

# An Interactive Method to Analysis of the Response of the Different Reinforcement Structures of a Door Opening of a Wind Tower

Mourad Belhadj<sup>1,2\*</sup>, Noureddine Lahbari<sup>2</sup>, Abdelkader Slimane<sup>3,4</sup>, Djamel Aouiche<sup>1,2</sup>

<sup>1</sup> Department of Mechanical, Faculty of Technology, University of Batna 2, P. O. B. 53, Road of Constatntine, 05078 Fesdis, Batna, Algeria

<sup>2</sup> Laboratory of Research in Applied Hydraulics (LRHYA), Department of Civil Engineering and Hydraulics, Faculty of Technology, University of Batna 2, P. O. B. 53, Route de Constatntine, 05078 Fesdis, Batna, Algeria

<sup>3</sup> Laboratoire de mécanique appliquée, Département de Génie Mécanique, Université des Sciences et de la Technologie d'Oran Mohamed Boudiaf, USTO-MB, P. O. B. 1505, El M'naouer, 31000 Oran, Algeria

<sup>4</sup> Laboratory of Materials and Reactive Systems (LMSR), Department of Mechanical Engineering, University of Sidi-Bel-Abbes, P. O. B. 89, Ben M'hidi, 22000 Sidi-Bel-Abbes, Algeria

\* Corresponding author, e-mail: [mourad2xw@yahoo.fr](mailto:mourad2xw@yahoo.fr)

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## Abstract

The aim of this work is to present a precise numerical calculus method capable to predict the behavior of a wind turbine mast, which is characterized by an open door in its lower part in order to facilitate the access to maintenance tasks. A parametric study had been conducted in this context. The structure studied of steel tower is considered a thin cylindrical shell with constant section and thickness along its studied height. The geometry of the tower had been modeled by non-linear shell type elements. Designers use interior reinforcements to avoid local buckling and minimize the disturbance of the distribution of stress in extreme conditions. The designs adopted in the models are proposed to achieve optimized results, the minimization of the mass, the maximization of the natural frequency and the rigidity at the end of the work. Many configuration had been considered in this study, the enhancement of the gap by using a panel with variable thickness value, by longitudinal stiffeners, by combined stiffeners and finally by a stiffened panel. A numerical model had been suggested to examine a cylindrical shell behavior in compression using the Abaqus software. The obtained results demonstrate the viability and performance of the proposed approach which perfectly meets the structural requirements of the wind tower. We have observed that the stiff plate model gives reliable results to stability under extreme load. On the other hand it is economically profitable is less material needed for manufacturing, which reduces the cost.

## Keywords

wind turbine tower, shell structures, stiffeners, finite element analysis buckling, numerical analysis

## 1 Introduction

This paper was written in reason of increased need for clean and renewable sources of electricity. Renewable energy comes from natural resources such as wind, rain, sun, tides, waves and geothermal heat. It should be noted that around 16 % of global energy consumption comes from renewable energies, 10 % from conventional biomass used mainly for heating and 3.4 % from hydropower. Sources of wind are among these sources. Wind energy is the only renewable energy that has developed rapidly in recent years. The main advantage of wind energy is its economic output. To extract energy from the wind, wind turbines are the most efficient way of converting the kinetic energy of the wind into

mechanical energy [1]. But in recent years, many damages have been caused by the failure of wind tower structures. The vast majorities of these damages are caused by strong winds, vibration of the structure or induced winds. Some failures are caused by stress and buckling loads. Various configurations and shapes of structures are designed. The best known model is the tubular conical column, which offers more stability to the tower compared to high wind loads [2]. Basic dimensions such as the thickness, height and diameter of the cylinders are relatively important in the design, especially to reduce the weight of the tower, making these towers safer and less sensitive to the wind load.

The structure of cylindrical shells used in mechanical and civil engineering are, in general, enhanced by stiffeners. Many studies are done to determine the actions of these tools on the cylindrical wall. An important bibliography is available in the archived researches.

Theoretical and experimental studies done to show the behavior of cylindrical shells stiffened or not, that treat cases of load of compression and lateral compression by bending moment. Donnell [3], Lundquist [4], Mossman and Robinson [5], Timoshenko and Gere [6], confirm Flügge analysis [7], comparing it to experimental results. However, does not precise Flügge restrictive hypotheses, and this critical strain adopted, in general by many authors. About thirty years later, Seide and Weingarten [8] come back to the problem. The loads buckling gotten are conditioned by the number of harmonics used in the interpolation of displacements but are also a function of the parameter  $R/t$ . similar results are also given by Öry [9]. The authors precise that the results can be modified by the cylinder length. In this case, for short cylinders, the axial wavelength can be blocked, leading increase in the critical moment.

**2 On the geometry of the structure**

The tower has a total height of 78 m and is formed as a cylindrical tube at the lower part and conical shape from height 18 m with an outside diameter of 4 m. For the purposes of moving the tower has been divided into four parts that are easily installed on the site. The sections are connected to each other by means of bolted flanges. The lower edge is fixed to the foundation by pre-tensioned anchors arranged partially in two concentric circles on both sides of the shell. The thickness of the casing of the steel tower is fixed at 40 mm along its studied height. Due to the safety requirements of the structure, high quality welding procedures are using. For the same reason, the tolerances for the manufacture of the stiffeners are of quality class A. The internal stiffeners plates and vertical and transverse sleepers have been designed to compensate for the effect of the local concentration of the stresses at the opening of the door. The details of geometries are shown in Fig. 1.

