Abstract

Double-diffusive convection in a horizontal layer of nanofluid in a porous medium is studied. The couple-stress fluid model is considered to describe the rheological behavior of the nanofluid and for porous medium Darcy model is employed. The model applied for couple stress nanofluid incorporates the effect of Brownian motion and thermophoresis. We have assumed that the nanoparticle concentration flux is zero on the boundaries which neutralizes the possibility of oscillatory convection and only stationary convection occurs. The dispersion relation describing the effect of various parameters is derived by applying perturbation theory, normal mode analysis method and linear stability theory. The impact of various physical parameters, like the couple stress parameter, medium porosity, solutal Rayleigh Number, thermo-nanofluid Lewis number, thermo-solutal Lewis number, Soret parameter and Dufour parameter have been examined on the stationary convection. It is observed that the couple stress parameter, thermo-nanofluid Lewis number, thermo-solutal Lewis number, Soret parameter and Dufour parameter have stabilizing effects on the stationary convection whereas the solutal Rayleigh number and Dufour parameter have very small effect on the system.

Keywords

nanofluid, porous medium, double-diffusive, convection, couple stress, Brownian motion, thermophoresis

1 Introduction

The onset of double-diffusive convection in nanofluid finds applications in the fields of chemical science, food processing, engineering and nuclear industries, geophysics, bioengineering and cancer therapy, movement of biological fluid, oceanography [1-9]. The word nanofluid was first proposed by Choi and Eastman [10]. Nanofluids are colloidal suspension of nanosized particles range between 1 to 100 nm [11] in the base fluid like water, oil, ethylene glycol etc. Nanoparticles are of materials such as metallic oxides (Al$_2$O$_3$, CuO), nitride ceramics (AlN, SiN), carbide ceramics (SiC, TiC), metals (Cu, Ag, Au) etc. have been used for the preparation of nanofluids.

Due to the small dimensions, the suspended nanoparticles can behave like a base fluid molecule, which helps us to reduce problems like particle clogging, sedimentation etc. The combination of highly stable and highly conductivity of the suspended nanoparticles which make them highly preferable for making heat transfer fluids [12-13]. The stable suspensions of small quantities of nanoparticles and increased heat transfer properties of nanofluids will possibly help us to design lighter, high performance thermal management systems and cost-effective liquid cooling systems.

A review article of heat transfer in nanofluid has been studied by Das and Choi [14] and a book [15]. Buongiorno [16] proposed a mathematical model for nanofluid based on the effects of Brownian motion and thermophoresis of suspended nanoparticles after analyzing the effect of seven slips mechanism, he concluded that in the absence of turbulent eddies, Brownian diffusion and thermophoresis are the dominant slip mechanisms.

Stokes [17] proposed and postulated the theory of couple-stress fluid. The application of couple-stress fluid is to the study of the mechanism of lubrication of synovial joints, which has become the main objective of scientific research and found that the synovial fluid in human joints behaves like a couple-stress fluid [18]. Sharma and Thakur [19] studied the couple-stress fluid heated from below in hydro magnetics and found that couple stress parameter has stabilizing effect on the stationary convection. Shivakumara et al. [20] studied the couple stress fluid under rotation in electro hydrodynamics.
and found that the couple stress fluid under rotation becomes destabilizing in the presence of couple stress for all the boundary conditions considered.

A convective transport in nanofluid was studied by [21-37]. In this paper, more realistic boundary conditions are used, see [38-39]. We assume that there is no flux at the plate and the nanoparticle flux value adjust accordingly. There is a need of changing the scale of dimensionless parameters. The basic solution of nanoparticle volume fraction is changed. The oscillatory convection does not exist and only stationary convection occurs.

Keeping in view of various applications of couple stress and nanofluid as mentioned above, our main aim in the present paper is to study the effect of cross-diffusion and couple stress parameter on the onset of convection in a horizontal layer of nanofluid saturating a porous medium. This problem is of triple-diffusion type as it contains heat, the nanoparticles and the solute.

