-voltage condition is hard to fulfil. Besides the voltage instability always happens at the heavy load condition, which means that the line power flow in the transmission lines must be very heavy.

## 2.2 Uncertainty Analysis of PV sensitivity

Compared with the centralized PV sensitivity calculation methods, the utilized measurements data could be the results of state estimation. So the errors of measurements can be eliminated as much as possible. However, with respect to the proposed distributed PVS, the utilized measurements do not go through the process of state estimation. So the uncertainties brought by instrumental errors should be considered. In this paper, we will analyse the uncertainty of the calculation result of PVS.

Assuming that the uncertainties of the voltage measurement devices are  $\sigma_{V_0}$  and the line active power flow is  $\sigma_{\mathbf{P}_{i0}}$ . The above parameters are determined by the instrumental errors. According to the law of error propagation, the uncertainty index of PVS,  $\sigma_{PVS}$ , can be expressed as the following equation.

$$\sigma_{PVS} = \sqrt{\left(\frac{\partial PVS}{\partial V_0} \cdot \sigma_{V_0}\right)^2 + \sum_{i=1, \dots, n} \left(\frac{\partial PVS}{\partial \mathbf{P}_{i0}} \cdot \sigma_{\mathbf{P}_{i0}}\right)^2} \quad (17)$$

According to Equation (14), the following expressions could be obtained.

$$\frac{\partial PVS}{\partial V_0} = 2G_0 - \sum_{i=1, \dots, n} \left( g_{i0} - \frac{\mathbf{P}_{i0}}{V_0^2} \right) \quad (18)$$

$$\frac{\partial PVS}{\partial \mathbf{P}_{i0}} = -\frac{1}{V_0} \quad (19)$$

Substitute Equation (18) and (19) into Equation (17), we could get the following equation.

$$\sigma_{QVS} = \sqrt{\left(2G_0 - \sum_{i=1, \dots, n} \left( g_{i0} - \frac{\mathbf{P}_{i0}}{V_0^2} \right)\right)^2 \cdot \sigma_{V_0}^2 + \sum_{i=1, \dots, n} \frac{\sigma_{\mathbf{P}_i}^2}{V_0^2}} \quad (20)$$

Equation (20) has shown the expression of the uncertainty of the index PVS. According to Equation (20), it could be discovered that the uncertainty isn't constant during the change of power system. When the operation condition of power system is close to the collapse point, the local voltage should be lower and the transmitted active power should be higher, which makes  $\sigma_{QVS}$  larger. However, for the detection of voltage instability, such an operation condition is in the most important interval. So in order to ensure the robustness of the proposed method, we should set up a security threshold of voltage instability to prevent the wrong judgement of the voltage stability.

### 2.3 Flowchart of the whole algorithm

The flowchart of the whole algorithm is shown in Table 1.

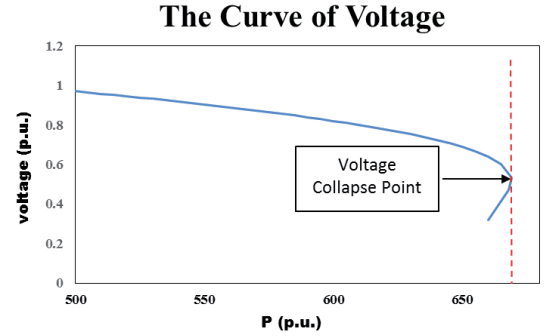
**Table 1** The Flowchart Of The Proposed Algorithm

| Algorithm: Distributed Voltage Stability Monitoring Method |  |
|--|--|
| <b>Step 1:</b>   | Obtain the transmission line parameters and real-time measurements;  |
| <b>Step 2:</b>   | Check the operation status of transmission lines and formulate the calculation expression of $PVS$ ;         |
| <b>Step 3:</b>   | Calculate the real-time $PVS$ accordingly;   |
| <b>Step 4:</b>   | Calculate the uncertainty index of $PVS$ ;   |
| <b>Step 5:</b>   | Judge the voltage stability condition and transfer the result to the local voltage stability control module. |

### 3 Case study

IEEE 39 bus system is used to investigate the performance of the proposed methods. As the increase of all the load demand in the system, the voltage magnitude will decrease gradually. Figure 2 has shown the curve of voltage magnitude of node 8 in the tested system. As is shown in the figure, there is a limit

