# CALCULATING RADIATION TEMPERATURE ASYMMETRY BY GRAPHICALLY DETERMINING THE SHAPE FACTOR 

László BÁnhidi and Ilona Frohner<br>Department of Building Service Engineering<br>Budapest University of Technology and Economics H-1521 Budapest, Hungary Phone: (+36-1) 463-2636,<br>e-mail:tanszek @epgep.bme.hu<br>frohner@witch.pmmf.hu<br>Received: Feb. 14, 2005


#### Abstract

The specific figure of radiation temperature asymmetry is highly influenced by the shape factor, the used calculation method and the type of the element examined. In addition to the regular calculation methods this paper presents a new drawing method that determines the shape factor based on geometrical surface ratios. This process has been further developed in a way that the used examination plane will always be adequately placed in line with the specific arrangement. To ensure the quick completion of the drawing a programme fitting a mathematical software package has been developed. It is good to use to calculate the values of radiation temperature asymmetry and provide their graphical presentation.


Keywords: thermal comfort, radiation temperature-asymmetry, shape factor.

## 1. Introduction

The modern designing of buildings requires that comfort criteria should be fulfilled. Among local discomfort factors the location-dependent values of radiation asymmetry can be determined by using various methods. Theoretical calculations and results can differ according to how the shape factor is determined.

The calculation of this factor varies in accordance with the shape of the examination body.

The two-dimensional graphical determination of the shape factor between and the surface element used in the norm as examination body is adequately accurate. In the following a new possibility of the drawing method in three-dimensional space will be presented by listing the required mathematical steps.

The graphic method is used in a special case to determine radiation asymmetry i.e. in the case of a plane dividing the space in a way that the two surfaces of extreme temperatures are located in two separate semi-spaces in all cases. To perform the traditional graphic drawing a software was developed that will be later described. The used Maple software is capable of graphically showing the steps of drawing in space and the final result, i.e. the location-dependent values of radiation temperature asymmetry in the examined room and three-dimensional space. The
software developed for simpler cases often occurring in heating technology helps to calculate radiation temperature asymmetry.

## 2. Determining Radiation Temperature Asymmetry

The radiation temperature asymmetry in the given room at the given examination point equals the difference between the radiation temperatures of the two semispaces divided according to the shape and position of the examination body.

$$
\begin{equation*}
\Delta t_{a}=t_{r a d, 1}-t_{r a d, 2} \tag{1}
\end{equation*}
$$

where:
$\Delta \mathrm{t}_{a} \quad$ - radiation temperature asymmetry $\left({ }^{\circ} \mathrm{C}\right)$;
$\mathrm{t}_{\text {rad, } 1}, \mathrm{t}_{\text {rad, } 2}$ - radiation temperature of the two semi-spaces $\left({ }^{\circ} \mathrm{C}\right)$.
The general definition of radiant temperature is as follows (1) : the homogenous temperature of surrounding surfaces that enables the same radiation heat exchange with the body located at the point as between the body located at the point and the surfaces of originally different temperatures.
Radiant temperature:

$$
\begin{equation*}
t_{r a d, i}=\sqrt[4]{\sum_{i=1}^{n} \varphi_{i, j} \cdot T_{j}^{4}}-273\left({ }^{\circ} \mathrm{C}\right) \tag{2}
\end{equation*}
$$

where:
$\mathrm{t}_{\text {rad, } i}$ - radiant temperature $\left({ }^{\circ} \mathrm{C}\right) ; i=1$, or 2 ;
$\varphi_{i, j}$ - the shape factor of surface $i$ for surface j ;
$\mathrm{T}_{j}$ - temperature of surface $j(\mathrm{~K})$.