**3 Phenomenology of shell under bending**

The geometry of a cylindrical shell is given by three dimensions, length  $L$ , radius  $R$  and thickness  $t$  of the structure, and generally characterized in dimensionless by two parameters:  $L/R$  and  $R/t$ . The first parameter  $L/R$  allows distinction between shells according to their height, the second  $R/t$  shows the difference between slim

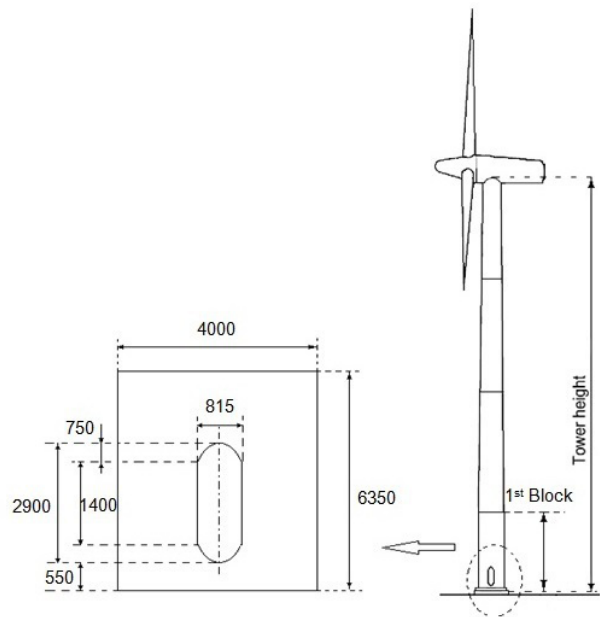


Fig. 1 Geometry of prototype tower (Dimensions in mm)

and thick shells. Whatever are these geometrical parameters  $R/t$  or  $L/R$ , two zones are distinguished when bending moment is applied on the cylinder, tension zone and compression zone.

The theoretical buckling or elastic instability of a shell submitted to moment  $M$  results when a particular value of loading, called critical moment. The deformations are not proportional to the applied forces and can become considerable, which lead to the collapse of the shell. This phenomenon presents some analogy with instable equilibrium of rigid solids in analytical mechanics. The buckling results essentially for compressed shells, by either compression, bending or torsion. But there can be also lateral buckling of a beam submitted to pure bending.

According to the intensity of applied force, we note several behaviors of compressed and bended shell. We say there is a buckling of shell submitted to sollicitation when, for a value called; instability sollicitation or critical load, the equilibrium of the structural becomes instable and often dangerous. The maximal moment supported by a shell must still less than the critical moment. When there is risk of buckling for a long shell, we must not exceed maximal load. This maximal moment is got by dividing the critical moment by security coefficient  $k$ .

The maximal strains of tension and compression are positioned on two opposite fibers. The maximal absolute value  $\sigma_F$ , called global strain of bending is given by Eq. (1):

$$\sigma_F = M_F / (\pi r^2 t). \tag{1}$$

For the thin shells, this kind of balancing is near to the reality if we still inside small displacements field, where the need that deformed configuration should be near to the initial configuration. This hypothesis is not valid during starting of buckling or in boundary conditions that allow the development of pre-critical non-linearity at the end of shell. In other hand, taking into account of plasticity phenomenon disturbs this balancing is caused by the redistributions of stresses leading to the weakening of the material. Applying a bending moment and achieving equilibrium will in any case result in a stress equal to  $\sigma_F$  on the most stressed fiber.

For a perfect medium-length cylinder without the effects of boundary conditions and oval pre-tensioning on buckling resistance, the critical bending stress is equal to bending for uniform axial compression and this would give the critical elasticity buckling moment.

$$M_{cr} = \pi r^2 t \sigma_{cl} = 1.814 \left[ \frac{E}{\sqrt{1-\nu^2}} \right] t^2 r \cong 1.9 E t^2 r \quad (2)$$

$$\sigma_{cl} = \left[ \frac{E}{3\sqrt{1-\nu^2}} \right] t/r \cong 0.605 E t/r \quad (3)$$

When rings are used to maintain cylinder circularity at moderate intervals, the cross section circular is very strongly constrained and the non-linear elastic buckling condition is very close to the linear bifurcation moment defined in Eq. (2). However, the value of the plastic moment given by Eq. (5), is less than 5 % of the critical elastic bending moment, not to mention the first moment of yield. The same situation is evident for a cylinder with  $r/t = 50$  and  $L/r = 4$ .

Then plasticity would definitely affect buckling behavior, depending on the geometry:

$$M_p = 4r^2 t \sigma_y \quad (4)$$

or

$$M_p = 4/3\pi \left[ (r+t/2)^3 - (r-t/2)^3 \right] \sigma_y. \quad (5)$$

A structure describes the relationship between buckling resistance and relative slenderness. For cylinders in global bending, the description can be written as follows. Relative elongation is defined in terms of the plastic moment  $M_p$  (theory of small elastic displacement-ideal plastic analysis, called MNA, Eq. (5)),  $M_{cr}$  linear elastic critical strength and LBA (Linear Bifurcation Analysis, Eq. (2)).

As an important reference resistance of the elastic-plastic buckling resistance evaluation, the definition of reference plastic resistance has sometimes been contested. For the current buckling problem, several of the analyses were carried out for a few perfect bendable shells. Results agreed well with predictions using Eq. (5).

So is adopted to represent the reference plastic resistance throughout this study. However, for relatively thin cylinders, this definition seems to pose some problems due to the complex bending and stretching behavior of the shell, which will be discussed later.

$$\lambda = \sqrt{(M_p/M_{cr})} \quad (6)$$

The dimensionless resistance parameter  $\chi$  is defined as

$$\chi = M_k/M_p, \quad (7)$$

where the characteristic strength  $M_k$  is determined by experience or from a geometrically and materially non-linear analysis of the imperfect structure called GMNIA, producing  $M_{GMNIA} = M_k$ .

