

Abstract

In Hungary the geothermal gradient is relatively high which means that there are high-temperature rocks underground. In theory the extraction of the heat of the rocks would be a perfect solution to ensure the heating of the country. The mathematical model written in the essay demonstrates that the exothermic system – consisted of two wells – is operable. The re-pressing was proved to be highly possible by many measurements, so the technological solution is ensured. In the USA scientists developed the hot dry stone method, when they make path for the fluid into the stones. Different system is used in France, when they re-pumping the fluid into the leaky stone.

Keywords

Geothermal heating

1 The origin of the geothermal energy

Approaching the depth of earth the temperature grows continuously. The rise in temperature was already noticed in the 17th century on the basis of the measurements taken in different mines and observations of English and French scholars. For the sake of the exploration of the water and the hydrocarbon, in the last century the exploration work gradually developed the technology of the deep-drilling that is why deeper drills could take place. In the depth of more than thousand meters they not only find water, oil and gas, but they also ascertained the existence of the geothermal energy, its physical parameters and they examined their conformation in the function of different geological, hydrogeological conditions.

The presence of the geothermal energy in every points of the earth can be ascertained but its extent is significantly influenced by local conditions. Therefore approaching the depth the measure of the warming is territorially changing. The index number of this warming's value called temperature gradient ($^{\circ}\text{C}/\text{km}$ or $^{\circ}\text{C}/\text{m}$), which shows how much $^{\circ}\text{C}$ rise in temperature belongs to the accretion of 1 km (or m) depth in the examined area. In the practice we use invent of the gradient (Fig. 1). In world average it is 30°C measured on earth radius but in some territory it is higher and for instance it is $50\text{--}60^{\circ}\text{C}$. The value is higher in the post vulcanic areas.

The origin of the geothermal energy can be attributed to the processes of the atomic fission, in which processes the earth functions as an atomic furnace and at this process the heat flows towards the surface. This ground heatflux's index number is 62 mWm^{-2} in average. In Hungary it is higher, 84 mWm^{-2} . We can say that this value is extremely low nevertheless it is sufficient to warm up the solid rock-mass very slowly. Practically the present geothermal energy production exploits these accumulated heat energy.

2 The method of the usage

However this accumulated heat energy can only be brought to surface from hot solid rocks with the help of a heat carrier. Up to this point it was only possible where in greater depth different liquid or gaseous substances take place, which absorbs

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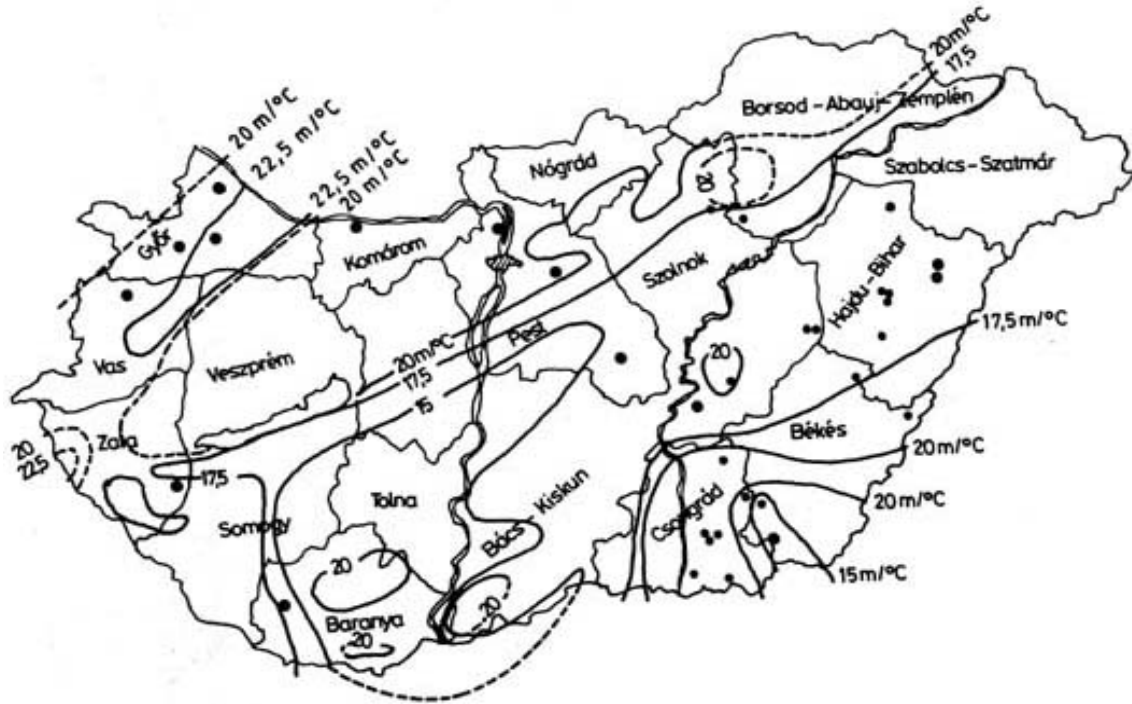