## 3. Basics of Calculating Radiant Heat Exchange

The specific total radiation power of a body for a radiating surface is defined as follows:

$$
\begin{equation*}
\dot{q}=\varepsilon \cdot \sigma \cdot T^{4} \quad\left[\frac{W}{m^{2}}\right] \tag{3}
\end{equation*}
$$

where:
$\varepsilon$ - emission factor of the body surface
$\sigma$ - Stefan-Boltzmann constant: $5.67 \cdot 10^{-8} \mathrm{~W} /\left(\mathrm{m}^{2} \cdot \mathrm{~K}^{4}\right)$
$T$ - surface temperature (K).
All surfaces in the room are regarded as grey radiant i.e. the optical characteristics of these surfaces are independent of the wavelength of radiation. According
to Lambert's radiation law for the direction dependence of the emitted radiation power the following applies:

$$
\begin{equation*}
\dot{q}=\varepsilon \cdot \sigma \cdot T^{4} \cdot \cos \beta \tag{4}
\end{equation*}
$$

The resultant radiation energy between two surfaces of different surface temperatures is the difference between the radiation emitted by both surfaces and absorbed by the other surface other than the examined one. By using the above simplifications and neglecting mutual reflections the heat flow for the surface unit between $A_{1} A_{2}$ surfaces where these surfaces are located at distance $r$ from each other (Fig. 1), is:

$$
\begin{equation*}
\stackrel{\bullet}{q}=\varphi_{1,2} \cdot \varepsilon_{1} \cdot \varepsilon_{2} \cdot \sigma \cdot\left(T_{1}^{4}-T_{2}^{4}\right) \quad\left[\frac{W}{m^{2}}\right] \tag{5}
\end{equation*}
$$



Fig. 1. The geometrical description of surfaces in the radiant heat exchange

In this equation all geometrical attributes are summed up in shape factor $\varphi_{1,2}$ [2].

In general the following equation applies:

$$
\begin{equation*}
\varphi_{1,2}=\frac{1}{\pi \cdot A_{1}} \cdot \int_{A_{1}} \int_{A_{2}} \frac{\cos \beta_{1} \cdot \cos \beta_{2}}{r^{2}} \mathrm{~d} A_{2} \mathrm{~d} A_{1} \tag{6}
\end{equation*}
$$

The meaning of the shape factor can be defined as follows:
The $\varphi_{1,2}$ value is the ratio of the radiation starting at surface $A_{1}$ and arriving at $A_{2}$ and all the radiations starting from surface $A_{1}$ [3].

The following correlation can be used to check the calculations for the shape factors. This correlation refers to the shape factors of all surfaces that are in visible
contact with surface $A_{1}$ and originates from the principle of energy conservation:

$$
\begin{equation*}
\sum_{i=1}^{N} \varphi_{1, i}=1 \tag{7}
\end{equation*}
$$

The double integral in Eq. (6) can be resolved analytically for only a few geometrical arrangements and numerical processes cannot be always used either. Processes based on statistical considerations may be used to determine it.

The main problem of the calculations concerning radiant heat exchange is the determination of shape factors and it is also the most important issue when defining radiation temperature asymmetry, owing to its complexity and time requirements.

To determine the shape factor is therefore a key issue that will be outlined for the basic cases and presented in more details only for flat surface elements.

## 4. Determining the Shape Factor According to the Form of the Examination Body

The shape factor of surfaces in space can be determined according to the shape of the examination body as follows (4):

### 4.1. Shape Factor between the Surface and the Surface Element

Surface element 1 radiates into the semi-space. Some of the radiation reaches surface 2. This proportion of the total radiation is marked $\varphi_{12}$. This is the shape factor of surface element 1 for surface 2:

$$
\begin{equation*}
\varphi_{1,2}=\frac{1}{\pi} \cdot \int_{A 2} \frac{\cos \beta_{1} \cdot \cos \beta_{2}}{r^{2}} \mathrm{~d} A_{2} \tag{8}
\end{equation*}
$$