### 3.1 Buckling forces (GMNIA)

The cylindrical bending shells were treated with medium length [10]. With a single local imperfection of vacuum welding away from the ends, the fine limits retained to remain circular. These boundary conditions effectively prevent the buckling of these shells. Calculations of imperfect elastic-plastic buckling have been performed using the Abaqus program [11]. The lowest determined strength of GMNIA was considered to be the best numerical prediction of elastic-plastic buckling resistance of imperfect structures. The bow length method described earlier in the non-linear finite element calculation was used to capture the breaking load in the load-displacement path. Due to the close relationship between buckling behavior of cylinders under axial compression and bending, it is natural to use the asymmetrical axis of weld imperfection [12] as the "worst" and "practically" type of imperfection for cylinders in global bending [13]. Imperfection was supposed to be a unique local characteristic, not interacting with other imperfections.

### 3.2 Buckling of the shell stiffened

The type of stiffened shells ruin is multiple because the shell can be ruined by global buckling, local or a combination of both. If the critical loads specific to the first two types of mating differ from each other, there is no interaction and, of course, the dominant mode of ruin is that of the lowest critical loads. If both phenomena occur at approximately the same load, the interaction of the two types of hardness can theoretically cause a significant reduction in the critical load. Modes of harassment interfere because of the non-linear relationships governing the post critical phase and produce a sudden drop in the post critical carrying strength of the shell [14, 15].

We present here a numerical study of cylindrical shells stability in cantilever with opening door (gap) at the bottom (in the lower part). Our work aims to make the shell slimmer that reflects the principal geometrical characteristics of the tower. The specimen can be classified as average slim shells affected by elastic effects and geometric imperfections. The displacement curves of load and strain measurement are presented and compared to numerical predictions by finite elements method, taking into account in the same time, plastic effects and geometric non-linearity. A good concordance between numerical and experimental results is found in terms of curves load-displacement. In reason of shape influence and the volume of the geometrical imperfections, the stiffness given is used to compensate the loss of force because of cutting [16].

We have examined three shells of same global geometry, the first one doesn't have an opening; the second one had a gap without stiffener near to the basis, the last one was enhanced with a frame in the opening.

The different parts of the specimen are represented on the Fig. 2. Every specimen has three parts. A transversal displacement is applied through the load cell to the end flange of the part.

Through the curves completed by Abaqus digital software, it is observed that after the application of the force, one finds a deformation appeared at the bottom of the cylinder and that after the formation of opening became a deformation in the middle of the opening. But after reinforcing the opening by a frame, we notice that the deformation is out of the weak area, which means that the reinforcement has succeeded. Compared to the experimental curves, we observe a similarity in all the curves.

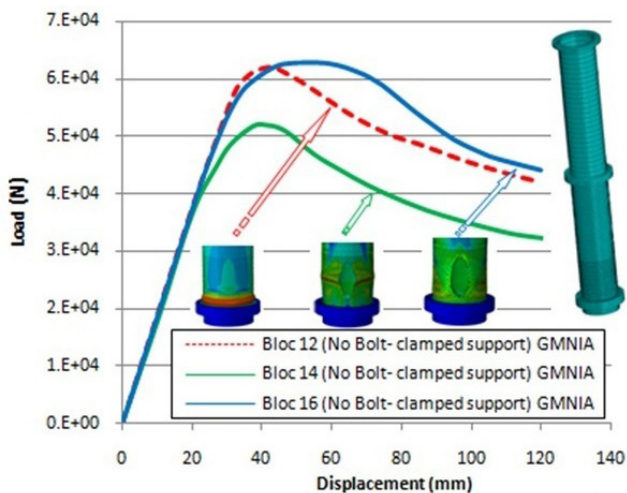


Fig. 2 Numerical load–displacement curves for Block 12, Block 14 and Block 16.

#### 4 Parametric studies

The stiffeners, which can be placed on the inner side of the shell wall, are frequently used to increase the axial strength of cylindrical shells [17].

The case of a cylindrical shell with stiffeners submitted to a moment is examined.

We studied twenty eight cases of different geometrical structures for strengthening the door opening of the steel door of the tower were analyzed and compared in parametric studies. Models with panel with thicknesses from 20 to 40 mm or longitudinal and combined stiffeners from 10 to 80 mm or stiffened panels were analyzed.

The wall thickness of the mast was constant at 40 mm and the height of the test section was maintained at 6350 mm [18].

In order to compare the results, we have used the model without opening and with door opening reinforced by frame with the same height and thickness of the mast. All models have the same diameters, heights and thicknesses, the details of geometries are shown in Fig. 3 to Fig. 5 and in Table 1.

#### 5 Material properties and boundary conditions

The boundary conditions were determined according to EN1993-1-6 [10], tank case with anchors on the bottom of the structure. The models are recessed at the cantilevered lower part, and the moment load has been applied to the model through a reference point coupled to the center of the section on top of the studied models, as shown in Fig. 6.

For all assembled elements of mast, Panel and stiffeners. The same global material, the property of S355J2G3 steel is used. In the elastic range, Young's modulus  $E = 205$  GPa and Poisson's ratio  $\nu = 0.3$  were used. As shown in the plastic true stress - the true stress is shown in Fig. 7, the yield strength is 355 MPa and the ultimate strength is 634 MPa. In the plastic domain, engineering stress - strains relationship has been transformed into true strain-true plastic strains relationship using Eqs. (8), (9):