Fig. 1. The geothermal gradient in Hungary

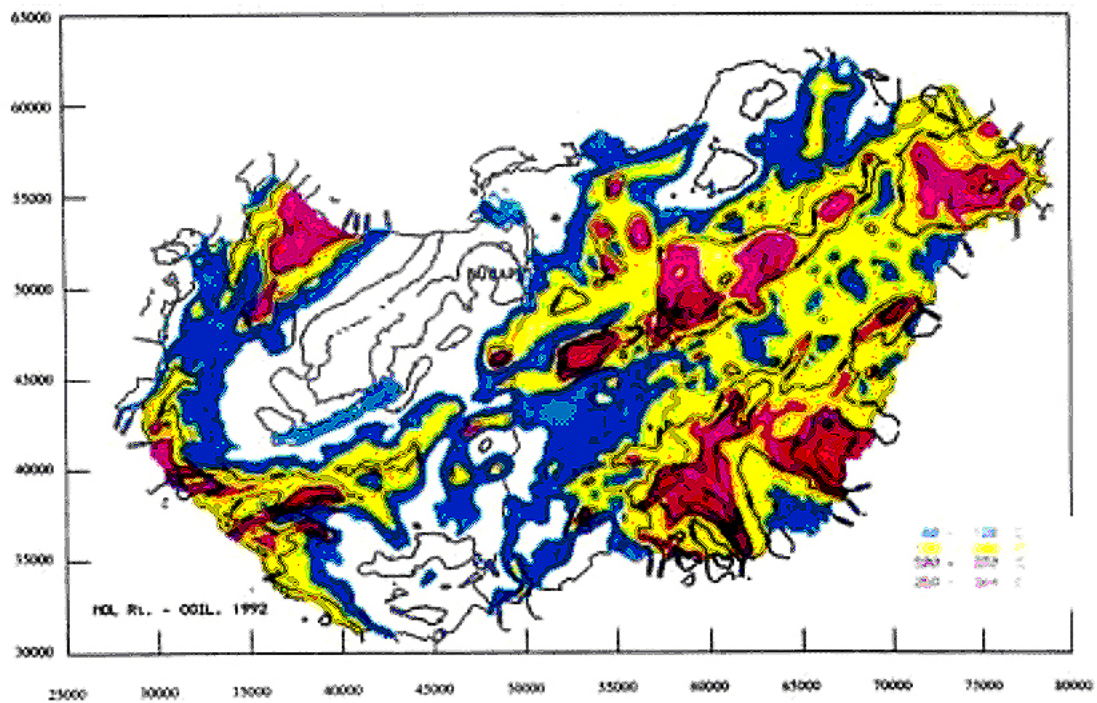


Fig. 2. The basement temperature

the temperature of the environment. The heat carrier could have appeared as a geyser, thermal water in a natural way of deep-drillings. These drillings can be different gases, oils but most frequently the water – being in different state – fills the part of the heat carrier. In the depth, the water takes place in the porous rocks, cracks and getting to surface being depended on its temperature, it presents in the form of 100 Celsius thermal-water. If there is no heat carrier medium, with the help of a producer and a pressing pair of wells can we exploit heat. As in Hungary more than one thousand prospector wells exist therefore we can develop this kind of system in almost every borders of any settlements.