### 4.2. Shape Factor between the Surface and the Surface

Surface element $1^{\prime}$ ' on surface 1 radiates into surface 2. The proportion of the radiation reaching surface 2 is characterized in an analogous way by $\varphi_{1^{\prime}, 2}$ described in section 4.1. If the shape factor for surface element $1^{\prime}$ is understood for all surface elements and locations of surface 1 a mean value can be created from the infinite


Fig. 2. Shape factor between the surface and the surface element
number of $\varphi_{1^{\prime}, 2}$ values. This is marked $\Phi_{1,2}$ being the mean shape factor of surface 1 for surface 2 is the following:

$$
\begin{equation*}
\Phi_{1,2}=\frac{1}{\pi \cdot A_{1}} \cdot \int_{A_{1}} \int_{A_{2}} \frac{\cos \beta_{1} \cdot \cos \beta_{2}}{r^{2}} \mathrm{~d} A_{2} \mathrm{~d} A_{1} \equiv \frac{1}{A_{1}} \int_{A_{1}} \varphi_{1,2} \mathrm{~d} A_{1} \tag{9}
\end{equation*}
$$



Fig. 3. Shape factor between the surface and the surface

### 4.3. Shape Factor between the Sphere Element and the Surface

Unlike the above two cases the sphere-shaped element marked 1 radiates not only to the semi-space but also to the entire space. The sphere-shaped element can simply be called a point.

Some of the emitted radiation reaches surface 2. This proportion of the total radiation is marked $\varphi_{1,2}^{o}$. This is the shape factor of sphere element 1 for surface 2 :

$$
\begin{equation*}
\varphi_{1,2}=\frac{1}{4 \cdot \pi} \cdot \int_{A_{2}} \frac{\cos \beta_{2}}{r^{2}} \mathrm{~d} A_{2} \tag{10}
\end{equation*}
$$



Fig. 4. Shape factor between the sphere element and the surface

## 5. Calculating the Shape Factor between the Surface and the Surface Element

### 5.1. Using Analytical Correlations

Among the available correlations the following enables us to calculate the shape factor between the surface element in the examination point and the surface, according to the arrangement seen in the Fig. 5.
$\varphi_{1,2}=\frac{1}{2 \cdot \pi}\left[\frac{\frac{a}{h}}{\sqrt{\left(\frac{a}{h}\right)^{2}+1}} \cdot \arctan \frac{\frac{b}{h}}{\sqrt{\left(\frac{a}{h}\right)^{2}+1}}+\frac{\frac{b}{h}}{\sqrt{\left(\frac{b}{h}\right)^{2}+1}} \cdot \arctan \frac{\frac{a}{h}}{\sqrt{\left(\frac{b}{h}\right)^{2}+1}}\right]$
$\varphi_{1,2}=\frac{1}{\pi}\left[\frac{\frac{a}{h}}{\sqrt{\left(\frac{a}{2 \cdot h}\right)^{2}+1}} \cdot \arctan \frac{\frac{b}{h}}{2 \cdot \sqrt{\left(\frac{a}{2 \cdot h}\right)^{2}+1}}+\frac{\frac{b}{h}}{\sqrt{\left(\frac{b}{2 \cdot h}\right)^{2}+1}} \cdot \arctan \frac{\frac{a}{h}}{2 \cdot \sqrt{\left(\frac{b}{h}\right)^{2}+1}}\right]$


Fig. 5. Shape factor in case of parallel surface and surface element


Fig. 6. Shape factor in case of perpendicular surface and surface element

The surface element is under a corner point of the surface in both cases.
To calculate radiation temperature asymmetry the shape factors of the boundary walls of different temperatures in the room can be determined using these equations.

In the case of various surface temperatures the calculations simply cannot be performed. Considering the large number of examination points in the given room this method is not useful to be applied owing to the high amount of time it requires.

Using analytical equations to determine shape factor and the radiation asymmetry is too complicated and can only be applied for a certain geometry (parallel and perpendicular surfaces). This criterion restricts the use of the equations in more complex room geometries.

