Thermal Field of a Solid Particle for Bi > 0 and Fluid Medium of a Countercurrent Heat Exchanger

Pavel Élesztős[1*], Ladislav Écsi[1], Roland Jančo[1], Jana Gabková[2]

[1] Institute of Applied Mechanics and Mechatronics, Faculty of Mechanical Engineering, Slovak University of Technology in Bratislava, Nám. slobody 17, 812 31 Bratislava, Slovak Republic
[2] Institute of Mathematics and Physics, Faculty of Mechanical Engineering, Slovak University of Technology in Bratislava, Nám. slobody 17, 812 31 Bratislava, Slovak Republic
* Corresponding author, e-mail: pavel.elesztos@stuba.sk

Abstract

This article is devoted to the solution of the thermal field of a particle for Bi > 0 with an ideal spherical shape and the behavior of the temperature in the fluid phase with countercurrent contact.

After establishing the underlying simplified assumptions and defining the initial and boundary conditions in the form of dimensionless criteria, a mathematical formulation of the problem is transformed into a suitable and solvable form. The formulation is then used in the analysis.

Keywords

non-stationary temperature field, heat exchangers, heating of grained material, heating particles, drying particles

1 Introduction

In the mathematical formulation of the problem, a few simplifications are assumed: The granular material is monodisperse, with an ideal spherical shape, existing in direct contact with the fluid of a countercurrent heat exchanger. The particles of the material are homogeneous and isotropic. The thermal-physical properties of the particles are not dependent on temperature. It is assumed that the heat transfer coefficient α is constant over the entire surface of the particle.

In the upper part of the exchanger a granular material enters with spherical shape of radius R, mass flow M, and an increasing solid phase temperature T_{s0} throughout the entire volume of the particle. In this cross section 0, the fluid phase exits with a mass flow M_f of known (output) temperature T_{f0}. The required variables are indicated in Fig. 1 in order to write the balance equations for the temperature. It is assumed that the countercurrent movement of the particles and fluid phase occur with respect to the law of piston flow, with ideal mixture of both phases assumed in the transverse direction of the heat exchanger's cross section. Heat exchange occurs only between the particles and the fluid phase while the whole heat exchanger is perfectly insulated. The thermal-physical variables of
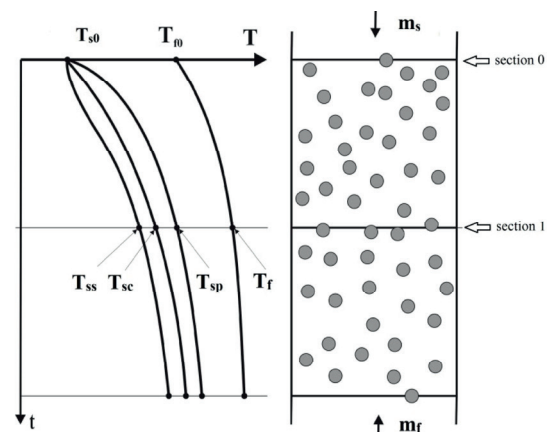

Fig. 1 Schematic of the contact in a countercurrent heat exchanger

the solid material and fluid phase (specific heat, coefficient of thermal conductivity) are assumed to be constant and independent of the temperature.

Applying the aforementioned assumptions and known parameters of the contacting phases within a cross section of the heat exchanger creates quasi-static thermal conditions. The article strives to analytically describe the thermal fields of these conditions.

