

Effect of MHD on Nanofluid Flow, Heat and Mass Transfer over a Stretching Surface Embedded in a Porous Medium

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Abstract

The steady, laminar, mixed convection, boundary layer flow of an incompressible nanofluid past over a semi-infinite stretching surface in a nanofluid –saturated porous medium with the effects of magnetic field and chemical reaction is studied. The governing boundary layer equations (obtained with the Boussinesq approximation) are transformed into a system of nonlinear ordinary differential equations by using similarity transformation. The effects of various physical parameters are analyzed and discussed in graphical and tabular form. Comparison with published results is presented and we found an excellent agreement with it. Mainly, it found firstly, that an increase in magnetic parameter M decreases both the local Nusselt number and local Sherwood number. Secondly, a great order of the chemical reaction increases the Nusselt and Sherwood numbers.

Keywords

mixed convection, boundary layer, nanofluid, magnetic field

1 Introduction

A nanofluid is a new innovation which drew the attention of most researchers in the course of last years. A nanofluid is composed of a fluid such as water and ethylene glycol and small particles of nanometric size (between 1 and 100 nm), which are generally metals (Al, Cu), oxides (Al_2O_3 , TiO_2 , and CuO), carbides (SiC), nitrides (AlN, SiN) or non-metals (graphite, nanotubes of carbon). The addition of these additives allows to improve the performances of heat-transfer of basic fluids.

Choi et al. [1] found that the additions of nanoparticles to a basic fluid can double its thermal conductivity. Other experimental studies as, Masuda et al. [2] and Minsta et al. [3] show that the addition of a small volume fraction of nanoparticles (less than 5 %) made an increase from 10 to 50 % of thermal conductivity of basic fluids with a remarkable improvement of coefficient of heat-transfer by convection. This improvement makes nanofluids a new promising technology, allowing improving the heat-transfer between the surfaces of exchanges and nanofluids in movement with different temperatures. Hybrid nanofluids have also proven its favorable impact because of these thermophysical properties, heat transfer rates and stability [4, 5].

A consequence of the nanofluid-area interaction is the development of a region in the nanofluid wherein the velocity varies from its zero value at the surface to the finite value of the external flow. Because the surface and nanofluid temperatures are different, there will also be a region in the nanofluid through which the temperature of the latter will vary between the value at the wall and that the outer flow. A thermal conductivity and heat-transfer coefficient by convection enhanced which make them potentially useful in many applications. Mustafa et al. [6] studied the flow of a nanofluid near a stagnation point towards a stretching surface with the consideration of the effects of Brownian motion and thermophoresis mechanism.

By considering others advanced nanofluid, Hayat et al. [7] investigates the three-dimensional flow of Powell-Eyring nanofluid with the same effects. Four parameters, namely the Brownian motion number, the buoyancy-ratio number, the thermophoresis number and the Lewis number have been

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identified by other researchers. Few of them are presented by Hayat et al. in [8, 9].

Alsaedi et al. [10] analyzed stagnation point of the flow of nanofluid near a permeable stretched surface with convective boundary condition. They have found good results compared to others. On the other hand, Zargartalebi et al. [11] studied the same problem but with varying thermo-physical properties. The volume fraction of the nanoparticles is assumed to be passively controlled with an isothermal exchange surface. The results show that the variation of various thermodynamic parameters induced substantial impression on the behavior of nanoparticles distribution. Khan and Pop [12], Hassani et al. [13], Makinde and Aziz [14] choose to study the boundary-layer flow of a nanofluid past a stretching sheet. It was found that the reduced Nusselt number is a decreasing function for each dimensionless parameter while the reduced Sherwood number is an increasing function for the specified values of Pr. An analysis is made by Vajravelu et al. [15] to study the heat-transfer by convection in a flow of the nanofluids (Ag-water and Cu-water) over a stretching sheet. The results indicate that the Ag-water decreases the boundary layer thickness more than that Cu-water.

The application of magnetohydrodynamics in the flow of nanofluids has received relatively an important consideration on the part of researchers. Hamad [16] examines the convective flow and heat transfer of an incompressible viscous nanofluid past a semi-infinite vertical stretching sheet in the presence of a magnetic field. It is noted that the momentum boundary layer thickness decreases while the thermal boundary layer thickness increases with increasing magnetic parameter. Kandasamy et al. [17] have studied theoretically the problem of steady MHD boundary-layer flow of a nanofluid past a vertical stretching surface in the presence of suction/injection. The boundary layer flow and heat transfer over a permeable stretching sheet due to a nanofluid with the effects of magnetic field, slip boundary condition and thermal radiation have been investigated by Wubshet and Bandari [18]. They found that the velocity profiles decrease with increasing M parameter.

Ruchika et al. [19] presented the MHD boundary layer flow and heat transfer analysis of nanofluid induced by a power-law shrinking sheet. Pal and Mandal [20] investigate the magnetohydrodynamic mixed convective heat transfer induced by a non-linear stretching and shrinking sheets in the presence of thermal radiation, viscous dissipation and Ohmic heating in the medium saturated by nanofluids. MHD boundary layer flow and heat transfer towards an exponentially stretching sheet embedded in a thermally stratified medium subject to suction are presented by Mukhopadhyay [21]. It is found that the fluid velocity decreases with increasing of magnetic parameter. MHD boundary layer flow and heat transfer of a water-based nanofluid over a nonlinear stretching sheet with viscous dissipation have been investigated numerically by Mabood et al. [22].

The results show that the skin friction coefficient increases, whereas the reduced Nusselt and Sherwood numbers decrease with magnetic parameter. Hayat et al. [23-29] have presented some interesting studies related mainly to specific rheology of the basic-fluid, MHD, mixed convection and others realistic boundary conditions with a combined configuration of them.

The combination of heat and mass transfer in the presence of a chemical reaction is a growing need in the chemical industries such as combustion systems, solar collectors. Patil et al. [30] studied the effects of chemical reaction on mixed convection flow of a polar fluid through a porous medium in the presence of internal heat generation. The effects of chemical reactions on unsteady MHD free convection and mass transfer for flow past a hot vertical porous plate with heat generation absorption through porous medium was studied by Singh and Kumar [31]. It is noted that temperature, velocity skin-friction coefficient increase and Nusselt number decreases for generative chemical reactions ($\gamma < 0$). However, destructive chemical reactions ($\gamma > 0$) have opposite effect on temperature, velocity, skin-friction coefficient and Nusselt number.

Motivated by the above studies, the present paper investigates the original configuration described by a mixed convection boundary-layer of a nanofluid flow near a semi-infinite stretching surface embedded in porous medium with the influence of magnetic field and chemical reaction. To the authors' knowledge, no studies have thus far been analyzed with regard to nanofluid on mixed convection in porous medium under magnetic field and chemical reaction effect. Here, the more representing model of Bouniorno for the nanofluid will be adopted.

This contribution is situated in the studies necessary for domains connected to the porous medium, such as the chemical industries where the unit operations which are doing in these complex areas. The use of a nanofluid in this domain and its beneficial effects as for its applicability are in full current events. We discuss the influence of the various non-dimensional parameters on velocity, temperature, concentration and nanoparticles volume fraction profiles, which are represented in through graphs and evaluate their effects on the heat and mass transfer rates.