### 5.2. Calculation of the Shape-factor with Help of the Approaching Equation

To calculate the shape factor the simpler correlation in Eq. (13) can be used that is derived from $E q$. (6) if surfaces $A_{1}$ and $A_{2}$ are very small compared to distance ' $r$ '. When calculating surface integrals, the mean values can replace angles $\beta$ and $\beta_{2}$ (see Fig. 1).

$$
\begin{equation*}
\varphi_{1,2} \approx \frac{A_{1}}{\pi} \cdot \frac{\cos \beta_{1} \cdot \cos \beta_{2}}{r^{2}} \tag{13}
\end{equation*}
$$

The shape factors used by GLÜCK in the heat technology room models [5] are based on this approach equation. As the condition $\sqrt{A_{1}} \ll s$ is often poorly fulfilled $10-20 \%$ errors may occur with reference to the amount $\varphi_{1,2}$ in Eq. (6), depending on the geometrical arrangement [6].

The shape factors calculated according to the approach equation in the room model calculations are therefore corrected in line with the principle of energy conservation Eq. (7).

### 5.3. The Drawing Method

The shape factors can be very accurately calculated from geometrical considerations using a numerical process which is absolutely stable and does not take a long time for computers to process.

The process is first illustrated by using the example of a two-dimensional model on Fig. 7. This process differs in its basic calculation steps and the position of the examination plane from the method that was used to calculate the room model developed with the Maple software.

Firstly, above the radiated surface element a so-called radiation semi-space is expanded that is a semi-circle in the two-dimensional model. Radiated surface element $\mathrm{dA}_{2}$ is the geometrical centre of the basic area of the semi-space. The radius of the semi-space can be chosen arbitrarily, yet the calculations become
much simpler if we choose $R=1$. Surface $A_{1}$ is then projected through the surface of the semi-space unto its basic area.

The shape factor is the surface proportion that lies between the basic area of the semi-space and the projected surface:

$$
\begin{equation*}
\varphi_{1,2}=\frac{A_{1}^{\prime \prime}}{2 \cdot R} \tag{14}
\end{equation*}
$$



Fig. 7. Determining the shape factor of surface $A_{1}$ for surface element $\mathrm{d} A_{2}$ in a twodimensional room model.

The computer technology application for the two-dimensional case can be traced back to the simplest basic functions of vector algebra, trigonometry and analytical algebra. This can be described in the following steps:

1. A local, two-dimensional polar coordinate system is posited over surface element $\mathrm{dA}_{2}$.
2. The coordinates for points ' B ' and ' $\mathrm{C}^{\prime}$ ' are transformed from the rectangular into the polar coordinate system $\left(\beta_{i}, \mathrm{R}_{i}\right)$.
3. Points ' $\mathrm{B}^{\prime}\left(\beta_{2} ; \mathrm{R}\right)$ and ' $\mathrm{C}^{\prime}\left(\beta_{1} ; \mathrm{R}\right)$ are determined.
4. Points ' B ' and ' C ' are projected onto the basic plane of the surface element: points ' $\mathrm{B}^{\prime}\left(\frac{\pi}{2} ; \mathrm{R} \cdot \sin \beta_{2}\right)$ and ' $\mathrm{C}^{\prime}\left(\frac{\pi}{2} ; \mathrm{R} \cdot \sin \beta_{1}\right)$ are determined.
5. The sought shape factor equals the ratio of lengths $\overline{B^{\prime \prime} C^{\prime \prime}}$ and 2 R .

$$
\begin{equation*}
\varphi_{1,2}=\frac{\overline{B^{" C} C^{"}}}{2 \cdot R} \tag{15}
\end{equation*}
$$

and can also be expressed as follows:

$$
\begin{equation*}
\varphi_{1,2}=\frac{1}{2} \cdot\left|\sin \beta_{2}-\sin \beta_{1}\right| \tag{16}
\end{equation*}
$$

In theory the above described idea can be used for a three-dimensional problem. The arbitrarily placed corner points of the room are first projected through the semi-sphere over the horizontal surface element (see Fig.8) onto the surface of the semi-sphere then the intersection points and the arcs are projected again perpendicularly unto the circle-shaped area. The examined surface may be curved but should have straight sides.