2 Mathematical formulation of the problem

We used the following notations in the mathematical description of the temperature fields of the solid and fluid phases:

a coefficient of temperature diffusivity [m^{2}s^{-1}]

c specific heat [J kg^{-1} K^{-1}]

C integration constant

E Young's modulus [MPa]

f function value

G shear modulus [MPa]

m thermal capacitance ratio of the contact phases [–]

M mass flow [kg s^{-1}]

r radius of the sphere [m]

R outer radius of the sphere [m]

t time [s]

T temperature [K]

α coefficient of heat transfer [W m^{-2} K^{-1}]

ρ dimensionless radial coordinate [–]

λ heat conductivity [W m^{-2} K^{-1}]

Θ dimensionless temperature [–]

Subscripts:

0   value with regard to the cross section 0

1   value with regard to the cross section 1

c calorimetric

f fluid phase

p variable value on the surface

s solid phase

The thermal field of a particle with spherical shape existing in the aforementioned conditions can be described in terms of the Fourier-Kirchhoff equation for heat conduction (in spherical coordinates) [7]

$$\frac{\partial T_s}{\partial t} = a\left(\frac{\partial^2 T_s}{\partial r^2} + \frac{2}{r}\frac{\partial T_s}{\partial r}\right). \tag{1}$$

The equation is solved with the following initial conditions

$$t = 0, \; T_s = T_{sc} = T_{s0}, \; T_f = T_{f0}. \tag{2}$$

In the process of heat exchange, the thermal profile is symmetric therefore,

$$r = 0, \quad \left(\frac{\partial T_s}{\partial r}\right)_{r=0} = 0. \tag{3}$$

Heat exchange on the surface of the particle is determined by Fourier's condition for heat conduction

$$r = R, \quad \left(\frac{\partial T_s}{\partial r}\right)_{r=R} = -\frac{\alpha}{\lambda}\left[T_f - (T_s)_{r=R}\right]. \tag{4}$$

The temperature of the fluid and solid phases are mutually related with the heat equation in the form:

$$-M_f c_f\left(T_{f0} - T_f\right) = M_s c_s\left(T_{sc} - T_{s0}\right). \tag{5}$$

The introduction of the mean calorimetric temperature T_{sc} at any arbitrary cross section in the heat exchanger results in a comparison between thermal capacities of a particle with constant temperature T_{sc} with the thermal capacity determined by the real temperature distribution over the particles

$$\frac{4}{3}\pi R^3 c_s \rho_s T_{sc} = \int_0^R 4\pi r^2 c_s \rho_s T_s dr. \tag{6}$$

3 Solution of the problem

A differential equation (Eq. (1)) in this form is solvable however it is not possible to introduce boundary and initial conditions. In order to introduce them into the equations, dimensionless variables are introduced to the calculation:

Identically defined dimensionless variables are often used for analytical solution of the temperature field in a couple problems in technical literature [5, 6]. The solution of such problems can be found for instance in [1-3], which also allow for the determination of the stress field of the solid phase [4]. The introduction of dimensionless variables also makes it easier to generalize the experimental results of heat transfer measurements [7].

After introducing these variables a system of equations similar to concurrent contact is obtained [1]:

$$\frac{\partial \Theta_s}{\partial Fo} = \frac{\partial^2 \Theta_s}{\partial \rho^2} + \frac{2}{\rho}\frac{\partial \Theta_s}{\partial \rho}, \tag{7}$$

$$1 + m\Theta_{sc} - \Theta_{sp} = -\frac{1}{Bi}\left[\frac{\partial \Theta_s}{\partial \rho}\right]_{\rho=1}, \tag{8}$$

$$\Theta_{sc} = -\frac{1}{m}\left(\Theta_{f0} - \Theta_f\right), \tag{9}$$

$$\Theta_f = 1 + m\Theta_{sc}, \tag{10}$$

$$\Theta_{sc} = 3\int_0^1 \rho^2 \Theta_s d\rho. \tag{11}$$

After further substitution into Fourier's method, it is possible to find a solution to Eq. (7) and after inserting the boundary and initial conditions for the solid phases thermal behavior a solution can be found in the form of an infinite series.