2 Mathematical analysis

Consider steady, laminar, MHD mixed convection, boundary layer flow of an incompressible nanofluid past over a semi-infinite stretching surface in a nanofluid –saturated porous medium as displayed in Fig. 1.

Introducing the Cartesian coordinate system, the x -axis measures the distance along the plate, and the y -axis measures the distance normally into the fluid. The surface of plate is maintained at uniform temperature, concentration and nanoparticle volume fraction T_w , C_w and ϕ_w , respectively, and these values are assumed to be greater than the ambient temperature, concentration and nanoparticle volume fraction, T_∞ , C_∞ , and ϕ_∞ respectively.

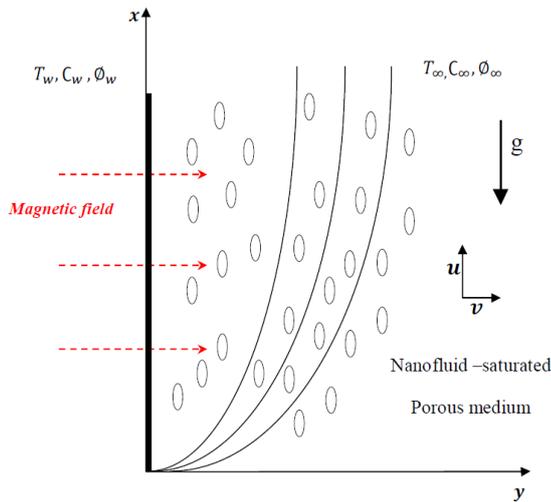


Fig. 1 Physical model and coordinate system.

The Oberbeck–Boussinesq approximation is employed. Homogeneity and local thermal equilibrium in the porous medium are assumed. We consider a porous medium whose porosity is denoted by ε and permeability by k . The Darcy velocity is denoted by ω .

In-line with these assumptions, the governing equations describing the conservation of mass, momentum, energy and concentration can be written as follows:

$$\nabla \cdot \omega = 0 \quad (1)$$

$$\frac{\rho_f}{\varepsilon} \frac{\partial \omega}{\partial t} = -\nabla p - \frac{\mu}{k} \omega - \delta B_0^2 \omega + g \left[\varphi \rho_p + (1-\varphi) \left\{ \rho_f \begin{pmatrix} 1 - \beta_T (T - T_\infty) \\ -\beta_c (C - C_\infty) \end{pmatrix} \right\} \right] \quad (2)$$

$$\begin{aligned} & (\rho c)_f \frac{\partial T}{\partial t} + (\rho c)_f \omega \cdot \nabla T \\ & = k_m \nabla^2 T + \varepsilon (\rho c)_p \left[D_B \nabla \varphi \cdot \nabla T + \left(\frac{D_T}{T_\infty} \right) \nabla T \cdot \nabla T \right] \end{aligned} \quad (3)$$

$$\frac{\partial \varphi}{\partial t} + \frac{1}{\varepsilon} \omega \cdot \nabla \varphi = D_B \nabla^2 \varphi + \left(\frac{D_T}{T_\infty} \right) \nabla^2 T \quad (4)$$

$$\frac{\partial C}{\partial t} + \frac{1}{\varepsilon} \omega \cdot \nabla C = D_m \nabla^2 C - K (C - C_w)^n \quad (5)$$

Here ρ_f , μ , β_T and β_c are the density, viscosity, volumetric volume expansion coefficient of the fluid and the analogous solutal coefficient respectively, while ρ_p is the density of the particles, δ and B_0 are the electrical conductivity and induced magnetic field, $\omega = (u, v)$ is the two-dimensional velocity vector and p , the pressure. The gravitational acceleration is denoted by g . In these above equations, T is the temperature field, C the concentration of species and φ the nanoparticles concentration fields. Eqs. (1)-(5) are based on the earlier model of Nield and Kuznetsov [32], with a modification included MHD. We

have introduced the heat capacity $(\rho c)_f$ of the fluid and $(\rho c)_p$ of the nanoparticle, and the effective thermal conductivity of the porous medium saturated with the nanofluid is k_m . The coefficients appeared in Eqs. (3)-(4) are the Brownian diffusion coefficient D_B and the thermophoretic diffusion coefficient D_T . D_m , K and n are the mass diffusivity, a dimensional chemical reaction parameter and order of the chemical reaction. The flow is assumed to be slow so that a Forchheimer quadratic drag term and an advective term do not appear in the momentum equation.

The boundary conditions are taken to be

$$v = 0, T = T_w, C = C_w, \varphi = \varphi_w \text{ at } y = 0 \quad (6a)$$

$$u = v = 0, T \rightarrow T_\infty, C \rightarrow C_\infty, \varphi \rightarrow \varphi_\infty \text{ at } y \rightarrow \infty \quad (6b)$$

Using the Oberbeck–Boussinesq approximation with an assumption that the nanoparticle concentration is dilute, the governing equations for this problem can be written as follows:

$$\begin{aligned} 0 = \nabla p - \frac{\mu}{k} \omega - \delta B_0^2 \omega \\ + g \left[\begin{aligned} & -(\rho_p - \rho_{f\infty})(\varphi - \varphi_\infty) \\ & + (1 - \varphi_\infty) \rho_{f\infty} \{ \beta_T (T - T_\infty) + \beta_c (C - C_\infty) \} \end{aligned} \right] \end{aligned} \quad (7)$$

We now make the standard boundary layer approximation, based on a scale analysis, and write the governing equations as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (8)$$

$$\frac{\partial p}{\partial x} = -\frac{\mu}{k} - \delta B_0^2 u + g \left[\begin{aligned} & (1 - \varphi_\infty) \rho_{f\infty} \beta_T (T - T_\infty) \\ & + \beta_c (C - C_\infty) - (\rho_p - \rho_{f\infty})(\varphi - \varphi_\infty) \end{aligned} \right] \quad (9)$$

$$\frac{\partial p}{\partial y} = 0 \quad (10)$$

$$\begin{aligned} u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_m \nabla^2 T \\ + \tau \left[D_B \frac{\partial \varphi}{\partial y} \cdot \frac{\partial T}{\partial y} + \left(\frac{D_T}{T_\infty} \right) \left(\frac{\partial T}{\partial y} \right)^2 \right] \end{aligned} \quad (11)$$

$$\frac{1}{\varepsilon} \left(u \frac{\partial \varphi}{\partial x} + v \frac{\partial \varphi}{\partial y} \right) = D_B \frac{\partial^2 \varphi}{\partial y^2} + \left(\frac{D_T}{T_\infty} \right) \frac{\partial^2 T}{\partial y^2} \quad (12)$$

$$\frac{1}{\varepsilon} \left(u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} \right) = D_m \frac{\partial^2 C}{\partial y^2} - K (C - C_\infty)^n \quad (13)$$

$$\text{where: } \alpha_m = \frac{k_m}{(\rho c)_f}, \tau = \frac{\varepsilon (\rho c)_p}{(\rho c)_f} \quad (14)$$

We can eliminate p from Eqs. (9) and (10) by cross-differentiation. At the same time, we can introduce a stream function Ψ defined by the Cauchy-Riemann equation:

$$u = \frac{\partial \Psi}{\partial y}, \quad v = -\frac{\partial \Psi}{\partial x} \quad (15)$$