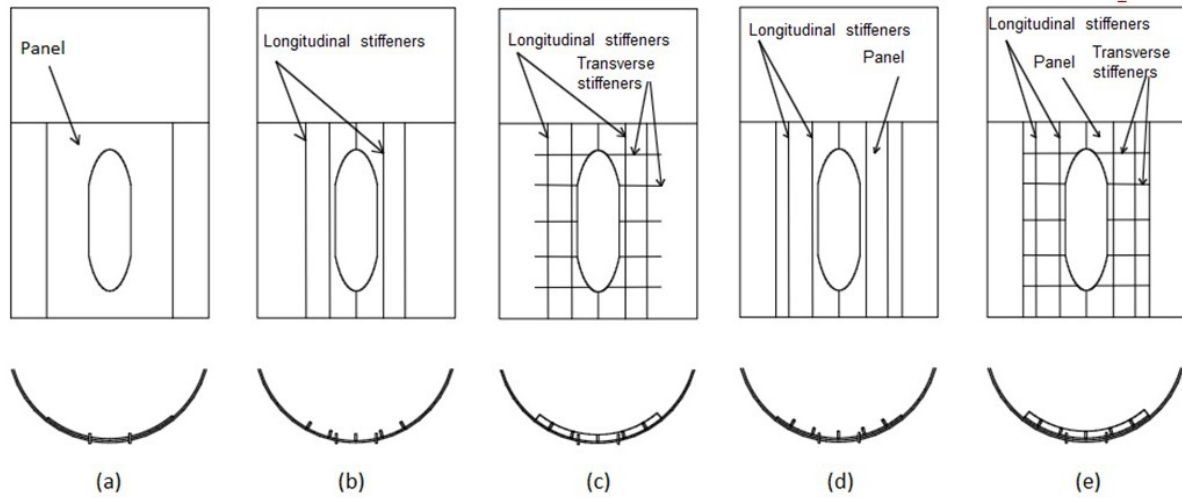
$$\sigma_t = \sigma_n (1 + \varepsilon_n) \tag{8}$$

$$\varepsilon_{pl} = \ln(1 + \varepsilon_n) - \sigma_t / E. \tag{9}$$

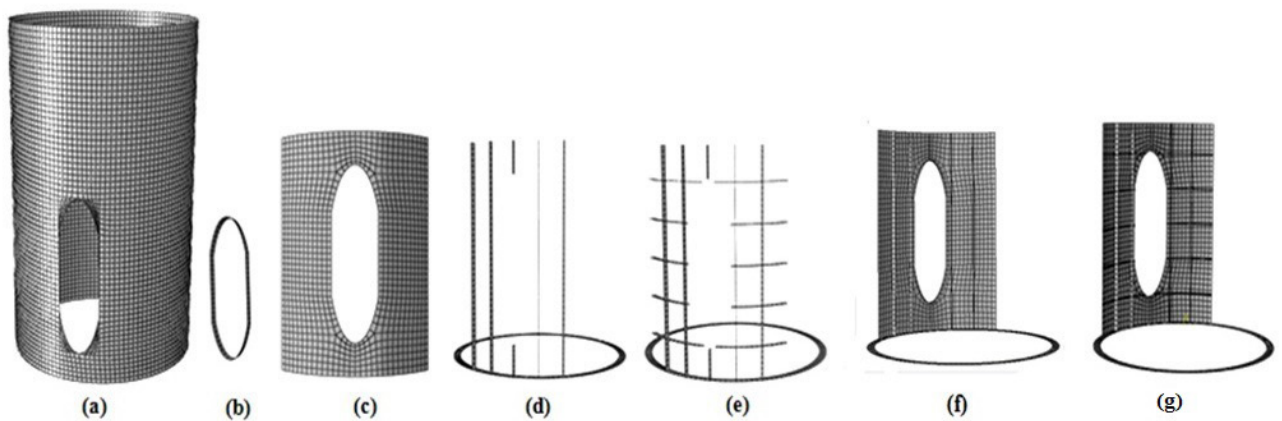
#### 6 Finite element models under Abaqus

This study uses the analysis of the effect of geometric imperfections on the buckling resistance of the wind turbine mast submitted to pure bending.

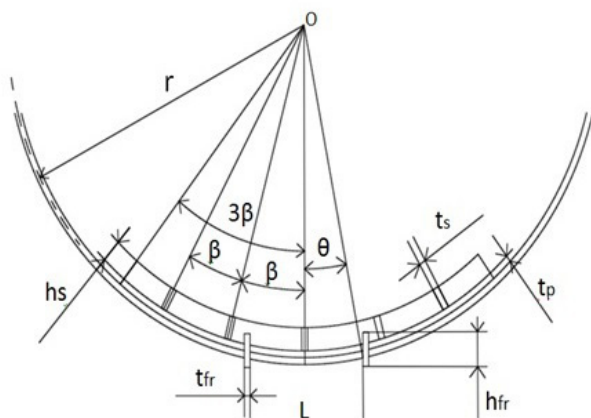
This method is used in particular to solve problems of elastic stability of shell structures. The resolution of shell



**Fig. 3** Stiffening types considered in this study: (a) type 1- Panel, (b) type 2- Longitudinal stiffeners, (c) Type 3- comb stringers, (d) Type 4- Panel and long stiffeners, (e) Type 5- Panel and comb stiffeners.



**Fig. 4** Details of finite element mesh in the area of cutout (peripheral frame, panel and stiffening)



**Fig. 5** Geometrical properties of the cross-section with a stiffened opening

hardness problems using finite element theory has seen its popularity increase as it uses a matrix formulation

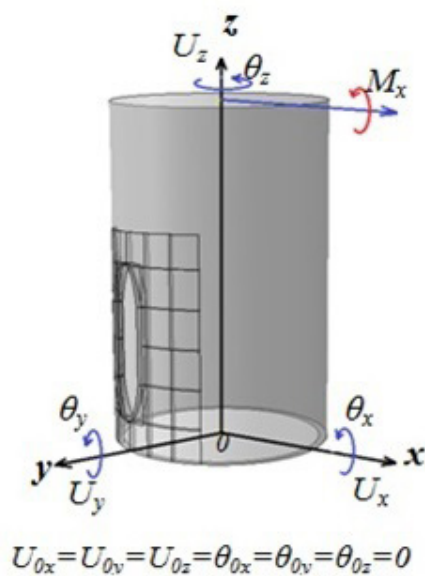
suitable for computer processing. The finite element method has the character of a piece of Rayleigh-Ritz technique; the shell is divided into a number of connected elements only in specific nodes, continuity and equilibrium being written in those nodes. Several shell-type elements are available in the Abaqus library. These elements are differentiated by the number of nodes per element and the number of degrees of freedom per node. Two families of elements are particularly interesting for buckling thin cylindrical shells. These are elements with linear and quadratic geometry.