Table 1 Dimensionless variables

$Bi = \dfrac{\alpha R}{\lambda_s}$	Biot number
$Fo = \dfrac{at}{R^2}$	Fourier number
$m = \dfrac{M_s c_s}{M_f c_f}$	ratio of thermal capacities of the contacting phases
$\rho = \dfrac{r}{R}$	dimensionless coordinate
$\Theta_s = \dfrac{T_s - T_{s0}}{T_{f0} - T_{s0}}$	relative temperature difference of the solid phase
$\Theta_{sc} = \dfrac{T_{sc} - T_{s0}}{T_{f0} - T_{s0}}$	average calorimetric relative temperature difference
$\Theta_{sp} = \dfrac{T_{sp} - T_{s0}}{T_{f0} - T_{s0}}$	surface relative temperature difference
$\Theta_f = \dfrac{T_f - T_{s0}}{T_{f0} - T_{s0}}$	relative temperature difference of the fluid phase.

$$\Theta_s = \frac{1}{1-m} - \sum_{i=1}^{\infty} \frac{2k_i^2 \left[\sin(k_i) - k_i \cos(k_i) \right]}{-6m \left[\sin(k_i) - k_i \cos(k_i) \right]^2 - \left[1 - \dfrac{\sin(k_i)\cos(k_i)}{k_i} \right] k_i^4} \frac{\sin(k_i \rho)}{\rho} e^{-k_i^2 Fo} \tag{12}$$

The mean calorimetric temperature of the solid phase becomes:

$$\Theta_{sc} = \frac{1}{1-m} - 3\sum_{i=1}^{\infty} \frac{2k_i^2 \left[\sin(k_i) - k_i \cos(k_i) \right]}{-6m \left[\sin(k_i) - k_i \cos(k_i) \right]^2 - \left[1 - \dfrac{\sin(k_i)\cos(k_i)}{k_i} \right] k_i^4} \left[\frac{\sin(k_i)}{k_i^2} - \frac{\cos(k_i)}{k_i} \right] e^{-k_i^2 Fo}. \tag{13}$$

And from this, using Eq. (10), the relative temperature difference of the fluid phase can be determined:

$$\Theta_f = \frac{1}{1-m} + 3m\sum_{i=1}^{\infty} \frac{2k_i^2 \left[\sin(k_i) - k_i \cos(k_i) \right]}{-6m \left[\sin(k_i) - k_i \cos(k_i) \right]^2 - \left[1 - \dfrac{\sin(k_i)\cos(k_i)}{k_i} \right] k_i^4} \left[\frac{\sin(k_i)}{k_i^2} - \frac{\cos(k_i)}{k_i} \right] e^{-k_i^2 Fo}. \tag{14}$$

In Eqs. (12)-(14), k_i is the root of the transcendental equation

$$\frac{k_i}{Bi} = -\frac{3m}{k_i} + \frac{k_i}{\left[1 - k_i \cot g(k_i) \right]}. \tag{15}$$

The solution of Eq. (15) differs to that of the ratio of conductive equivalents of both contact phases and also obtains a differing temperature behavior of the contact phase. If $m < 1$ all roots of the transcendental equation are real numbers. The equation is represented in Fig. 2 for

the ratio of thermal capacities of both phases $m = 0.5$ and Biot number $Bi = 3$.

This graphical representation was performed using a software Mathematica considering the first 25 roots of Eq. (15) for $m = 0.5$ a Biot number of $Bi = 3$ obtaining the solution for dimensionless relative temperature difference of the solid phase $\Theta_s(\rho, Fo)$ which is represented in Fig. 3. Fig. 4 represents the behavior of dimensionless relative temperature difference of the fluid phase and behavior of the relative temperature difference of the solid phase at the center and surface of the spherical particle.