2 Mathematical Model and Governing Equations

We consider an infinite horizontal porous medium layer of thickness \( d \), bounded by the planes \( z = 0 \) and \( z = d \) of a couple stress nanofluid as shown in Fig. 1. The gravity force \( g = (0, 0, -g) \) is acted aligned in the \( z \) direction of the layer which is heated and soluted from below. Let \( T_0, C_0 \) and \( \varphi_0 \) be the constant values of temperature, concentration and the volumetric fraction of nanoparticles at the lower boundary and \( T_T, C_T \) and \( \varphi_T \) be the constant values of temperature, concentration and the volumetric fraction of nanoparticles at the upper boundary. We know that keeping a constant volume fraction of nanoparticles at the horizontal boundaries will be almost impossible in a realistic situation.

![Fig 1 Physical Configuration](image)

Let \( \rho_0, \varepsilon, \mu, \mu_C, p \) and \( q(u, v, w) \), denote respectively, the reference density, medium porosity, viscosity, the material constant responsible for couple stress property known as the couple stress viscosity, pressure, and Darcy velocity vector, respectively. \( T \) is the temperature, \( \varphi \) is the volume fraction of nanoparticles, \( \rho_\varphi \) is the density of nanoparticles, \( \alpha_T \) is the coefficient of thermal expansion and \( \alpha_C \) is analogous to solute concentration. The equations of continuity and motions for couple-stress nanofluid saturating a porous medium by applying Oberbeck-Boussinesq approximation [13, 17, 21, 22] are

\[
\nabla \cdot \mathbf{q} = 0, \\
0 = -\nabla p - \frac{1}{k_1}(\mu - \mu_\varepsilon) \nabla^2 \mathbf{q} + g\left(\rho_\varphi \rho_p + \rho_s (1-\varphi) \left\{ (1-\alpha_T (T-T_0) - \alpha_C (C-C_0)) \right\} \right).
\]

(2)

Let \( D_B \) be the thermophoretic diffusion coefficient and \( D_a \) is the Brownian diffusion coefficient. Assuming that the solute does not affect the transport of the nanoparticles. The equation of continuity [16, 24] for the nanoparticles in the presence of thermophoresis and absence of chemical reaction is

\[
\left( \frac{\partial}{\partial t} + \frac{1}{\varphi} \mathbf{q} \cdot \nabla \right) \varphi = D_B \nabla^2 \varphi + \frac{D_a}{T} \nabla^2 T.
\]

(3)

The thermal energy equation for a nanofluid is

\[
(\rho c)_w \frac{\partial T}{\partial t} + (\rho c)_\varphi \mathbf{q} \cdot \nabla T = k \nabla^2 T + \varepsilon (\rho c)_p \\
D_B \nabla \varphi \cdot \nabla T + \frac{D_a}{T} \nabla^2 \nabla T + \rho C_D \nabla^2 C,
\]

(4)

where \( c, c_p, k \) and \( D_{CT} \) be the fluid specific (at constant pressure), the heat capacity of the material constituting nanoparticles, the thermal conductivity of the porous medium and the diffusivity of Dufour type, respectively.

Let \( D_B \) and \( D_{CT} \) be respectively, the solute diffusivity and diffusivity of Soret type and it is assumed that the nanoparticles do not affect the transport of the fluid. The conservation equation for solute concentration [24] is

\[
\left( \frac{\partial}{\partial t} + \frac{1}{\varphi} \mathbf{q} \cdot \nabla \right) C = D_B \nabla^2 C + D_{CT} \nabla^2 T.
\]

(5)

We assume that the temperature to be constant and the thermodiffact of nanoparticles flux to be zero at the boundaries [27]. The appropriate boundary conditions [38, 39] are

\[
w = 0, \quad \frac{\partial w}{\partial z} = 0, \quad T = T_0, \quad C = C_0, \quad D_B \frac{\partial \varphi}{\partial z} + \frac{D_a}{T_1} \frac{\partial T}{\partial z} = 0 \quad \text{at } z = 0,
\]

\[
w = 0, \quad T = T_\sigma, \quad C = C_\sigma, \quad \frac{\partial w}{\partial z} = 0, \quad D_B \frac{\partial \varphi}{\partial z} + \frac{D_a}{T_1} \frac{\partial T}{\partial z} = 0 \quad \text{at } z = d.
\]

(6)

(7)