Equation (8) is satisfied identically so the following four coupled similarity equations become:

$$\frac{\partial^2 \Psi}{\partial y^2} = \left(1 + \frac{\delta B_0^2}{\mu}\right) \left[\frac{(1-\varphi_\infty)\rho_\infty kg \left(\beta_T \frac{\partial T}{\partial y} + \beta_c \frac{\partial C}{\partial y}\right)}{\mu} - \frac{(\rho_p - \rho_{f\infty})gk}{\mu} \frac{\partial \varphi}{\partial y} \right] \quad (16)$$

$$\begin{aligned} \frac{\partial \Psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \Psi}{\partial x} \frac{\partial T}{\partial y} &= \alpha_m \nabla^2 T \\ &+ \tau \left[D_B \frac{\partial \varphi}{\partial y} \cdot \frac{\partial T}{\partial y} + \left(\frac{D_T}{T_\infty}\right) \left(\frac{\partial T}{\partial y}\right)^2 \right] \end{aligned} \quad (17)$$

$$\frac{1}{\varepsilon} \left(\frac{\partial \Psi}{\partial y} \cdot \frac{\partial \varphi}{\partial x} - \frac{\partial \Psi}{\partial x} \cdot \frac{\partial \varphi}{\partial y} \right) = D_B \frac{\partial^2 \varphi}{\partial y^2} + \left(\frac{D_T}{T_\infty}\right) \frac{\partial^2 T}{\partial y^2} \quad (18)$$

$$\frac{1}{\varepsilon} \left(\frac{\partial \Psi}{\partial y} \cdot \frac{\partial C}{\partial x} - \frac{\partial \Psi}{\partial x} \cdot \frac{\partial C}{\partial y} \right) = D_m \frac{\partial^2 C}{\partial y^2} - K(C - C_\infty)^n \quad (19)$$

The system of equations can be simplified by introducing the following similarity transformations:

$$\begin{aligned} \eta &= \frac{y}{x} Pe_x^{1/2}, \quad f(\eta) = \frac{\Psi}{\alpha_m Pe_x^{1/2}}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \\ \varnothing(\eta) &= \frac{C - C_\infty}{C_w - C_\infty}, \quad S(\eta) = \frac{\varphi - \varphi_\infty}{\varphi_w - \varphi_\infty} \end{aligned} \quad (20)$$

The governing equations (Eqs. (16) - (19)) can be reduced to:

$$f'' = \frac{Ra_x}{Pe_x} \left[\theta' + Nc\varnothing' - NrS' \right] \frac{1}{(1+M)} \quad (21)$$

$$\theta'' + \frac{1}{2} f\theta' + NbS'\theta' + Nt(\theta')^2 = 0 \quad (22)$$

$$S'' + \frac{1}{2} Lef'S' + \frac{Nt}{Nb}\theta'' = 0 \quad (23)$$

$$\frac{1}{Le_m}\varnothing'' + \frac{1}{2} f\varnothing' - \gamma\varnothing'' = 0 \quad (24)$$

The transformed boundary conditions are:

$$\eta = 0, \quad f = 0, \quad \theta = 1, \quad \varnothing = 1, \quad S = 1 \quad (25a)$$

$$\eta \rightarrow \infty, \quad f' = 1, \quad \theta = 0, \quad \varnothing = 0, \quad S = 0, \quad (25b)$$

Where prime denote differentiation with respect to η . The involved physical parameters are defined as:

$$\begin{aligned} Nr &= \frac{(\rho_p - \rho_{f\infty})(\varphi_w - \varphi_\infty)}{\rho_{f\infty}\beta_T(T_w - T_\infty)(1 - \varphi_\infty)}, \\ Nb &= \frac{\varepsilon(\rho c)_p D_B(\varphi_w - \varphi_\infty)}{(\rho c)_f \alpha_m}, \\ Nt &= \frac{\varepsilon(\rho c)_p D_T(T_w - T_\infty)}{(\rho c)_f \alpha_m T_\infty}, \\ Nc &= \frac{\beta_c(C_w - C_\infty)}{\beta_T(T_w - T_\infty)}, \\ Ra_x &= \frac{(1 - \varphi_\infty)\rho_{f\infty}kg\beta_T(T_w - T_\infty)x}{\mu\alpha_m}, \\ Pe_x &= \frac{u_\infty x}{\alpha_m}, \quad Le = \frac{\alpha_m}{\varepsilon D_B}, \\ Le_m &= \frac{\alpha_m}{\varepsilon D_m}, \quad M = \frac{\delta B_0^2 k}{\mu}, \quad \gamma = \frac{(C_w - C_\infty)^{n-1} \varepsilon k x^2}{\alpha_m Pe_x} \end{aligned} \quad (26)$$

Where Nr , Nb , Nt , Nc , Ra_x , Pe_x , Le , Le_m , M and γ represent the buoyancy ratio parameter, the Brownian motion parameter, the thermophoresis parameter, the regular double-diffusive buoyancy ratio, the local Darcy-Rayleigh number, the local Peclet number, nanofluid Lewis number, modified Lewis number, magnetic parameter and dimensionless chemical reaction parameter. We note that porosity ε is absorbed into the Nb , Nt , Le and Le_m parameters and therefore it is not explicitly simulated in this study.

The physical quantities of interest are the local Nusselt number (Nu_x) and the local Sherwood number (Sh_x) which are defined as:

$$Nu_x = \frac{xq_w}{k_m(T_w - T_\infty)}, \quad Sh_x = \frac{xq_m}{D_m(\varphi_w - \varphi_\infty)} \quad (27)$$

Where q_w and q_m are the heat flux and the mass flux at the wall, respectively. Using Eq. (20), we obtain dimensionless version of these key design quantities:

$$(Pe_x)^{-1/2} Nu_x = -\theta'(0), \quad (Pe_x)^{-1/2} Sh_x = -\varnothing'(0). \quad (28)$$

Here $(Pe_x)^{-1/2} Nu_x$ and $(Pe_x)^{-1/2} Sh_x$ are identified as the reduced Nusselt number and reduced Sherwood number which are represented by $-\theta'(0)$ and $-\varnothing'(0)$, respectively.

3 Results and discussion

First and in order to test the accuracy of our results, those obtained are compared with the results of Puneet Rana et al. [33] for local Nusselt number $Nu_x/Pe_x^{1/2} = -\theta'(0)$ and reduced Sherwood number $Sh_x/Pe_x^{1/2} = -\varnothing'(0)$. Neglecting the effects of M , Nc , Le_m , n and γ in the present developed equations, we

notice that the comparison shows an excellent agreement, as presented in Table 1.

The results obtained shows the influences of the non dimensional governing parameters, namely M , Ra_x/Pe_x , Nt , Nb , Nr , Nc , Le , Le_m , n and γ on velocity, temperature, concentration and nanoparticle volume fraction profiles.

Fig. 2 presents typical profiles for the velocity, temperature, nanoparticle volume fraction and concentration of the nanofluid for different values of magnetic strength in the presence and

absence of chemical reaction. In both cases, the velocity of the fluid decreases, whereas the temperature, nanoparticle volume fraction and concentration of the nanofluid increase. The effect of a transverse magnetic field is to give rise to a resistive-type force called the Lorentz force. This force has the tendency to slow down the motion of the fluid and to increase its temperature, nanoparticle volume fraction and concentration profiles. It shows particularly that the effect of increasing γ leads to decrease in velocity and concentration of the species in the nanofluid.

