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RESEARCH ARTICLE

Characterization and implementation of the viscoelastic properties of an EPDM rubber into FEA for energy loss prediction

László Pálfi / *Károly* Váradi
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Abstract

This paper investigates the efficiency of different generalized Maxwell models based on DMTA measurements in predicting the amount of dissipated energy. Generalized Maxwell models combined with Mooney-Rivlin model have been used to describe the material behaviour of the studied EPDM 75 IRH rubber. For verifying the material models FE simulations were carried out.

It can be concluded that the 15-term generalized Maxwell model produces inaccurate results. Improving the material model to 40-term the simulations show a small underestimation of the energy loss in the lower frequency range. Using 40-term Maxwell model fitted to the loss factor, the underestimation is reduced.

Keywords

DMTA · FE analysis · generalized Maxwell model · viscoelascity · rubber

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László Pálfi

Department of Machine and Product Design, BME, H-1111 Budapest, Műegyetem rkp 3., Hungary
e-mail: palfi.laszlo@gt3.bme.hu

Károly Váradi

Department of Machine and Product Design, BME, H-1111 Budapest, Műegyetem rkp 3., Hungary
e-mail: varadik@eik.bme.hu

1 Introduction

Rubbers and elastomeric materials are used broadly in the mechanical engineering. Tires, v-belts, belts, rollers, special bearings are made of rubber. The internal damping of rubbers can be used in some application, but the same phenomenon can cause unfavourable energy dissipation during rolling or cyclic bending of rubbery elements. This is the reason why it is necessary to investigate how to predict the rubber internal friction.

To calculate the exact strains during mechanical loading of rubber or elastomeric elements is very complicated because of the complexity of the viscoelastic material models. One of the most essential parts in numerical calculation of energy loss is the material model. It should be able to describe, as accurately as possible, the frequency dependencey of the measured storage modulus as well as the measured loss factor. The generalized Maxwell model can be a suitable selection because it is available in most commercial finite element software. The problem with using such a material model is that its parameters are commonly determined from a fit to the measured storage modulus master curve. It is necessary to note that good correspondence between the measured and the fitted storage modulus curve does not assure an equally good correspondence in case of the loss factor curve. Exact numerical solution for the energy loss in the rubber can not be predicted as long as the material model is unable to depict either the storage modulus or the loss factor curve with a reasonable correspondence.

The aim of this study is to determinate the viscoelastic material properties of an EPDM rubber (with the hardness of 75 IRH) material using DMTA measurement and compare the different material models with each other in order to determine the applicability of the frequently used generalized Maxwell model. The linear viscoelastic properties of the rubber were determined by DMTA measurements. On the basis of DMTA measurements, master curves were constructed and n-term Maxwell-models were fitted to the different master curves at different temperatures. To describe the incompressibility and the non-linear behavior of the rubber the well known Mooney-Rivlin material model was used [1, 2].

2 Experimentals

DMTA measurements were performed in order to characterize the viscoelastic material models. The material model was used in a simulated DMTA test by FEA to model the dynamic behavior and dissipated energy in the rubber. The measurements were carried out at TU Kaiserslautern in the IVW (Institut für Verbundwerkstoffe GmbH). The details of the measurements are written in the following chapters.

2.1 Material

Peroxide cured EPDM rubber was used with a ∼65 phr carbon black content (CB N347), IRH 75 (referred further on as EPDM 75) was investigated. The samples were cut from the hot pressed 6.3 mm thick rubber sheets for the DMTA measurements.

2.2 DMTA measurement

The DMTA tests were prepared with equipment GABO Eplexor 100N (Ahlden, Germany). During a DMTA test, the specimen is subjected to dynamic sinusoidal load in a way that temperature and frequency sweeps were done in the course of the test. During the test the complex modulus (E^*) of the material, its storage (E') and loss (E'') parts, as well as its mechanical loss factor (tan(δ)) were recorded. DMTA tests were performed on three specimens of identical size and shape. The dynamic load can be regarded as a superposition of a constant load and a sinusoidally varying load. The load of DMTA sample can be realized by prescribing a given force (stress) or displacement (strain). The static (tensile) strain was $\varepsilon_s = 0.06581\%$ while the sinusoidally varying strain was $\varepsilon = 0.01\%$. In the course of the test, the values of E', E'' and tan(δ) were measured in a frequency range between 1 and 100 Hz. Then the measurement was repeated on the same frequency range by changing the temperature. The temperature was changed from -100^oC to $+100^o$C with 5^oC increments.