Fig. 8. Geometric definition of the shape factor of a square surface with straight sides for flat surface element $\mathrm{d} A_{2}$ in a three-dimensional room model [2]

Using this drawing process (Fig. 8) the shape factor of surface ABCD equals the surface ratio of ' A ' 'B' ' C ' ' D ' and $\pi \cdot R^{2}$ semi-space area. In theory this calculation provides us with an adequately accurate figure. There are some geometrical arrangements, however, where the errors may be larger than in the case of using the approach Eq. (12).

The reason for this is that the projection of straight sidelines unto the basic area of the semi-sphere results in elliptical sections rather than straight lines (see Fig. 9). The shape factor of surface ABCD can be calculated with adequate accuracy if its projection unto the basic area of the semi-space and the area of the projection are corrected with these elliptical sections.


Fig. 9. Double projection of the AB straight surface side.

The calculations required to determine the shape factor using the drawing process are presented in Fig. 9 with the example of the AB sideline:

A local three-dimensional polar coordinate system is posited over the surface element at point $M$.

Points $A$ and $B$ are projected unto the basic area of the semi-sphere through its surface. Points $A^{\prime \prime}$ and $B$ " are determined analogously to the two-dimensional case.

By projecting the straight section AB to the basic plane arcs are created. To correct the elliptical arc it is assumed that points $A^{\prime \prime}$ and $B^{\prime \prime}$ are located on the sought ellipse (Fig. 10). The greater axis of the ellipse is the same as the intersection line of plane formed by points ABM and its length is $2 \cdot \mathrm{R}$. Further calculations are made easier if a two-dimensional rectangular coordinate system is posited on the basic area of the semi-sphere. Its horizontal axis equals the major axis of the ellipse and its origin is the centre of the basic circle of the semi-sphere.

As the coordinate and the ellipse axes coincide and the major axis of the ellipse is also known, the minor axis of the ellipse can be determined from the normal shape of the ellipse:

$$
\begin{equation*}
Y=\sqrt{\frac{y^{2}}{1-\left(x^{2} / R^{2}\right)}} \tag{17}
\end{equation*}
$$

To do calculations coordinates for points A" or B" can be inserted in Eq. 17 thus the sought ellipse can be fully defined.

Surface $S$ of the ellipse slice with A"B" chord can be calculated as follows:

$$
\begin{equation*}
S=\frac{R \cdot Y}{2} \cdot\left(\arccos \frac{x_{B}}{R}-\arccos \frac{x_{A}}{R}\right)-\frac{1}{2} \cdot(\bar{a} \times \bar{b}) \tag{18}
\end{equation*}
$$

$(\bar{a} \times \bar{b})$ is the vector product of the location vectors of points $A^{\prime \prime}$ and $B^{\prime \prime}$ (Fig. 10).

The surface of the $A^{\prime \prime} B^{\prime \prime} C^{\prime \prime} D^{\prime \prime}$ square of straight sides needs to be corrected with the ellipse cross section surfaces of all four sides.

The shape factor of $A B C D$ surface is the following surface ratio:

$$
\begin{equation*}
\varphi_{12}=\frac{\left.\overline{\left(A^{\prime \prime} B^{\prime \prime} C^{\prime \prime} D^{\prime \prime}\right.}\right)_{\mathrm{corr}}}{\pi \cdot R^{2}} \tag{19}
\end{equation*}
$$



Fig. 10. Calculating the ellipse chord on the basic circle of the semi-sphere formed by the surface side $\overline{A B}$

The process was compared with the analytical case documented in the VDI atlas for two geometrical cases. The results are the same. In the case of shape factors $\varphi_{1,2}<10^{-3}$ approach Eq. ((13)) was used to save time using the computer. The resulting error is negligible [6].

