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RESEARCH ARTICLE

Comparison of Methods for Calculation of Core-Form Power Transformer's Core Temperature Rise

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Abstract

The temperature rise calculation of core-type power transformers is an essential question in the design process however this is neglected during the preliminary optimization process. The methods of the calcula- tion methods are classified and introduced shortly and they applicability is examined for the preliminary design process. For comparison of the methods performance, two core arrangements were examined, the Roth's transformer and a 50 MVA three-phase transformers' core. The result shows that the FEM method provides the most accurate solution however according to its simplicity, the Ryder's method has better performance in the preliminary design stage.

Keywords

Power transformer, Optimization, Finite Element Method

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1 Introduction

The first step in a transformer design process (Fig. 1 illustrates this process [1, 2]) is to obtain the main dimensions of the most economic transformer design. It is widely accepted that the core geometry, and the flux density in the columns determine these main parameters [3, 4]. Therefore a fast and accurate calculation method of the core parameters is necessary, to achieve these optimal design variables from thousands of possible solutions.

The modern core design process is based not only on the fulfillment of the electrical requirements, but also necessary to limit the core temperature to prevent the components and the oil from damage itself. For larger cores with a diameter of approx. 0.6 m or more [5], cooling ducts in different arrangements (Fig. 2) are necessary to be applied to limit the surface and the maximum core temperature. Generally, the accepted value of this temperature difference between the core interior (maximum) and the surface is about 15 to 20 °C [4, 6–9]. This requires an accurate method to determine the temperature profile in these complex shaped core layout.This problem can be solved precisely by using three-dimensional finite element method with the anisotropic thermal material properties taken into account. These algorithms have a crucial role in the final design validation (Fig. 1), but the usage of these methods in the early design stage when not every key parameter is known, is very laborious and inaccurate [7, 8]. Due to the importance of the core heating problem many researchers proposed various methods from the beginning of the last century. However, most of these previous techniques do not consider the exact core geometry, the anisotropies or the differences in the boundary layer as well as the FEM based calculations. But some of them are very simple and fast which allows to use them in the optimization stage (Fig. 1) to obtain the most economical design parameters.

Several publications have appeared in recent years documenting a bibliographic analysis in the field of transformer design [10–13]. However, most of the previous studies do not take into account the core heat generation problem in the preliminary stage of the transformer design. This paper reviews the earlier work concerned the calculation and modeling of heat