Introducing non-dimensional variables as

\[
(x', y', z') = \left( \frac{x, y, z}{d} \right), \quad (u', v', w') = \left( \frac{\mu_v, \nu, \omega}{k_f} \right) d,
\]

\[
t' = \frac{Tk_f'}{\sigma d T'}, \quad p' = \frac{pd^2}{\mu k_f}, \quad \varphi' = \frac{\varphi - \varphi_0}{\phi_\varepsilon - \phi_0}, \quad T' = \frac{T - T_0}{T_0 - T_1},
\]

\[
C' = \frac{C - C_1}{C_\sigma - C_1},
\]

and

\[
\frac{\partial T'}{\partial t'} + \frac{1}{\varphi'} \mathbf{q}' \cdot \nabla T' = \frac{k}{d} \nabla^2 T' + \varepsilon (\rho c)_p \\
D_B \frac{\partial \varphi'}{\partial z'} + \frac{D_a}{T_1} \frac{\partial T'}{\partial z'} + \rho C_D \frac{T'}{d} \nabla^2 C'
\]

(8)
where $\kappa = \frac{k}{(pc)}_f$ is thermal diffusivity of the fluid and $\sigma = \frac{(pc)}{\rho c T_f}$ is the thermal capacity ratio. Thereafter, dropping the dashes (') for convenience.

In non-dimensional form, Eqs. (1)-(8) can be written as

$$\nabla q = 0, \quad (9)$$

$$0 = -\nabla p - (1 - \eta \nabla^2) q - \rho \alpha_0 T + \frac{R_s}{Le} C e - \eta \phi e, \quad (10)$$

$$\frac{1}{\varepsilon} \frac{\partial \phi}{\partial t} + \frac{1}{\varepsilon} q \nabla \phi = \frac{1}{L_n} \nabla^2 \phi + \frac{N_b}{L_n} \nabla^2 \phi, \quad (11)$$

$$\frac{\partial T}{\partial t} + q \nabla T = \nabla^2 T + \frac{N_b}{L_n} \nabla \phi \nabla T + \frac{N_b}{L_n} T \nabla \nabla T + N_c \nabla^2 C, \quad (12)$$

$$\frac{1}{\varepsilon} \frac{\partial C}{\partial t} + \frac{1}{\varepsilon} q \nabla C = \frac{1}{L_n} \nabla^2 \phi + N_{ct} \nabla^2 T, \quad (13)$$

$$w = 0, \quad \frac{\partial w}{\partial z} = 0, \quad T = 1, \quad C = 1, \quad \frac{\partial \phi}{\partial z} + N_b \frac{\partial T}{\partial z} = 0 \quad \text{at} \quad z = 0, \quad (14)$$

$$w = 0, \quad \frac{\partial w}{\partial z} = 0, \quad T = 0, \quad C = 0, \quad \frac{\partial \phi}{\partial z} + N_b \frac{\partial T}{\partial z} = 0 \quad \text{at} \quad z = 1. \quad (15)$$

Where

- $Le = \frac{\kappa}{D_s}$; (thermo-solutal Lewis number)
- $Ln = \frac{\kappa}{\kappa_s}$; (thermo-nanofluid Lewis number)
- $F = \frac{\mu}{\mu_s}$; (couple-stress parameter)
- $Ra = \frac{g \rho \alpha_0 a_d (T_0 - T_f)}{\mu \kappa}$; (thermal Rayleigh number)
- $Rs = \frac{g \rho \alpha_0 a_d (C_0 - C)}{\mu D_s}$; (solutal Rayleigh number)
- $Rm = \frac{\rho \omega \phi + \rho(1 - \omega) \eta \phi e}{\mu \kappa}$; (Density Rayleigh number)
- $Rn = \frac{\rho \omega \phi + \rho(1 - \omega) \eta \phi e}{\mu \kappa}$; (nanoparticle Rayleigh number)
- $N_b = \frac{D_b (T_0 - T_f)}{D_s (\phi - \phi_e)}$; (modified diffusivity ratio)
- $N_s = \frac{(pc)}{(pc)}_f (\phi - \phi_e)$; (modified particle-density ratio)
- $N_c = \frac{D_c (C_0 - C)}{\kappa_s (T_0 - T_f)}$; (Dufour parameter)
- $N_{ct} = \frac{D_{ct} (C_0 - C)}{\kappa_s (T_0 - T_f)}$; (Soret parameter).