The DMTA measurement results, considering the storage modulus (E') of the investigated EPDMA rubber, are shown in Fig. 3 in a logarithmic scale. Each curve characterizes the measured storage modulus in the observed range of frequency at different temperatures. In the course of constructing the master curves for different temperatures, the storage modulus curves pertaining to the -50^{o}C, room temperature and 150^{o}C were selected as the reference curves. Conforming to the experiences at low temperatures the storage modulus curves show increasing character as the frequency risesw, converging to a limit value, as the temperature decreases (Fig. 1).

3 Material model

Based on the previous chapters, three different material models were constructed for modelling the DMTA measurement. A 15-term and a 40-term Maxwell model fitted to the E' and a 40-term Maxwell model fitted to the loss factor was created from the DMTA results.

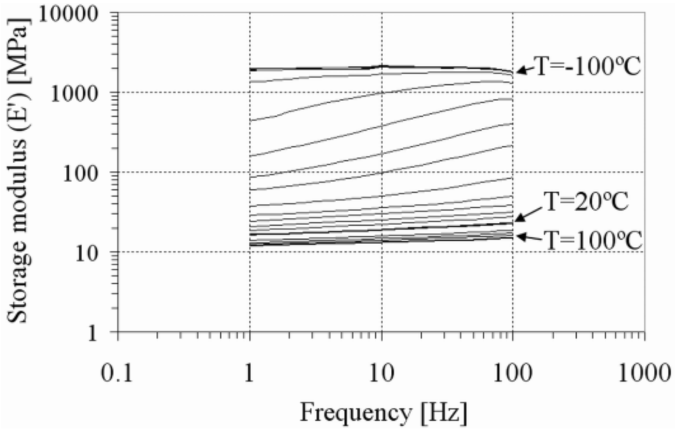

Fig. 1. Results of the frequency sweep at various temperatures for storage modulus (E')

For constructing the master curves of the investigated EPDM rubber for different temperatures the WLF and Arrhenius equation [3–6] was used. In this way, one can get the master curve of the observed material in a broad frequency range considering the previously selected reference temperature. With the help of this method one can cover such a broad frequency range that can not be measured by any measuring device.

The curves in Fig. 2 represent the master curves pertaining to the different reference temperatures (T=-50, 20 and 150^{o}C), respectively, and indicate the storage (E') and the loss moduli (E'') as a function of frequency. It can be seen that the value of the storage modulus increases as the frequency rises and converges to a maximum value, while the loss modulus (E'') does not increase continuously as the frequency rises, but after reaching a maximum value it begins to decrease and approaches a plateau value [7].

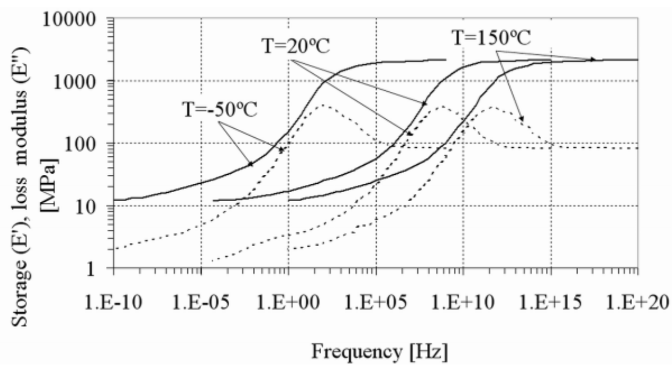

Fig. 2. Master curve of the storage (E', continuous lines) and loss moduli (E", dotted lines); reference temperature T_{ref}=-50, 20 and 150^{o}C

4 Fitting of the generalized Maxwell model to the master curve

After the master curve construction different generalized Maxwell models (Fig. 3) were created. The relaxation modulus of the generalized Maxwell model is described in the software

"ViscoData" [8]:

$$E(t; T_g) = E_\infty + \sum_{i=1}^{n} (E_i) \cdot e^{-\frac{t}{\tau_i}} \tag{1}$$

and for E' and E":

$$E'(\omega, T_g) = E_o \left[1 - \sum_{i=1}^{N} e_i \frac{1}{1+\omega^2 \tau_i^2} \right], \tag{2}$$

$$E"(\omega, T_g) = E_o \cdot \sum_{i=1}^{N} e_i \frac{\omega \tau_i}{1+\omega^2 \tau_i^2}, \tag{3}$$

where , E_∞ is the modulus at infinitely high time, E_0 is the modulus at infinitely high frequency, e_i is the i-th non-dimensional energy parameter of the model and τ_i is the i-th relaxation time.