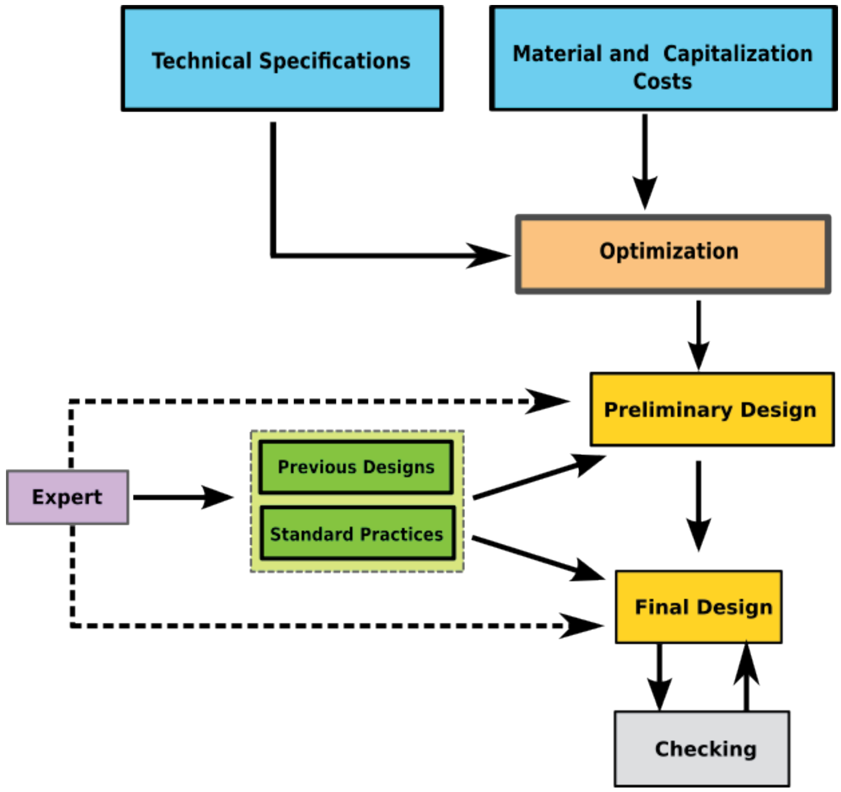

Fig. 1 Position of the optimization method in a design process [2]

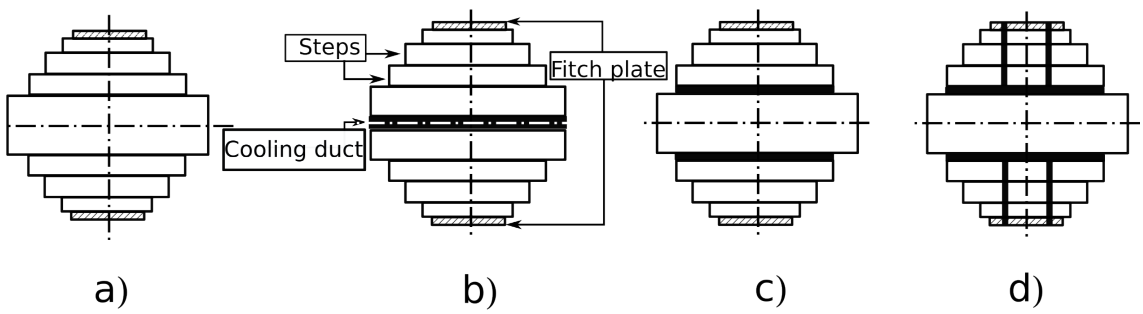

Fig. 2 Typical cooling duct arrangements.

flow in anisotropic solids and compares they performance on two examples with a measurement and a FEM based calculation.

2 Methods

2.1 Thermal Resistances

This is one of the oldest and the latest approach for the core temperature calculation, founded in the literature. This method firstly proposed by Gotter [14] and lastly mentioned by Ryder who demonstrated that this method with some modifications can be used accurately in the optimization process [4, 5, 8, 15]. Using the electrical analogy, the losses distributed through the core may be replaced by a point source laying at the center of the considered part. The core material and the cooling liquid boundary can be represented as separate thermal resistances (illustrated in Fig. 3). Because of the anisotropy of the heat conductivity in the laminated core material, the maximum core temperature is calculated by the resultant resistance of the two parallel connected branches. The method replaces the examined part of the core geometry by an equivalent rectangle (illustrated on Fig. 4) where the direction of the heat-flow is possible in the longitudinal or in the transversal directions only (in Fig. 3).

This means that a 2 x 1 dimension approximation of the two-dimensional Poisson-equation of the steady-state heat flow can be used due to an arbitrary core layout with 10 % precision due to the effective rectangle normalization [4, 5, 15]. We can derive the thermal resistance in the core from Fourier's law to find the temperature distribution within this block. From this, the following equation describes the thermal resistance (illustrated on Fig. 4) in the core is:

$$R_{L_{iron}} = \frac{2b}{8 \cdot k_L \cdot a}, \tag{1}$$

$$R_{T_{iron}} = \frac{2a}{8 \cdot k_T \cdot b}, \tag{2}$$

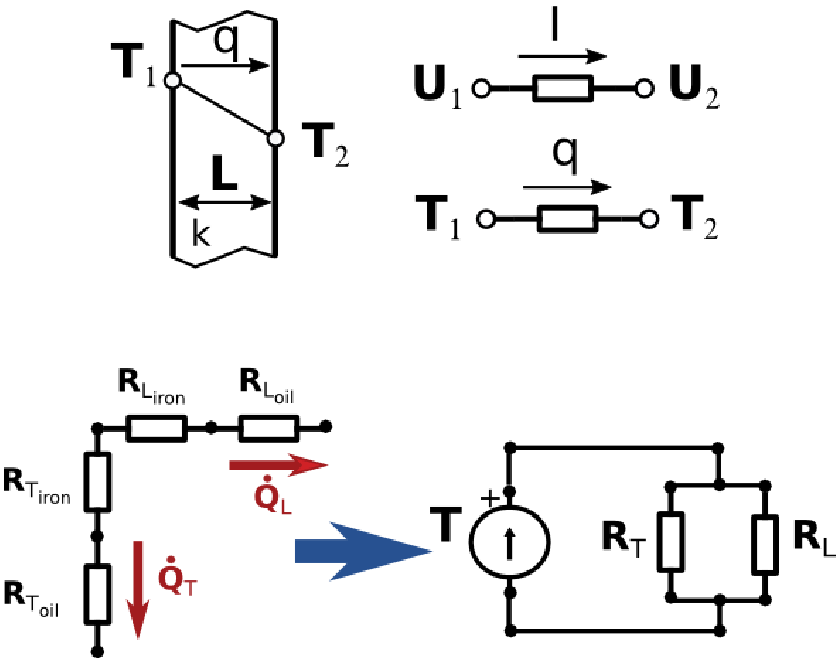

Fig. 3 Shows the electrical analogy and Gotter's thermal resistance model [14]

where k_L and k_T means the heat conductivity in the co-laminar and the trans-laminar dimensions in [W/m °C], a and b are the width and height dimensions of the equivalent rectangle (Fig. 4) [8, 15].

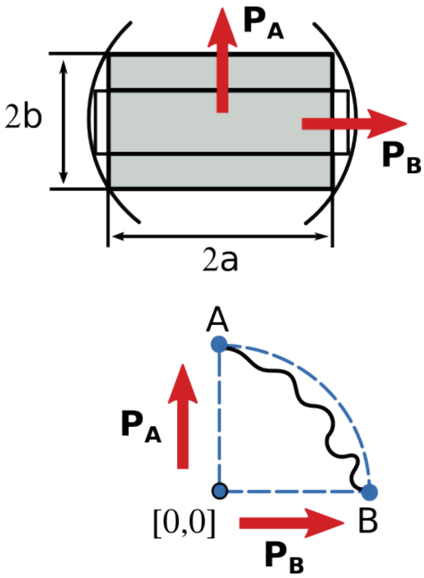

Fig. 4 The effective rectangle for an arbitrary core part geometry for anisotropic thermal calculations. The core parts bounded by cooling ducts or oil.

The thermal resistance at the oil boundary layer depends on the heat transfer rate, oil temperature, the type of the oil and the duct size. Experimental evidence of the temperature-dissipation relationship for surfaces immersed in transformer oil has been given e.g. by Taylor [16]. Ryder derived an expression from the oil properties to calculate the oil boundary layer thermal resistance. In the case of natural oil flow, he found the following simple expression [8]:

$$R_{oil} = \frac{1}{0.1029}\left(\frac{\beta \cdot \rho_L^2 \cdot \Theta \cdot c_P \cdot g \cdot k_L^2}{\mu}\right)^{-1/3}, \tag{3}$$

where β is the cubic expansivity of the oil [1/K], ρ_L is the density of the cooling liquid [kg/m^3], Θ is the temperature gradient in the oil boundary layer in [K], c_P is the specific heat capacity of constant pressure [J/kg/K], g is the acceleration due to gravity in [m/s^2] and k_L the thermal conductivity of the cooling liquid [W/m/K]. The surface heat conductivity can be derived from here by the Fourier's law. Another approach derives the surface heat conductivity (h) in a much more complicated way from the Grasshof and Prandtl numbers [4, 17].