Using equations given in Eq. (16) into Eqs. (9)-(15), linearizing the resulting equations by neglecting nonlinear terms that are product of prime quantities and dropping the primes (”) for convenience, we obtained

$$\nabla q = 0, \quad (17)$$

$$0 = -\nabla p - (1 - \eta \nabla^2) q - \rho \alpha_0 T + \frac{R_s}{Le} C e - \eta \phi e, \quad (18)$$

$$\frac{1}{\varepsilon} \frac{\partial \phi}{\partial t} + \frac{1}{\varepsilon} w = \frac{1}{L_n} \nabla^2 \phi + \frac{N_b}{L_n} \nabla^2 T, \quad (19)$$

$$\frac{\partial T}{\partial t} - w
abla^2 T + \frac{N_b}{L_n} \left( \frac{\partial T}{\partial z} - \frac{\partial \phi}{\partial z} \right) - 2N_b^2 \frac{\partial T}{\partial z} + N_c \nabla^2 C, \quad (20)$$

$$\frac{1}{\varepsilon} \frac{\partial C}{\partial t} - \frac{1}{\varepsilon} w = \frac{1}{L_n} \nabla^2 C + N_{ct} \nabla^2 T, \quad (21)$$

$$w = 0, \quad T = 0, \quad C = 0, \quad \frac{\partial \phi}{\partial z} + N_b \frac{\partial T}{\partial z} = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad z = 1. \quad (22)$$

Note that as the parameter $Rm$ is not involved in Eqs. (17)-(21) it is just a measure of the basic static pressure gradient. In the absence of couple stress parameter and nanoparticles, the above system of Eqs. (17)-(21) reduced to the well-known equations for the double-diffusive Rayleigh-Bénard problem.

The seven unknowns $u, v, w, p, T, C$ and $\phi$ can be reduced to four by operating Eq. (18) with $\varepsilon$-curl curl, together with Eq. (17), which yields

$$\left(1 - \eta \nabla^2\right) \nabla^2 w = \frac{R_s}{Le} \nabla^2 T + \frac{R_s}{Le} \nabla \nabla \phi, \quad (23)$$

where $\nabla^2 w$ is the two-dimensional Laplace operator on the horizontal plane, that is $\nabla^2 w = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2}$.

The differential equations (23), (19)-(21) and the boundary conditions Eq. (22) constitute an eigenvalue problem which can be solved by the method of normal mode analysis method.

4 Normal Modes Analysis Method

We express the disturbances into normal modes of the form

$$[w, T, C, \phi] = [W(z), \Theta(z), \Gamma(z), \Phi(z)] \exp(\text{i}rx + \text{i}sy + \text{i}t), \quad (24)$$

where $r, s$ are the wave numbers in the $x$ and $y$ direction, respectively, and $p$ is the growth rate of the disturbances.

Substituting Eq. (24) into Eqs. (23) and (19)-(22), we obtain the following eigenvalue problem

$$\left(1 - \eta \left( D^2 - a^2 \right) \right) \left( D^2 - a^2 \right) W + \omega^2 R_a \Theta + \frac{R_s}{Le} \omega^2 \Gamma - \omega^2 R_n \Phi = 0, \quad (25)$$

$$\frac{1}{L_n} \left[ N_{ct} \left( D^2 - a^2 \right) \right] \Theta + \frac{1}{Le} \left[ D^2 - a^2 \right] \Phi = 0, \quad (26)$$
\[ W + \left( D^2 + \frac{N_a}{L_n} D - \frac{2N_a N_b}{L_n} D - \omega^2 - \frac{P}{\sigma} \right) \Phi = \frac{N_a}{L_n} D \Phi + N_{tc} \left( D^2 - \omega^2 \right) \Gamma = 0, \]  
\[ (27) \]

\[ \frac{1}{\varepsilon} W - \frac{N_a}{L_n} \left( D^2 - \omega^2 \right) \Theta - \left( \frac{1}{L_n} \left( D^2 - \omega^2 \right) - \frac{P}{\sigma} \right) \Phi = 0, \]  
\[ (28) \]

where \( D \) and \( \omega^2 = r^2 + s^2 \) is the dimensionless horizontal wave number.