## 6. Using the Graphic Method for the Thermally Active Surfaces of the Room

The idea to determine the shape factor by a graphic method originates from 1935 and was developed by Eckert, an optician. The connection of the shape factor and the radiant heat emission cannot be regarded as independent of the mechanism of seeing. The ammount of the radiant heat emission to the given receptor surface largely depends on the solid angle that the receptor surface takes up of the radiant space of the radiant surface i.e. the degree of the sight contact. Later the drawing
method to calculate radiant heat emission was included in several specialist books yet other calculation methods were used to do the specific calculations.

The examination element of the processes to determine shape factor so far has been elemenary size and a flat surface in almost all cases. Unlike the other methods the drawing method uses the semisphere-shaped space over the surface element to determine the shape factor.

As the human body has a finite surface and is made up of curved surfaces rather than flat ones drawing with a sphere of R radius as a complementary examination element seems correct.

Building simulation software can calculate the shape factor, radiation temperature asymmetry and the operative temperature with adequate accuracy at any point of the room. Access to the building simulation software, however, is limited for many designers. This is one of the reasons why a new solution was sought to calculate the shape factor.

Our objective is to develop a programme that provides adequately accurate figures and requires simpler computers. As calculations have been made to verify the determination of the shape factor by the drawing method and the accuracy of the calculation results the method can be regarded as adequate. The rapidly developing mathematical software offer a possibility to resolve the given task. The drawing method has been performed using Maple, a mathematical programme with mathematical correlations that somewhat differ from the above listed steps.

### 6.1. Position of the Surface Element and the Examination Plane

To determine the shape factor of the given surface the drawing method uses a semisphere above the surface element. The plane containing the basic circle of the semi-sphere and the surface element is the examination plane that, in most of the cases, is located angularly to the boundary walls of the room rather than horizontally or vertically. The ,cold' and 'warm' surfaces (e.g. window and radiator, or cooling ceiling and hot window surface) are separated by this plane.

This method in all cases divides the space into two semi-spaces where one contains the cold surface and some of the neutral surfaces while the other holds the hot surface and some of the neutral surfaces. The examination plane holding the basic circle of the semi-sphere is in all cases located at an equal distance from the surfaces of extreme temperatures in the room.

The plane dividing the room into the two 'warm' and 'cold' semi-spaces are defined by point $P$ and points $K, K 2$ in all examination points $P(x, y, z)$ (Fig.11). Points $K 1$ and $K 2$ are the bisectors of the two shortest straight sections (among the 16 sections) connecting the corner points of the two surfaces of extreme temperatures.

By positing the examination plane in this way and thus supplementing the mathematical steps of the drawing method we can be sure that the two semi-spaces only include 'warm' and 'cold' surfaces (Figs. 11 and 12). This requirement for
positing the examination plane has not been fulfilled in the known methods of calculating radiation temperature asymmetry.

In the event this requirement is not fulfilled and the examination plane intersects with the 'warm' or the 'cold' surface both semi-spaces would contain ,cold' and ,warm' surfaces as well. As a result warm and cold surface pieces in both semispaces partly 'eliminate' each other during difference formation depending on the sizes of the radiation factors when performing the calculations of the radiation temperature asymmetry (Fig. 13).