Table 1 Comparison of results for the reduced Nusselt number and Sherwood number when $M=0$; $Nt = 0.5$; $Nr=0.5$; and $\alpha=\pi/6$.

Le	Ra_x/Pe_x	$(Pe_x)^{-1/2} Nu_x = -\theta'(0)$				$(Pe_x)^{-1/2} Sh_x = -\phi'(0)$			
		Nb=0.5		Nb=1.0		Nb=0.5		Nb=1.0	
		Puneet [33]	Present study	Puneet [33]	Present study	Puneet [33]	Present study	Puneet [33]	Present study
5	0.3	0.3331	0.3300	0.2179	0.2224	1.3424	1.3465	1.3856	1.3895
	0.5	0.3453	0.3421	0.2260	0.2311	1.3952	1.3998	1.4439	1.4480
	1	0.3742	0.3708	0.2559	0.2516	1.5194	1.5250	1.5803	1.5850
15	0.3	0.3169	0.3123	0.1982	0.2012	2.3935	2.4031	2.4089	2.4173
	0.5	0.3300	0.3252	0.2066	0.2098	2.4992	2.5094	2.5187	2.5273
	1	0.3609	0.3554	0.2366	0.2299	2.7461	2.7575	2.7741	2.7835

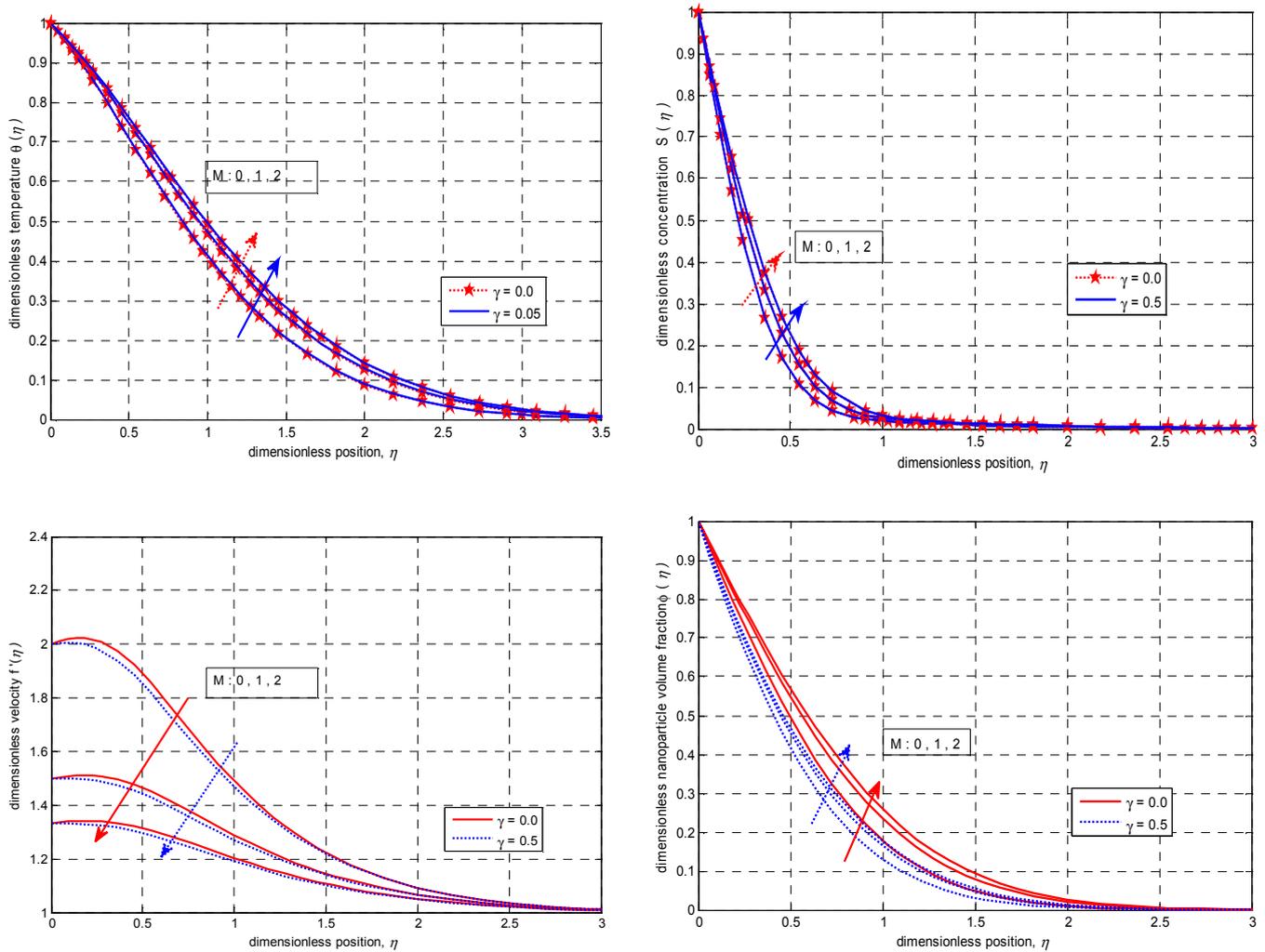


Fig. 2 Effects of M on velocity, temperature, concentration and nanoparticle volume fraction profiles for $Ra_x/Pe_x = 1$, $Nt = 0.2$, $Nb = 0.5$, $Nr = 0.5$, $Nc = 0.5$, $Le = 10$, $Le_m = 2$, $n = 1$.