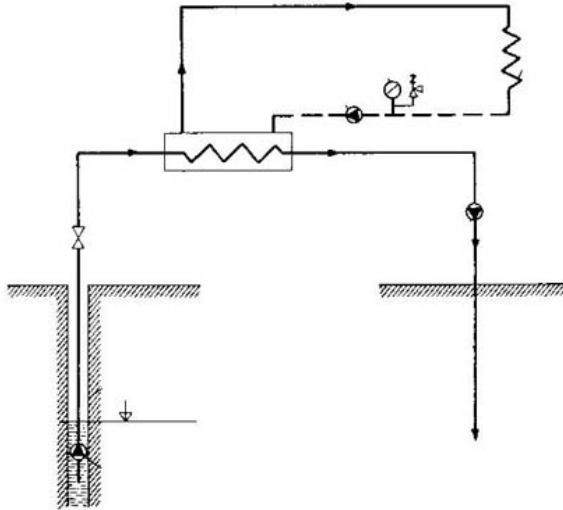


Fig. 3. Two wells system model

3 The mathematical model

After that we have to examine that one producer and re-pressing pair of wells how we storage heater medium affect the geothermal and hydrogeological's relation. The selected area is average 20-25 meter thick Upper-Pannon sandstone (700 mD) with good permeability. Minimal hydraulic gradients to be developing regional size flow systems in this good permeability layer. The thickness of layer is in negligible next on horizontal extent. As follows the leaking water is available two dimensions with good approach.

The flowing layerwater is incompressible, the developing velocity field is rotary free. In the vertical level the linear partial equation is describable with the XY linear and orthogonal coordinate system and the v_x, v_y components of leaking velocity:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \quad (1)$$

$$\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} = 0 \quad (2)$$

The first equation means the incompressibility, the second equation means the rotary free. If these equations to be realized in every points of the layer, to be exist two scalar equation which satisfy the LAPLACE criteria and one another harmonic pairs.

v_x, v_y velocity components are represent with these F_1 and F_2 function:

$$v_x = \frac{\partial F_1}{\partial x} = \frac{\partial F_2}{\partial y} \quad (3)$$

$$v_y = \frac{\partial F_1}{\partial y} = \frac{\partial F_2}{\partial x} \quad (4)$$

The third and the fourth equation are the CAUCHY-RIEMANN differential equation-pair. These have got infinite solutions. The flow complex potential is:

$$P = F_1(x, y) + i \cdot F_2(x, y) \quad (5)$$

This is a regular complex variable function, and its real part the F_1 velocity potential, F_2 flow image is the imaginary part. The leaking velocity's conjugal is the P function z numeral derivation:

$$\bar{v}(z) = \frac{dP}{dz} = v_x - i \cdot v_y \quad (6)$$

In the knowledge of these equations any parameters are calculable. The Fig. 4 shows the position of the wells. The wells' distance is $2 \cdot d$, the pressing wells label is $-m$, the re-pressing wells label is $+m$.