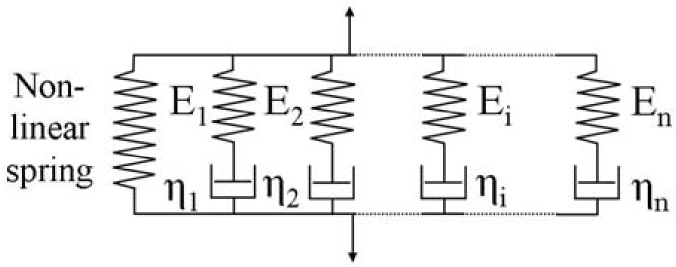

Fig. 3. The n-term generalized Maxwell model (E_i – elastic modulus of the i-th linear spring, η_i – viscosity of the i-th dashpot)

Recall that the master curves of the loss modulus and the loss factor are required to check the accuracy of the resulting generalized Maxwell model.

In the lack of a viscoelastic model which can approximate both the storage modulus vs. frequency and the loss factor vs. frequency curves, an acceptable accuracy between simulated and measured results could not be produced. For this purpose different generalized Maxwell models were created to assure the material model is able to handle the viscoelastic nature of the observed rubber.

The ViscoData software [8], was used to fit 15- and 40-term generalized Maxwell models to the storage modulus and 40-term generalized Maxwell models to the loss factor master curve produced to observe the effect of the different Maxwell models in a broad frequency range at three different temperatures. In Fig. 4 the continuous lines show the measured curves, while the dashed lines show storage modulus and loss factor vs. frequency functions obtained by the fitted generalized Maxwell models. Figs. 4a) c) and e.) show the measured and fitted storage modulus vs. frequency curves, while Figs. 4b) d) and f) show the loss factor vs. frequency curves in case of the 15- and 40-term generalized Maxwell models fitted to the storage modulus and 40-term generalized Maxwell model fitted to the loss factor, respectively. Even if the correspondence between the measurement and the fitted models is good in case of the storage modulus (Fig. 4a)), there is a huge fluctuation in the loss factor taking into consideration the 15-term generalized Maxwell model (Fig. 4b)). The fluctuation will disappear if the number of Maxwell elements is increased up to 40, nevertheless unfortunately the loss factor stays underestimated in the lower frequency range (Fig. 4d)). In view of this it can be concluded that 15-term Maxwell model is unable to reproduce the behavior of the loss factor. Observing the 40-term Maxwell model considering the loss factor vs. frequency curve determined from the measurement, one can see that the description of the loss factor curve will not be adequate in the lower frequency range, even if the storage modulus is sufficient at any frequencies. For a reliable material model, not only a good correspondence between the measured and modelled storage modulus is needed but an acceptable accuracy is necessary in case of the loss factor vs. frequency curve as well. Without a reliable material model, incorrect viscoelastic loss can be calculated by FE technique.

A 40-term generalized Maxwell model was directly fitted to the loss factor curve by manual modification [9] to improve the viscoelastic material model. Figs. 4e) and f) show the measured storage modulus vs. frequency and loss factor vs. frequency curves of the 40-term generalized Maxwell model fitted to the loss factor at the observed three different temperature. It can be concluded that the correspondence with the storage modulus curve will not be as good in the whole frequency range as in the case of adjusting the material model to the measured storage modulus master curve but it can be seen that the material model can describe the measured tan(δ) curve with acceptable accuracy (Fig. 4f)).

4.1 Non-linear material model

The mechanical behavior of rubber-like materials is principally characterized by a non-linear stress-strain curve and with time and temperature dependency. The non-linear stress-strain behavior was modelled with the commonly used Mooney-Rivlin model with two parameters. They were defined on the basis of [9, 10], using the following equations:

$$E_0 = 6 \cdot (C_{01} + C_{10}), \tag{4}$$

$$\frac{C_{01}}{C_{10}} = \frac{1}{4}, \tag{5}$$

where E_0 is the glassy modulus. The two parameters used for the calculations in case of the DMTA test simulation were C_{10}=283.33 MPa and C_{01}=72.33 MPa in case of the 15- and 40-term generalized Maxwell models fitted to the storage modulus, based on the measured glassy modulus of the observed EPDM that is E_0=2134 MPa; and, in case of the generalized Maxwell model fitted to the loss factor, their values were E_0=3050 MPa, C_{10}=406.66 MPa and C_{01}=106.66 MPa, respectively. The difference is caused by the fact that in the course of fitting to the loss factor, the value of the storage modulus also changes (see section 3.1 and in more detailed [9]).