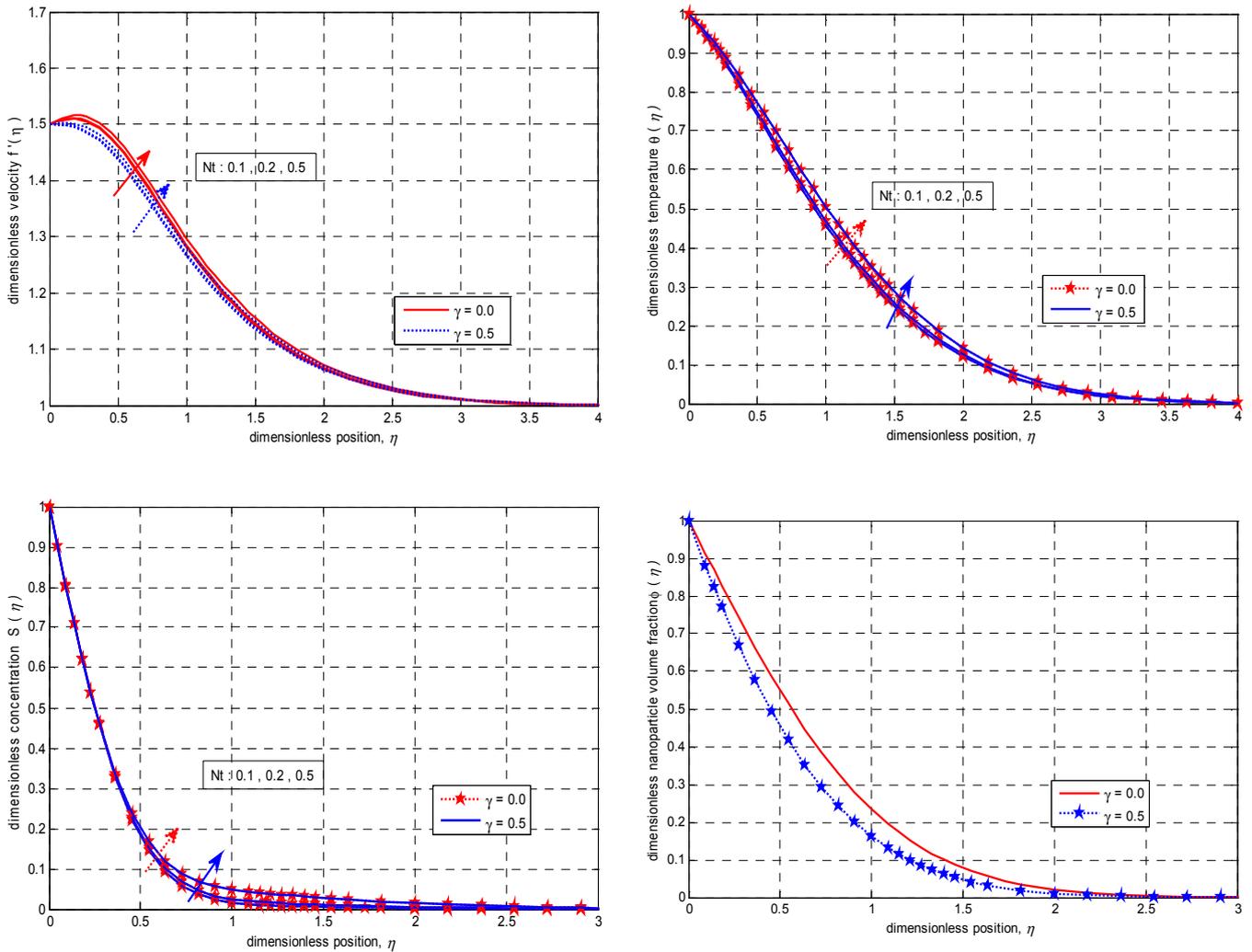


Fig. 3 Effects of Nt on the velocity, temperature, concentration and nanoparticle volume fraction profiles for $M = 1, Ra_x/Pe_x = 1, Nb = 0.5, Nr = 0.5, Nc = 0.5, Le = 10, Le_m = 2, n = 1$.

Fig. 3 displays the effect of thermophoretic particle deposition Nt on velocity, temperature and nanoparticle volume fraction profiles in the presence and absence of chemical reaction. If $\gamma = 0.0$, then it is observed that the velocity, temperature and concentration of species increase with the increasing of thermophoretic parameter. If $\gamma = 0.5$, then the velocity also increases.

The temperature gradient generates thermophoretic force which creates a fast flow away from this surface. In this way more heated fluid is moved away from the surface and consequently, the temperature increase when Nt increases. We can note that the presence of chemical reaction decrease the nanoparticle concentration whereas there are no significant change in this concentration when Nt increases.

In Fig.4, it is viewed that the velocity and temperature increase with increase of Brownian motion of the nanoparticles and the nanoparticle volume fraction decrease whereas there are no significant changes in concentration profiles. It is due to the fact that an increase in Nb leads to increase the random motion of nanoparticles which enhances the particle collision

resulting increase of temperature that last in result is account for the fall in concentration. It is interesting to note that in the presence of chemical reaction the velocity and nanoparticles concentration decrease.

The effects of nanofluid Lewis number parameter on the velocity, temperature and nanoparticle volume fraction for selected values of parameters is shown in Fig.5. For $\gamma=0.0$ as the parameter Le increase, the velocity increase but the temperature and nanoparticle volume fraction decrease.

On the other hand, there are no significant changes in the concentration when Le increases. It is interesting to note that the presence of chemical reaction affect only the velocity whose decrease with increasing of γ parameter.

The variation of the dimensionless heat transfer rates $(Nu_x/Pe_x^{1/2})$ and mass transfer rates $Sh_x/Pe_x^{1/2}$ for different values of the parameters M, n, Nc and Le_m by fixing other governing parameters are displayed in Table 2 and 3.

It is possible to see through these tables that as Magnetic field parameter increase, the heat transfer rate (local Nusselt number and mass transfer rates (Sherwood number)) are