So the formula that describes the thermal resistance on Fig. 3 in the oil boundary layer is as follows:

$$R_{L_{oil}} = \frac{1}{2 \cdot h_L \cdot a}, \tag{4}$$

$$R_{T_{oil}} = \frac{2a}{8 \cdot h_T \cdot b}, \tag{5}$$

where h_L and h_T describes the core surface thermal conductivity in [W/m^2/K]. The maximum temperature rise in the center of the core, on the plate edges and surfaces can be calculated according Ohm's and Kirchoff 's laws [4, 5, 15]. Because of the step-lap, additional losses are generated in the transformer joints [15, 18]. As Ryder showed, the performance of this method can be significantly improved if the temperature rises are calculated to the L- and T-joints separately.

2.2 Analytical solution of the Poisson-equation

Weh and Delvecchio [17, 19] mentioned an analytical solution for the two-dimensional problem of the temperature distribution in circular cores without ducts. The solution is derived for the case of uniform heat generation and oil temperature, and uniform thermal resistance around the core circumference. This method is an approximative solution of the steady-state heat conduction, Poisson equation [17]:

$$\Delta T = -\frac{q_v}{\vec{k}}, \tag{6}$$

where T in [K] is the temperature, q_v is the loss per unit volume in [W/m^3], and $\vec{k}$ is the heat conductivity vector in [W/m/K]. Because the core is made of thin insulated electrical steel sheets, it has an anisotropy in the thermal conductivity. For the simplicity q_v is a constant in this calculation. So in rectangular coordinates the Eq. (6) is in this situation:

$$k_x \frac{\partial^2 T}{\partial x^2} + k_y \frac{\partial^2 T}{\partial y^2} + q_v = 0, \tag{7}$$

where the thermal conductivities are different in the two directions. The solution can be written in the next form:

$$T = A + B \cdot x + C \cdot x^2 + D \cdot y + E \cdot y^2, \tag{8}$$

which satisfies the next boundary conditions:

- $$\frac{\partial T}{\partial x} = \frac{\partial T}{\partial y} \tag{9}$$

  at the center of the rectangle, this is required by the symmetry.

- $$-k_x \cdot \frac{\partial T}{\partial x} = h \cdot (T - T_{oil}) \tag{10}$$

  at $x = \mathrm{a}$, $y = 0$.

- $$-k_y \cdot \frac{\partial T}{\partial y} = h \cdot (T - T_{oil}) \tag{11}$$

  at $x = 0$, $y = \mathrm{b}$. These equations mean that the convective boundary conditions are satisfied exactly at the surface points of the axes. This means that the calculated area is not exactly a rectangle. A circle shape or an arbitrary shape, e.g. a stepped core also can be used. (Fig. 4).

$$\alpha = \frac{h \cdot a}{2k_x}, \tag{12}$$

$$\beta = \frac{h \cdot b}{2k_y}, \tag{13}$$

$$T(x,y) - T_{oil} = \frac{q_v}{h} \cdot \frac{1 - \frac{\alpha}{\alpha+1} \cdot (x/a)^2 - \frac{\beta}{1+\beta} \cdot (y/b)^2}{1/\left(a(1+\alpha)\right) + 1/\left(b(1+\beta)\right)} \tag{14}$$

2.3 Boundary valued problems - Higgins' Method

This method [20] is a boundary-value problem in the mathematical theory of the heat. Similar formulas derived by [21] which was the first exact solution of the two dimensional heat conduction problem. This formula used for laminated, rectangle shaped cores (like Fig. 5). The basis of the solution is a method of separation of variables. This method works with non-linear boundary conditions, too. Buchholz and Roth [22, 23] made some improvements and further restrictions on boundary conditions. The Higgins' formula is more general than these predecessors, it can encompass such practically cases when a cooling duct is applied inside the core. It covers non-uniform heat-generation with different thermal resistances an all sides for the restricted case of uniform oil temperatures for a two dimensional problem. Higgins' formula [20] to calculate the maximum of the two dimensional core heat problem is the following:

$$T_{\max} = 4C'' \cdot \sum_{i=1}^{\infty} \sin \frac{m_i a}{m_i^2 \left(2m_i a + \sin(2m_i a)\right)} \cdot \left(1 - \left(\cosh(n_1 b) + K_3 n_i \sinh(n_i b)\right)^{-1}\right) \tag{15}$$

where $T_{\max}$ is the maximum temperature drop in [K], k is the ratio of the thermal conductivities $k = k_y / k_x$, $C'' = q_v / k, m_i$ and n_i is calculated from the roots of the next two equations (Fig. 6):

$$\tan(m_i a) = \frac{1}{K_i m_i} \tag{16}$$

$$n_i = m_i / k^{0.5} \tag{17}$$

This formula is applicable to determine the temperature distribution in windings, bus bars of rectangular cross sections, too.

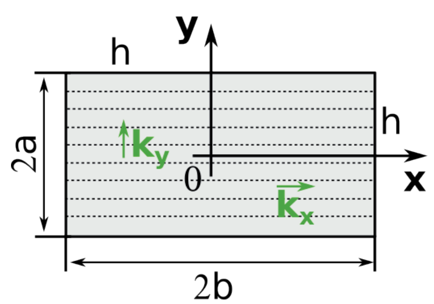

Fig. 5 Rectangular core geometry for anisotropic thermal calculations.

2.4 Finite Difference and Finite Element Method

Using the advanced 3D simulation techniques is essential for the verification of some critical aspects of design and performance parameters of large power transformers. After some simplification they can be used effectively in the everyday transformer design. Hence there are a lot of FEM and FD based solution found in the literature for core loss calculation. But few of them pay attention to the thermal aspects in the core temperature rise [13, 18, 24–28]. This heat transfer problem can be solved by using three-dimensional finite element or finite difference thermal formulations with the anisotropic thermal material properties taken into account [7, 28–31]. This FEM based methods has shown very good accuracy in the range of 2 °C for most of the cases with a greater accuracy than the original empirical methods [29]. These simplified, 2D methods are appropriate for an everyday transformer design practice, but they can be inaccurate when this check should be made in the preliminary design process (Fig. 1). However, this study [29] does not take into account pre-existing, new empirical solutions such as Ryder's model [8], which gives similar accuracy with much more simple method.

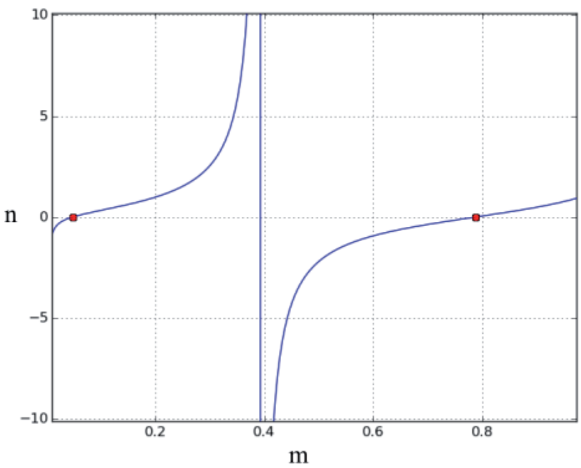

Fig. 6 The first two roots of the Eq. (16).

In this paper we examined two-dimensional FEM and FD methods to calculate the arbitrary shaped temperature distribution. An Ansys model (as seen on Fig. 7 or Fig. 8) was created with third type boundary conditions to model the heat convection [32]. A FD method with equidistant grid is also implemented [33] and compared to the previously showed methods and a measurement.

2.5 Method of Functional Approximation

This method solved accurately the temperature in an arbitrary shaped geometry, this method assumes that the boundary condition is linear and the physical properties of the core are independent from the temperature. The basis of this method is a transformation of the Poisson-equation of the heat conduction into Laplace's equation. The coefficients of these functions are evaluated iteratively by using the boundary condition at several points on the boundary [5].