Fig. 11. Positing the plane dividing the space into two semi-spaces

### 6.2. Radiation Semi-spaces in the Various Cases of Determining Radiation Temperature Asymmetry

In international practice three cases have been described to calculate radiation temperature asymmetry according to the shape of the examination body:

1. Surface element: In norm MSZ CR 1752-2000 the examination element is the surface element for which the threshold values are specified. The calculation needs to be performed at point(s) 0.6 m high from the floor of the room if the surface element is placed vertically or horizontally. The radiation temperature
asymmetry is the greater value of the $\Delta \mathrm{t}_{a}$ calculated as the difference of the radiation temperatures on the upper/lower and right and left sides.
2. Cube element: In Glück's heat technology model the examination body is a cube element that is located in the centres of the screen grid located at a height of 1.3 m in two basic positions. In the first position the vertical sides of the cube are placed parallel to the walls of the room (mostly perpendicular to each other) then they are turned $45^{\circ}$ around the vertical axis. The cube symbolizes the human head and its position. As the human head does not have a heat emitting surface towards the plane of the floor the calculations for the radiation temperature only need to be carried out for the five sides. The difference between the two extreme radiation temperatures is the determining figure. The calculation is then made for the turned position of the cube. The two give the $\Delta \mathrm{t}_{a}$ value and the greater will be the final figure. This is suitable to ensure a more accurate determination of the radiation temperature asymmetry between surfaces of extreme surface temperatures perpendicular to each other. This reflects the reality as in practice wall surfaces of extreme surface temperatures are rarely parallel.
3. Sphere element (point): DIN 1946/2 uses a sphere element as the examination body to calculate radiation temperature asymmetry and the calculations are made with correlations given in closed forms.


Fig. 12. Positing the examination plane if radiator located under window


Fig. 13. Radiation in semi-spaces for surface element and cube element

In all three cases the examination plane that divides the space into two semispaces at the given examination point most often intersects with and divides the thermally active surface between the two semi-spaces and the surfaces of extreme temperatures are often located in the same semi-space.

This is valid for the cube element too on case of two perpendicular sides (Fig. 14).

## 7. Using the Drawing Method to Determine the Shape Factor and the Radiation Temperature Asymmetry with the Help of the Maple Mathematical Software

Thanks to the recent rapid development of the mathematical softwares we now have the chance to easily solve and calculations that had been considered too complicated. The Maple software package models these graphic processes and makes calculations easy to track.

The examined room of arbitrary dimensions have been modelled using the Maple software package to ensure the graphic presentation of the room and its thermally active surfaces. With the help of the drawing method the shape factors were determined at a number of points at a desired height and distance in the room for all the boundary walls, including the active ones. Instead of the correction required owing to the elliptical arcs, Eq. (6) was used.

The software is capable of creating animations therefore the calculated values of radiation asymmetry can be displayed at the $P(x, y, z)$ examination points at the given height in the room i.e. at the intersection points of a grid.


Fig. 14. The position of radiant semi-space if the examination body is a cube element. The part of the examined surface marked AB is used in both semi-spaces to the determine radiation temperatures weighted by the shape factor calculated for the semi-space in question. This fact reduces the value of the difference when calculating for the difference of the two radiation temperatures.

The steps of the drawing method of the software to determine the shape factor are the following (see Fig. 8):

1. Writing up the equation for $\bar{n}$ normal vector plane including points $P, K 1$ and $K 2$.
2. Determining the equation for P centred sphere of a unit radius.
3. Writing up the equations of right lines $A P, B P, C P, E P, F P, G P, H P, J P$ (corner points of the hot surface: $A, B, C, E$, corner points of the cold surface: $F, G, H, J)$.
4. The intersection points of the sphere and the straight lines make up the coordinates for projection points $A^{\prime}, B^{\prime}, C^{\prime}, E^{\prime}, F^{\prime}, G^{\prime}, H^{\prime}, J^{\prime}$.
5. Writing up the equations for straight lines crossing, $A^{\prime}$ or $B^{\prime}$ or $C^{\prime}$ or $E^{\prime}$ with direction vector $\bar{n}$.
6. The intersection points of the plane and the straight lines make up the projection points $A^{\prime \prime}, B^{\prime \prime}, C^{\prime \prime}, E^{\prime \prime}, F^{\prime \prime}, G^{\prime \prime}, H^{\prime \prime}, J^{\prime \prime}$.
7. Determining the area of the rectangle closed by the projection points and the shape factors of both thermally active surfaces.
8. Determining the shape factor of the other boundary walls as specified in 1-7.