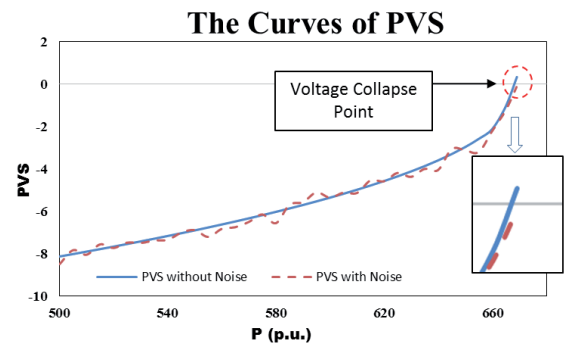
of the increase of the load demand and the voltage magnitude is very low at the limit point. If the power consumption is beyond the limit, the voltage will collapse and the long-term voltage stability will happen. In this part, we will use the proposed voltage stability monitoring method to detect the voltage instability.



**Fig. 2** The curve of voltage magnitude

In Figure 3, there are two curves of PVS. One of them is calculated without the perturbation of noise and the other one has considered the noise. According to both of the curves, along with the decline of voltage, the value of PVS grows gradually. And the value reaches close to zero at the voltage collapse point.

However, at the enlarged view of the curves, it could be found that the curve of PVS without noise has exceeded zero but the one with noise hasn't. The perturbation of noise has interference the judgement of voltage instability. In order to enhance the robustness of the proposed method, we should set up a security threshold to eliminate the disturbance of noises. In Figure 4, we have shown the uncertainty index of PVS. As is shown in the figure, the uncertainty index increases during the process. The value of uncertainty index reaches 4 times of the starting point at the voltage collapse point.



**Fig. 3** The curves of PVS

Another scenario is that one of the transmission lines is tripped off and the voltage magnitude will drop suddenly. The curve of voltage is shown in Figure 5. And the curves of PVS is shown in Figure 6. There are two curves in the figure. If we detect the topological change immediately after the fault of transmission line, the value of PVS will be the blue curve.



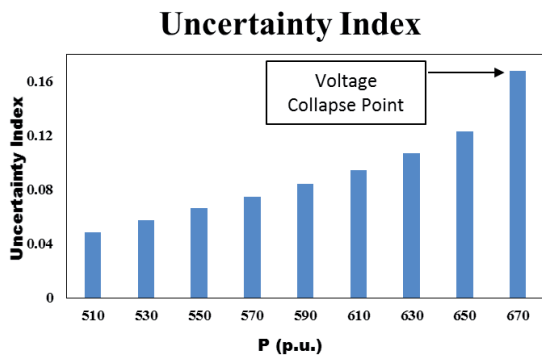


Fig. 4 The curve of uncertainty index

Or the value of PVS will be the red one. It could be found from the figure that there is an error between the curves, but as the operation point moves closer to the voltage collapse point, the error becomes smaller. According to Equation (16), the error term is  $g_{n0}V_0$ . During the simulation process,  $g_{n0}$  stays constant and  $V_0$  keeps decreasing. So the value of error term becomes smaller during the process.

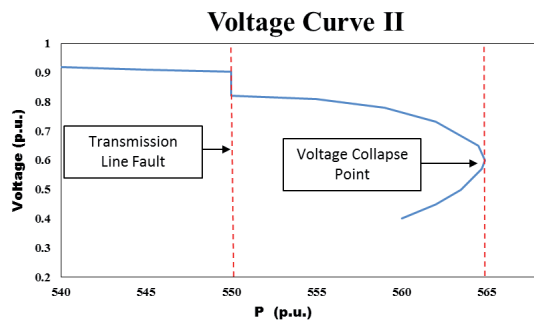


Fig. 5 The curve of voltage magnitude II

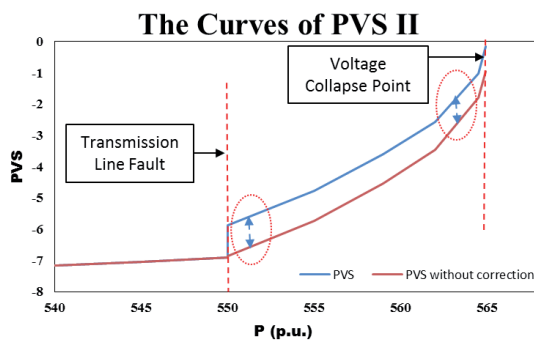


Fig. 6 The curves of PVS II

## 4 Conclusion

In this paper, we have introduced a distributed voltage stability monitoring method. First, we proposed a PVS calculation method based on local measurements and transmission line parameters. Then we analyzed the uncertainty of PVS led by the errors of measurements. The effectiveness of the proposed methods is tested by simulation work on the IEEE 39 nodes system.

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