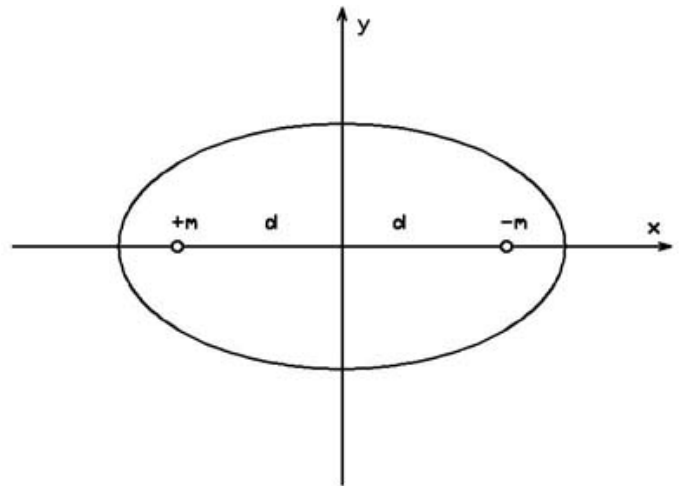


Fig. 4. The wells' position

The v_{0x} velocity's direction is same the x axis direction. It is the undisturbed and the shortest flow picture. The complex potential's function:

$$P = v_{0x} \cdot z + \frac{m}{2 \cdot \pi} \cdot \ln(z + d) - \frac{m}{2 \cdot \pi} \cdot \ln(z - d) \quad (7)$$

Near the wells the velocity field is:

$$\bar{v} = v_{0x} + \frac{m}{2 \cdot \pi} \left[\frac{1}{z + d} - \frac{1}{z - d} \right] \quad (8)$$

After simplification:

$$v_x = v_{0x} + \frac{m}{2 \cdot \pi} \left[\frac{1}{z + d} - \frac{1}{z - d} \right] = v_{0x} + \frac{m}{2 \cdot \pi} \cdot \frac{d}{d^2 - x^2} \quad (9)$$

The minimum of velocity is:

$$v_{\min} = v_{0x} + \frac{m}{d \cdot \pi} \quad (10)$$

The average of velocity is:

$$v_{av} = \frac{1}{2\pi} \int_{-d}^{+d} \left(v_{0x} + \frac{m}{2 \cdot \pi} \cdot \frac{d}{d^2 - x^2} \right) dx \quad (11)$$

The average time while the water circulated once is:

$$y_{av} = \frac{2 \cdot d}{v_{av}} \quad (12)$$

The energy balance is:

$$\begin{aligned} \int_V \frac{\partial}{\partial t} [(1-p) \cdot \zeta_r \cdot c_r \cdot T_r + p \cdot \zeta_w \cdot c_w \cdot T_w] dV = \\ = \int_A [(1-p) \cdot \lambda_r + p \cdot \lambda_w] grad T d\bar{A} \end{aligned} \quad (13)$$

p – porosity, c_r – specific heat of rock, c_w – specific heat of water, T – temperature, ζ – density, λ – thermal conductivity.

The heat energy from the pressing well is:

$$E_p = \zeta_{wp} \cdot c_w \cdot t_{wp} \cdot m \quad (14)$$

The heat energy into the re-pressing well is:

$$E_r = \zeta_{wr} \cdot c_w \cdot t_{wr} \cdot m \quad (15)$$

The gain heat energy is:

$$E_{gain} = E_p - E_r \quad (16)$$

4 Summary

The gain heat energy is $2 \cdot 3 \cdot 10^{11}$ kJ/year if the average time while the water circulated once is 0.8-1.3 year and $m=1$ million m^3 /year and $t_{vp} - t_{vr}=50-60$ °C. The re-pressing was proved to be highly possible by many measurements, so the technological solution is ensured. This mathematical model works individual wells and many wells system. The gain is positive, and with the clean electrical energy from wind turbines, we got really clean heat energy for the buildings.

References

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Appendix

- x, y – linear and orthogonal coordinate system
- v_x, v_y – components of leaking velocity
- F_1 – velocity potential
- F_2 – flow image
- d – wells' distance
- m – mass flow
- y_{av} – average time
- p – porosity
- c_r – specific heat of rock
- c_w – specific heat of water
- T – temperature
- ζ – density
- λ – thermal conductivity
- E – energy