The examination plane intersects with and divides the side walls, the ceiling and floor between the two cold and hot semi-spaces in all cases.

After summing the shape factor for both semi-spaces the software corrects them in accordance with Eq. (7). Finally, it calculates the radiation temperature and the radiation temperature asymmetry and shows them in space.

## 8. Summary

The results can be summed as follows:
a) After specially positing the examination plane dividing the room into two semispaces the values of the radiation temperature asymmetry differ from the figures obtained through methods used in designing. To illustrate this Tablel and Figs. 15 and 16 are presented. They contain the values of radiation temperature asymmetry created by the heating panel on the ceiling (a $2.2 \mathrm{~m} \times 2.2 \mathrm{~m}$ heating panel in a room of $4.7 \mathrm{~m} \times 6.0 \mathrm{~m} 2.4 \mathrm{~m}$ ) at different heights ( $h=0.6 \mathrm{~m}$ and 1.3 m ) in the case of various floor temperatures and examination bodies.

The second and third columns of the Table 1 contain the temperature of the heating panel and the mean temperature of the surrounding walls. Indices used in the other columns are the following: $s, e=$ surface elements; $k, l-u=$ lower and upper sides of the cube; draw $=$ drawing method.

Table 1. The values of radiation temperature asymmetry in the case of a heating panel on the ceiling

|  | $\mathrm{t}_{\text {panel }}$ <br> $\left({ }^{\circ} \mathrm{C}\right)$ | $\mathrm{t}_{\text {tall }}$ <br> $\left({ }^{\circ} \mathrm{C}\right)$ | $\Delta \mathrm{t}_{s, e}$ <br> $\left({ }^{\circ} \mathrm{C}\right)$ | $\Delta \mathrm{t}_{c, l-f u}$ <br> $\left({ }^{( } \mathrm{C}\right)$ | $\Delta \mathrm{t}_{\text {draw }}$ <br> $\left({ }^{\circ} \mathrm{C}\right)$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $h=0.6 \mathrm{~m}$ | $h=1.3 \mathrm{~m}$ | $h=0.6 \mathrm{~m}$ | $h=1.3 \mathrm{~m}$ |
| 1. | 34 | 24.2 | 4.5 | 7.6 | 5.92 | 8.73 |
| 2. | 43 | 23.4 | 9.2 | 15.4 | 12.079 | 17.56 |
| 3. | 52 | 22.9 | 14.1 | 23.1 | 18.25 | 26.26 |
| 4. | 63 | 22.2 | 20.4 | 32.7 | 26.15 | 36.93 |
| 5. | 69 | 21.5 | 23.6 | 38.2 | 30.66 | 43.11 |

The diagrams clearly show that the drawing method gives us figures that are higher by a few degrees than the surface element or the cube element placed at the same height. The difference increases with the temperature of the heating panel.
b) Separating thermally active surfaces in space into cold and hot surfaces on one side can give us a more accurate figure for radiation temperature asymmetry. It should also be investigated to what degree the low value of shape factors influence the calculated asymmetry value.
c) The new drawing method is adequate to calculate the shape factor and can be well used in practice too.
d) The drawing method and the applied equations are suitable to determine the values of the radiation temperature asymmetry at a desired number of points in the room.
e) Calculations can be performed by using in the software and the final result is easy to interpret thanks to the graphic presentation.


Fig. 15. Radiation temperature asymmetry in the case of examination bodies placed at a height of $h=0.6 \mathrm{~m}$


Fig. 16. Radiation temperature asymmetry in the case of examination bodies placed at a height of $h=1.3 \mathrm{~m}$

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