Using the analytical formula, in the case of a fault-free axis symmetric shell under uniform axial loading, was described by Kim and Kim [19]. They compared the critical stress with those obtained by the finite element computations under Abaqus using the S4R element. This was the most accurate element; the numerical simulations presented in this work are performed using this element.

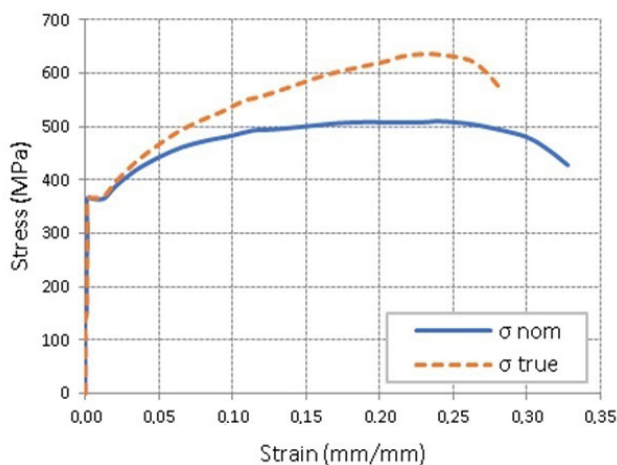
**Table 1** Dimension of the different studied models

The thickness variable	1 <sup>st</sup> Case		2 <sup>nd</sup> Case	3 <sup>rd</sup> Case	4 <sup>th</sup> Case	5 <sup>th</sup> Case	6 <sup>th</sup> Case
	SNO	SO	Frame	Panel	Stiff comb	Panel/Stiff long	Panel/Stiff comb
Shell	40 mm	40 mm	40 mm	40 mm	40 mm	40 mm	40 mm
Thickness of opening frame ( $t_f$ )	/		40 mm	40 mm	40 mm	40 mm	40 mm
Thickness of reinforcement panel ( $t_p$ )	/		/	20÷40 mm	/	10÷20 mm	10÷20 mm
Thickness of longitudinal stiffeners ( $t_{s1}$ )	/		/	/	40÷80 mm	20÷40 mm	40÷80 mm
Thickness of transverse stiffeners ( $t_{s2}$ )	/		/	/	40÷80 mm	/	40÷80 mm

$\theta$ : Door opening angle ( $2\theta = 0.436$  rad)  
 $\beta$ : Angle between the longitudinal stiffeners ( $2\beta = 0.52$  rad and  $4\beta = 1.047$  rad)  
 the angle of the shell between two ends:  $6\beta = 1.57$  rad)



**Fig. 6** Boundary conditions



**Fig. 7** Stress–Strain curves of the steel material S355

**7 Results and discussions**

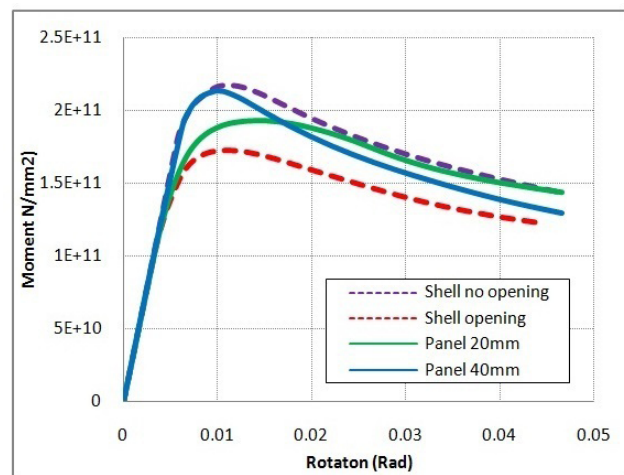
In this study, the ultimate load of a wind turbine mast with an  $R$ -radius,  $L$ -length, cantilevered wind turbine with a local Von Mises criterion of uniaxial strength  $\sigma_0$  is determined.

The geometries of the shells are shown in Fig. 6, with the loading corresponding to a  $M_{cr}$  moment. By way of comparison, we have calculated the ultimate load predicted under bending shell models whose ruin mechanism consists of an area located at the part of the door opening.

We have implemented static approaches on these geometrical structures. The calculated ultimate loads, dimensionless in relation to that of the flexural shell model, are shown in Fig. 8 to Fig. 11 for different reinforcement models. Fig. 7 shows the case of reinforcement of opening by a panel. When increasing the value of the thickness up to 40 mm is noted, it is almost identical to the curve of shell without opening. But there is a divergence between the two curves from almost the top.

Fig. 8 shows the opening reinforcement model with combined stiffeners. It is found that in the thickness of 80 mm there is a correspondence at the top with the curve of shell without opening. But there is a divergence and decrease in the value of the moment in large proportions.