5 FE modelling of the DMTA measurement

The FE models were constructed using the MSC.MARC 2007(r1) FEA software. Geometry of the FE model was cre-

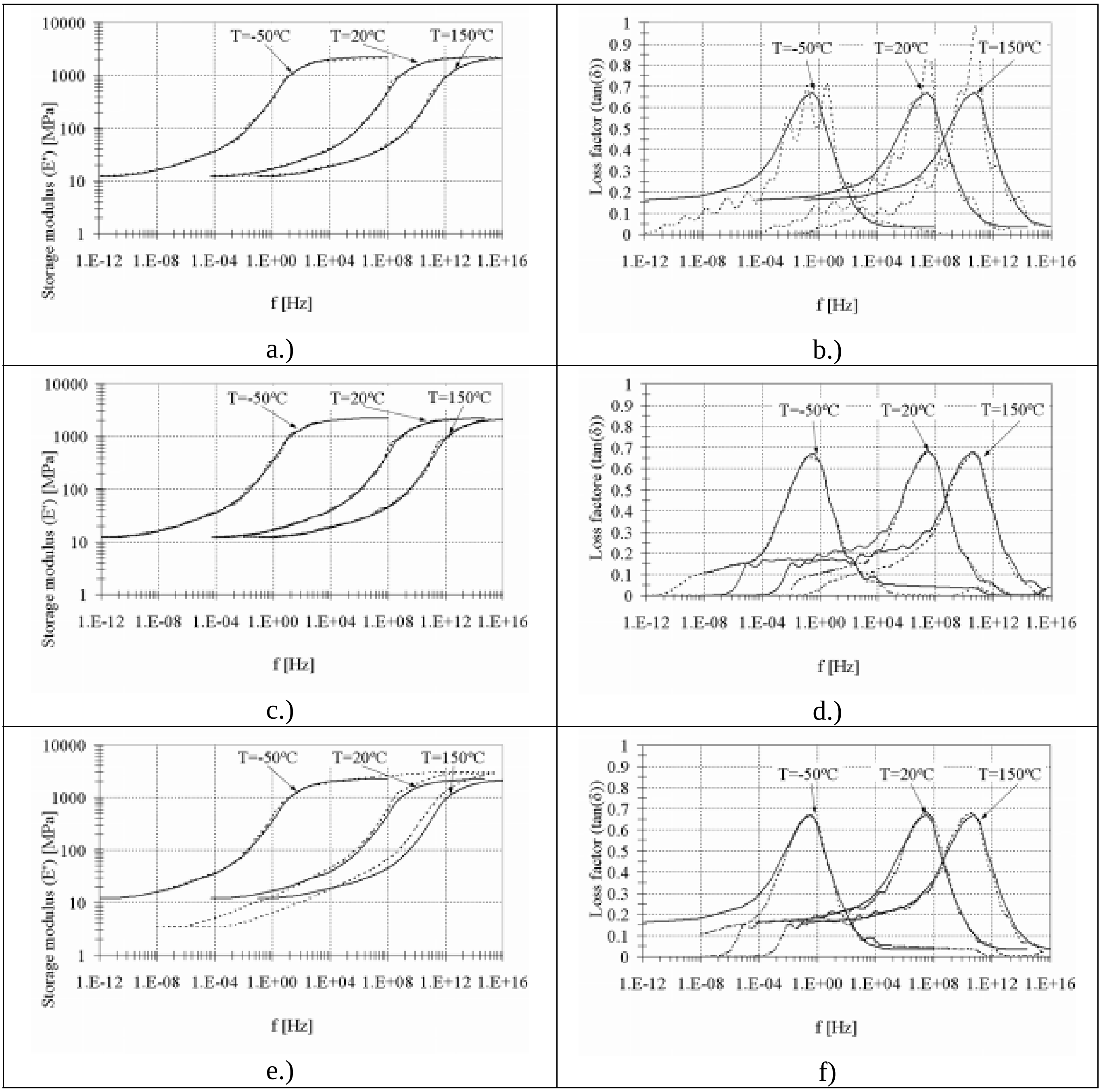

Fig. 4. Comparison of the measured (continuous lines) and simulated (dotted lines) storage modulus and loss factor vs. frequency curves at three different temperatures (T=-50, 20, 150^{o}C):
a) storage modulus vs. Frequency, 15-term Maxwell model fitted to the storage modulus,
b) loss factor vs. Frequency, 15-term Maxwell model fitted to the storage modulus,
c) storage modulus vs. Frequency, 40-term Maxwell model fitted to the storage modulus,
d) loss factor vs. Frequency, 40-term Maxwell model fitted to the storage modulus,
e) storage modulus vs. Frequency, 40-term Maxwell model fitted to the loss factor,
f) loss factor vs. Frequency, 40-term Maxwell model fitted to the loss factor

ated according to the sample in the DMTA measurement using a 1/8 model due to the symmetry conditions (Fig. 5). The FE model was built up from 8-node incompressible Herrmann elements [10]. The applied loads were $\varepsilon_s = 0.06581\%$ in static case and $\varepsilon = 0.01\%$ in dynamic case. The studied frequency range was f=10^{-5}-10^{+15} Hz. The result of the simulation was the shift (Δt) between the stress and the strain responses. The phase shift (δ) loss can be calculated from the shift by [7]:

$$\delta = \Delta t \cdot 2 \cdot \pi \cdot f \tag{6}$$

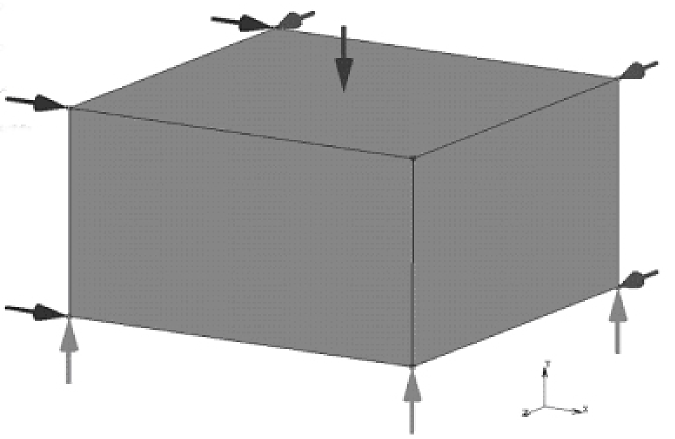

Fig. 5. 1/8 FE model of the measured DMTA sample geometry

The FE results for the loss factor follow the n-term generalized Maxwell models at every frequency. The FE calculation results compared to the measurements can be seen in Fig. 6, Fig. **??** and Fig. **??** using the 15- and 40-term generalized Maxwell model fitted to the storage modulus and 40-term generalized Maxwell model fitted to the loss factor, respectively. In these figures the continuous lines pertaining to the measurement, while the dashed lines correspond to the fitted generalized Maxwell model. Black points, rectangles and triangles shows the simulated results at $T = -50$, 20 and 150^oC, respectively.

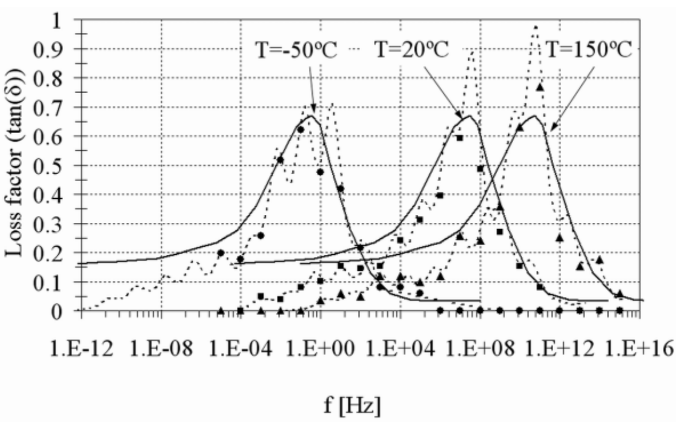

Fig. 6. The measured (continuous lines) and the fitted (dashed lines) loss factor curves and the FE simulated loss factor values at three different temperatures in case of the 15-term generalized Maxwell model fitted to the storage modulus master curve

It can be concluded that in each cases the generalized Maxwell models follow the master curves of the fitted loss factor.