In the case where the ratio between thermal capacities of the contacting phases are $m > 1$ then the character of the temperature changes over the length of the heat exchanger. In this case the first root of the transcendental equation (Eq. (15)) is an imaginary number. Equation (15) for $m = 2$ and $Bi = 3$ is represented in Fig. 5.
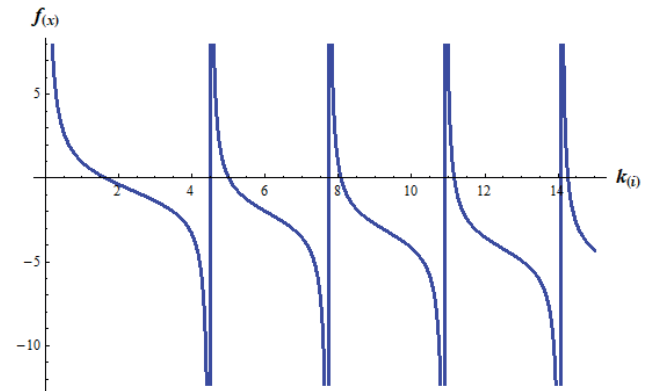

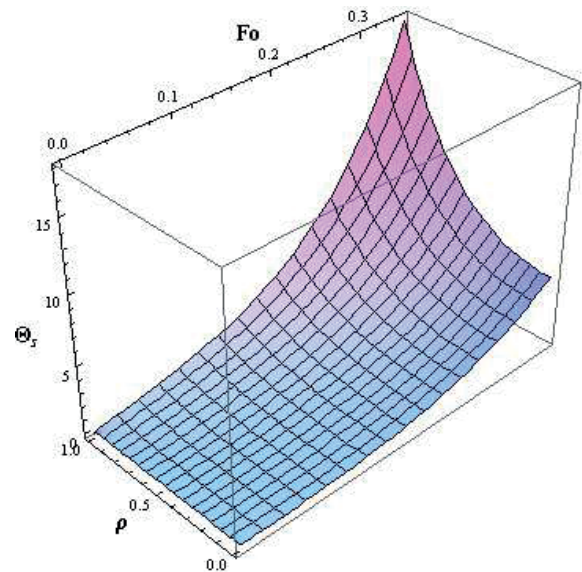**Fig. 2** Roots of the transcendental equation for $m = 0.5$ and $Bi = 3$

Fig. 3 Behavior of the relative temperature difference of the solid phase Θ_s dependent on the radius ρ and Fo number
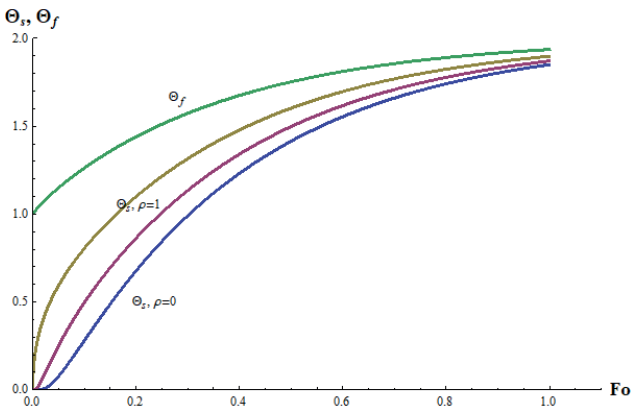
Fig. 4 Behavior of the temperature of spherical particles for a chosen radius and temperature of the fluid phase at $m = 0.5$ and $Bi = 3$
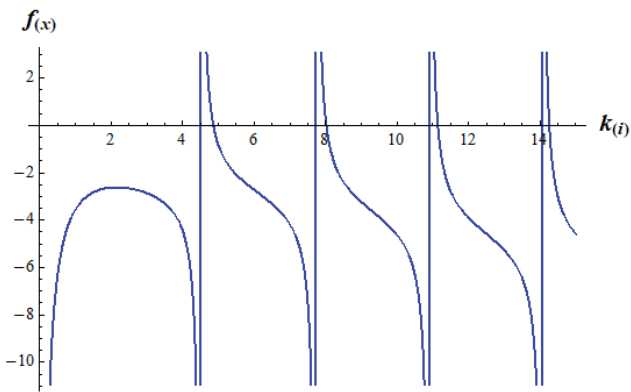