Fig. 9 shows the case of reinforcement of the opening with a panel stiffened by longitudinal crosspieces. We found that there is a correspondence with the curve of shell without



**Fig. 8** Moment-Rotation Curves of shell with reinforced opening by panel

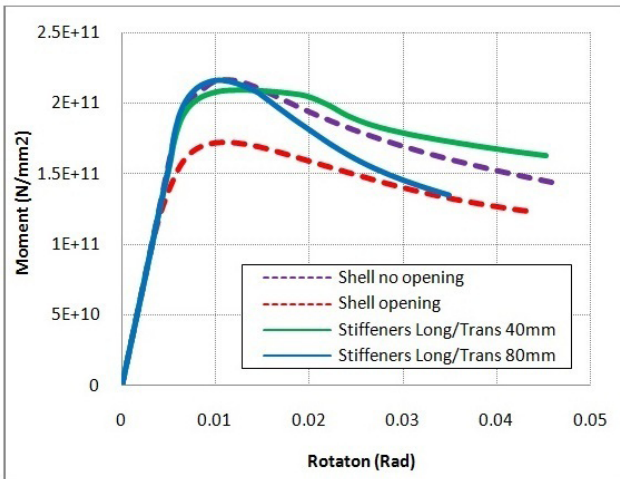


Fig. 9 Moment-Rotation Curves of shell with reinforced opening by comb stiffeners

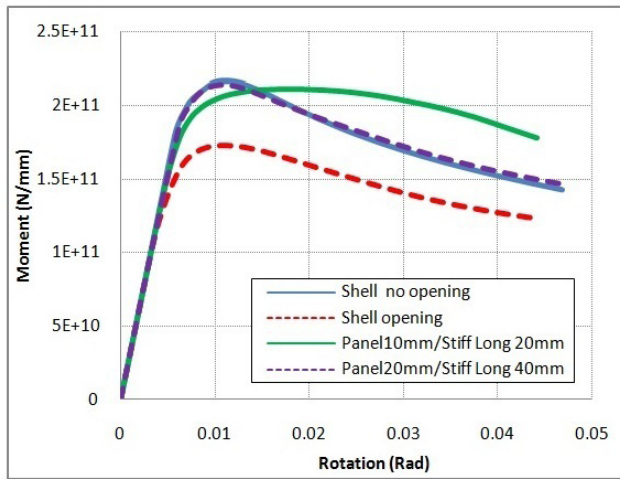


Fig. 10 Moment-Rotation Curves of shell with reinforced opening by panel and stiffeners longitudinal

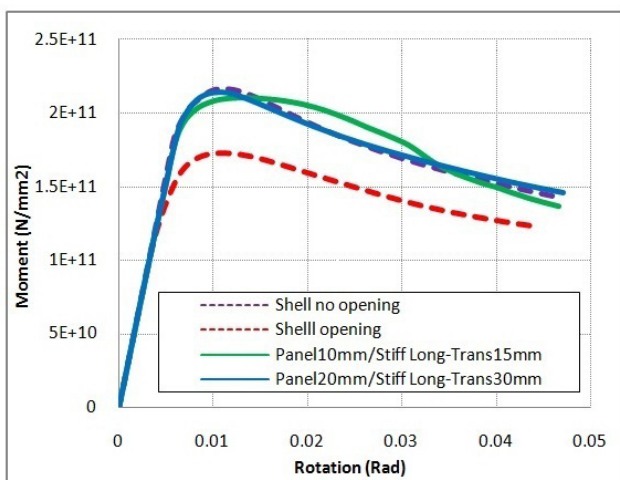


Fig. 11 Moment-Rotation Curves of shell with reinforced opening by panel and comb stiffeners

opening in the case of the value of the thickness of the panel is 20 mm and longitudinal stiffeners 40 mm with a slight divergence in the rise, in the case of the panel thickness 10 mm and longitudinal stiffeners 20 mm, there is semi-stability in the curve without any noticeable convergence.

Fig. 10 shows the model of an opening reinforcement with a stiffened panel. It is noted that there is a convergence with the curve of shell without opening in the case of the thickness value of the panel is 20 mm and combined stiffeners 30 mm. In the case of the thickness of the panel 10 mm and combined stiffeners of 15 mm, there is a case of divergence up and down in the irregular curve.

Fig. 12 compares the curves of the dimensionless moments of the forms proposed with the different types of reinforcement of the opening. It is observed that the curve of the panel stiffened by longitudinal stiffeners or more combined stiffeners corresponds to the curve in the form of shell without opening.

The fatigue calculation procedure for the design of shell was based on the nominal stress and the structure was analyzed for moment cases applied to the shell. From the results are constrained in Fig. 13 to Fig. 16.

The deformation and stress concentration developed at about the gate opening of the tower. The maximum stress appears out of the opening is  $7.06 \times 10^2$  MPa. In cases where the values are higher, there is a flexural shell type ruin mechanism for which the deformation is located near the door opening.

Fig. 12 to Fig. 16 show that the reinforcement is acceptable and useful in case of deformation above and outside the door opening area. In some cases, the deformations appear and they are concentrated at the center of the height of the door opening, such as Figs. 13 and 14, when the value of the thickness is insufficient. If the acceptable reinforcement thickness is reached the deformation and the maximum stress appears in the upper part of the opening of the door, as indicated in the last two cases Figs. 15 and 16.

### 8 Modal analysis based on four models

Using Abaqus software to get modal analysis results, the Table 2 shows the difference between the four models of door opening reinforcement structures. In order to easily explain the results, we can divide the 5 series into 2 groups, and each group contains 2 and 3 values.

From the results, we found that the modes values of models 1, 2 and 3 are convergent, compared to the values of model 4 which show larger values, especially the first