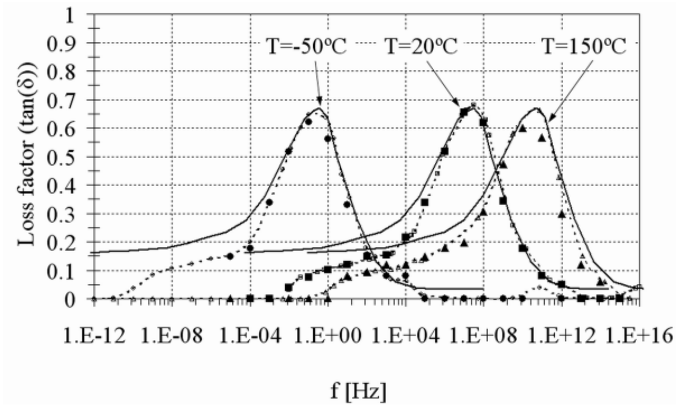

Fig. 7. The measured (continuous lines) and the fitted (dashed lines) loss factor curves and the FE simulated loss factor values at three different temperatures in case of the 40-term generalized Maxwell model fitted to the storage modulus master curve

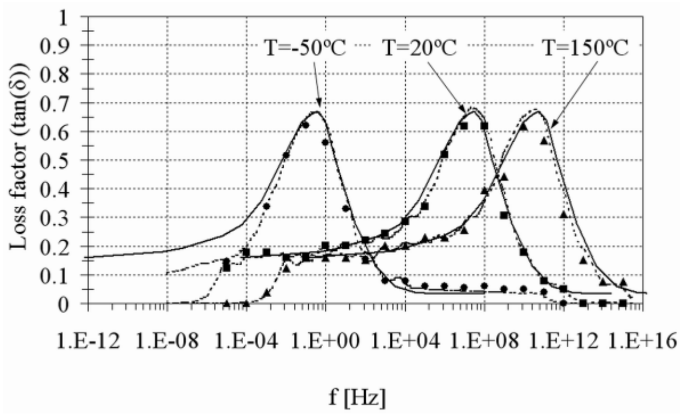

Fig. 8. The measured (continuous lines) and the fitted (dashed lines) loss factor curves and the FE simulated loss factor values at three different temperatures in case of the 40-term generalized Maxwell model fitted to the loss factor master curve

The 15-term generalized Maxwell model can not describe the time- and temperature- dependent viscoelastic behavior due to large fluctuation of the loss factor. By increasing the number of the Maxwell elements, the finite element simulation produces better correspondence with the measured loss factor in accordance with the material model, but it still underestimates the dissipated energy at lower frequency ranges due to the poor agreement to the measured loss factor. If the viscoelastic material model is fitted to the loss factor by trial and error technique, the loss factor will show an acceptable correspondence with the measured loss factor curve, but we have to note that due to the smaller underestimation of the storage modulus the estimation of the dissipated energy will be slightly higher than the real value. It can be concluded that the generalized Maxwell model is an effective tool to simulate the time- and temperature dependent material behavior of rubber.

6 Conclusion

The time- and temperature-dependent material behavior of the investigated EPDM 75 rubber was measured by DMTA equipment. Using the temperature-time equivalence principle, master curves were constructed from the measurement results considering three different temperatures ($T = -50, 20$ and 150 ^{o}C). After that, three different generalized Maxwell models were constructed. Using the Maxwell models, a series of FE models has been created to investigate the dynamic time- and temperature-dependent behavior of the rubber in a broad frequency range.

Based on the results of the FE simulation of the DMTA test, it can be concluded that the 15-term generalized Maxwell model having a high fluctuation of the tan(δ) master curve, produces inaccurate results. Improving the material model to 40 term, the Maxwell model becomes more realistic considering the entire frequency range, but it have to be noted that at each temperature in the lower frequency range, energy loss is underestimated.

The storage modulus in the case of the 40-term Maxwell model fitted to the loss factor master curve shows a smaller underestimation. Therefore, for modelling accurately the energy loss of EPDM 75 rubber in the entire frequency range 40-term generalized Maxwell model fitted to the loss factor master curve is suggested.

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