2.6 Electrical Analog Methods

Kayan [34] used a metalized conducting paper to determine of two-dimensional equipotential patterns of Poissonian fields. These fields created by uniformly or non-uniformly distributed sources for steady state or transient heat-flow conditions. Simmons [35] extended this method by feeding in currents at multiple points on the surface. The physical properties and the surface heat transfer coefficient cannot be a function of time or temperature within the limits of the problem. Time dependent source functions of the exponential type may be simulated by this method. Potential measurements are converted to temperatures by use of a standard resistance.

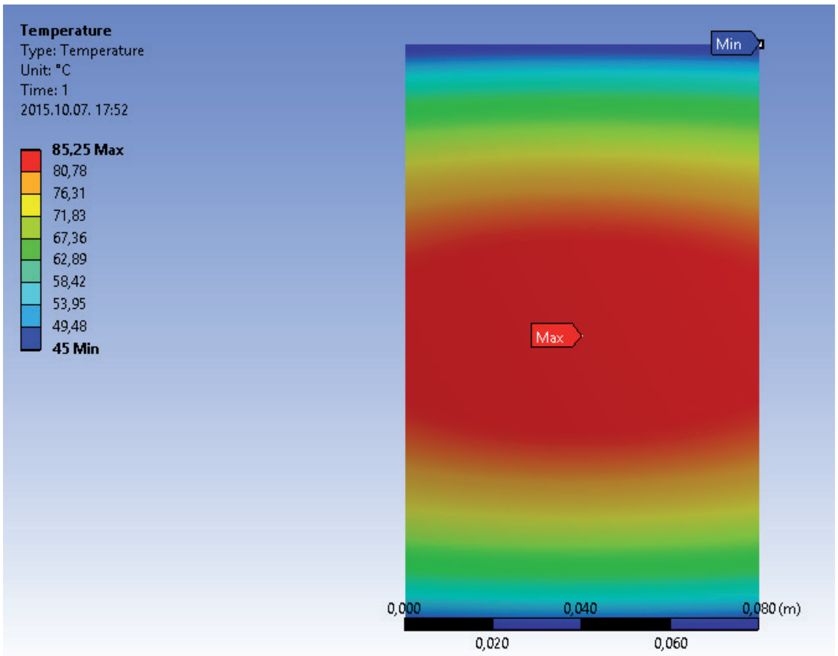

Fig. 7 2D finite element model from Roth's transformer.

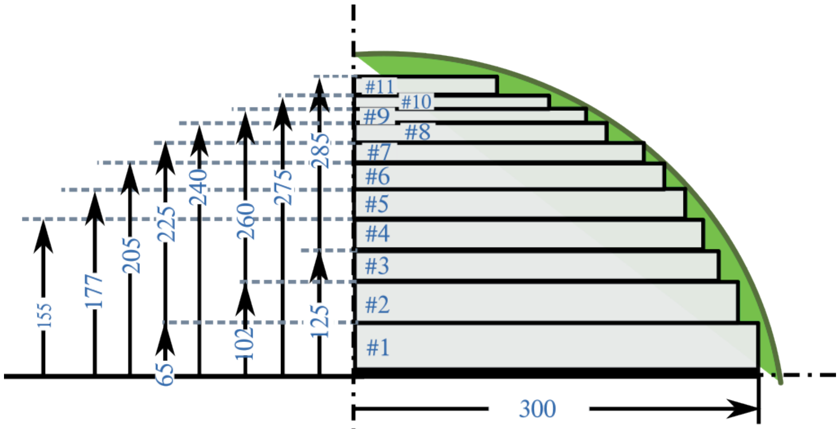

Fig. 8 Quarter of the of the examined transformers' core cross section.

Birke and Palmer described a method, which uses modified electrical conducting paper [5, 36], the paper could be made orthotropic with the electrical conductivity ratio made equal to the ratio of the principal thermal conductivities of the laminated iron core. The heat transfer from iron surfaces to oil at the boundaries could clearly be simulated by Kayan's technique of using extension strips of length. Besides being laborious, this procedure made it difficult to maintain the ratio constant within reasonable limits.