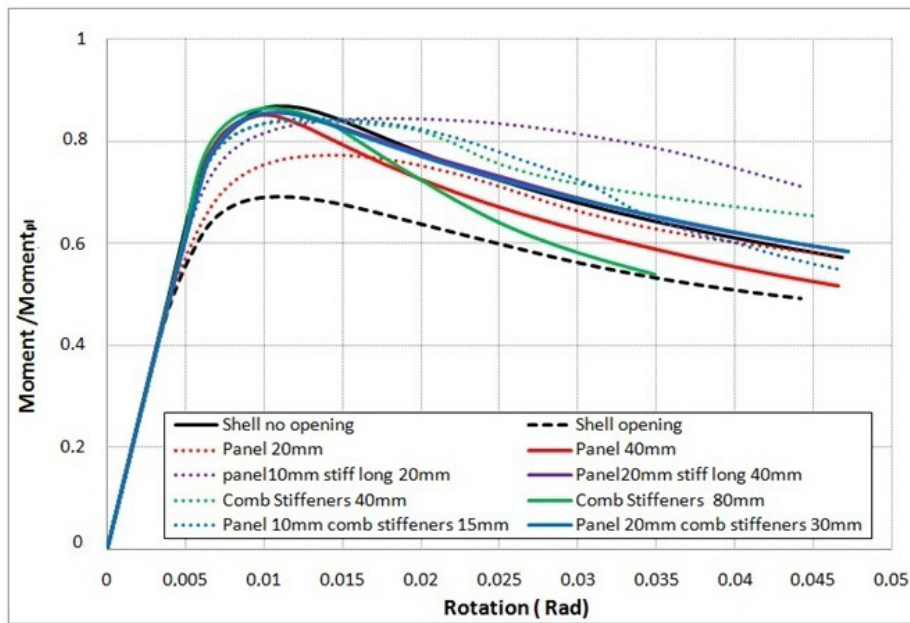


Fig. 12 comparison of reinforcement stiffened shell

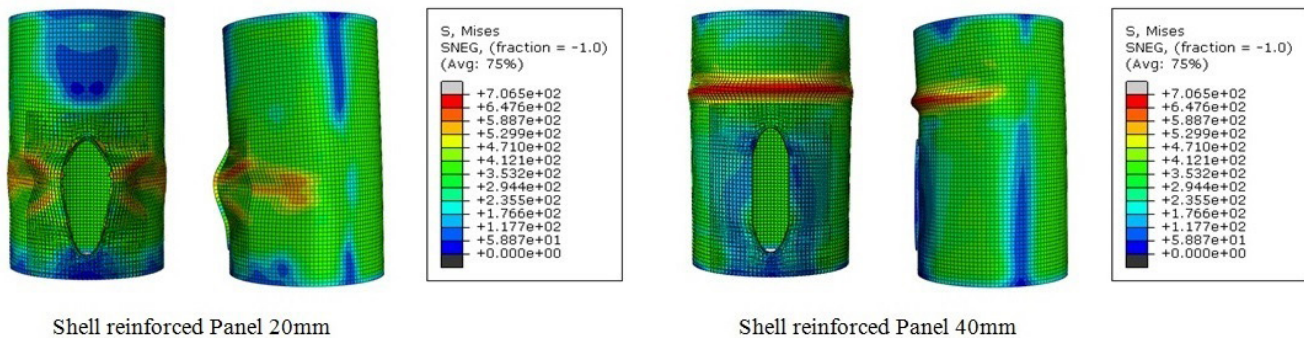


Fig. 13 Mode of instability and stress distribution Von Mises to shell

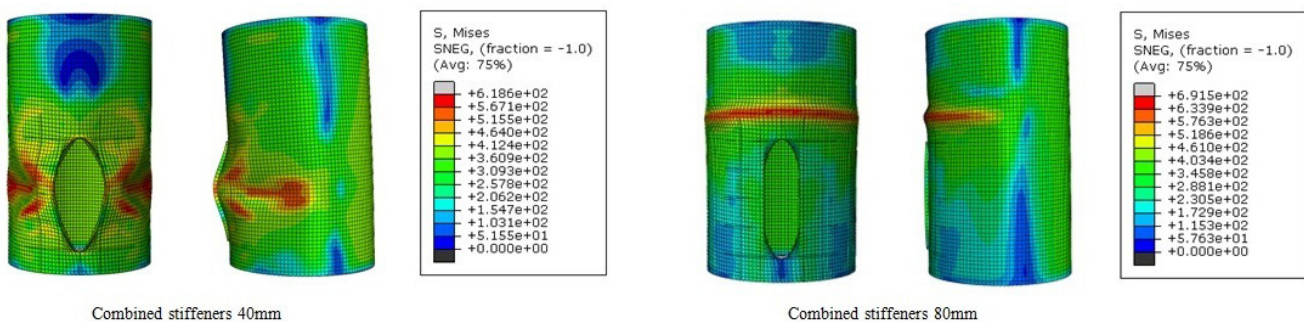


Fig. 14 Mode of instability and stress distribution Von Mises to comb stiffeners 40–80 mm

two values. Therefore the frequency values and the results are shown in Fig. 17. We can say that the door opening reinforced by a plate, stiffened plate or stiffeners combined did not affect any damage on the tower.

### 9 Conclusion and recommendations

The purpose of this study is to analyze the effect of geometrical imperfections on the buckling resistance of a

wind-mast subjected to pure bending. In order to do this; numerical simulation using the Abaqus calculation code considering the S4R element was conducted. The calculation is based on the ultimate load predicted by flexural shell models whose ruin mechanism consists of an area located at the interior part of the door.

Reinforcement of the opening area of the wind tower is the only solution to compensate for the lack of material



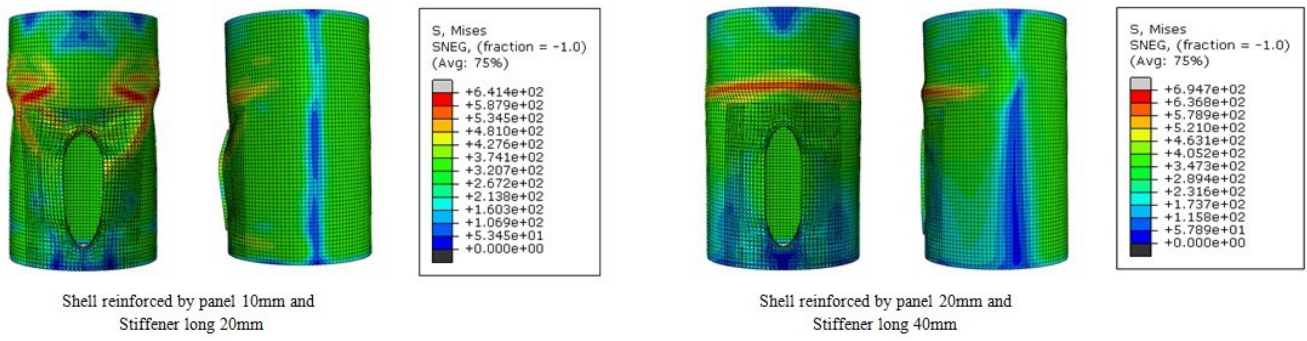


Fig. 15 Mode of instability and stress distribution Von Mises to shell stiffeners longitudinal