Considering solutions \( W, \Theta, \Gamma \) and \( \Phi \) of the form
\[ W = W_0 \sin (\pi z), \quad \Theta = \Theta_0 \sin (\pi z), \quad \Gamma = \Gamma_0 \sin (\pi z), \quad \Phi = \Phi_0 \sin (\pi z). \]  
\[ (30) \]

Substituting Eq. (30) into Eqs. (25)-(28) and integrating each equation from \( z = 0 \) to \( z = 1 \), we obtain the following matrix equations
\[ \begin{bmatrix}
1 + \eta J^2 & -\omega^2 Ra & 0 & -\omega^2 Rn \\
-1 & \frac{J^2 N_{ct}}{Le} & 0 & 0 \\
-1 & J^2 + p & -\frac{J^2 N_{tc}}{Le} & 0 \\
\frac{1}{\varepsilon} & \frac{N_a}{L_n} J^2 & 0 & \frac{J^2}{Le} + \frac{p}{\sigma} \\
\end{bmatrix}
\begin{bmatrix}
W_0 \\
\Theta_0 \\
\Gamma_0 \\
\Phi_0 \\
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}, \]  
\[ (31) \]

where \( J^2 = \pi^2 + a^2 \) is the total wave number.

The linear system Eq. (31) has a non-trivial solution if and only if
\[ Ra = \frac{1}{\varepsilon \sigma J^2 + p \sigma + \sigma N_{tc} J^2 Le} \times \]
\[ \frac{1}{\varepsilon} \left( 1 + \eta J^2 \right) J^4 \left( J^2 + p \right) \left( \sigma J^2 + p \right) + LeN_{tc} N_{ct} J^4 \]
\[ + Rn \sigma \left( J^2 + p Ln \right) + \frac{Rn \sigma}{J^2 + p Ln} \times \]
\[ (J^2 + p) \left( J^2 + p Ln + \varepsilon N_{tc} J^2 \right) \]  
\[ (32) \]

Equation (32) is the dispersion relation representing the effect of couple-stress parameter, medium porosity, thermosolutal Lewis number, thermo-nanofluid Lewis number, solutal Rayleigh Number, nanoparticle Rayleigh number, kinematic modified diffusivity ratio, Soret and Dufour parameter on double-diffusive convection in a layer of couple stress nanofluid saturating a porous medium.

### 5 The Stationary Convection

For stationary convection, we put \( p = 0 \) in Eq. (32), reduces it to
\[ Ra = \frac{1}{\varepsilon + N_{tc} Le} \times \]
\[ \left\{ \frac{1}{\varepsilon} \left( \pi^2 + \omega^2 \right) \left( 1 + \eta \left( \pi^2 + \omega^2 \right) \right) \right\} \left( 1 + \frac{LeN_{tc} N_{ct}}{\varepsilon} \right) \]
\[ + \frac{Rn \left( N_{ct} e - 1 \right) - Rn \left( N_{tc} + LnN_{ct} \right)}{LeN_{tc} \left( N_{tc} + LnN_{ct} \right)} \]  
\[ (33) \]

Equation (33) represents the thermal Rayleigh number as a function of the non-dimensional wave number \( \omega \) corresponding to the parameters \( \eta, \varepsilon, N_{tc}, N_{ct}, Ra, Ln, Rn, Le, N_{s} \).

Equation (33) is identical to that obtained by [24, 28, 29]. In the following discussion, negative values of the solute gradient parameter \( N_{s} \) do not appear and the diffusivity ratio parameter \( N_{s} \) appears only in association with the nanoparticle Rayleigh number \( Rn \). This implies that the nanofluid cross-diffusion terms approach to be dominated by the regular cross-diffusion term.

If we neglect the Dufour and Soret parameters \( N_{tc}, N_{ct} \), Eq. (33) reduces to
\[ Ra = \frac{1}{\varepsilon} \left( \pi^2 + \omega^2 \right) \left( 1 + \eta \left( \pi^2 + \omega^2 \right) \right) - \frac{1}{\varepsilon} \]
\[ \left( \frac{Rn \left( N_{ct} e - 1 \right) - Rn \left( N_{tc} + LnN_{ct} \right)}{LeN_{tc} \left( N_{tc} + LnN_{ct} \right)} \right) \]  
\[ (34) \]

which is identical with the result derived in [24, 28, 29]. In the absence of the solute gradient parameter \( Rn \), Eq. (33) reduces to
\[ Ra = \frac{1}{\varepsilon} \left( \pi^2 + \omega^2 \right) \left( 1 + F \left( \pi^2 + \omega^2 \right) \right) \]  
\[ (35) \]

Equation (35) is identical with the results derived in [26-29].