3 Comparison of the Selected Methods

To compare the performance of solution techniques, which are discussed in the previous section, five different methods from each group were selected and analysed on two different core temperature calculation problems. The first one is a temperature rise in a single phase transformer tested and measured by Roth [22]. The second comparison was executed on a leg (Fig. 8) of a modern three phase, 50 MVA power transformer's core (Fig. 9).

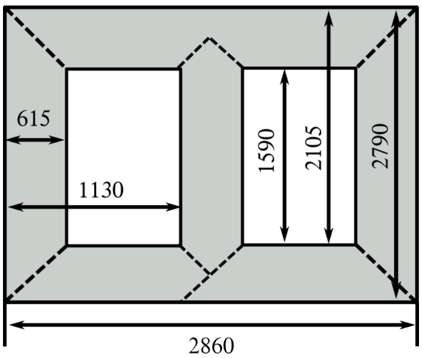

Fig. 9 Schematic view of the dimensions of the examined 3 phase 3 legged transformer core.

3.1 Roth's transformer

Roth measured a single-phase transformer with rectangular cross section [22, 37], the width of the rectangle is $2a = 8$ cm, and the height of the rectangle is $2b = 13.5$ cm, the laminations are parallel to edges $2a$. The measured maximum temperature is $T_{max} = 85$ °C above temperature of ambient medium, which is air. The core loss is $q = 0.0678$ W/cm^3. It is to be expected that in this instance no appreciable error is introduced by the neglect of non-uniformity of core loss due to temperature distribution. Small error is introduced because the non-uniformity of the magnetic field and eddy current losses in the transformer core are neglected. The heat conductivity in the x direction is (parallel to a) $k_x = 0.32$ W/cm/K; similarly for direction y $k_y = 0.005$ W/cm/K.

The results calculated with the five different and described methods can be seen in Table 1 for comparison. The result calculated with the Ansys model is shown on Fig. 7.

Table 1 Comparison of the calculation results on Roth's transformer core (degrees in Kelvin). The measured temperature gradient was 85 K.

	ΔT_{max} [K]	δ [%]	$\Delta T\ (a, 0)$[K]	$\Delta T\ (0, b)$[K]
Del Vecchios et al.	88.2	+3.8	86.8	32.3
Gotter	78.8	-7.3	77.4	34.3
Higgins	85.26	0.3	-	-
FD	82.9	-2.47	81.44	45
FEM	85.2	0.23	83.6	46.2

Surprisingly, the results show the simplest method, namely the Gotter's one, provides accurate solution within 10 %, however this method does not require computers, it can be solved by traditional 'paper and pencil' method. The most obvious finding is the result of FEM method is most accurate, this is within the measurement error. The accuracy of Higgins' method was unexpected because its result was closer to the measured value than the FD method's one.

3.2 A 50 MVA transformer's core

The selected methods are compared on a three phase three-legged 50 MVA transformers' core (Fig. 9). This core is built-up of M1 grade electrical steel sheets. The turn voltage