Fig. 5 Roots of the transcendental equation for $m = 2$ and $Bi = 3$

Fig. 6 Behavior of the relative temperature difference of the solid phase Θ_s dependent on the radius ρ and Fo number

Fig. 7 Behavior of dimensionless relative temperature difference of the solid phase Θ_s for chosen radiuses ρ and fluid phases Θ_f dependent on Fo number

In these cases, Eqs. (12)-(14) also allow for the solution of quasi-static thermal fields of a granular material $\Theta_s = f(m, \rho, Fo)$ and temperature of the fluid phase $\Theta_f = f(Fo)$ over the entire length of the heat exchanger for the given boundary and initial conditions.

A graphical representation of the case where $m = 2$ and $Bi = 3$ can be seen in Fig. 6.

Fig. 7 shows the behavior of particle temperature Θ_s for chosen parameters of the radius ρ, as well as the behavior of the fluid phase temperature Θ_f dependent on time Fo.

4 Analysis of the results

The analytical results provide the temperature distribution in a spherical particle along the height of the heat exchanger. For the heat capacity ratio of the phases $m < 1$ the temperature of the particles is gradually brought to equilibrium in the overall volume of the exchanger. The temperature gradient in the particles steadily decreases 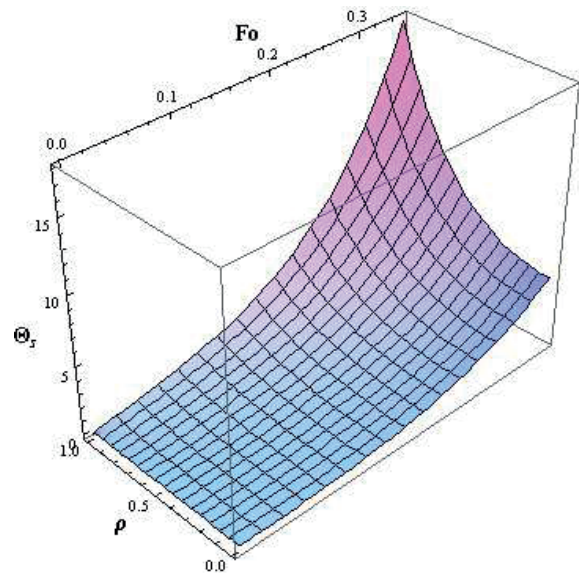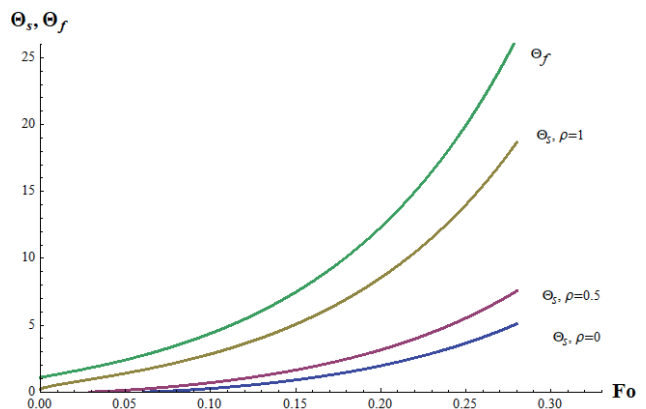with time and finally the particle temperature reaches the temperature of the fluid phase. In case, when the heat capacity ratio of the phases $m > 1$ and $Fo > 0$ the temperature gradient in the particles increases with time and as a result, the temperature of the solid phase never reaches the temperature of the fluid phase. During heating, cooling and drying processes it is therefore always necessary to carry out a detailed thermal analysis in order to determine the temperature distribution over the volume of the exchanger. Increasing temperature gradients in particles may lead to overheating/overcooling of their near surface layers and eventually cause inadmissible thermal stresses in these areas.

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