Due to the absence of opposing buoyancy forces, the oscillatory convection does not exist. So we consider only the case of stationary convection.

### 6 Results and Discussions

To depict the stability characteristics, the dispersion relation Eq. (33) is analyzed numerically and graphs have been plotted. According to the definition of nanoparticle Rayleigh number \( Rn \), this corresponds to negative value of \( Rn \) for heavy nanoparticles (\( \rho_{p} > \rho \)). In the following discussion, negative values of \( Ra \) are presented.

The variations of thermal Rayleigh number \( Ra \) with the wave number \( \omega \) for different values of the couple stress parameter \( \eta = 0.2, \eta = 0.4 \) and \( \eta = 0.6 \) is plotted in Fig. 2 and it is found that the thermal Rayleigh number \( Ra \) increases with the
increase of couple stress parameter. Thus couple stress parameter stabilizes the stationary convection.

In Fig. 3, the variations of thermal Rayleigh number $Ra$ with the wave number $\omega$ for three different values of the solutal Rayleigh number, namely, $Rs = 1000$, $5000$ and $9000$ is plotted and it is noticed that the thermal Rayleigh number slightly increases with the increase in solutal Rayleigh number so the solutal Rayleigh number has very small effect on the stability of the system. In Fig. 4, the variations of thermal Rayleigh number $Ra$ with the wave number $\omega$ for three different values of the thermo-nanofluid Lewis number, namely, $Ln = 300$, $600$ and $900$ is plotted and it is observed that thermal Rayleigh number increases slightly with the increase in thermo-nanofluid Lewis number so the thermo-nanofluid Lewis number has slightly stabilizing effect on the system.

The variations of thermal Rayleigh number $Ra$ with the wave number $\omega$ for three different values of the Soret parameter, namely, $N_{CT} = 10$, $20$, $30$ which shows that thermal Rayleigh number increases with the increase in Soret parameter. Thus, Soret parameter has stabilizing effect on the system.

The variations of thermal Rayleigh number $Ra$ with the wave number $\omega$ for three different values of Dufour parameter, namely $N_{TC} = 5$, $45$ and $85$ is plotted in Fig. 7 and it is observed
that thermal Rayleigh number increases with the increase in Dufour parameter so the Dufour parameter has stabilizing effect on the onset of stationary convection in a layer of couple-stress nanofluid. The system becomes more stable when the values of Soret and Dufour parameters are equal.

In Fig. 8, the variations of thermal Rayleigh number $Ra$ with the wave number $\omega$ for three different values of the medium porosity, namely $\varepsilon = 0.2$, $0.4$, $0.6$ which shows that thermal Rayleigh number increases with the increase in medium porosity. Thus medium porosity has stabilizing effect on the system.

The results obtained in Figs. 2 to 8 are in good agreement with the result obtained by [23-35].

7 Conclusions
The onset of double-diffusive convection in a porous medium layer of couple stress nanofluid in a more realistic boundary conditions has been investigated which comprises the effects of thermophoresis and Brownian motion. It is found that the couple stress parameter has no effect on the basic solution. Nanofluid cross-diffusion terms approach to be dominated by the regular cross-diffusion term. The couple stress parameter, thermo-nanofluid Lewis number, thermo-solutal Lewis number, Soret parameter and Dufour parameter have stabilizing effects on the stationary convection as shown in Figs. 2, 4, 5, 6 and 8, respectively. The solutal Rayleigh number and Dufour parameter have very small effect on the system as can be seen in the Figs. 2 and 7, respectively. The system becomes more stable when the values of Soret and Dufour parameters are equal. Oscillatory convection does not exist under the more realistic boundary conditions.

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References
https://doi.org/10.18869/acadpub.jafm.68.236.25048

https://doi.org/10.18869/acadpub.jafm.73.240.27475

https://doi.org/10.1016/j.asej.2015.11.023

https://doi.org/10.1080/01457632.2018.1470298

https://doi.org/10.1016/j.ijheatmasstransfer.2013.09.026

https://doi.org/10.1016/j.asej.2015.05.005