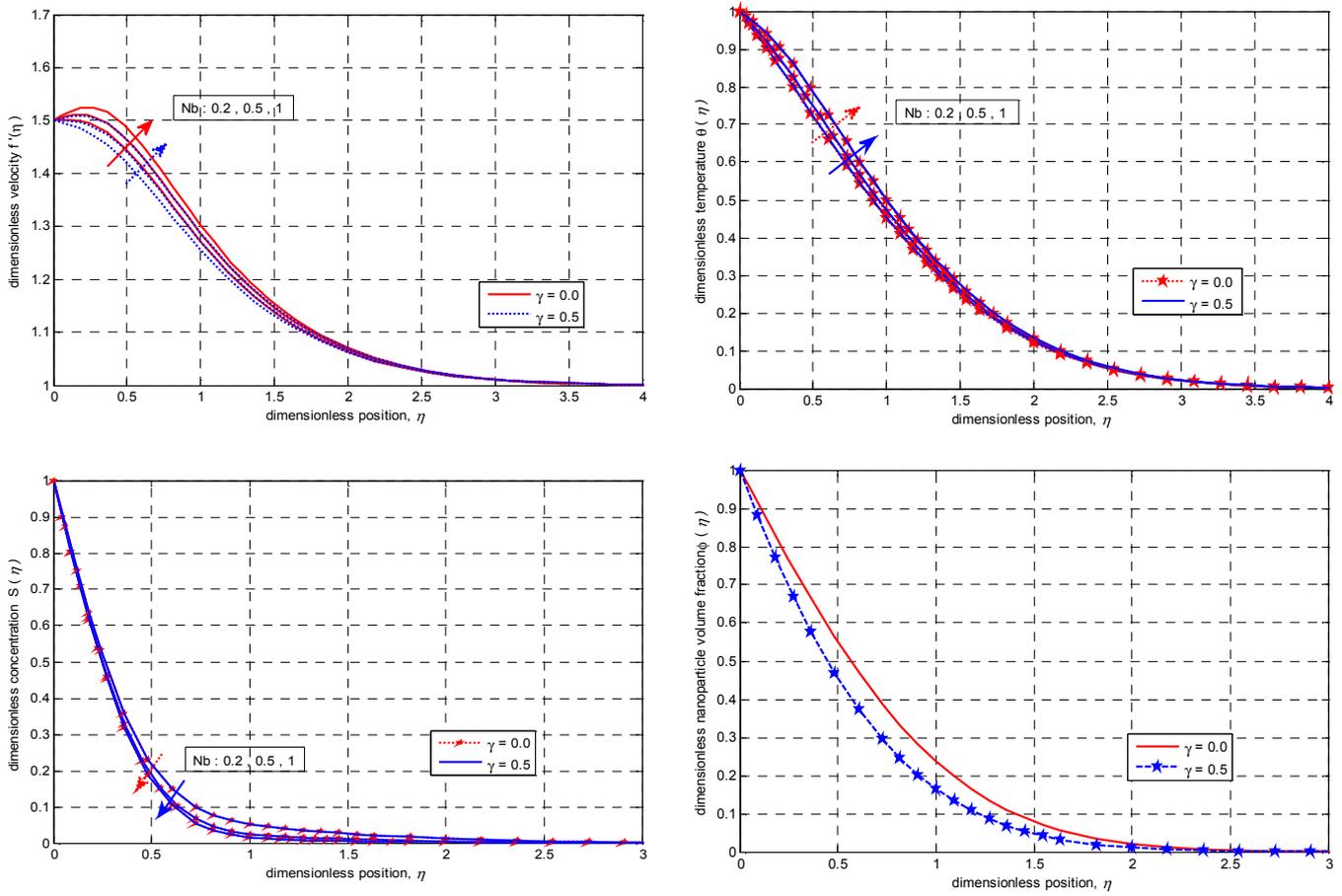


Fig. 4 Effects of Nb on velocity, temperature, concentration and nanoparticle volume fraction profiles for $M = 1$, $Ra_x/Pe_x = 1$, $Nt = 0.2$, $Nr = 0.5$, $Nc = 0.5$, $Le = 10$, $Le_m = 2$, $n = 1$.

Table 2 Computation showing the values local Nusselt number $-\theta'(0)$ at $Nr = 0.5$, $Ra_x/Pe_x = 1$, $Nt = 0.2$, $Nb = 0.5$, $Le = 10$ and $\gamma = 0.5$ for different values of Le_m , M , n and Nc .

Le_m	M	$(Pe_x)^{-1/2} Nu_x = -\theta'(0)$								
		$Nc=0.2$			$Nc=0.5$			$Nc=1$		
		$n=1$	$n=2$	$n=3$	$n=1$	$n=2$	$n=3$	$n=1$	$n=2$	$n=3$
0	0	0.4450	0.4450	0.4450	0.4809	0.4809	0.4809	0.5358	0.5358	0.5358
	1	0.3962	0.3962	0.3962	0.4165	0.4165	0.4165	0.4484	0.4484	0.4484
	2	0.3785	0.3785	0.3785	0.3927	0.3927	0.3927	0.4154	0.4154	0.4154
1	0	0.4347	0.4355	0.4358	0.4567	0.4585	0.4592	0.4915	0.4945	0.4956
	1	0.3904	0.3909	0.3911	0.4025	0.4037	0.4041	0.4221	0.4242	0.4250
	2	0.3745	0.3748	0.3750	0.3828	0.3837	0.3841	0.3965	0.3981	0.3988
2	0	0.4320	0.4328	0.4330	0.4500	0.4517	0.4524	0.4786	0.4816	0.4826
	1	0.3889	0.3894	0.3895	0.3988	0.4000	0.4004	0.4149	0.4170	0.4178
	2	0.3734	0.3738	0.3739	0.3803	0.3812	0.3815	0.3915	0.3931	0.3937
3	0	0.4305	0.4312	0.4314	0.4462	0.4478	0.4484	0.4712	0.4740	0.4750
	1	0.3880	0.3885	0.3887	0.3967	0.3978	0.3982	0.4108	0.4128	0.4135
	2	0.3728	0.3732	0.3733	0.3789	0.3797	0.3800	0.3886	0.3902	0.3908

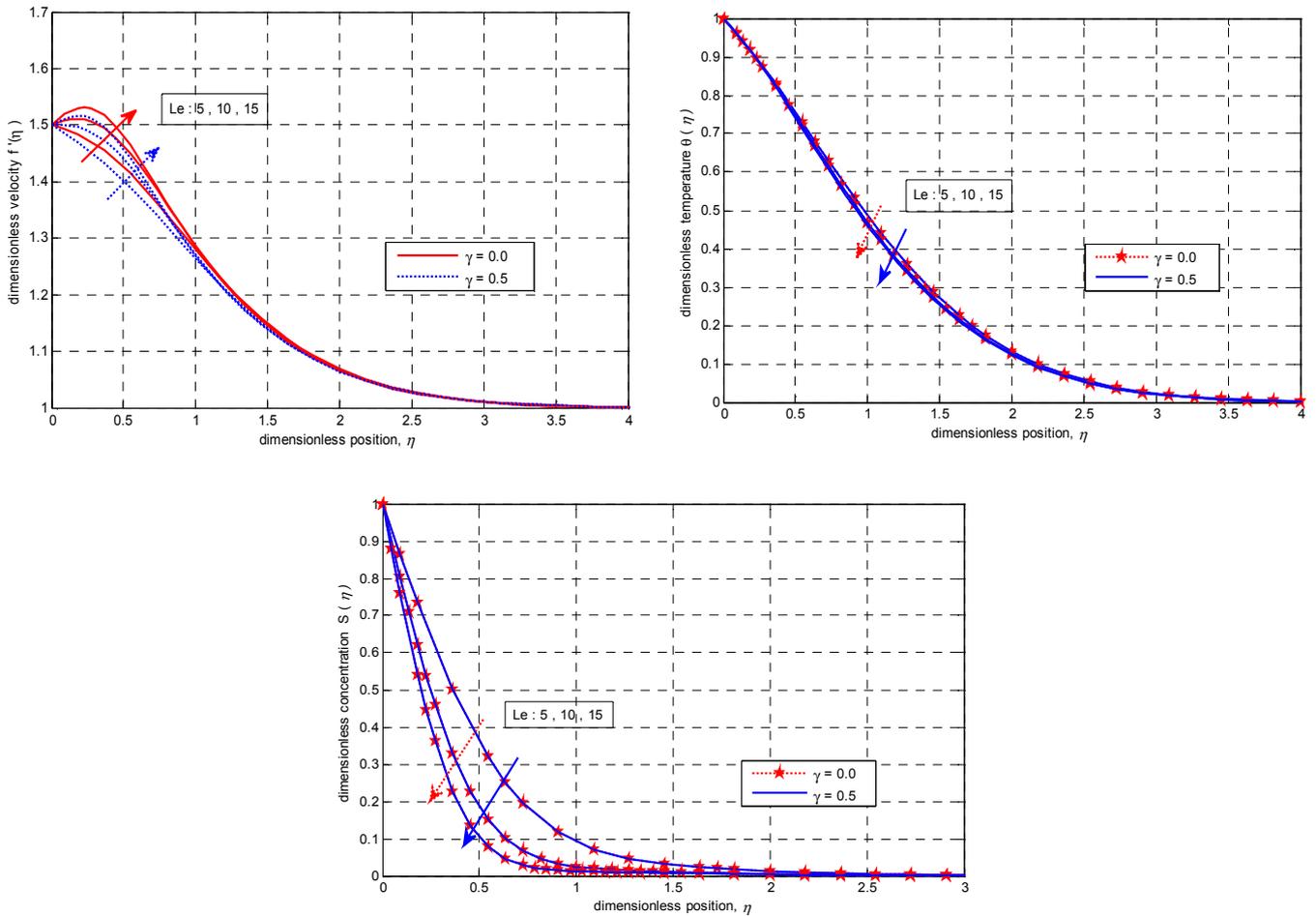


Fig. 5 Effects of Le on velocity, temperature and nanoparticle volume fraction profiles for $M = 1$, $Rax/Pe_x = 1$, $Nb = 0.5$, $Nt = 0.2$, $Nr = 0.5$, $Nc = 0.5$, $Lem = 2$, $n = 1$.