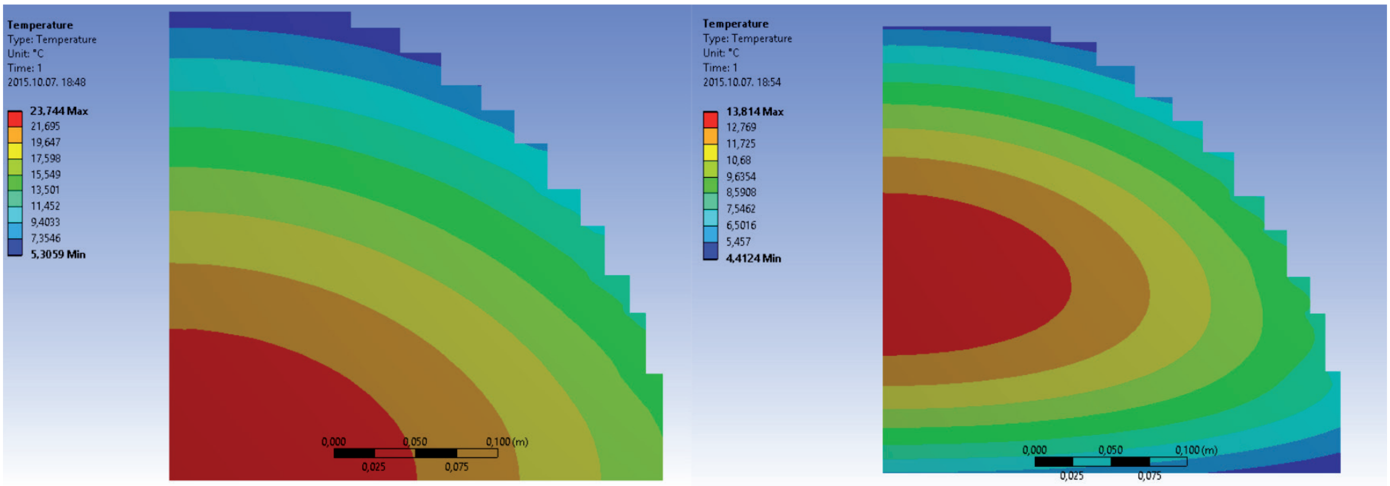

Fig. 10 Finite element solution for the quarter of the of the 50 MVA transformers' core cross section. The left-side picture shows the solution for a transformer core without a cooling duct, case a) in the Fig. 2. In the right-side picture a cooling duct considered in the horizontal direction, the case b) in Fig. 2.

is U_T = 96 V, the peak value of the flux density in the core is B_0 = 1.63 T. The no-load loss of the employed material is p_0 = 0.739 W/kg. Step-lap joints is used in this design, it consists of 6 group of lamination, the stacking factor is about s_f = 0.96. Rounded type yokes employed to the upper and the lower yokes. The shape and the cross section of the yoke and the columns are the same (Fig. 8). The cross section area is A = 2662 mm^2. The generated heat density is assumed homogeneous in this calculation.

The results calculated with the five different and described method can be seen in Table 2 for comparison. The result calculated with the Ansys model is shown on Fig. 10.

Table 2 Comparison of the calculation results using each method (degrees in kelvin) in a normed replacement rectangle cross section.

	ΔT_{max}	$\Delta T\,(a, 0)$	$\Delta T\,(0, b)$
Del Vecchios et al.	14.4	10.1	6
Gotter	13.87	10.0	5.9
Higgins	16.22	-	-
FD	15.6	11.13	6.8
FEM	16.1	11.45	7.1

The results of the calculations of Roth's transformer shows the FEM methods practically same result as the measurement. This corresponds to [29]. Therefore the FEM method's result will be the base of the comparison for this complex arrangement (Fig. 8). Compared the results of two simple (Del Vecchios' and Gotter's) methods to FEM methods' one it can be stated the accuracy these methods are similar to the previous arrangement and the result of the Gotter's method provides lower values. In this three phase arrangement, the result of Higgins' method is also closest to the FEM method's one, however this is almost as complicated as the FEM.

4 Conclusion

The paper presents an overview of the temperature rise calculation methods in the case of core-form power transformers. In this general overview, the methods from the first exact analytical solutions to the state of the art FEM are described. In addition, four widely used calculation methods are compared by two practical examples.

The first example is a rectangle shaped transformer core. The results show the difference between the measured and the FEM calculated values is less than the uncertainty of the temperature measurement. Higgins' method is also very precise, but it requires as complex computation as the FEM or FD solutions. However, it can be derived only for rectangular shapes. Gotter's thermal resistance solution is very simple and fast. The precision of this method is about 10 %, and it can be easily improved by Ryder's method.

The second example is an arbitrary shaped core where the FEM was the basis of the comparison. The results indicate that the simple methods provide decent result with much less computation time. Therefore in a final design process a FEM method based calculation gives the best accuracy, but in the preliminary design stage, the Gotter's thermal resistive method can be the most efficient.

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