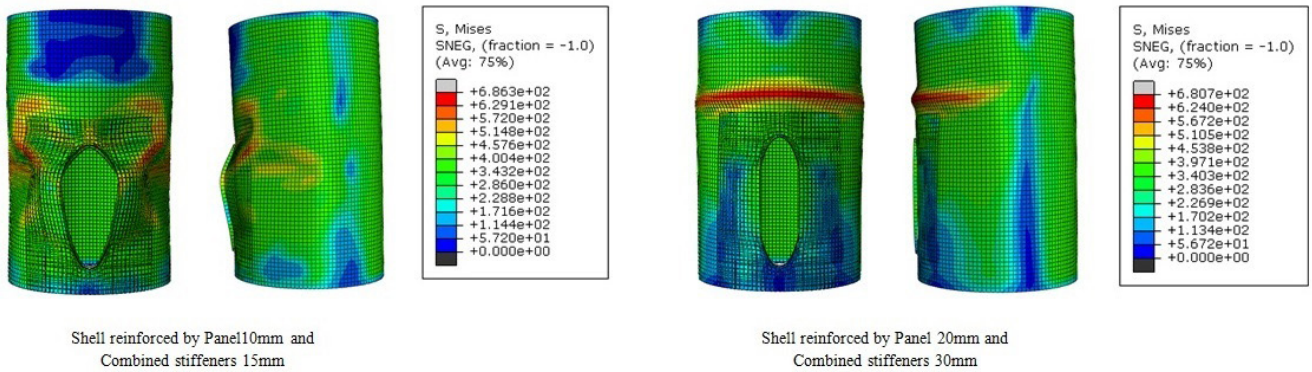


Fig. 16 Mode of instability and stress distribution of Von Mises for stiffened shell

Table 2 Results of the five modals of the different studied models

Set/Frequency	Model 1	Model 2	Model 3	Model 4
Set 1	6.556 E+11	7.227 E+11	6.011 E+11	8.025 E+11
Set 2	6.998 E+11	7.402 E+11	6.384 E+11	8.111 E+11
Set 3	9.606 E+11	1.051 E+12	9.195 E+11	1.127 E+12
Set 4	1.004 E+12	1.175 E+12	9.920 E+11	1.251 E+12
Set 5	1.351 E+12	1.354 E+12	1.341 E+12	1.364 E+12

in the cut of the door. This makes it the weakest, most sensitive and most vulnerable area, especially in extreme conditions. Consequently, we present this parametric study based on the comparison of all the proposed solutions; the panel, combined stiffeners, stiffened panel by longitudinal stiffeners or by combined stiffeners. Comparing and analyzing the curves and areas of maximum stress values presented in the figures. We found that the optimal model and the best solution in terms of economic cost and ease of manufacture, in that the panel stiffened by longitudinal stiffeners members through the desired results. Then by the combined stiffeners, panel and finally by the panel stiffened by the combined stiffeners. The results of analysis moment-rotation indicate that the tower is reaching the limit of plasticity charged before approaching the point elastic bifurcation. The collapse from the buckling of the

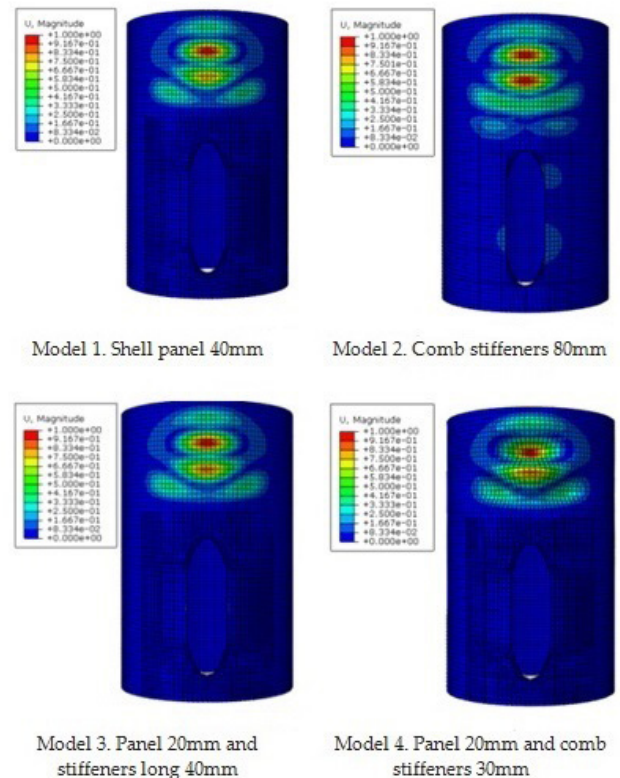


Fig. 17 First modes of buckling for different reinforcements of the opening

shell on the door opening, where the plastic stresses of Von Mises are components of the meridian compression and circumferential stresses.

From this study, it is clear that the limit load evolves towards a value close to the flexural shell model when the elongation values increase, while it is low for moderate elongation values and for different reinforcement geometries.

This work does not claim to be complete to solve all the problems of buckling instability of the wind tower

under extreme conditions. To further improve the work presented and make it more fruitful, it is recommended to further develop the following points:

- study of the possibility of coupling between modes of vibration and modes of buckling of the tower wind subjected to seismic areas;
- introduction of initial imperfections and taking into account the sensitivity of the shell structures to these imperfections.

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