Table 3 Computation showing the values local Sherwood number $-S'(0)$ at $Nr = 0.5$, $Rax/Pe_x = 1$, $Nt = 0.2$, $Nb = 0.5$, $Le = 10$ and $\gamma = 0.5$ for different values of Le_m , M , n and Nc .

		$(Pe_x)^{-1/2} Sh_x = -\phi'(0)$								
Le_m	M	$Nc=0.2$			$Nc=0.5$			$Nc=1$		
		$n=1$	$n=2$	$n=3$	$n=1$	$n=2$	$n=3$	$n=1$	$n=2$	$n=3$
0	0	2.3884	2.3884	2.3884	2.5791	2.5791	2.5791	2.8697	2.8697	2.8697
	1	2.1104	2.1104	2.1104	2.2196	2.2196	2.2196	2.3908	2.3908	2.3908
	2	2.0093	2.0093	2.0093	2.0862	2.0862	2.0862	2.2086	2.2086	2.2086
1	0	2.3677	2.3697	2.3706	2.5318	2.5361	2.5381	2.7861	2.7931	2.7962
	1	2.0980	2.0993	2.0999	2.1904	2.1935	2.1949	2.3375	2.3428	2.3453
	2	2.0005	2.0015	2.0019	2.0650	2.0674	2.0684	2.1690	2.1732	2.1752
2	0	2.3595	2.3620	2.3631	2.5126	2.5181	2.5205	2.7512	2.7602	2.7641
	1	2.0932	2.0949	2.0957	2.1790	2.1829	2.1847	2.3162	2.3230	2.3261
	2	1.9971	1.9984	1.9989	2.0569	2.0598	2.0612	2.1535	2.1589	2.1614
3	0	2.3537	2.3565	2.3577	2.4990	2.5052	2.5079	2.7262	2.7365	2.7409
	1	2.0899	2.0918	2.0927	2.1711	2.1755	2.1774	2.3011	2.3089	2.3123
	2	1.9948	1.9962	1.9968	2.0512	2.0546	2.0561	2.0595	2.1488	2.1515

decreasing. We can also note that as order of the chemical reaction parameter n increase, the heat transfer rate and the mass transfer rates is increasing.

4 Conclusions

In this study, magneto-hydrodynamic, mixed convection, boundary layer flow of nanofluid over a semi-infinite stretching surface in porous medium is studied. Effects of different parameters on the velocity, temperature, concentration and nanoparticle volume fraction profiles are investigated. The main results of present study are given below:

- As the magnetic parameter M increases, the velocity decrease whereas the temperature, concentration and nanoparticle volume fraction increase.
- A rise in Brownian motion number Nb and /or thermophoresis number Nt lead to increase the velocity near the wall, temperature in the whole boundary layer and little influence on concentration, far the wall.
- As the regular nanofluid Lewis number parameter increase, the velocity increase but the temperature and nanoparticle volume fraction decrease.
- An increase in magnetic parameter M decreases both the local Nusselt number and local Sherwood number.
- A great order of the chemical reaction increases the Nusselt and Sherwood numbers.

The present study of mixed convection with presence of magnetic field can be used in many applications, for example in next-generation heat exchangers technology, in chemical industries involving the nanofluid in a porous medium, and all processes which are affected with heat enhancement concept. The combined effect of thermophoresis particle deposition with Brownian motion on nanofluids due to magnetic field is of great interest worldwide for basic and applied research. A future works can be conducted with complex nanofluids under complicated rheological behavior in porous medium.

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References

- [1] Choi, S. U. S., Zhang, Z. G., Yu, W., Lockwood, F.E., Grulke, E.A. "Anomalous thermal conductivity enhancement in nano-tube suspensions." *Applied Physics Letters*. 79, pp. 2252-2254. 2001. <https://doi.org/10.1063/1.1408272>
- [2] Masuda, H., Ebata, A., Teramea, K., Hishinuma, N. "Altering the thermal conductivity and viscosity of liquid by dispersing ultra-fine particles." *Netsu Bussei*. 7(4), pp. 227-233. 1993.
- [3] Minsta, H. A., Roy, G., Nguyen, C. T., Doucet, D. "New temperature dependent thermal conductivity data for water-based nanofluids." *International Journal of Thermal Sciences*. 48, pp. 363-371. 2009. <https://doi.org/10.1016/j.ijthermalsci.2008.03.009>
- [4] Hamza, M. H., Sidik, N. A. C., Ken, T. L., Mamat, R., Najafi, G. "Factors affecting the performance of hybrid nanofluids." *International Journal of Heat and Mass Transfer*. 115, pp. 630-646. 2017. <https://doi.org/10.1016/j.ijheatmasstransfer.2017.07.021>
- [5] Siddik, N.A. C., Muhammad, M. J., Wan, M. A. A. J., Isa, M. A. "A review on preparation methods, stability and applications of hybrid nanofluids." *Renewable and Sustainable Energy Reviews*. 80, pp. 1112-1122. 2017. <https://doi.org/10.1016/j.rser.2017.05.221>
- [6] Mustafa, M., Hayat, T., Pop, I., Asghar, S., Obaidat, S. "Stagnation-point flow of a nanofluid towards a stretching sheet." *International Journal of Heat and Mass Transfer*. 54, pp. 5588-5594. 2011. <https://doi.org/10.1016/j.ijheatmasstransfer.2011.07.021>
- [7] Hayat, T., Ullah I., Muhammad, T., Alsaedi, A., Shehzad, S. A. "Three-dimensional flow of Powell-Eyring nanofluid with heat and mass flux boundary conditions." *Chinese Physics B*. B, 25-7. 2016. <https://doi.org/10.1088/1674-1056/25/7/074701>
- [8] Hayat, T., Ullah I., Muhammad, T., Alsaedi, A. "Thermal and solutal stratification in mixed convection three-dimensional flow of an Oldroyd-B nanofluid." *Results in Physics*. 7, pp. 3797-3805. 2017. <https://doi.org/10.1016/j.rinp.2017.09.051>
- [9] Hayat, T., Ullah I., Muhammad, T., Alsaedi, A., Waqas, M., Ahmad, B. "Three-dimensional mixed convection flow of Sisko nanoliquid." *International Journal of Mechanical Sciences*. 133, pp. 273-282. 2017. <https://doi.org/10.1016/j.ijmecsci.2017.07.037>
- [10] Alsaedi, A., Awais, M., Hayat, T. "Effects of heat generation/absorption on stagnation point flow of nanofluid over a surface with convective boundary conditions." *Commun Nonlinear Sci Numer Simulat*. 17, pp. 4210-223. 2012. <https://doi.org/10.1016/j.cnsns.2012.03.008>
- [11] Zargartalebi, H., Ghalambaz, M., Noghrehabadi, A., Chamkha, A. "Stagnation-point heat transfer of nanofluids toward stretching sheets with variable thermo-physical properties." *Advanced Powder Technology*. 26, pp. 819-829. 2015. <https://doi.org/10.1016/j.appt.2015.02.008>
- [12] Khan, W.A., Pop, I. "Boundary-layer flow of a nanofluid past a stretching sheet." *International Journal of Heat and Mass Transfer*. 53, pp. 2477-2483. 2010. <https://doi.org/10.1016/j.ijheatmasstransfer.2010.01.032>
- [13] Hassani, M., Mohammad Tabar, M., Nemati, H., Domairry, G., Noori, F. "An analytical for solution for boundary layer flow of nanofluid past a stretching sheet." *International Journal of Thermal Sciences*. 50, pp. 2256-2263. 2011. <https://doi.org/10.1016/j.ijthermalsci.2011.05.015>
- [14] Makinde, O. D., Aziz, A. "Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition." *International Journal of Thermal Sciences*. 50, pp. 1326-1332. 2011. <http://dx.doi.org/10.1016/j.ijthermalsci.2011.02.019>
- [15] Vajravelu, K., Prasad, K.V., Lee, J., Lee, C., Pop, I., Van Gorder, R. A. "Convective heat transfer in the flow of viscous Ag-water and Cu-water nanofluids over a stretching surface." *International Journal of Thermal Sciences*. 50, pp. 843-851. 2011. <https://doi.org/10.1016/j.ijthermalsci.2011.01.008>
- [16] Hamad, M. A. A. "Analytical solution of natural convection flow of a nanofluid over a linearly stretching sheet in the presence of magnetic field." *International Communications in Heat and Mass Transfer*. 38, pp. 487-492. 2011. <https://doi.org/10.1016/j.icheatmasstransfer.2010.12.042>

- [17] Kandasamy, R., Loganathan, P., Puvi Arasu, P. "Scaling group transformation for MHD boundary-layer flow of a nanofluid past a vertical stretching surface in the presence of suction/injection." *Nuclear Engineering and Design*. 241, pp. 2053-2059. 2011.
<https://doi.org/10.1016/j.nucengdes.2011.04.011>
- [18] Ibrahim, W., Shankar, B. M. "MHD boundary layer flow and heat transfer of a nanofluid past a permeable stretching sheet with velocity, thermal and solutal slip boundary conditions." *Computers & Fluids*. 75, pp. 1-10. 2013.
<https://doi.org/10.1016/j.compfluid.2013.01.014>
- [19] Dhanai, R., Rana, P., Kumar, L. "Multiple solutions of MHD boundary layer flow and heat transfer behavior of nanofluids induced by a power-law stretching/shrinking permeable sheet with viscous dissipation." *Powder Technology*. 273, pp. 62-70. 2015.
<https://doi.org/10.1016/j.powtec.2014.12.035>
- [20] Pal, D., Mandal, G. "Hydromagnetic convective-radiative boundary layer flow of nanofluids induced by a non-linear vertical stretching/shrinking sheet with viscous-Ohmic dissipation." *Powder Technology*. 279, pp. 61-74. 2015.
<https://doi.org/10.1016/j.powtec.2015.03.043>
- [21] Mukhopadhyay, S. "MHD boundary layer flow and heat transfer over an exponentially stretching sheet embedded in a thermally stratified medium." *Alexandria Engineering Journal*. 52, pp. 259-265. 2013.
<https://doi.org/10.1016/j.aej.2013.02.003>
- [22] Mabood, F., Khan, W. A., Ismail, A. I. M. "MHD boundary layer flow and heat transfer of nanofluids over a nonlinear stretching sheet: A numerical study." *Journal of Magnetism and Magnetic Materials*. 374, pp. 569-576. 2015.
<https://doi.org/10.1016/j.jmmm.2014.09.013>
- [23] Hayat, T., Ullah I., Alsaedi, A., Ahmad, B. "Radiative flow of Carreau liquid in presence of Newtonian heating and chemical reaction." *Results in Physics*. 7, pp. 715-722. 2017.
<https://doi.org/10.1016/j.rinp.2017.01.019>
- [24] Hayat, T., Ullah I., Alsaedi, A., Ahmad, B. "Modeling tangent hyperbolic nanoliquid flow with heat and mass flux conditions." *The European Physical Journal Plus*, 132, pp. 112. 2017.
<https://doi.org/10.1140/epjp/i2017-11369-0>
- [25] Hayat, T., Ullah I., Alsaedi, A., Farooq, M. "MHD flow of Powell-Eyring nanofluid over a non-linear stretching sheet with variable thickness." *Results in Physics*. 7, pp. 189-196. 2017.
<https://doi.org/10.1016/j.rinp.2016.12.008>
- [26] Hayat, T., Ullah I., Muhammad, T., Alsaedi, A. "A revised model for stretched flow of third grade fluid subject to magneto nanoparticles and convective condition." *Journal of Molecular Liquids*. 230, pp. 608-615. 2017.
<https://doi.org/10.1016/j.molliq.2017.01.074>
- [27] Hayat, T., Ullah I., Muhammad, T., Alsaedi, A. "Magneto hydrodynamic (MHD) three-dimensional flow of second grade nanofluid by a convectively heated exponentially stretching surface." *Journal of Molecular Liquids*. 220, pp. 1004-1012. 2016.
<https://doi.org/10.1016/j.molliq.2016.05.024>
- [28] Hayat, T., Ullah, I., Ahmed B., Alsaedi, A. "MHD mixed convection flow of third grade liquid subject to non-linear thermal radiation and convective condition." *Results in Physics*. 7, pp. 2804-2811. 2017.
<https://doi.org/10.1016/j.rinp.2017.07.045>
- [29] Hayat, T., Ullah I., Muhammad, T., Alsaedi, A. "Radiative three-dimensional flow with Soret and Dufour effects." *International Journal of Mechanical Sciences*. 133, pp. 829-837. 2017.
<https://doi.org/10.1016/j.ijmecsci.2017.09.015>
- [30] Patil, P. M., Chamkha, A. J., Roy, S. "Effects of chemical reaction on mixed convection flow of a polar fluid through a porous medium in the presence of internal heat generation." *Meccanica*. 47, pp. 483-499. 2012.
<https://doi.org/10.1007/s11012-011-9443-z>
- [31] Singh, K. D., Kumar, R. "Effects of chemical reactions on unsteady MHD free convection and mass transfer for flow past a hot vertical porous plate with heat generation-absorption through porous medium." *Indian Journal of Physics*. 84(1), pp. 93-106. 2010.
<https://doi.org/10.1007/s12648-010-0005-3>
- [32] Nield, D. A., Kuznetsov, A. V. "The Cheng-Minkowycz problem for natural convection boundary-layer flow in a porous medium saturated by a nanofluid." *International Journal of Heat and Mass Transfer*. 52, pp. 5792-5795. 2009.
<https://doi.org/10.1016/j.ijheatmasstransfer.2009.07.024>
- [33] Puneet, R., Bhargava, R., Bég, O. A. "Numerical solution for mixed convection boundary layer flow of a nanofluid along an inclined plate embedded in a porous medium." *Computers and Mathematics with Applications*. 64, pp. 2816-2832. 2012.
<https://doi.org/10.1016/j.camwa.2012.04.014>